NYU Tandon Bridge 2021 Winter
NetID: kl4227

Graded 47/48

Question 7:
Exercise 3.1.1: Set membership and subsets - true or false.
Use the definitions for the sets given below to determine whether each statement is true or false:

A = { x ∈ Z: x is an integer multiple of 3 }
B = { x ∈ Z: x is a perfect square }
C = { 4, 5, 9, 10 }
D = { 2, 4, 11, 14 }
E = { 3, 6, 9 }
F = { 4, 6, 16 }

An integer x is a perfect square if there is an integer y such that x = y^2.

(a)

27 ∈ A
Solution.
True. ■

(b)

27 ∈ B
Solution.
False. ■

(c)

100 ∈ B.
Solution.
True. ■

(d)

E ⊆ C or C ⊆ E.
Solution.
False. ■

(e)

E ⊆ A
Solution.
True. ■

(f)

A ⊂ E
Solution.
False. ■

(g)

E ∈ A
Solution.
False. ■

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Exercise 3.1.2: Set membership and subsets - true or false, cont.
Use the definitions for the sets given below to determine whether each statement is true or false:

A = { x ∈ Z: x is an integer multiple of 3 }
B = { x ∈ Z: x is a perfect square }
C = { 4, 5, 9, 10 }
D = { 2, 4, 11, 14 }
E = { 3, 6, 9 }
F = { 4, 6, 16 }

An integer x is a perfect square if there is an integer y such that x = y2.

(a)

$15 \subset A$

Solution.

False. ■

(b)

$\{15\} \subset A$

Solution.

True. ■

(c)

$\emptyset \subset A$

Solution.

True. ■

(d)

$A \subseteq A$

Solution.

True. ■

(e)

$\emptyset \in B$

Solution.

False. ■

Exercise 3.1.5: Expressing sets in set builder notation.
Express each set using set builder notation. Then if the set is finite, give its cardinality. Otherwise, indicate that the set is infinite.

(b)

{ 3, 6, 9, 12, ... }

Solution.

{ x ∈ N: x is a multiple of 3 and x > 0 }; the set is infinite. ■

(d)

{ 0, 10, 20, 30, ..., 1000 }

Solution.

{ x ∈ N: x is a multiple of 10 and x <= 1000 }; the cardinality is 101■

Exercise 3.2.1: Sets of sets - true or false.

Let $X = \{1, \{1\}, \{1, 2\}, 2, \{3\}, 4\}$. Which statements are true?

(a)

$2 \in X$

Solution.

True. ■

(b)

$\{2\} \subseteq X$

Solution.

True. ■

(c)

$\{2\} \in X$

Solution.

False. ■

(d)

$3 \in X$

Solution.

False. ■

(e)

$\{1, 2\} \in X$

Solution.

True. ■

(f)

$\{1, 2\} \subseteq X$

Solution.

True. ■

(g)

$\{2, 4\} \subseteq X$

Solution.

True. ■

(h)

$\{2, 4\} \in X$

Solution.

False. ■

(i)

$\{2, 3\} \subseteq X$

Solution.

False. ■

(j)

$\{2, 3\} \in X$

Solution.

False. ■

(k)

$|X| = 7$

Solution.

False. ■

Question 8:
Exercise 3.2.4: A subset of a power set.

(b)

Let $A = \{1, 2, 3\}$. What is $\{X \in P(A): 2 \in X\}$?
Solution.

$P(A)$: $\{\cancel{\emptyset}, \cancel{\{1\}}, \{2\}, \cancel{\{3\}}, \{1, 2\}, \{2, 3\}, \cancel{\{1, 3\}}, \{1, 2, 3\}\}$.

$= \{\{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$.
■

Question 9:
Exercise 3.3.1: Unions and intersections of sets.
Define the sets A, B, C, and D as follows:

A = {-3, 0, 1, 4, 17}
B = {-12, -5, 1, 4, 6}
C = {x ∈ Z: x is odd}
D = {x ∈ Z: x is positive}

For each of the following set expressions, if the corresponding set is finite, express the set using roster notation. Otherwise, indicate that the set is infinite.

