STAT 4080/5080 Experimental Design

Project Report

Paper Airplane Experiment

By

Tomiwa Omotesho, Ashra Regmi, Daniel Tullis, Mehmet Atay

Bowling Green State University

1. Introduction

This project report is based on a paper airplane experiment that was conducted to determine the impact factors such as plane design, paper type and payload have on the total flight distance of the paper plane. A paper airplane experiment is a great way to learn the basics of an experimental design, using readily available materials such as papers, paper clips and measurement tape, to simulate how a paper airplane flies. Paper plane experiment allows us to simulate the scientific experiment of real airplanes. The exploration of experimental ideas and procedures on how paper airplanes work is also encouraged in many world competitions for paper airplanes . Thus, these activities regarding real-life simulations are intuitive, interactive, and instructive.

2. Description of Data Collection

In the paper airplane experiment, we had a three-factor randomized complete block design with fixed effects, as described in our proposal. The three factors of interest in our experiment were plane type (Basic Dart, Stable), paper type (A4 and Construction) and payload (No Load and Load, in the form of two binder paper clips). All of the three factors were categorical. A total of 24 trials were made (2 levels x 2 levels x 2 levels x 3 blocks/operators) in the randomized order designed for the experiment. The total distance (in feet) the paper airplanes traveled perpendicular to the operator's standing position, as illustrated in appendix A were measured as the response. The response measurements were taken from the nose (tip) of the paper airplanes when the planes came to a complete stop as shown in appendix B.

The experiment was performed at the S200 Eppler Gym, Bowling Green State University on April 08, 2022. A picture of the gym is shown in appendix C. The only change made to the experiment from our final proposal was that we replaced the Sprinter paper airplane design with the Stable design. The reason for this plane design change was that the Sprinter paper airplane nose dived and collapsed immediately at the feet of the operators without achieving any significant flight distance. The Stable design was chosen to replace the Sprinter design because it shares similar body and wing characteristics with the Sprinter design. The Stable design has a shorter body and wider wings when compared to the Basic Dart design. Figures 1 and 2 show the paper airplanes used in the experiment. The tools used for the experiment included A4 (white) and construction papers (purple), binder paper clips, 25 ft measuring tape, ruler and painter's tape. Some of the tools used can be seen in appendix B and D. The data collection was completed in two hours. Table 1 shows the data collected during the experiment.

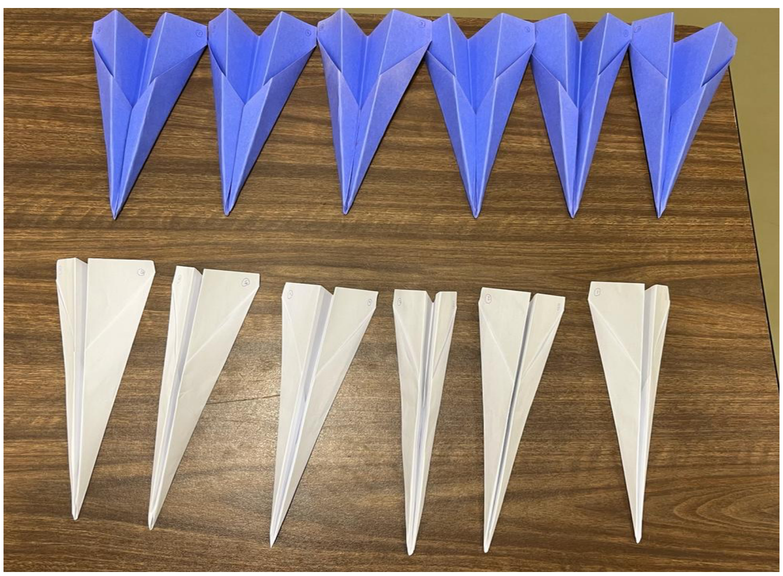


Figure 1: Basic Dart design

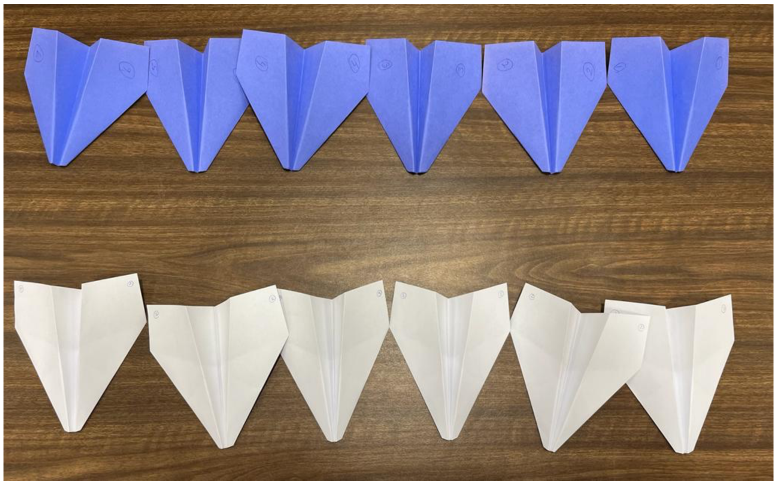


Figure 2: Stable design

Table 1: Flight distance of paper airplane (ft) for all factor combinations in each block.

Plane Type	Block	No Load		Load	
		Paper Type		Paper Type	
		A4	Construction	A4	Construction
Basic Dart	Ashra	44.4	21.96	39.14	28.75
	Daniel	38.79	28.48	36.2	36.94
	Mehmet	35	27.69	34.15	33.75
Stable	Ashra	10	14.48	17.08	28.32
	Daniel	11.25	13.58	14.71	25.02
	Mehmet	9.5	14.12	20.46	33.27

3. Discussion of Analysis

After the data was collected, data analysis was conducted using R software.

3.1 ANOVA Table

As seen from the ANOVA table, Plane Type and Payload, as well as all three two-factor interactions are significant at (α =0.05).

Table 2: ANOVA test results of the paper airplane experiment

Source of variation	**Degrees of freedom (df)**	**Sum of squares (SS)**	**Mean square (MS)**	**F value**	**Pr(>F)**
Block	2	1.0	0.5	0.042	0.959459
Plane Type	1	1559.4	1559.4	129.231	1.87e-08
Paper Type	1	0.8	0.8	0.064	0.803302
Payload	1	257.0	257.0	21.299	0.000401
PlaneType:PaperType	1	383.2	383.2	31.756	6.15e-05
PlaneType:Payload	1	118.5	118.5	9.817	0.007333

PaperType:Payload	1	116.8	116.8	9.677	0.007666
PlaneType:PaperType:Payload	1	2.1	2.1	0.173	0.683695
Residuals	14	168.9	12.1		

3.2. Assumption Checks

2.2.1 Normality Assumption

The normal probability plot for the residual paper airplane flight distance is represented in figure 3. The normal probability plot is approximately linear, with little deviations from the normal line, so we assumed that the data is normally distributed.

