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CHECKERS GAME AS A MARKOV CHAIN PROCESS

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MOTIVATION

- Checkers is a game that has been played for centuries and is still enjoyed by millions of people worldwide.

- By analyzing the moves made during a game of checkers using a Markov Chain model, we believe we can gain valuable insights into the strategy and decision-making process of players.

introduction

CHECKERS

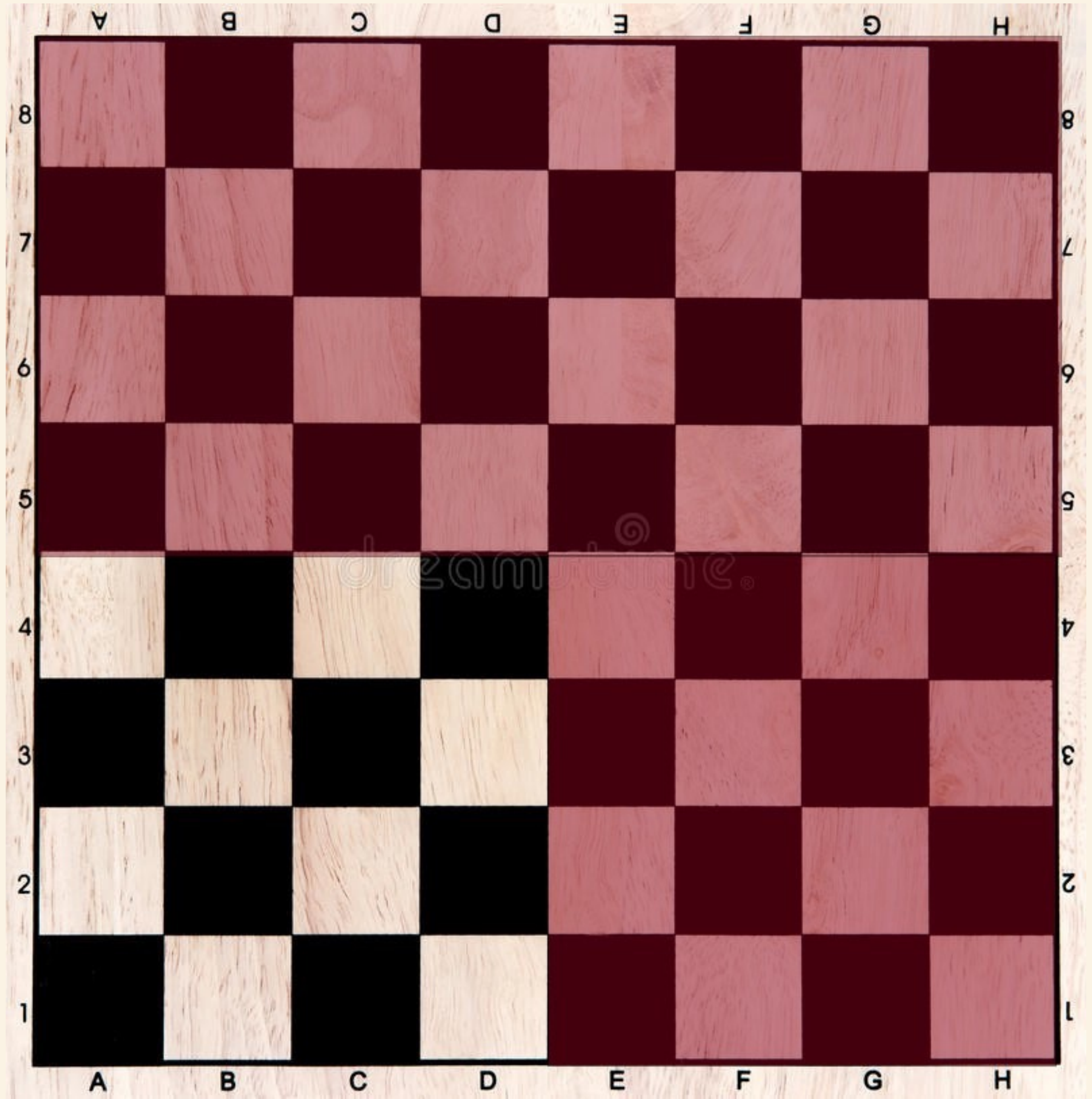
- **Checkers** is also known as **draughts.**
- It is a strategy board game for two players.
- It involves diagonal moves of pieces and mandatory captures by jumping over opponent pieces.
- The most popular form is **American Checkers**, which is played on a 8x8 board.
- Other variations include **International draughts, Canadian Checkers and Russian draughts .**
- **Ordinary piece and King piece.**

LET'S WATCH RULES

- 2 player game.
- Goal is to capture all opponents pieces or make it impossible for them to move a piece.
- 12 piece setup in 3 rows at bottom.
- Each row has 4 pieces.
- Each piece should be placed on dark square.
- Play by moving pieces diagonally.
- A piece cannot be moved onto white square or backwards.
- Capture by jumping diagonally.
- A piece that goes all the way to the opposite side of board becomes a King.

