Question 1:

A. **Convert the following numbers to their decimal representation. Show your work.**

1. $10011011_2 = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2^7 \\ 2^6 \\ 2^5 \\ 2^4 \\ 2^3 \\ 2^2 \\ 2 \\ 1 \end{bmatrix} = 155_{10}$

2. $456_7 = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 7^2 \\ 7 \\ 1 \end{bmatrix} = 237_{10}$

3. $38A_{16} = \begin{bmatrix} 3 & 8 & 11 \end{bmatrix} \begin{bmatrix} 16^2 \\ 16 \\ 1 \end{bmatrix} = 907_{10}$

4. $2214_5 = \begin{bmatrix} 2 & 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 5^3 \\ 5^2 \\ 5 \\ 1 \end{bmatrix} = 309_{10}$

B. **Convert the following numbers to their binary representation:**

1. $69_{10} = 1000101_2$

Consider remainder of 1 to be the 1 in the corresponding bit in the binary representation and remainder of 0 to be 0. Starting from the top, the first remainder belongs to the least significant bit, and the last remainder belongs to the most significant bit.

$$\frac{69}{2} = 34R1$$
$$\frac{34}{2} = 17$$
$$\frac{16}{2} = 8R1$$
$$\frac{8}{2} = 4$$
$$\frac{4}{2} = 2$$
$$\frac{2}{2} = 1$$
$$\frac{1}{2} = R1$$

2. $485_{10} = 111100101_2$

Using the same logic as above.

$$\frac{485}{2} = 242R1$$
$$\frac{242}{2} = 121$$

$\frac{121}{2} = 60R1$
$\frac{60}{2} = 30$
$\frac{30}{2} = 15$
$\frac{15}{2} = 7R1$
$\frac{7}{2} = 3R1$
$\frac{3}{2} = 1R1$
$\frac{1}{2} = R1$

3. $6D1A_{16} = 0110110100011010_2$

   Convert each digit in the hexadecimal representation to its equivalent 4 digits in the binary representation and concatenate the results to yield the final answer.

   $6_{16} = 0110_2$
   $\frac{6}{2} = 3$
   $\frac{3}{2} = 1R1$
   $\frac{1}{2} = R1$

   $D_{16} = 1101_2$
   $\frac{13}{2} = 6R1$
   $\frac{6}{2} = 3$
   $\frac{3}{2} = 1R1$
   $\frac{1}{2} = R1$

   $1_{16} = 0001_2$
   $\frac{1}{2} = R1$

   $\mathrm{A}_{16} = 1010_2$
   $\frac{10}{2} = 5$
   $\frac{5}{2} = 2R1$
   $\frac{2}{2} = 1$
   $\frac{1}{2} = R1$

C. Convert the following numbers to their hexadecimal representation:

1. $1101011_2 = 6B_{16}$

Convert each 4 digit in the binary representation to its equivalent decimal, and then convert to hexadecimal. Lastly, concatenate the results to yield the final answer.

$$[0 \quad 1 \quad 1 \quad 0]\begin{bmatrix}2^3\\2^2\\2\\1\end{bmatrix} = 6$$

$$[1 \quad 0 \quad 1 \quad 1]\begin{bmatrix}2^3\\2^2\\2\\1\end{bmatrix} = 11 = B$$

2. $895_{10} = 37F_{16}$

$$\frac{895}{16} = 55R15_{10} = 55RF_{16}$$

$$\frac{55}{16} = 3R7$$

$$\frac{3}{16} = R3$$

Question 2:
Solve the following, do all calculation in the given base. Show your work.

1. $7556_8 + 4515_8 = 14303_8$

	7	5	6	6
+	4	5	1	5
			~~6~~ 7	~~11~~ 3
			1	0
		~~5~~ 6	~~8~~ 0	3
		5	0	0
1	~~7/8~~ 0	~~11~~ 3	0	3
	4	0	0	0
1	4	3	0	3

2. $10110011_2 + 1101_2 = 11000000_2$

	1	0	1	1	0	0	1	1
+					1	1	0	1
						1	~~1/2~~ 0	~~2~~ 0
							0	0
						1	0	0
						1	0	0
					1	~~2~~ 0	0	0
					1	0	0	0
	1	~~0~~ 1	~~1/2~~ 0	~~1/2~~ 0	~~2~~ 0	0	0	0
=	1	1	0	0	0	0	0	0