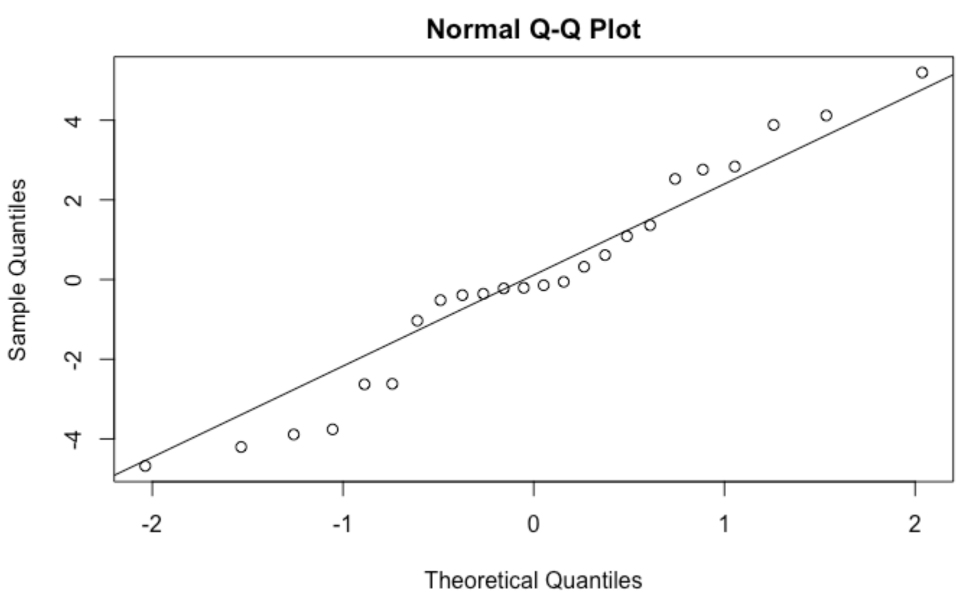


Figure 3: Normal probability plot of paper airplane flight distance

3.2.2 Constant Variance

For the residuals vs plane type, residuals vs paper type and residuals vs payload plots, we saw that each level of the three factors seemed to have approximately the same variance (constant variance). Likewise, for the residuals vs fitted flight distances plot, we did not observe a clear pattern between the fitted flight distances and the residuals. Therefore, we concluded that the constant variance assumption was satisfied for all four plots in figure 4.

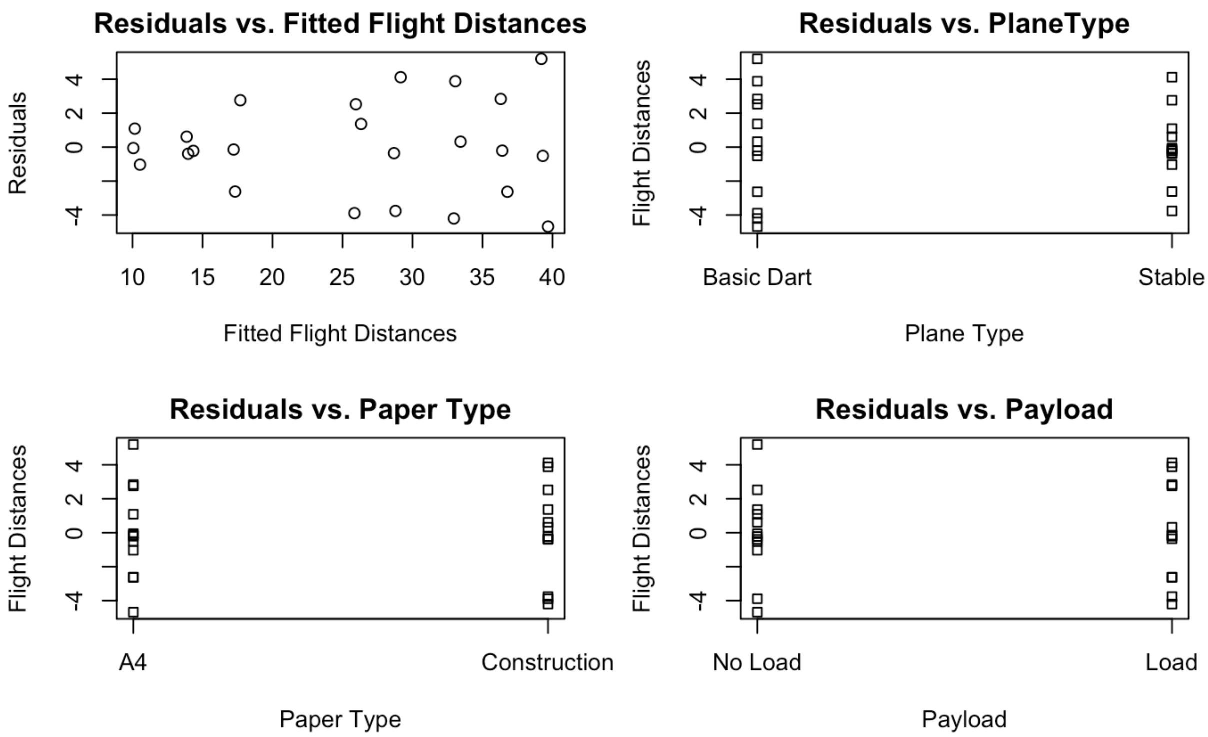


Figure 4: Constant variance plots

3.3 Two-factor interactions

From the ANOVA analysis, we observed that the two factor interaction between all the factors were significant. The two-factor interaction plots in figure 5 also show that there are significant two-factor interactions between Plane type, Paper type and Payload.