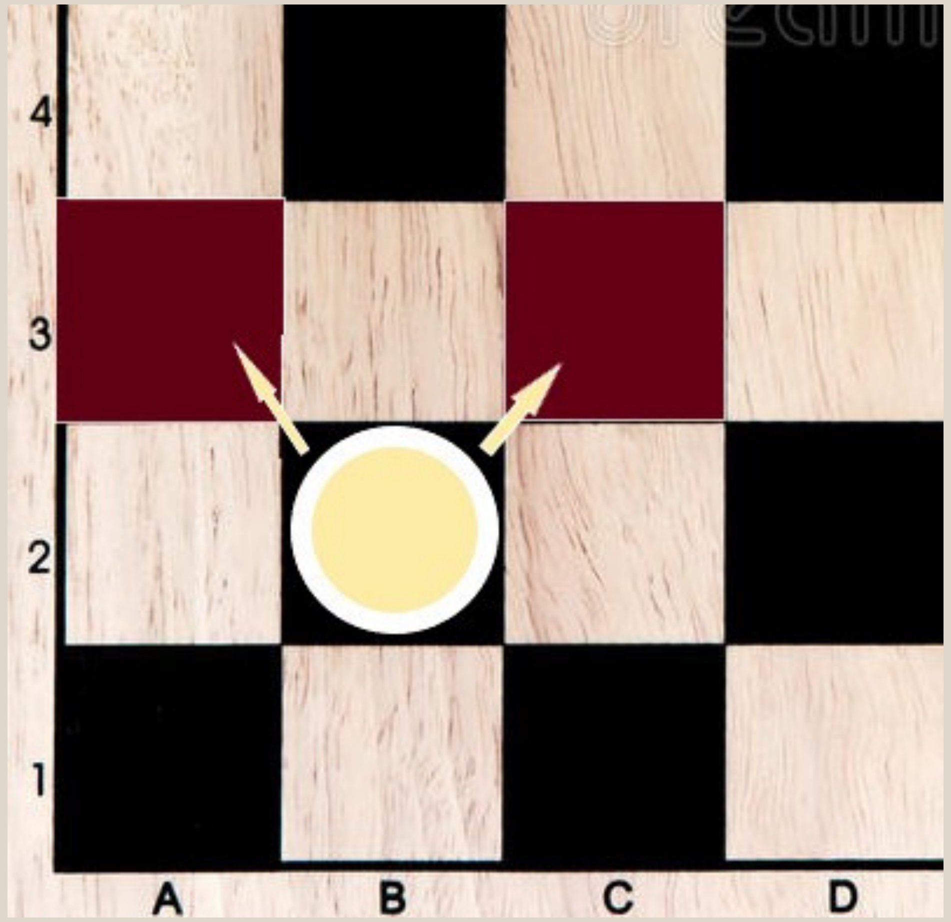
CHECKERS BOARD

- A typical Checkers game is played using a 8x8 board.
- For the purpose of this study, we considered just a quarter of the 8x8 board.
- This consideration was done to reduce the size of our state space.

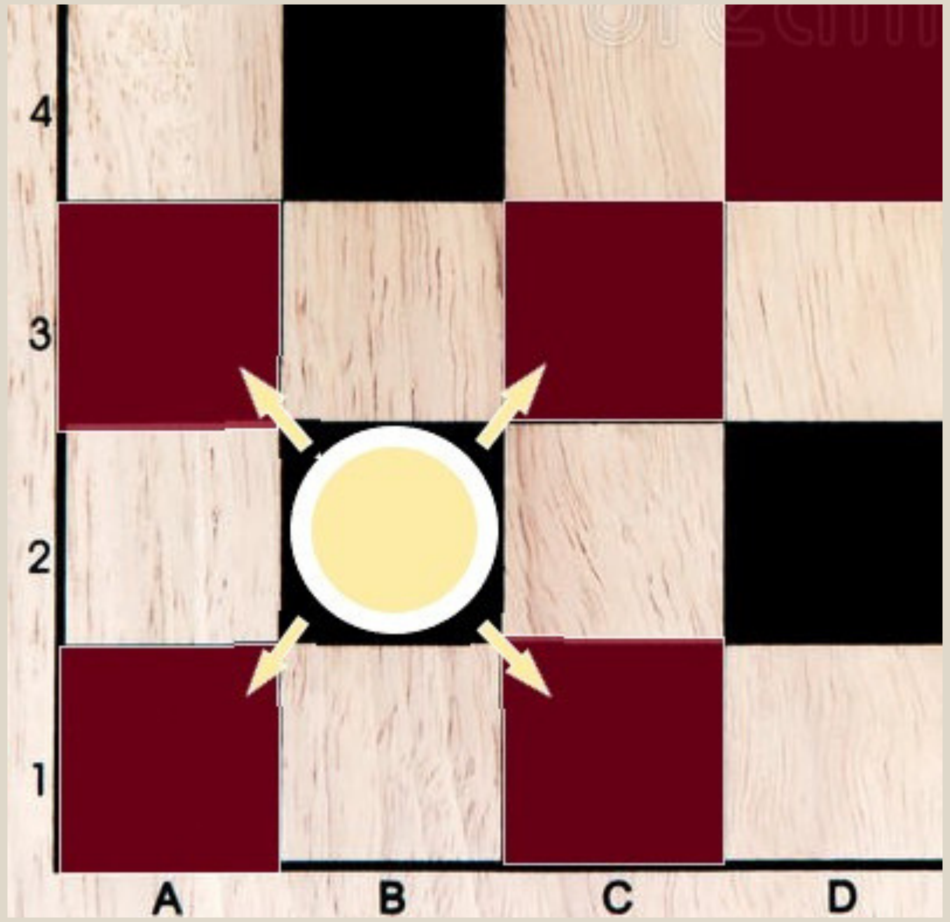


MARKOV PROCESS VERIFICATION

- The Checkers game models a **Markov Chain.**
- The next state Xn + 1 is conditionally independent of the past (Xo,..... Xn-1), given the present state Xn.