3. $7A66_{16} + 45C5_{16} = C02B_{16}$

	7	A	6	6
+	4	5	C	5
			6	B
			C	0
		~~A~~ B	~~18~~ 2	B
		5	0	0
	~~7~~ 8	~~16~~ 0	2	B
	4	0	0	0
	C	0	2	B

4. $3022_5 - 2433_5 = 34_5$

	3	0	2	2
-	2	4	3	3
	3	0	~~2~~ 1	4
			3	0
	~~3~~ 2	~~0~~ 4	3	4
		4	0	0
	2	0	3	4
	2	0	0	0
			3	4

Question 3:

A. Convert the following numbers to their 8-bits two's complement representation. Show your work.

1. $124_{10} = 01111100_2$
To convert the positive number in the decimal representation to the binary representation, the number is calculated with 2's division as normal, and 0 is concatenated to the result as the first bit
$\frac{124}{2} = 62$
$\frac{62}{2} = 31$
$\frac{31}{2} = 15R1$
$\frac{15}{2} = 7R1$
$\frac{7}{2} = 3R1$
$\frac{3}{2} = 1R1$
$\frac{1}{2} = R1$

2. $-124_{10} = 10000100_2$
Using the result from the preceding question; subtract the prior answer from 100000000_2 and yield the negative -124. Alternatively, keeping the least significant 1 and flip the rest of the bit that are more significant than it will get the same result.

	1	0	0	0	0	0	0	0	0
-		0	1	1	1	1	1	0	0
		1	0	0	0	0	1	0	0

3. $109_{10} = 01101101_2$
$\frac{109}{2} = 54R1$
$\frac{54}{2} = 27$
$\frac{27}{2} = 13R1$
$\frac{13}{2} = 6R1$
$\frac{6}{2} = 3$
$\frac{3}{2} = 1R1$
$\frac{1}{2} = R1$

4. $-79_{10} = 10110001_2$
Using the same logic as question 2.
$79_{10} = 01001111$

	1	0	0	0	0	0	0	0	0
-		0	1	0	0	1	1	1	1
		1	0	1	1	0	0	0	1

$\frac{79}{2} = 39R1$

$$\frac{39}{2} = 19R1$$
$$\frac{19}{2} = 9R1$$
$$\frac{9}{2} = 4R1$$
$$\frac{4}{2} = 2$$
$$\frac{2}{2} = 1$$
$$\frac{1}{2} = R1$$

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

Determine the sign of the number by checking the most significant bit; 0 to be positive and 1 to be negative. If the number is negative, convert the number to its corresponding positive binary number. Convert the positive binary number to decimal by summing its weights, and assigned the sign according to the most significant bit.

1. $00011110_{8\text{ bit 2's comp}} = 30_{10}$

$$[0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0]\begin{bmatrix} 2^6 \\ 2^5 \\ 2^4 \\ 2^3 \\ 2^2 \\ 2 \\ 1 \end{bmatrix} = 30_{10}$$

2. $11100110_{8\text{ bit 2's comp}} = -26_{10}$

	1	0	0	0	0	0	0	0	0
-		1	1	1	0	0	1	1	0
		0	0	0	1	1	0	1	0

$$[0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 2^6 \\ 2^5 \\ 2^4 \\ 2^3 \\ 2^2 \\ 2 \\ 1 \end{bmatrix} = 26_{10}$$

3. $00101101_{8\text{ bit 2's comp}} = 45_{10}$

$$[0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1]\begin{bmatrix} 2^6 \\ 2^5 \\ 2^4 \\ 2^3 \\ 2^2 \\ 2 \\ 1 \end{bmatrix} = 45_{10}$$

4. $10011110_{8\text{ bit 2's comp}} = -98_{10}$

	1	0	0	0	0	0	0	0	0
-		1	0	0	1	1	1	1	0
		0	1	1	0	0	0	1	0

$$[1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0] \begin{bmatrix} 2^6 \\ 2^5 \\ 2^4 \\ 2^3 \\ 2^2 \\ 2 \\ 1 \end{bmatrix} = 98$$

Question 4:

Exercise 1.2.4: Writing truth tables.
Write a truth table for each expression.