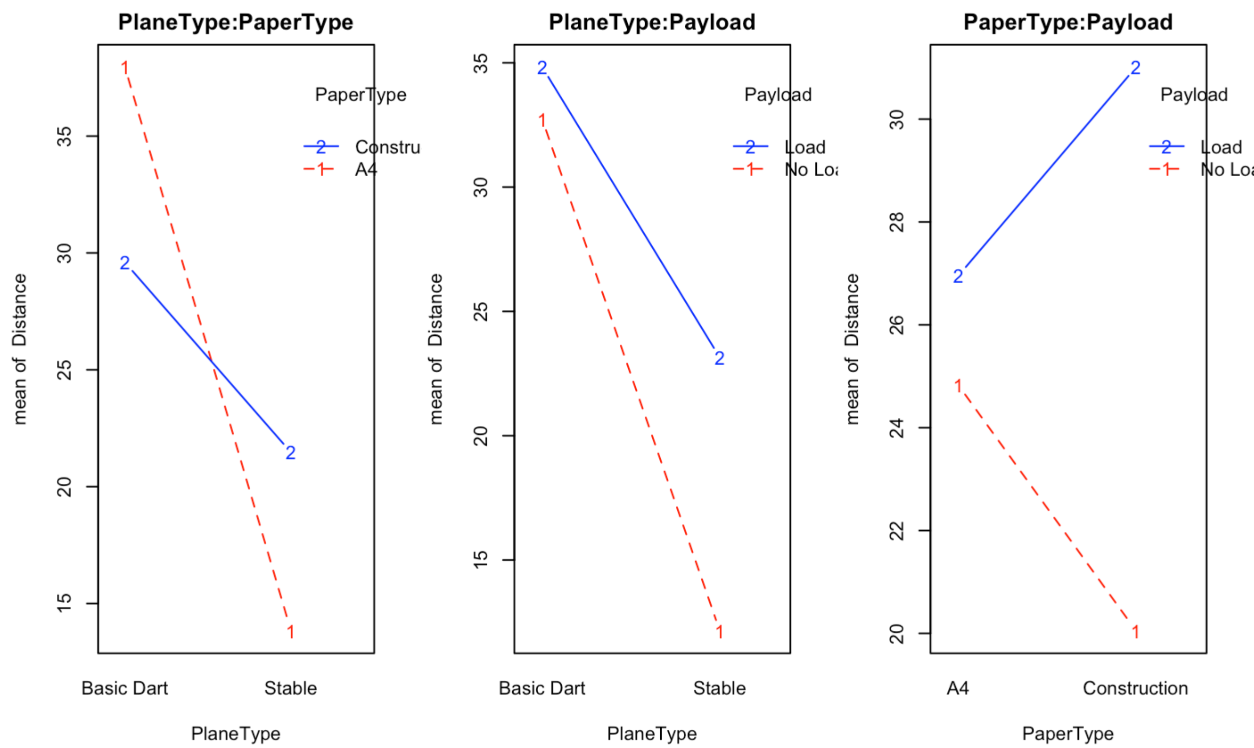


Figure 5: Two-factor interaction plots

3.4 Contrasts

We carried out six contrasts based on the fact that we observed interactions in our experiment. The contrasts were done to achieve a family-wise confidence interval of 95%. The results are shown in table 3.

Table 3: Summary of contrasts

		Difference	**Lower CI**	**Upper CI**
Paper Type=A4	Basic Dart - Stable	24.11333	18.655222	29.57144
Paper Type=Construction	Basic Dart - Stable	8.13000	2.671889	13.58811
Payload = No Load	Basic Dart - Stable	20.56500	15.106889	26.02311
Payload = Load	Basic Dart - Stable	11.67833	6.220222	17.13644
Paper Type=A4	No Load - Load	-2.133333	-7.591445	3.324778
Paper Type=Construction	No Load - Load	-10.956667	-16.414778	-5.498555

At family wise confidence interval of 95%:

There is a significant difference between the mean flight distance of Basic Dart and Stable for both levels of Paper Type. The Basic Dart achieves a farther flight distance.

There is a significant difference between the mean flight distance of Basic Dart and Stable for both levels of Payload. The Basic Dart achieves a farther flight distance.

There is no significant difference in the means of No Load and Load when the Paper Type is fixed as A4.

There is a significant difference in the means of No Load and Load when the Paper Type is fixed as Construction.

3.5 Data Snooping

From the treatment means, we observed that the Basic Dart - A4 - No Load treatment combination achieved the best flight distance followed by the Basic Dart - A4 and Load treatment. We decided to compare the contrast between the two treatment means with the Scheffe multiplier to account for the data snooping.

The Scheffe test indicated that at a family-wise confidence interval of 95%, the mean flight distance of the Basic Dart - A4 - No Load treatment is between 9.593 less than and 15.393 more than the mean flight distance of the Basic Dart - A4 - Load treatment. We concluded that there is no significant difference between both treatments.

3.6 Relative Efficiency

Blocking in this three-factor randomized complete block design with fixed effects was not really worthwhile, as we obtained a relative efficiency of 0.904. We would have needed 0.904 times of data in a three-factor completely randomized design to achieve the same power as the three-factor randomized complete block design in this experiment.

4. Conclusion and Summary

What we expected to find:

We expected blocking to make more of a difference within our experiment. We thought that different operators would have an effect on the distance the plane traveled. As the different operators would have been due to throwing style, throwing force, release point, etc.. This did not appear to be the case and it was a surprise to us. Each operator also made their own planes. We thought this may cause a significant difference between the operators and the distance their planes flew. This also did appear to be the case with our experiment. Our relative efficiency was 0.904. This means that blocking was not worthwhile for our experiment. If we ran the same experiment as a Completely Randomized Design (CRD), we would only need 22 observations. We also expected that adding a payload to a paper airplane would cause the plane to travel less distance. This was not the case. The load did not cause the planes to travel less distance. The biggest thing that we expected was that the Basic Dart would fly farther than the Stable. Looking at the two factor interactions we were able to see that the Basic Dart flew further than The Stable. The Stable was more impacted by the factors than the Basic Dart was. The different levels of factors had more of an impact on the mean flight distance (in feet) for the Stable. We did not think about this before conducting the experiment. We were not sure how the different factors would impact each plane's flight distance.

What we would change:

If we were running this experiment again, there are a few things that we would change. Firstly, we would include more levels for each of our factors. For plane design, we would have included a minimum of three different plane designs. For payload, we would increase the level of payload to see if there was a significant difference between each load. This could be done by having four levels of payload consisting of, no binder clips, one binder clip, two binder clips, and three binder clips. For paper type, we would include a minimum of three different paper types. We would try to find paper types of varying weight and thickness. Secondly, we would not block if we were to run the experiment again. The relative efficiency was 0.904 and this means that only 0.904 times the data would be required for a Completely Randomized Design (CRD).

5. Statement of Appreciation for the Best Learning Experience

This experiment provided us with a unique learning experience. Conducting this experiment allowed us to understand how the data collection process works. We were able to learn the process of designing and running an experiment. We had to consider all the details of the experiment including the materials we used, location we conducted the experiment, and efficient sample size. It was a great opportunity to learn how to implement the material we have been learning in class into a real world example. Working with the data after the experiment was like a homework problem. We did not find too much trouble with the analysis itself. Creating the report and presentation was a new process for us. We believe that the report and presentation will help give us an idea of what real world experiments will be like. The people we will be presenting to (Management, CFO'S, CEO'S, etc.) will not care about the whole experiment. They will be interested in the results, the accuracy of our results, and how well we stayed within the budget. We realized that only necessary information should be included in our presentation. We should not discuss how we did the analysis, but rather the results and their implications.