ORDINARY PIECE



KING PIECE

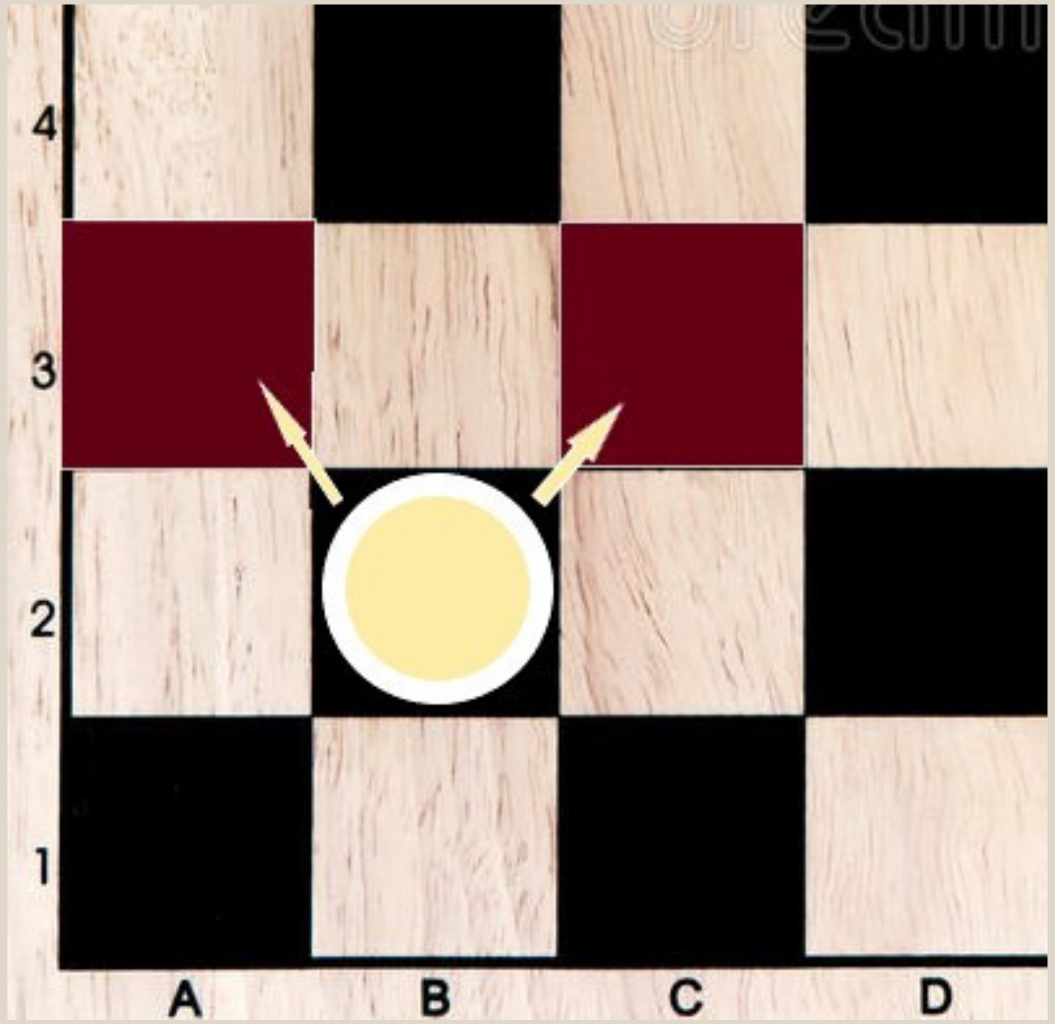
INDEX & STATE SPACE

- **Index space:**

  {1, 2, 3, ...}

- **State space:**

  {A1, A3, B2, B4, C1, C3, D2, D4}