(b) $\neg(p \vee q)$

p	q	$\neg(p \vee q)$
T	T	F
T	F	F
F	T	F
F	F	T

(c) $r \vee (p \wedge \neg q)$

r	p	q	$r \vee (p \wedge \neg q)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	F

Exercise 1.3.4: Truth tables for logical expressions with conditional operations.
(b)$(p \to q) \to (q \to p)$

p	q	$(p \to q) \to (q \to p)$
T	T	T
T	F	T
F	T	F
F	F	T

(d)$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T
T	F	T
F	T	T
F	F	T

Question 5:

Exercise 1.2.7: Expressing a set of conditions using logical operations.
Consider the following pieces of identification a person might have in order to apply for a credit card:
B: Applicant presents a birth certificate.
D: Applicant presents a driver's license.
M: Applicant presents a marriage license.
Write a logical expression for the requirements under the following conditions:

(b)The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.
$(B \wedge D) \vee (B \wedge M) \vee (D \wedge M)$

(c)
Applicant must present either a birth certificate or both a driver's license and a marriage license.
$B \vee (D \wedge M)$

Exercise 1.3.7: Expressing conditional statements in English using logic.
Define the following propositions:
s: a person is a senior
y: a person is at least 17 years of age
p: a person is allowed to park in the school parking lot
Express each of the following English sentences with a logical expression:

(b)A person can park in the school parking lot if they are a senior or at least seventeen years of age.
$(s \vee y) \rightarrow p$

(c)Being 17 years of age is a necessary condition for being able to park in the school parking lot.
$p \rightarrow y$

(d)A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.
$p \leftrightarrow (y \wedge s)$

(e)Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.
$p \rightarrow (y \vee s)$

Exercise 1.3.9: Translating English propositions into logical expressions.
Use the definitions of the variables below to translate each English statement into an equivalent logical expression.
y: the applicant is at least eighteen years old
p: the applicant has parental permission
c: the applicant can enroll in the course

(c)The applicant can enroll in the course only if the applicant has parental permission.
$c \rightarrow p$

(d)Having parental permission is a necessary condition for enrolling in the course.
$c \rightarrow p$

Question 6:

Exercise 1.3.6: Expressing English sentences in if-then form.
Give an English sentence in the form "If...then...." that is equivalent to each sentence.

(b)Maintaining a B average is necessary for Joe to be eligible for the honors program.
If Joe is eligible for the honors program, then he maintains a B average.

(c)Rajiv can go on the roller coaster only if he is at least four feet tall.
If Rajiv can go on the roller coaster, then he is at least four feet tall.

(d)Rajiv can go on the roller coaster if he is at least four feet tall.
If Rajiv is at least four feet tall, then he can go on the roller coaster.

Exercise 1.3.10: Determining if a truth value of a compound expression is known given a partial truth assignment.
The variable p is true, q is false, and the truth value for variable r is unknown. Indicate whether the truth value of each logical expression is true, false, or unknown.

(c)$(p \lor r) \leftrightarrow (q \land r)$
False. The expression is false regardless of the value of r; $(q \land r)$ is always false, and $(p \lor r)$ is always true.

(d)$p \land r) \leftrightarrow (q \land r)$
Unknown. If r is true, then the expression is false. If r is false, then the expression is true.

(e)$p \to (r \lor q)$
Unknown. If r is true, then the expression is true. If r is false, then the expression is false.

(f)$(p \land q) \to r$
True. The expression is true regardless of the value of r; the hypothesis is false.

Question 7:

Exercise 1.4.5: Logical equivalence of two English statements.
Define the following propositions:
j: Sally got the job.
l: Sally was late for her interview
r: Sally updated her resume.
Express each pair of sentences using a logical expression. Then prove whether the two expressions are logically equivalent.

(b)
If Sally did not get the job, then she was late for interview or did not update her resume.
If Sally updated her resume and was not late for her interview, then she got the job.

$$\neg j \to (l \lor \neg r)$$
$$(r \land \neg l) \to j$$

Logically equivalent.

j	l	r	$\neg j \to (l \lor \neg r)$	$(r \land \neg l) \to j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

(c)
If Sally got the job then she was not late for her interview.
If Sally did not get the job, then she was late for her interview.

$$j \to \neg l$$
$$\neg j \to l$$

Not Logically equivalent.

j	l	$j \to \neg l$	$\neg j \to l$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

(d)
If Sally updated her resume or she was not late for her interview, then she got the job.
If Sally got the job, then she updated her resume and was not late for her interview.