6.1 Original Proposal

STAT 4080/5080 Experimental Design Project Proposal

Paper Airplane Experiment

By

Tomiwa Omotesho, Ashra Regmi, Mehmet Atay, Daniel Tullis

Bowling Green State University

Goal: The goal of this experiment is to determine if the design of the paper airplane, type of paper and payload of the plane impact the total flight distance of the paper airplane.

Response: The flight distance (in feet) achieved by the paper airplane.

Table 1: Factors for the Experiment

Factors (These factors are chosen as categorical variables)	**Levels**
A = Plane Design	2 (1 = Basic Dart, 2 = Sprinter)
B = Paper Type	2 (1 = A4, 2 = Construction)
C = Payload	2 (1 = No Load, 2 = Load)

Narrative

The flight distance that a paper airplane achieves based on the plane design, paper type and payload is an important measure of the success of the paper airplane. The total distance (in feet) the paper airplane travels in a straight line from the operator's standing position will be measured as the response.

We are interested in investigating how the design of the paper airplane, type of paper, and payload (weight added to the plane) impact the flight distance. If there is any impact, we expect it to be with all three factors. Our experiment will have three group members (operators) as blocks. We have chosen two paper airplane designs with significant differences in body/wing shape for the experiment. Basic Dart design has a longer body and narrow wings whereas the Sprinter design has shorter body and wider wings. Similarly, two levels of payload of the airplane will be used; no load added to the paper airplane and load in the form of 2 paper clips attached to the center of gravity of the paper airplane. Two paper types; A4 paper and construction paper will be used. A4 paper is lighter whereas construction paper is heavier. Since, all the three factors and their levels are deliberately chosen for the experiment, we expect to see reasonable differences in the response values for different combinations of factors.

With three operators (Daniel, Ashra and Mehmet) performing each of the eight treatments once, we will have 24 observations in three blocks. Each of the operators has varying throwing capabilities, therefore, blocking is needed in our experiment. The order that the three blocks are to be run, as well as the order of the treatments within each block was determined with the R code 1. The data table on page 5 shows the throwing sequence. To determine the standard deviation, we performed a pilot study and carried out 10 trials with the same treatment (paper design = Basic Dart, paper type = A4, payload = No Load) and the same operator (Ashra), as shown in the R code 2. We obtained a standard deviation of 2.949 and variance of 8.696. With constant variance assumed, the variation determined from our pilot study is the same as that for any other treatment within any block.

We would like to be able to detect a difference of 4 feet between the means of any two levels of each factor, with a reasonably high probability of 0.80 and type 1 error at 0.05. The non-centrality parameter for each factor, $\lambda = 11.0391$. With the factors having two levels each ($a = b = c = 2$), DA = DB = DC = 4, and $\sigma^2 = 8.696$, we confirm that three blocks are sufficient, and we achieve a power of 0.87 for each factor. The desired power of 0.80 is obtained for each factor as shown in the R code 2. Thus, it appears we will have an adequate amount of data. The data will be analyzed as a three-factor randomized complete block design with fixed effects.

To complete this experiment we need; a large indoor vacant area (to prevent the paper airplanes from colliding with obstacles such as walls and to minimize wind effect), a measuring tape (to measure the flight distance of the paper airplane in feet), A4 and construction papers (to make the paper airplanes), paper clips (for adding load to the plane) and painter's tape (to mark out distances at intervals of 10 feet from the operator's standing position). We plan to allocate a minimum of two hours for data collection. It should take at least five minutes to perform each trial (throwing + waiting for landing + distance measurement + recording + preparation for next trial). We plan to make separate sets of paper airplanes for each operator to maintain consistency. When an operator is performing a set of eight treatments, with one trial at a time, the other three group members will work on measuring, reporting the distance and preparing the materials for subsequent trials. We plan to gather data in a single day. Assuming we get this proposal approved on April 03, we plan to proceed to South Eppler Gym on April 04 to collect the data.

R code used to randomize the experiment (R code 1)

```
library('psych')

set.seed(0)

flight_distance <- rep(" ", 24)

blocks <-sample(c("Ashra","Mehmet", "Daniel"),3)

data_table <- data.frame(block.random(24,c(plane_design=2,paper_type=2, payload=2)))

data_table$blocks <- c(rep((blocks),each=8))

data_table$plane_design <- ifelse(data_table$plane_design==1, "Basic Dart", "Sprinter")

data_table$paper_type <- ifelse(data_table$paper_type==1, "A4", "Construction")

data_table$payload <- ifelse(data_table$payload==1, "No Load", "Load")
```

data_table <- cbind(data_table, flight_distance)

R code used to determine the number of blocks and calculate the power for the experiment (R code 2)

Sd_paperplane <- sd(c(31.9375, 36.708333, 37.380208333, 32.546875, 35.473958333, 34.90625, 41.21875, 38.994791667, 39.03125, 34.791667))

sd_paperplane

[1] 2.948947

<u>For Plane Design (Factor A)</u>

a <- 2 # number of levels of plane design

b <- 2 # number of levels of paper type

c <- 2 # number of levels of payload

alpha <- .05

sigsq <- sd_paperplane^2 # our sigma^2 estimate

blocks <- 2:10 # block size for plane design comparisons

DA <- 4 # desired diff in means to detect with prob 1-beta

Fcrit <- qf(1-alpha,a-1,(a*b*c-1)*(blocks-1)) # value at which we reject H0

lam <- (blocks*b*c)*(DA^2)/(2*sigsq) # non-centrality parameter (ncp)

beta <- pf(Fcrit,a-1, (a*b*c-1)*(blocks-1),ncp=lam)

power <- 1-beta

blocksforA <- cbind(blocks, lam, Fcrit,beta,power) # output for plane design

blocksforA[2,]

```
   blocks        lam      Fcrit       beta      power
3.0000000 11.0391901  4.6001099  0.1297354  0.8702646
```

For Paper Type (Factor B)

```
blocks <- 2:10 # block size for paper type comparisons

DB <- 4 # desired diff in means to detect with prob 1-beta

Fcrit <- qf(1-alpha,b-1,(a*b*c-1)*(blocks-1)) # value at which we reject H0

lam <- blocks*a*c*(DB^2)/(2*sigsq)# non-centrality parameter (ncp)

beta <- pf(Fcrit,b-1,(a*b*c-1)*(blocks-1),ncp=lam)

power <- 1-beta

blocksforB <- cbind(blocks, lam, Fcrit,beta,power)# output for paper type

blocksforB[2,]

 blocks       lam      Fcrit       beta      power
 3.0000000 11.0391901  4.6001099  0.1297354  0.8702646
```