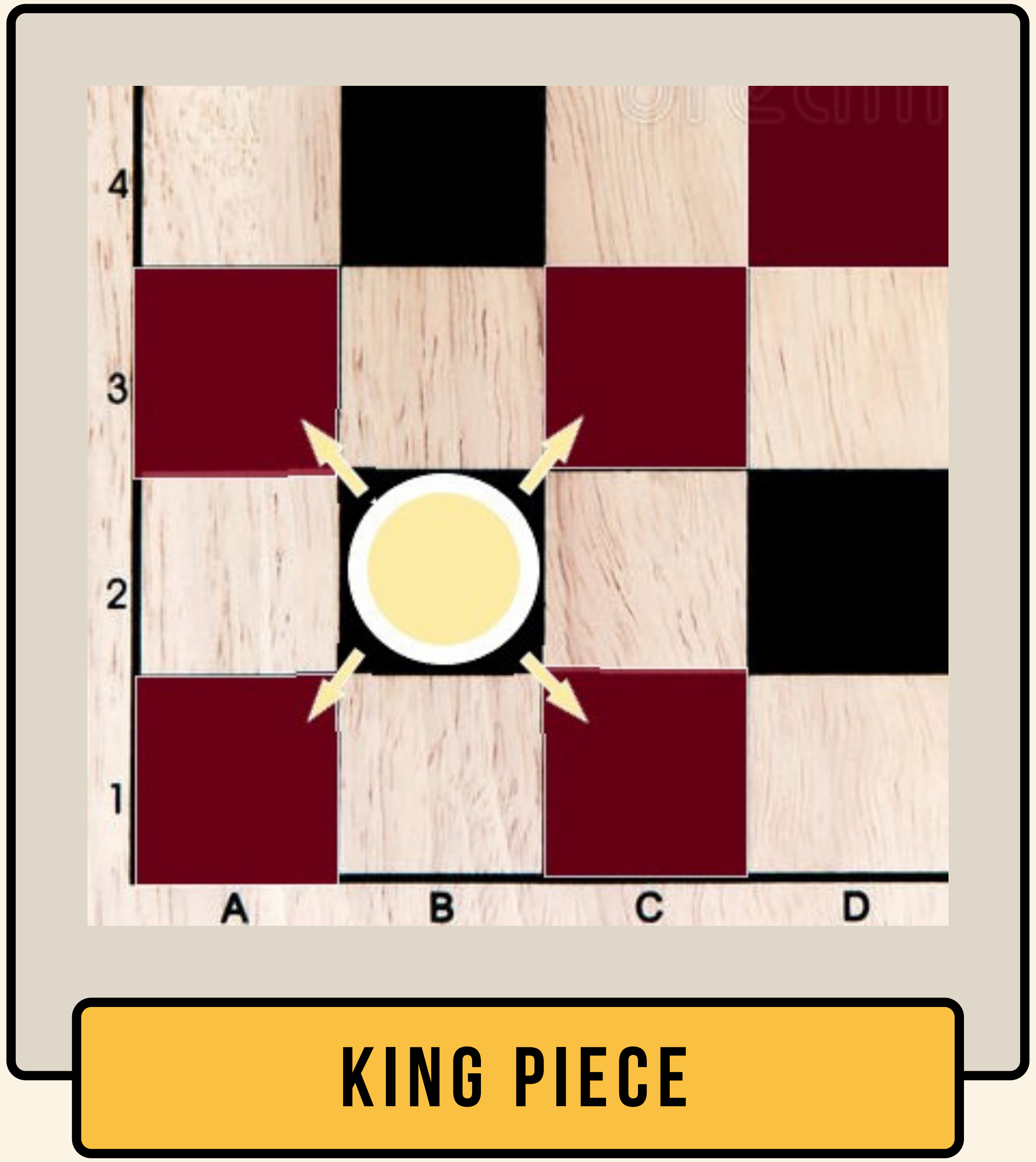


ORDINARY PIECE

INDEX & STATE SPACE

- **Index space:**
  {1, 2, 3, ...}

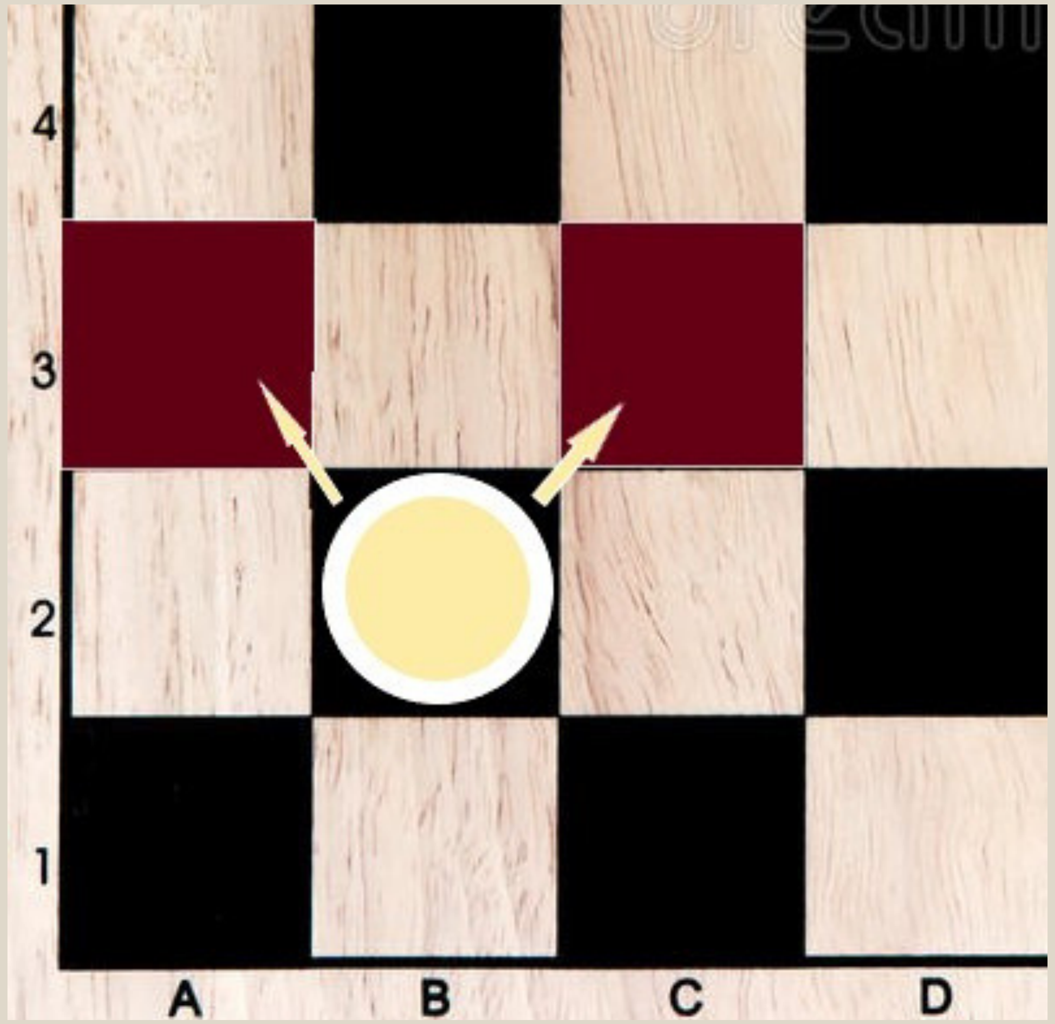
- **State space:**
  {A1, A3, B2, B4, C1, C3, D2, D4}