(c)

A ∩ C
Solution.
{-3, 1, 17}. ■

(d)

A ∪ (B ∩ C)
Solution.
(B ∩ C): {-5, 1}

A ∪ (B ∩ C): {-5, -3, 0, 1, 4, 17}. ■

(e)

A ∩ B ∩ C
Solution.
A ∩ B: {1, 4}

A ∩ B ∩ C: {1}. ■

Exercise 3.3.3: Unions and intersections of sequences of sets, part 2.
Use the following definitions to express each union or intersection given. You can use roster or set builder notation in your responses, but no set operations.

- $A_i = \{i^0, i^1, i^2\}$ (Recall that for any number x, $x^0 = 1$.)
- $B_i = \{x \in \mathbf{R} : -i \leq x \leq 1/i\}$
- $C_i = \{x \in \mathbf{R} : -1/i \leq x \leq 1/i\}$

(a)

$\bigcap_{i=2}^{5} A_i$

Solution.
{1, 2, 4} ∩ {1, 3, 9} ∩ {1, 4, 16} ∩ {1, 5, 25} = {1} ■

(b)

$\bigcup_{i=2}^{5} A_i$

Solution.
{1, 2, 4} ∪ {1, 3, 9} ∪ {1, 4, 16} ∪ {1, 5, 25} = {1, 2, 3, 4, 5, 9, 16, 25} ■

(e)

$\bigcap_{i=1}^{100} C_i$

Solution.

$\{x \in \mathbf{R}\colon -\frac{1}{1} \leq x \leq \frac{1}{1}\} \cap \{x \in \mathbf{R}\colon -\frac{1}{2} \leq x \leq \frac{1}{2}\} \cap \ldots \cap \{x \in \mathbf{R}\colon -\frac{1}{100} \leq x \leq \frac{1}{100}\}$
$= \{x \in \mathbf{R}\colon -\frac{1}{100} \leq x \leq \frac{1}{100}\}$ ■

(f)

$\bigcup_{i=1}^{100} C_i$

$\{x \in \mathbf{R}\colon -\frac{1}{1} \leq x \leq \frac{1}{1}\} \cup \{x \in \mathbf{R}\colon -\frac{1}{2} \leq x \leq \frac{1}{2}\} \cup \ldots \cup \{x \in \mathbf{R}\colon -\frac{1}{100} \leq x \leq \frac{1}{100}\}$
$= \{x \in \mathbf{R}\colon -\frac{1}{1} \leq x \leq \frac{1}{1}\}$ ■

Exercise 3.3.4: Power sets and set operations.
Use the set definitions A = {a, b} and B = {b, c} to express each set below. Use roster notation in your solutions.

(b)

P(A ∪ B)

Solution.

{∅, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}. ■

(d)

P(A) ∪ P(B)

Solution.

{∅, {a}, {b}, {c}, {a, b}, {b, c}}. ■

Question 10:
Exercise 3.5.1: Cartesian product of three small sets.
The sets A, B, and C are defined as follows:
A = {tall, grande, venti}
B = {foam, no-foam}
C = {non-fat, whole}
Use the definitions for A, B, and C to answer the questions. Express the elements using n-tuple notation, not string notation.
(b)
Write an element from the set B × A × C.
Solution.
One example: (foam, tall, non-fat) ■

(c)
Write the set B × C using roster notation.
Solution.
{(foam, non-fat), (foam, whole), (no-foam, non-fat), (no-foam, whole)} ■

Exercise 3.5.3: Cartesian product - true or false.
Indicate which of the following statements are true.

(b)
$\mathbf{Z}^2 \subseteq \mathbf{R}^2$
Solution.
True. If (c, d) ∈ $\mathbf{Z}^2$, then c and d are both elements of **Z**. Since $\mathbf{Z} \subseteq \mathbf{R}$, then c and d are also elements of **R**. Therefore (c, d) ∈ $\mathbf{R}^2$. ■

(c)
$\mathbf{Z}^2 \cap \mathbf{Z}^3 = \emptyset$
Solution.
True. The elements in $\mathbf{Z}^2$ are pairs, and the elements in $\mathbf{Z}^3$ are triples. Therefore the two sets have no elements in common. ■
(e)
For any three sets, A, B, and C, if A ⊆ B, then A × C ⊆ B × C.
Solution.
True. If (c, d) ∈ A × C, then c is an element of **A**, and d is an element in **C**. Since **A** ⊆ **B**, then c is also an elements of **B**. Therefore (c, d) ∈ B × C. ■

Exercise 3.5.6: Roster notation for sets defined using set builder notation and the Cartesian product.
Express the following sets using the roster method. Express the elements as strings, not n-tuples.