For Payload (Factor C)

```
blocks <- 2:10 # block size for weight type comparisons

DC <- 4 # desired diff in means to detect with prob 1-beta

Fcrit <- qf(1-alpha,c-1,(a*b*c-1)*(blocks-1)) # value at which we reject H0

lam <- blocks*a*b*(DC^2)/(2*sigsq) # non-centrality parameter (ncp)

beta <- pf(Fcrit,c-1,(a*b*c-1)*(blocks-1),ncp=lam)

power <- 1-beta

blocksforC <- cbind(blocks, lam, Fcrit,beta,power) # output for payload

blocksforC[2,]

 blocks       lam      Fcrit       beta      power
 3.0000000 11.0391901  4.6001099  0.1297354  0.8702646
```

Data Table

Table 2: Data Table for the Experiment

Blocks	**Plane Design**	**Paper Type**	**Payload**	**Flight Distance (ft)**
Daniel	Sprinter	A4	Load	
Daniel	Sprinter	Construction	Load	
Daniel	Basic Dart	Construction	No Load	
Daniel	Basic Dart	A4	No Load	
Daniel	Basic Dart	A4	Load	
Daniel	Basic Dart	Construction	Load	
Daniel	Sprinter	Construction	No Load	
Daniel	Sprinter	A4	No Load	
Ashra	Basic Dart	Construction	No Load	
Ashra	Basic Dart	A4	No Load	
Ashra	Sprinter	A4	Load	
Ashra	Sprinter	Construction	Load	
Ashra	Basic Dart	A4	Load	
Ashra	Basic Dart	Construction	Load	
Ashra	Sprinter	Construction	No Load	
Ashra	Sprinter	A4	No Load	
Mehmet	Basic Dart	A4	Load	
Mehmet	Basic Dart	Construction	Load	
Mehmet	Sprinter	Construction	Load	

Mehmet	Basic Dart	Construction	No Load	
Mehmet	Sprinter	Construction	No Load	
Mehmet	Sprinter	A4	No Load	
Mehmet	Basic Dart	A4	No Load	
Mehmet	Sprinter	A4	Load	

References

Puspita, A. N. G., Ambarwati, L., Ludfiyanti, E., Tejonugroho, D. P., & Zen, Y. (2020). Effect of paper weight, paper length, and nose of paper plane on aircraft mileage in paper airplane game. *UI Proceedings on Science and Technology*, *2*.

Montgomery, D.C. (2020), *Design and Analysis of Experiments*, 10th Edition, Wiley.

Fold 'N Fly. (2022). *Paper Airplane Designs*. Retrieved March 26, 2022, from https://www.foldnfly.com/#/1-1-1-1-1-1-1-1-2.html.

6.2 Final Approved Proposal

STAT 4080/5080 Experimental Design Project Proposal

Paper Airplane Experiment

By

Ashra Regmi, Mehmet Atay, Tomiwa Omotesho, Daniel Tullis

Group 5

Bowling Green State University

Goal: The goal of this experiment is to determine if the design of the paper airplane, type of paper and payload of the plane impact the total flight distance of the paper airplane.

Response: The flight distance (in feet) achieved by the paper airplane.

Table 1: Factors for the Experiment

Factors (These factors are chosen as categorical variables)	**Levels**
A = Plane Design	2 (1 = Basic Dart, 2 = Sprinter)
B = Paper Type	2 (1 = A4, 2 = Construction)
C = Payload	2 (1 = No Load, 2 = Load*)

***Load will be the equivalent of two, standard binder paper clips.**

Narrative

The flight distance that a paper airplane achieves based on the plane design, paper type and payload is an important measure of the success of the paper airplane. The total distance (in feet) the paper airplane travels perpendicular to the operator's standing position as shown in figure 1, will be measured as the response. This measurement will be taken from the nose (tip) of the paper airplane when the plane comes to a complete stop.

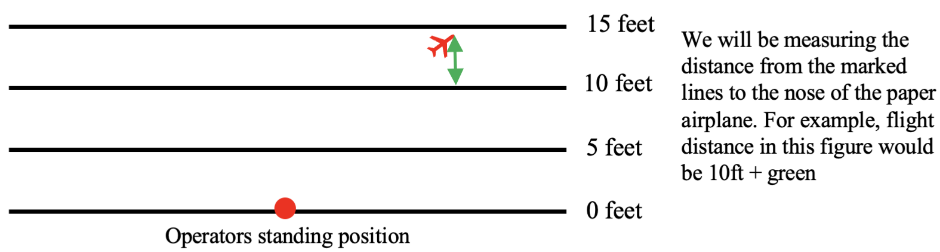


Figure 1. Flight distance measurement for paper airplane

We are interested in investigating how the design of the paper airplane, type of paper, and payload (weight added to the plane) impact the flight distance. If there is any impact, we expect it to be with all three factors. Our experiment will have three group members (operators) as blocks. We have chosen two paper airplane designs with significant differences in body/wing shape for the experiment. Basic Dart design has a longer body and narrow wings whereas the Sprinter design has shorter body and wider wings as shown in figures 2 and 3.

Figure 2. Basic Dart Design Figure 3. Sprinter Design

Similarly, two levels of payload of the airplane will be used; no load added to the paper airplane and load in the form of two binder paper clips attached to the midpoint of the base of the airplane as shown in figures 4, 5, 6, and 7. Two paper types; A4 paper and construction paper will be used. A4 paper is lighter whereas construction paper is heavier. Since, all the three factors and their levels are deliberately chosen for the experiment, we expect to see reasonable differences in the response values for different combinations of factors.

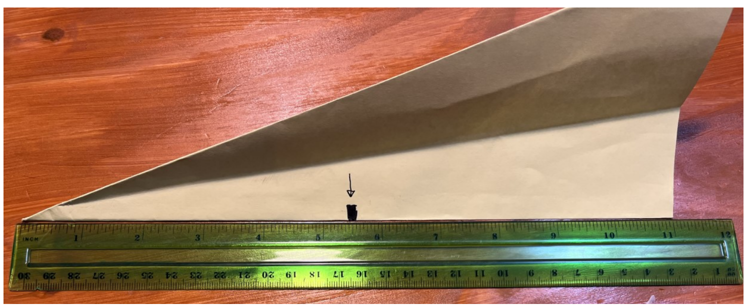


Figure 4. Midpoint of the base of the basic dart

Figure 5. Two binder paper clips added to the midpoint of the base of the basic dart