PROBABILITY MATRICES

$$
\begin{array}{c}
\\
A1 \\ A3 \\ B2 \\ B4 \\ C1 \\ C3 \\ D2 \\ D4
\end{array}
\begin{array}{c}
\begin{array}{cccccccc}
A1 & A3 & B2 & B4 & C1 & C3 & D2 & D4
\end{array} \\
\begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 \\
0 & 0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\end{array}
$$

ORDINARY PIECE

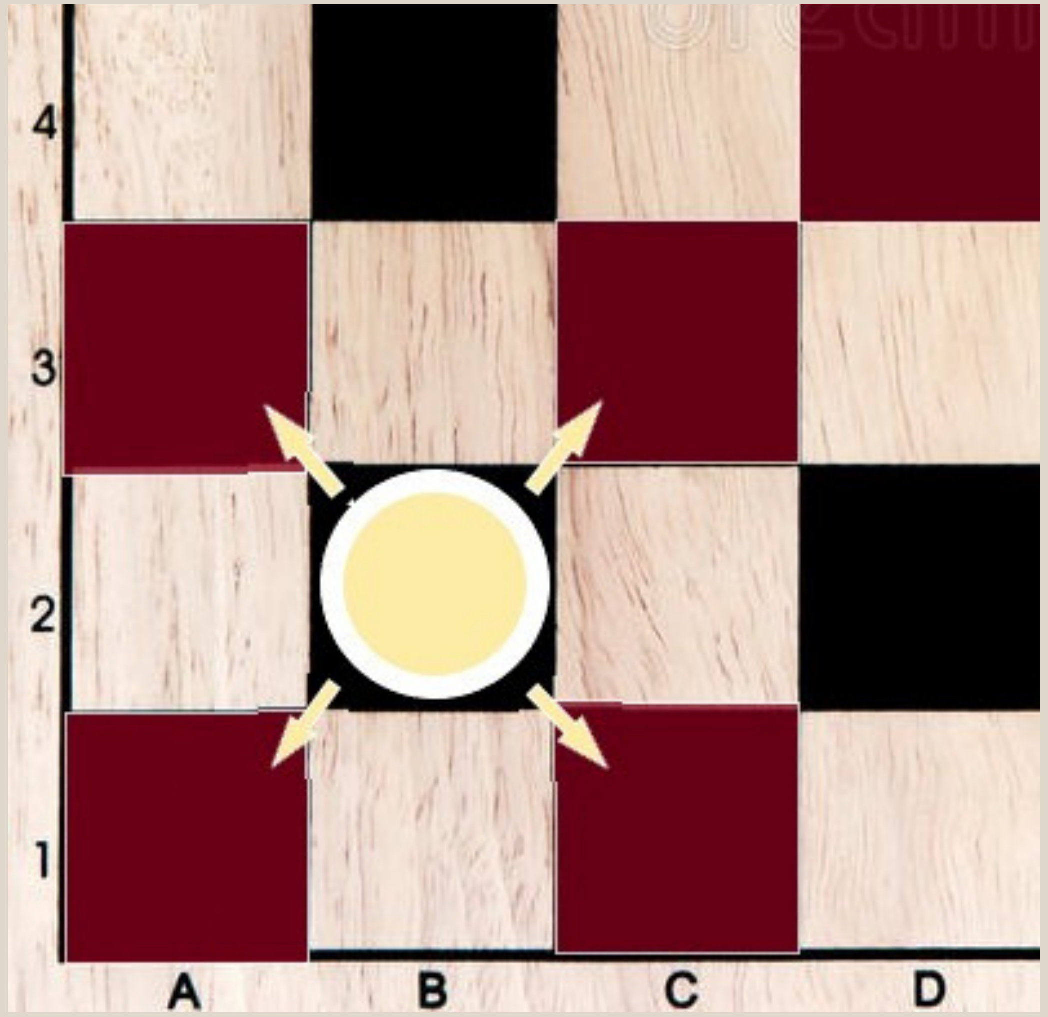


ORDINARY PIECE

PROBABILITY MATRICES

$$
\begin{array}{c c}
 & \begin{array}{cccccccc} A1 & A3 & B2 & B4 & C1 & C3 & D2 & D4 \end{array} \\
\begin{array}{c} A1 \\ A3 \\ B2 \\ B4 \\ C1 \\ C3 \\ D2 \\ D4 \end{array} &
\begin{bmatrix}
0 & 0 & 1/3 & 0 & 0 & 1/3 & 0 & 1/3 \\
0 & 0 & 1/3 & 1/3 & 1/3 & 0 & 0 & 0 \\
1/5 & 1/5 & 0 & 0 & 1/5 & 1/5 & 0 & 1/5 \\
0 & 1/3 & 0 & 0 & 0 & 1/3 & 1/3 & 0 \\
0 & 1/3 & 1/3 & 0 & 0 & 0 & 1/3 & 0 \\
1/5 & 0 & 1/5 & 1/5 & 0 & 0 & 1/5 & 1/5 \\
0 & 0 & 0 & 1/3 & 1/3 & 1/3 & 0 & 0 \\
1/3 & 0 & 1/3 & 0 & 0 & 1/3 & 0 & 0
\end{bmatrix}
\end{array}
$$