(d)
{xy: where x ∈ $\{0\} \cup \{0\}^2$ and y ∈ $\{1\} \cup \{1\}^2$}
Solution.
{01, 011, 001, 0011}. ■

(e)
{xy: x ∈ {aa, ab} and y ∈ $\{a\} \cup \{a\}^2$}
Solution.
{aaa, aaaa, aba, abaa}. ■

Exercise 3.5.7: Cartesian products, power sets, and set operations.
Use the following set definitions to specify each set in roster notation. Except where noted, express elements of Cartesian products as strings.
A = {a}
B = {b, c}
C = {a, b, d}

(c)

$(A \times B) \cup (A \times C)$
Solution.
{aa, ab, ac, ad}. ■

(f)

$P(A \times B)$
Solution.
{∅, {ab}, {ac}, {ab, ac}}. ■

(g)

$P(A) \times P(B)$. Use ordered pair notation for elements of the Cartesian product.
Solution.
{(∅, ∅), (∅, {b}), (∅, {c}), (∅, {b, c}), ({a}, ∅), ({a}, {b}), ({a}, {c}), ({a}, {b, c})}. ■

Question 11:
Exercise 3.6.2: Proving set identities.
Use the set identities given in the table to prove the following new identities. Label each step in your proof with the set identity used to establish that step.
(b)

$(B \cup A) \cap (\bar{B} \cup A) = A$
Solution.

$(B \cup A) \cap (\bar{B} \cup A)$	
$(B \cap \bar{B}) \cup A$	Distributive law
$\phi \cup A$	Complement law
A	Identity law

■

(c)

$\overline{A \cap \bar{B}} = \bar{A} \cup B$
Solution.

Com(A) union Com(Com(B)): De Morgan's law
Com(A) union B: Double Complement law

$\overline{A \cap \bar{B}}$	
$\cancel{\bar{A} \cup B}$	~~De Morgan's law~~

■

Exercise 3.6.3: Showing set equations that are not identities.
A set equation is not an identity if there are examples for the variables denoting the sets that cause the equation to be false. For example A ∪ B = A ∩ B is not an identity because if A = {1, 2} and B = {1}, then A ∪ B = {1, 2} and A ∩ B = {1}, which means that A ∪ B ≠ A ∩ B.
Show that each set equation given below is not a set identity.
(b)

A - (B ∩ A) = A
Solution.
If A = {a, b}, and B = {a}, then A - (B ∩ A) = {b}, which is not equal to A. ■

(d)

(B - A) ∪ A = A
Solution.
If A = {a}, and B = {a, b, c}, then (B - A) ∪ A = {a, b, c}, which is not equal to A. ■

Exercise 3.6.4: Proving set identities with the set difference operation.
The set subtraction law states that A - B = A ∩ B. Use the set subtraction law as well as the other set identities given in the table to prove each of the following new identities. Label each step in your proof with the set identity used to establish that step.

(b)

$A \cap (B - A) = \phi$

Solution.

$A \cap (B - A)$	
$A \cap (B \cap \bar{A})$	Set subtraction law
$A \cap (\bar{A} \cap B)$	Commutative law
$(A \cap \bar{A}) \cap B$	Associative law
$\phi \cap B$	Complement law
ϕ	Domination law

■

(c)

$A \cup (B - A) = A \cup B$

Solution.

$A \cup (B - A)$	
$A \cup (B \cap \overline{A})$	Set subtraction law
$(A \cup \overline{A}) \cap (A \cup B)$	Distributive law
$U \cap (A \cup B)$	Complement law
$A \cup B$	Identity law

■