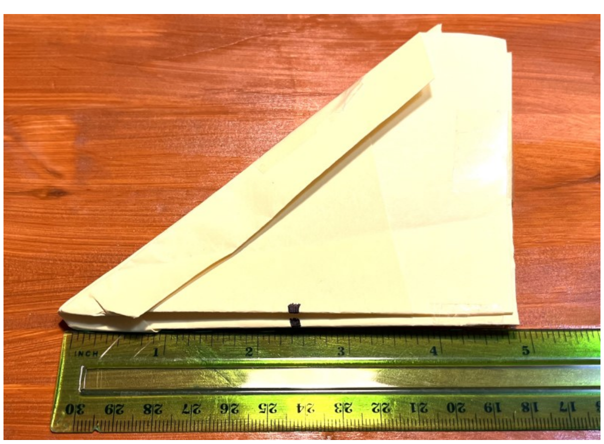


Figure 6. Midpoint of the base of the sprinter

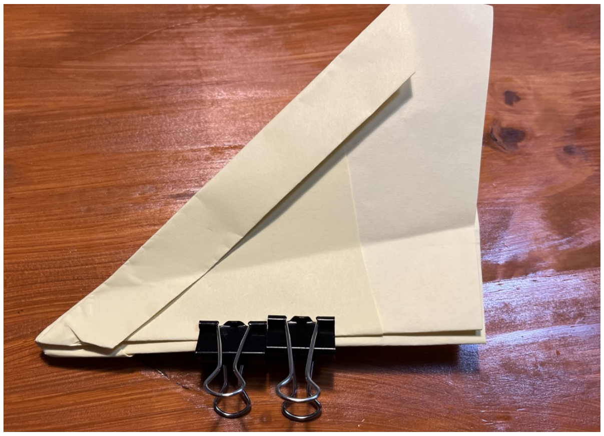


Figure 7. Two binder paper clips added to the midpoint of the base of the sprinter

With three operators (Daniel, Ashra and Mehmet) performing each of the eight treatments once, we will have 24 observations in three blocks. Each of the operators has varying throwing capabilities, therefore, blocking is needed in our experiment. The order that the three blocks are to be run, as well as the order of the treatments within each block was determined with the R code 1. The data table on page 8 shows the throwing sequence. To determine the standard deviation, we performed a pilot study and carried out 10 trials with the same treatment (paper design = Basic Dart, paper type = A4, payload = No Load) and the same operator (Ashra), as shown in the R code 2. The distances from the floor markers to the final position of the paper airplanes was measured in inches for accuracy when taking readings and then converted to feet (without any approximation done). We used a 25ft (300 inches) tape measure. A standard deviation of 2.949 feet and variance of 8.696 feet was obtained using the data from the pilot study. With constant variance assumed, the variation determined from our pilot study is the same as that for any other treatment within any block.

We would like to be able to detect a difference of 4 feet between the means of any two levels of each factor, with a reasonably high probability of 0.80 and type 1 error at 0.05. The non-centrality parameter for each factor, **λ =** 11.0391. With the factors having two levels each (a = b = c = 2), DA = DB = DC = 4, and σ^2 = 8.696, we confirm that three blocks are sufficient, and we achieve a power of 0.87 for each factor. The desired power of 0.80 is obtained for each factor as shown in the R code 2. Thus, it appears we will have an adequate amount of data. The data will be analyzed as a three-factor randomized complete block design with fixed effects.

To complete this experiment we need; a large indoor vacant area (to prevent the paper airplanes from colliding with obstacles such as walls and to minimize wind effect), a measuring tape (to measure the flight distance of the paper airplane in feet to the nearest sixteenth of an inch), A4 and construction papers (to make the paper airplanes), binder paper clips (for adding load to the plane), ruler (to determine the midpoint of the paper airplanes) and painter's tape (to mark out distances at intervals of 5 feet from the operator's standing position). We plan to allocate a minimum of two hours for data collection. It should take at least five minutes to perform each trial (throwing + waiting for landing + distance measurement + recording + preparation for next trial). 24 paper airplanes will be made in total for the experiment, with each operator making paper airplanes for his/her eight trials. While an operator is performing a set of eight treatments, with one trial at a time, the other three group members will work on measuring, and recording the distance. We plan to gather data in a single day. Assuming we get this proposal approved on April 07, we plan to proceed to South Eppler Gym on April 08 to collect the data.

R code used to randomize the experiment (R code 1)

```
library('psych')

set.seed(0)

flight_distance <- rep(" ", 24)

blocks <-sample(c("Ashra","Mehmet", "Daniel"),3)

data_table <- data.frame(block.random(24,c(plane_design=2,paper_type=2, payload=2)))

data_table$blocks <- c(rep((blocks),each=8))

data_table$plane_design <- ifelse(data_table$plane_design==1, "Basic Dart", "Sprinter")

data_table$paper_type <- ifelse(data_table$paper_type==1, "A4", "Construction")

data_table$payload <- ifelse(data_table$payload==1, "No Load", "Load")

data_table <- cbind(data_table, flight_distance)
```

R code used to determine the number of blocks and calculate the power for the experiment (R code 2)

Sd_paperplane <- sd(c(31.9375, 36.708333, 37.380208333, 32.546875, 35.473958333, 34.90625, 41.21875, 38.994791667, 39.03125, 34.791667))

sd_paperplane

[1] 2.948947

<u>For Plane Design (Factor A)</u>

a <- 2 # number of levels of plane design

b <- 2 # number of levels of paper type

c <- 2 # number of levels of payload

alpha <- .05

sigsq <- sd_paperplane^2 # our sigma^2 estimate

blocks <- 2:10 # block size for plane design comparisons

DA <- 4 # desired diff in means to detect with prob 1-beta

Fcrit <- qf(1-alpha,a-1,(a*b*c-1)*(blocks-1)) # value at which we reject H0

lam <- (blocks*b*c)*(DA^2)/(2*sigsq) # non-centrality parameter (ncp)

beta <- pf(Fcrit,a-1, (a*b*c-1)*(blocks-1),ncp=lam)

power <- 1-beta

blocksforA <- cbind(blocks, lam, Fcrit,beta,power) # output for plane design

blocksforA[2,]

```
  blocks      lam       Fcrit    beta      power

 3.0000000 11.0391901  4.6001099  0.1297354  0.8702646
```

For Paper Type (Factor B)

```
blocks <- 2:10 # block size for paper type comparisons

DB <- 4 # desired diff in means to detect with prob 1-beta

Fcrit <- qf(1-alpha,b-1,(a*b*c-1)*(blocks-1)) # value at which we reject H0

lam <- blocks*a*c*(DB^2)/(2*sigsq) # non-centrality parameter (ncp)

beta <- pf(Fcrit,b-1,(a*b*c-1)*(blocks-1),ncp=lam)

power <- 1-beta

blocksforB <- cbind(blocks, lam, Fcrit,beta,power)# output for paper type

blocksforB[2,]

 blocks       lam         Fcrit     beta       power

 3.0000000 11.0391901  4.6001099  0.1297354  0.8702646
```