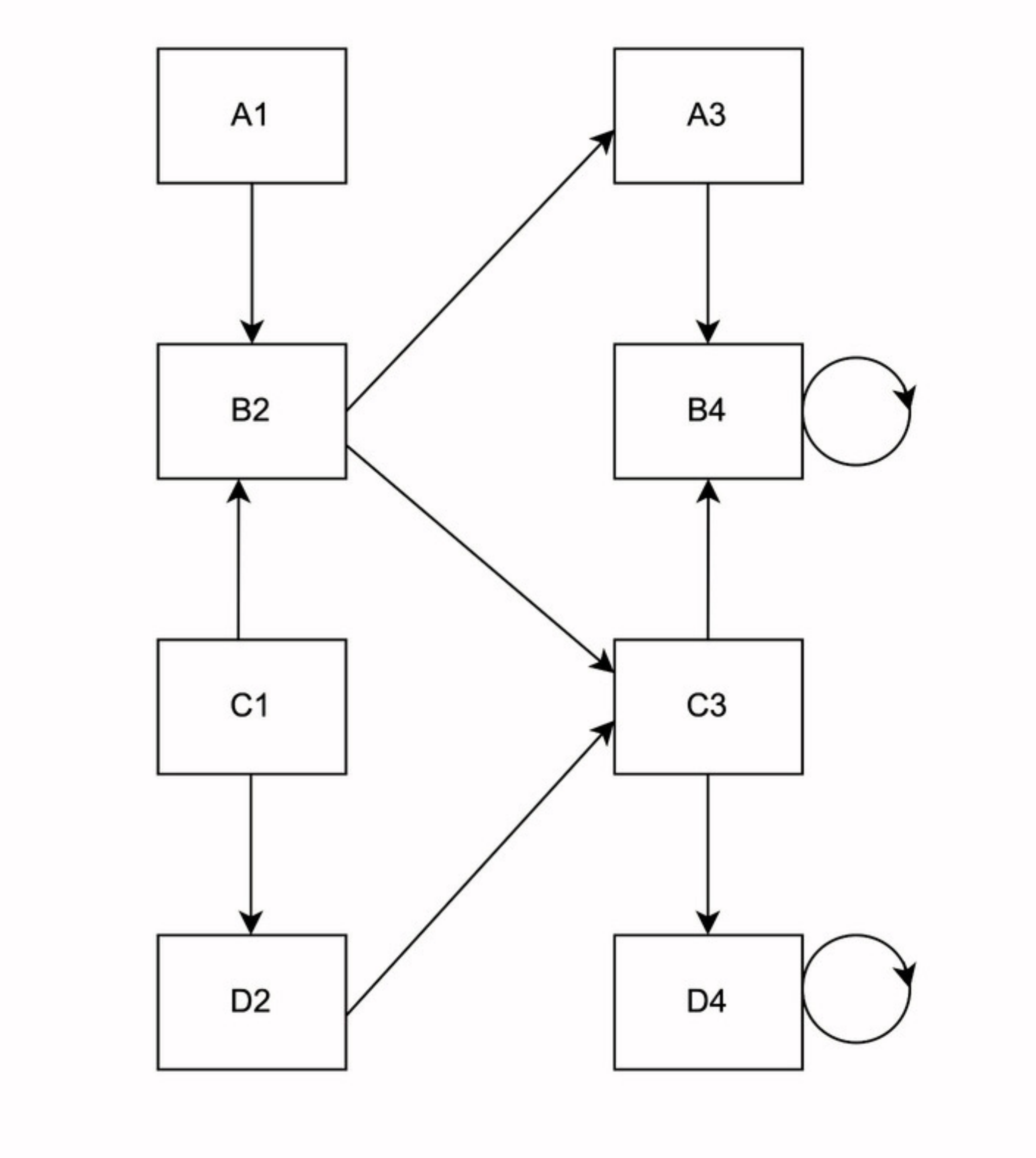
KING PIECE



KING PIECE

MARKOV PROCESS

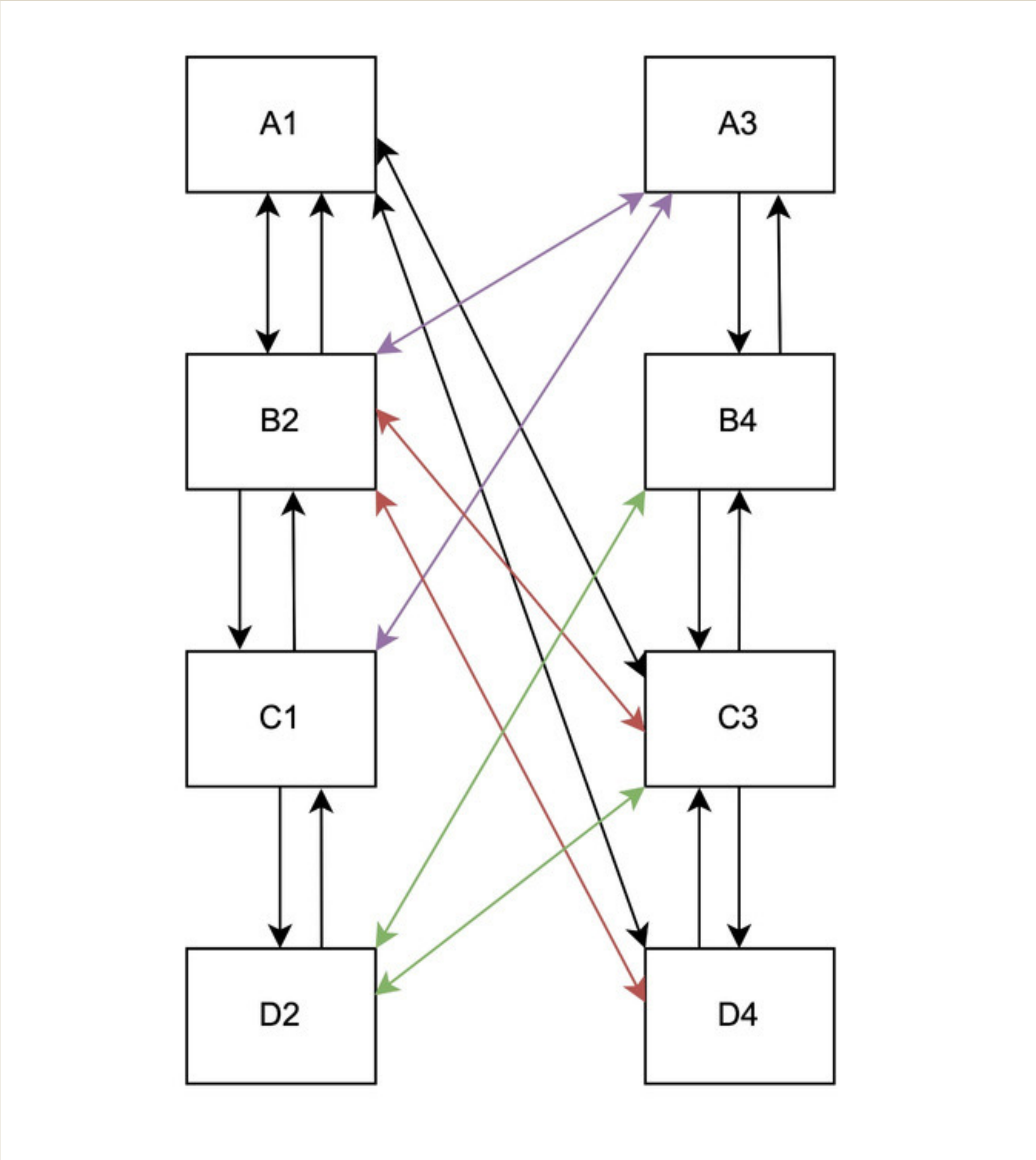
- There are 8 classes.
- Every state on board is a class of its own.
- A1, A3, B2, C1, C3, D2 are transient.
- B4 and D4 are recurrent.
- B4 and D4 are also absorbing states.
- Reducible Markov Chain.
- Periodicity is undefined for A1, A3, B2, C1, C3, D2.
- B4 and D4 are A-periodic



ORDINARY PIECE

MARKOV PROCESS

- There is only 1 class.
- Every state on board belongs to the same class.
- All states in the class are recurrent
- There are no absorbing states
- Irreducible Markov Chain.
- All states are A-periodic.