$$(r \lor \neg l) \to j$$
$$j \to (r \lor \neg l)$$

Not Logically equivalent.

j	l	r	$(r \lor \neg l) \to j$	$j \to (r \lor \neg l)$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	T
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T

Question 8:

Exercise 1.5.2: Using the laws of logic to prove logical equivalence.
Use the laws of propositional logic to prove the following:
(c) $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

$(p \to q) \wedge (p \to r)$	
$(\neg p \vee q) \wedge (\neg p \vee r)$	Conditional identities
$\neg p \vee (q \wedge r)$	Distributive laws
$p \to (q \wedge r)$	Conditional identities

(f) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

$\neg(p \vee (\neg p \wedge q))$	
$\neg p \wedge \neg(\neg p \wedge q)$	De Morgan's laws
$\neg p \wedge (p \vee \neg q)$	De Morgan's laws
$(\neg p \wedge p) \vee (\neg p \wedge \neg q)$	Distributive laws
$F \vee (\neg p \wedge \neg q)$	Complement laws
$\neg p \wedge \neg q$	Identity laws

(i) $(p \wedge q) \to r \equiv (p \wedge \neg r) \to \neg q$

$(p \wedge q) \to r$	
$\neg(p \wedge q) \vee r$	Conditional identities
$(\neg p \vee \neg q) \vee r$	De Morgan's laws
$\neg p \vee (\neg q \vee r)$	Associative laws
$\neg p \vee (r \vee \neg q)$	Commutative laws
$(\neg p \vee r) \vee \neg q$	Associative laws
$\neg(p \wedge \neg r) \vee \neg q$	De Morgan's laws
$(p \wedge \neg r) \to \neg q$	Conditional identities

Exercise 1.5.3: Using the laws of logic to prove tautologies.
Use the laws of propositional logic to prove that each statement is a tautology.
(c) $\neg r \vee (\neg r \to p)$

$\neg r \vee (\neg r \to p)$	
$\neg r \vee (r \vee p)$	Conditional identities
$(\neg r \vee r) \vee p$	Associative laws
$T \vee p$	Complement laws
T	Domination laws

(d) $\neg(p \to q) \to \neg q$

$\neg(p \to q) \to \neg q$	
$\neg(\neg p \vee q) \to \neg q$	Conditional identities
$(p \wedge \neg q) \to \neg q$	De Morgan's laws
$\neg(p \wedge \neg q) \vee \neg q$	Conditional identities
$(\neg p \vee q) \vee \neg q$	De Morgan's laws
$\neg p \vee (q \vee \neg q)$	Associative laws
$\neg p \vee T$	Complement laws
T	Domination laws

Question 9:

Exercise 1.6.3: Translating mathematical statements in English into logical expressions.
Consider the following statements in English. Write a logical expression with the same meaning. The domain of discourse is the set of all real numbers.

(c)There is a number that is equal to its square.
$\exists x(x = x^2)$

(d)Every number is less than or equal to its square.
$\forall x(x \leq x^2)$

Exercise 1.7.4: Translating quantified statements from English to logic, part 3.
In the following question, the domain of discourse is a set of employees who work at a company. Ingrid is one of the employees at the company. Define the following predicates:
S(x): x was sick yesterday
W(x): x went to work yesterday
V(x): x was on vacation yesterday

(b)Everyone was well and went to work yesterday.
$\forall x(\neg S(x) \land W(x))$

(c)Everyone who was sick yesterday did not go to work.
$\forall x(S(x) \rightarrow \neg W(x))$

(d)Yesterday someone was sick and went to work.
$\exists x(S(x) \land W(x))$

Question 10:

Exercise 1.7.9: Determining whether a quantified logical statement is true, part 1.
The domain for this question is the set {a, b, c, d, e}. The following table gives the value of predicates P, Q, and R for each element in the domain. For example, Q(c) = T because the truth value in the row labeled c and the column Q is T. Using these values, determine whether each quantified expression evaluates to true or false.

	P(x)	Q(x)	R(x)
a	T	T	F
b	T	F	F
c	F	T	F
d	T	T	F
e	T	T	T

(c)$\exists x((x = c) \rightarrow P(x))$
~~False.~~ True

(d)$\exists x(Q(x) \land R(x))$
True. Example: e.

(e)$Q(a) \land P(d)$
True.