For Payload (Factor C)

```
blocks <- 2:10 # block size for weight type comparisons

DC <- 4 # desired diff in means to detect with prob 1-beta

Fcrit <- qf(1-alpha,c-1,(a*b*c-1)*(blocks-1)) # value at which we reject H0

lam <- blocks*a*b*(DC^2)/(2*sigsq) # non-centrality parameter (ncp)

beta <- pf(Fcrit,c-1,(a*b*c-1)*(blocks-1),ncp=lam)

power <- 1-beta
```

```
blocksforC <- cbind(blocks, lam, Fcrit,beta,power) # output for payload

blocksforC[2,]

 blocks       lam          Fcrit    beta      power

 3.0000000 11.0391901  4.6001099  0.1297354  0.8702646
```

Data Table

Table 2: Data Table for the Experiment

Blocks	**Plane Design**	**Paper Type**	**Payload**	**Flight Distance (ft)**
Daniel	Sprinter	A4	Load	
Daniel	Sprinter	Construction	Load	
Daniel	Basic Dart	Construction	No Load	
Daniel	Basic Dart	A4	No Load	
Daniel	Basic Dart	A4	Load	
Daniel	Basic Dart	Construction	Load	
Daniel	Sprinter	Construction	No Load	
Daniel	Sprinter	A4	No Load	
Ashra	Basic Dart	Construction	No Load	
Ashra	Basic Dart	A4	No Load	
Ashra	Sprinter	A4	Load	
Ashra	Sprinter	Construction	Load	

Ashra	Basic Dart	A4	Load	
Ashra	Basic Dart	Construction	Load	
Ashra	Sprinter	Construction	No Load	
Ashra	Sprinter	A4	No Load	
Mehmet	Basic Dart	A4	Load	
Mehmet	Basic Dart	Construction	Load	
Mehmet	Sprinter	Construction	Load	
Mehmet	Basic Dart	Construction	No Load	
Mehmet	Sprinter	Construction	No Load	
Mehmet	Sprinter	A4	No Load	
Mehmet	Basic Dart	A4	No Load	
Mehmet	Sprinter	A4	Load	

References

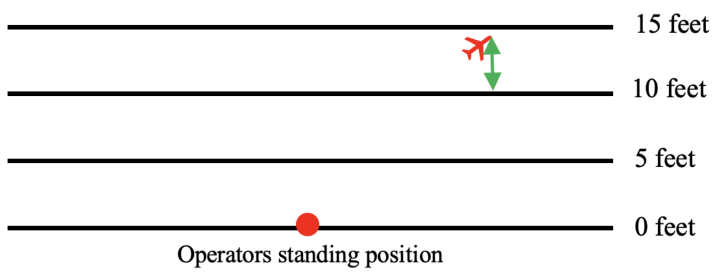
Puspita, A. N. G., Ambarwati, L., Ludfiyanti, E., Tejonugroho, D. P., & Zen, Y. (2020). Effect of paper weight, paper length, and nose of paper plane on aircraft mileage in paper airplane game. *UI Proceedings on Science and Technology*, *2*.

Montgomery, D.C. (2020), *Design and Analysis of Experiments*, 10th Edition, Wiley.

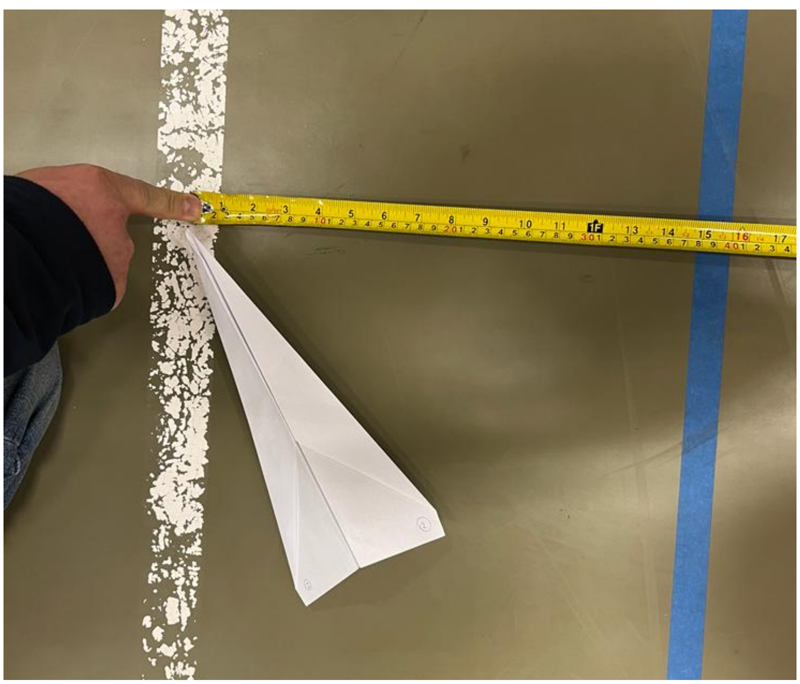
Fold 'N Fly. (2022). *Paper Airplane Designs*. Retrieved March 26, 2022, from https://www.foldnfly.com/#/1-1-1-1-1-1-1-1-2.html.

Appendices

Appendix A: Illustration of the response measurement



Appendix B: Typical response measurement during the experiment.



Appendix C: S200 South Eppler Gym at Bowling Green State University

Appendix D: Binder paper clip and rulers used for the experiment

Appendix E: R code used for analysis of the experiment

 # Data entry

```
Distance = c(44.4, 21.96, 38.79, 28.48, 35, 27.69, 10, 14.48, 11.25, 13.58, 9.5, 14.12, 39.14,
28.75, 36.2, 36.94, 34.15, 33.75, 17.08, 28.32, 14.71, 25.02, 20.46, 33.27)

Block = (rep(rep(c("Ashra", "Daniel", "Mehmet"),each=2),4))
PlaneType = rep(rep(c("Basic Dart", "Stable"),each =6),2)
PaperType = rep(c("A4", "Construction"),12)
Payload = rep(c("No Load", "Load"), each = 12)
```

 # ANOVA test

```
threefactorial=data.frame(Distance, Block = factor(Block),
PlaneType=factor(PlaneType),PaperType=factor(PaperType),Payload=factor(Payload))
ANOVA = aov(Distance ~ Block + PlaneType*PaperType*Payload, threefactorial)
summary(ANOVA)


PlaneType=factor(PlaneType)
PaperType=factor(PaperType)
Payload=factor(Payload)
Payload=relevel(Payload, "No Load")
```

 # Mean matrix

```
meanvec=numeric(8)
```

```
count=1
for(i in c("Basic Dart","Stable")) {
     for (k in c("No Load", "Load")) {
       for(j in c("A4","Construction")) {
       meanvec[count]=  mean(Distance[PlaneType==i & PaperType==j & Payload==k])
       count = count +1
       }
     }
}
meanmat=matrix(meanvec,2,4,byrow=T)

colnames(meanmat)=c("PaperType=A4,Payload=No
Load","PaperType=Construction,Payload=No Load",
            "PaperType=A4,Payload=Load","PaperType=Construction,Payload=Load")
     rownames(meanmat)=c("PlaneType=Basic Dart","PlaneType=Stable")
Meanmat
```