KING PIECE

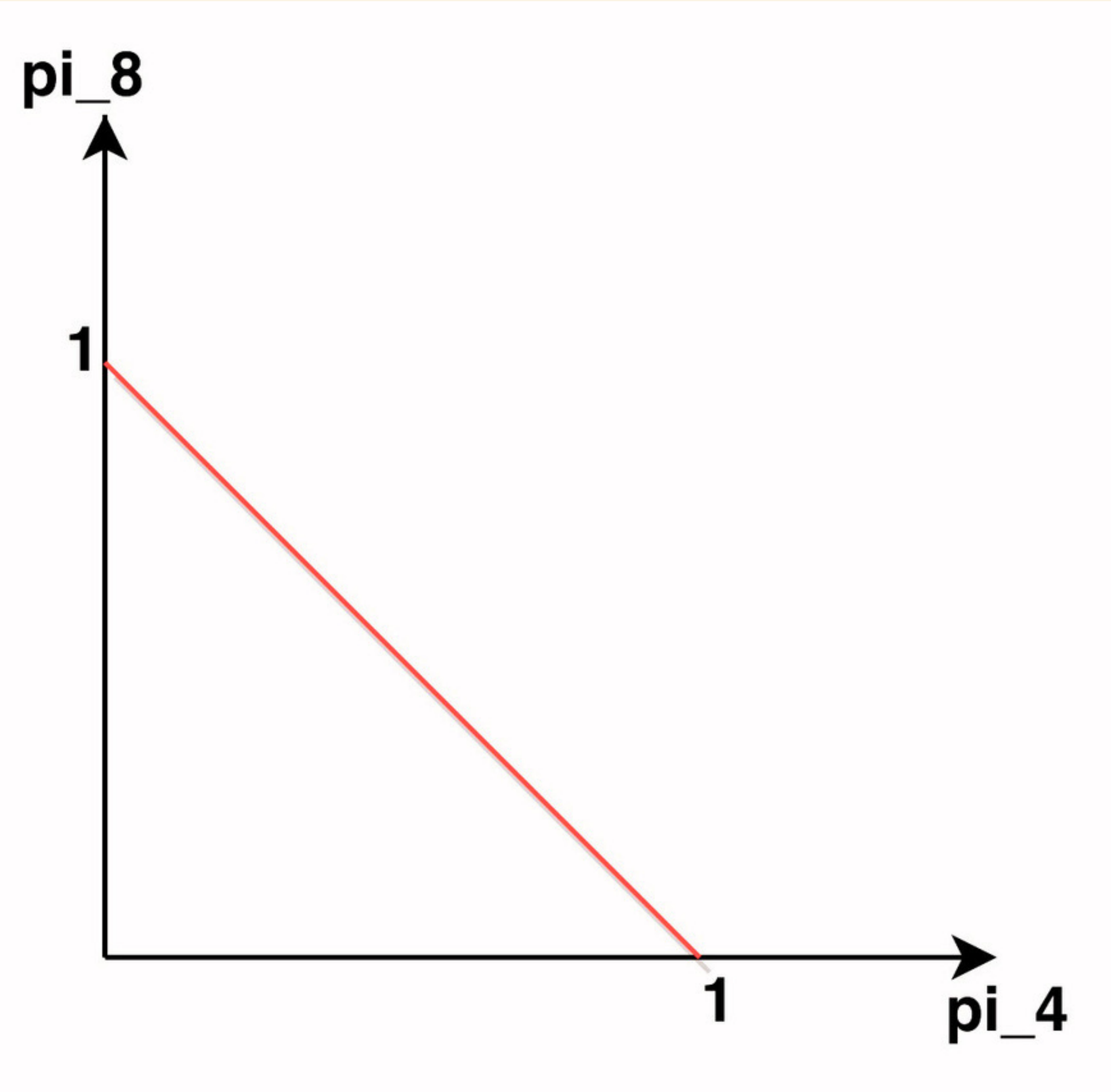
LIMITING DISTRIBUTION FOR ORDINARY PIECE

$$\begin{array}{ccccccccc} & \text{A1} & \text{A3} & \text{B2} & \text{B4} & \text{C1} & \text{C3} & \text{D2} & \text{D4} \\ \pi = & [\pi_1 & \pi_2 & \pi_3 & \pi_4 & \pi_5 & \pi_6 & \pi_7 & \pi_8] \end{array}$$

Since the Markov Chain is reducible and not a-periodic, there will be many solutions.

$$\boldsymbol{\pi} = \boldsymbol{\pi} * \boldsymbol{P}$$
$$\pi_1 = 0$$
$$\pi_2 = 1/2\pi_3$$
$$\pi_{3=}\pi_1 + 1/2\pi_5$$
$$\pi_4 = \pi_2 + \pi_4 + 1/2\pi_6$$
$$\pi_{5=0}$$
$$\pi_6 = 1/2\pi_3 + \pi_7$$
$$\pi_{7=}1/2\pi_5$$
$$\pi_8 = 1/2\pi_6 + \pi_8$$
$$\sum_{i=1}^{8} \pi_i = 1$$

LIMITING DISTRIBUTION FOR ORDINARY PIECE



$$\pi_1 = \pi_2 = \pi_3 = \pi_5 = \pi_6 = \pi_7 = 0$$
$$\pi_4 + \pi_8 = 1$$

$$\begin{array}{cccccccc} \text{A1} & \text{A3} & \text{B2} & \text{B4} & \text{C1} & \text{C3} & \text{D2} & \text{D4} \\ \end{array}$$
$$\pi = \begin{bmatrix} 0 & 0 & 0 & \pi_4 & 0 & 0 & 0 & \pi_8 \end{bmatrix}$$

LIMITING DISTRIBUTION FOR KING PIECE

$$\begin{array}{cccccccccc} & & \text{A1} & \text{A3} & \text{B2} & \text{B4} & \text{C1} & \text{C3} & \text{D2} & \text{D4} \\ \pi = & [& \pi_1 & \pi_2 & \pi_3 & \pi_4 & \pi_5 & \pi_6 & \pi_7 & \pi_8] \end{array}$$

Since the Markov Chain is irreducible and a-periodic, there will be an unique solution.

$$\boldsymbol{\pi} = \boldsymbol{\pi} * \boldsymbol{P}$$

$$\pi_1 = 1/5\,\pi_3 + 1/5\,\pi_6 + 1/3\,\pi_8$$

$$\pi_2 = 1/5\,\pi_3 + 1/3\,\pi_4 + 1/3\,\pi_5$$

$$\pi_3 = 1/3\,\pi_1 + 1/3\,\pi_2 + 1/3\,\pi_5 + 1/5\,\pi_6 + 1/3\,\pi_8$$

$$\pi_4 = 1/3\pi_2 + 1/5\,\pi_6 + 1/3\,\pi_1$$

$$\pi_5 = 1/3\,\pi_2 + 1/5\,\pi_3 + 1/3\,\pi_7$$

$$\pi_6 = 1/3\,\pi_1 + 1/5\,\pi_3 + 1/3\,\pi_4 + 1/3\,\pi_7 + 1/3\,\pi_8$$

$$\pi_7 = 1/3\,\pi_4 + 1/3\,\pi_5 + 1/5\,\pi_6$$

$$\pi_8 = 1/3\,\pi_1 + 1/5\,\pi_3 + 1/5\,\pi_6$$

$$\sum_{i=1}^{8} \pi_i = 1$$

LIMITING DISTRIBUTION FOR KING PIECE

A1	π_1	$= 0.08333333$
A3	π_2	$= 0.14000000$
B2	π_3	$= 0.16666667$
B4	π_4	$= 0.12666667$
C1	π_5	$= 0.12666667$
C3	π_6	$= 0.16666667$
D2	π_7	$= 0.14000000$
D4	π_8	$= 0.05000000$

CONCLUSION

- The Markov chain process is a powerful tool for analyzing checkers games.

- By understanding the state space, transitions, probability matrix, stationary distribution, and absorbing states, players can make more informed decisions and improve their chances of winning.

FUTURE WORKS

- Analyzing the Markov process of the full 8x8 Checkers board.
- Considering the opponent pieces in making the Probability Transition Matrix.

THANK YOU. 😊

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