NYU Computer Science Bridge to Tandon Course Winter 2021

Homework 6

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Question 5

Use the definition of Θ in order to show the following:

(a) $5n^3 + 2n^2 + 3n = \Theta(n^3)$

solution:
By the definition of Θ: Let f and g be two functions $\mathbb{Z}^+$ to $\mathbb{Z}^+$. $f = \Theta(g)$ if $f = O(g)$ and $f = \Omega(g)$. We will show that $5n^3 + 2n^2 + 3n = O(n^3)$ and $5n^3 + 2n^2 + 3n = \Omega(n^3)$.

Select $n_0 = 1$ and $c = 10$ such that for $n \geq n_0$, $5n^3 + 2n^2 + 3n \leq cn^3$.
When $n \geq n_0 = 1$, $2n^3 \geq 2n^2$ and $3n^3 \geq 3n$. Thus for $n \geq 1$, $10n^3 = 5n^3 + 2n^3 + 3n^3 \geq 5n^3 + 2n^2 + 3n$ and $f = O(n^3)$.

Select $n_0 = 1$ and $c = 5$ such that for $n \geq n_0$, $f(n) = 5n^3 + 2n^2 + 3n \geq cn^3$.
When $n \geq n_0 = 1$, $2n^2 \geq 1$ and $3n \geq 1$. Thus for $n \geq 1$, $5n^3 + 2n^2 + 3n \geq 5n^3$ and $f = \Omega(n^3)$. $\blacksquare$

(b) $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

solution:
By the definition of Θ: Let f and g be two functions $\mathbb{Z}^+$ to $\mathbb{Z}^+$. $f = \Theta(g)$ if $f = O(g)$ and $f = \Omega(g)$. We will show that $\sqrt{7n^2 + 2n - 8} = O(n)$ and $\sqrt{7n^2 + 2n - 8} = \Omega(n)$.

Select $n_0 = 1$ and $c = \sqrt{16}$ such that for $n \geq n_0$, $\sqrt{7n^2 + 2n - 8} \leq cn = \sqrt{(cn)^2}$.
When $n \geq n_0 = 1$, $2n^2 \geq 2n$ and $8n^2 > -8$. Thus for $n \geq 1$, $16n^2 = 7n^2 + 2n^2 + 8n^2 > 7n^2 + 2n - 8 > 0$, $\sqrt{16}n = \sqrt{16n^2} > \sqrt{7n^2 + 2n - 8}$, and $f = O(n)$.

When $n \geq 0$, $2n \geq 0$, $\sqrt{7n^2 + 2n - 8} \geq \sqrt{7n^2 - 8}$. Select $c = \sqrt{\frac{7}{2}}$, $\sqrt{\frac{7}{2}n^2 + (\frac{7}{2}n - 8)} \geq \sqrt{\frac{7}{2}n^2}$ as long as $(\frac{7}{2}n^2 - 8) \geq 0$. We have $(\frac{7}{2}n^2 - 8) \geq (\frac{7}{2}n^2 - 8n)$ because $n \geq 0$. Then we have $(\frac{7}{2}n^2 - 8n) = n(\frac{7}{2}n - 8)$. Since $n \geq 0$, we only need to make sure $(\frac{7}{2}n - 8)$ is geater than 0, then we can guarantee that $(\frac{7}{2}n^2 - 8) \geq 0$ and $\sqrt{7n^2 + 2n - 8} \geq \sqrt{\frac{7}{2}n^2}$. Thus we select $n_0 = \frac{16}{7}$ such that for $n \geq n_0$, $f(n) = \sqrt{7n^2 + 2n - 8} \geq cn$ and $f = \Omega(n)$. $\blacksquare$