 # Assumption Checking

 # Normality assumption check

```
res=ANOVA$residuals
qqnorm(res)
qqline(res)
fits=ANOVA$fitted.values
```

 # Constant variance assumption check

```
par(mfrow=c(2,2))
plot(fits,res, xlab="Fitted Flight Distances",ylab="Residuals",
    main="Residuals vs. Fitted Flight Distances")
stripchart(res~PlaneType,vertical=T,xlab="Plane Type",ylab="Flight Distances",
        main="Residuals vs. PlaneType")
stripchart(res~PaperType,vertical=T,xlab="Paper Type",ylab="Flight Distances",
        main="Residuals vs. Paper Type")
stripchart(res~Payload,vertical=T,xlab="Payload",ylab="Flight Distances",
        main="Residuals vs. Payload")
par(mfrow=c(1,1))
```

 # Two-factor interactions

```
par(mfrow=c(1,3))
PlaneType=factor(PlaneType)
PaperType=factor(PaperType)
Payload=factor(Payload)

interaction.plot(PlaneType,PaperType,Distance,type="b",main="PlaneType:PaperType",col=c("
Red","blue"))
```

```
interaction.plot(PlaneType,Payload,Distance,type="b",main="PlaneType:Payload",col=c("Red",
"blue"))
interaction.plot(PaperType,Payload,Distance,type="b",main="PaperType:Payload",col=c("Red",
"blue"))
par(mfrow=c(1,1))
```

 # AB - Contrast for Plane Type for each level of Paper Type

```
a=2
b=2
c=2
n=3
mse=12.1
dfe=(a*b*c-1)*(n-1)
alpha=.05
tmult=qt(1-alpha/(6), dfe)
t=numeric(2)
tmultnames=numeric(2)
t[1] = mean(c(meanmat[1,1],meanmat[1,3])) - mean(c(meanmat[2,1],meanmat[2,3]));
tmultnames[1]=paste0("Paper Type=A4,          Basic Dart - Stable")
t[2] = mean(c(meanmat[1,2],meanmat[1,4])) - mean(c(meanmat[2,2],meanmat[2,4]));
tmultnames[2]=paste0("Paper Type=Construction, Basic Dart - Stable")
se=sqrt(mse*(1/(2*n)+1/(2*n)))
lower=t-tmult*se
```

```
upper=t+tmult*se

t_out = cbind(lower,t,upper)

rownames(t_out) = tmultnames

t_out
```

 # AC Contrast for Plane Type for each level of Payload

```
a=2

b=2

c=2

n=3

mse=12.1

dfe=(a*b*c-1)*(n-1)

alpha=.05

tmult=qt(1-alpha/(6), dfe)

t=numeric(2)

tmultnames=numeric(2)

t[1] = mean(c(meanmat[1,1],meanmat[1,2])) - mean(c(meanmat[2,1],meanmat[2,2]));

tmultnames[1]=paste0("Payload=No Load,    Basic Dart - Stable")

t[2] = mean(c(meanmat[1,3],meanmat[1,4])) - mean(c(meanmat[2,3],meanmat[2,4]));

tmultnames[2]=paste0("Payload=Load,       Basic Dart - Stable")

se=sqrt(mse*(1/(2*n)+1/(2*n)))

lower=t-tmult*se

upper=t+tmult*se
```

```
t_out = cbind(lower,t,upper)

rownames(t_out) = tmultnames

t_out
```

 # CB Contrast for Payload for each level of Paper Type

```
a=2

b=2

c=2

n=3

mse=12.1

dfe=(a*b*c-1)*(n-1)

alpha=.05

tmult=qt(1-alpha/(6), dfe)

t=numeric(2)

tmultnames=numeric(2)

t[1] = mean(c(meanmat[1,1],meanmat[2,1])) - mean(c(meanmat[1,3],meanmat[2,3]));

tmultnames[1]=paste0("PaperType=A4,           No Load - Load")

t[2] = mean(c(meanmat[1,2],meanmat[2,2])) - mean(c(meanmat[1,4],meanmat[2,4]));

tmultnames[2]=paste0("PaperType=Construction,    No Load - Load")

se=sqrt(mse*(1/(2*n)+1/(2*n)))

lower=t-tmult*se

upper=t+tmult*se

t_out = cbind(lower,t,upper)
```

```
rownames(t_out) = tmultnames
t_out
```

 # Data Snooping - Compare PlaneType = Basic Dart, PaperType = A4, Payload = No Load combination to PlaneType = Basic Dart, PaperType = A4, Payload = Load combination

```
#Scheffe
a=2; b=2; c=2; n=3 # where n is the number of blocks
mse=12.1
dfe=(a*b*c-1)*(n-1)
sch=sqrt((a*b*c-1)*qf(1-alpha,a*b*c-1,dfe))
BasicDart_A4_NoLoad=mean(with(threefactorial,Distance[PlaneType=="Basic Dart" & PaperType=="A4" & Payload=="No Load"]))
BasicDart_A4_Load=mean(with(threefactorial,Distance[PlaneType=="Basic Dart" & PaperType=="A4" & Payload=="Load"]))
pe=BasicDart_A4_NoLoad-BasicDart_A4_Load
se=sqrt(2*mse/n)
c(pe-sch*se, pe, pe+sch*se)
```

 # Relative efficiency calculation

```
a <- 8 # a is the number of treatments
b <- 3 # b is the number of blocks
ms_block <- 0.5
mse <- 12.1
var_rbd <- mse
```

```
var_crd <- (((b-1)*ms_block) + (b*(a-1)*mse))/((a*b)-1)

dfe_rbd <- (a-1)*(b-1)

dfe_crd <- a*(b-1)

cf <- ((dfe_rbd +1)*(dfe_crd +3))/((dfe_rbd + 3)*(dfe_crd + 1))

RE <- (cf*var_crd)/(var_rbd)

RE
```

References

1. The Red Bull Paper Wings [homepage on the internet]. Retrieved April 6, 2022, from http://paperwings.redbull.com/Rules.html
2. Puspita, A. N. G., Ambarwati, L., Ludfiyanti, E., Tejonugroho, D. P., & Zen, Y. (2020). Effect of paper weight, paper length, and nose of paper plane on aircraft mileage in paper airplane game. *UI Proceedings on Science and Technology*, *2*.
3. Montgomery, D.C. (2020), *Design and Analysis of Experiments*, 10th Edition, Wiley.
4. Fold 'N Fly. (2022). *Paper Airplane Designs*. Retrieved March 26, 2022, from https://www.foldnfly.com/#/1-1-1-1-1-1-1-1-2.html.