(f)$\forall x ((x \neq b) \rightarrow Q(x))$
True.

(g)$\forall x (P(x) \lor R(x))$
False. Counter-example: c.

(h)$\forall x (R(x) \rightarrow P(x))$
True.

(i)$\exists x(Q(x) \lor R(x))$
True.

Exercise 1.9.2: Truth values for statements with nested quantifiers - small finite domain.
The tables below show the values of predicates P(x, y), Q(x, y), and S(x, y) for every possible combination of values of the variables x and y. The row number indicates the value for x and the column number indicates the value for y. The domain for x and y is {1, 2, 3}.

P	1	2	3
1	T	F	T
2	T	F	T
3	T	T	F

Q	1	2	3
1	F	F	F
2	T	T	T
3	T	F	F

S	1	2	3
1	F	F	F
2	F	F	F
3	F	F	F

(b) $\exists x\ \forall y\ Q(x, y)$
True. Example: $x = 2$.

(c) $\exists x\ \forall y\ P(y, x)$
True. Example: $x = 1$.

(d) $\exists x\ \exists y\ S(x, y)$
False. There is no such x and y that $S(x, y)$ are true.

(e) $\forall x\ \exists y\ Q(x, y)$
False. Counter-example: $x = 1$.

(f) $\forall x\ \exists y\ P(x, y)$
True.

(g) $\forall x\ \forall y\ P(x, y)$
False. Counter-example: $x = 1$, $y = 2$.

(h) $\exists x\ \exists y\ Q(x, y)$
True. Example: $x = 2$, $y = 1$.

(i) $\forall x\ \forall y\ \neg S(x, y)$
True.

Question 11:

Exercise 1.10.4: Mathematical statements into logical statements with nested quantifiers.
Translate each of the following English statements into logical expressions. The domain of discourse is the set of all real numbers.

(c)There are two numbers whose sum is equal to their product.
$\exists x \exists y((x + y) = (x * y))$

(d)The ratio of every two positive numbers is also positive.
$\forall x \forall y(((x > 0) \wedge (y > 0)) \rightarrow (\frac{x}{y} > 0))$

(e)The reciprocal of every positive number less than one is greater than one.
~~$\forall x((x > 0) \wedge \left(\frac{1}{x} < 1\right)) \rightarrow (x > 1))$~~ for all ((0 < x < 1) -> (1/x > 1))

(f)There is no smallest number.
$\forall x \exists y(x > y)$

(g)Every number besides 0 has a multiplicative inverse.
$\forall x \exists y((x \neq 0) \rightarrow (\frac{1}{x} = y)$

Exercise 1.10.7: Statements with nested quantifiers: English to logic, part 3.
The domain of discourse is a group working on a project at a company. One of the members of the group is named Sam. Define the following predicates.
P(x, y): x knows y's phone number. (A person may or may not know their own phone number.)
D(x): x missed the deadline.
N(x): x is a new employee.
Give a logical expression for each of the following sentences.

(c)There is at least one new employee who missed the deadline.
$\exists x(N(x) \wedge D(x))$

(d)Sam knows the phone number of everyone who missed the deadline.
$\forall x(D(x) \rightarrow P(Sam, x))$

(e)There is a new employee who knows everyone's phone number.
~~$\exists x \forall y(N(x) \rightarrow P(x, y))$~~ for some x and all y (N(x) ^ P(x, y))

(f)Exactly one new employee missed the deadline.
~~$\exists x(N(x) \rightarrow \left(D(x) \wedge \forall y(N(y) \wedge (x \neq y) \rightarrow \neg D(x)\right)))$~~
for some x and all y (N(x) ^ D(x) ^ (((x != y) ^ N(y)) -> ~D(y)))

Exercise 1.10.10: Statements with nested quantifiers: variables with different domains.
The domain for the first input variable to predicate T is a set of students at a university. The domain for the second input variable to predicate T is the set of Math classes offered at that university. The predicate T(x, y) indicates that student x has taken class y. Sam is a student at the university and Math 101 is one of the courses offered at the university. Give a logical expression for each sentence.

(c)Every student has taken at least one class besides Math 101.
~~$\forall x \exists y(((y \neq Math\ 101) \rightarrow T(x, y)) \wedge T(x, Math\ 101))$~~

(d)There is a student who has taken every math class besides Math 101.
~~$\exists x \forall y(T(x, y))$~~

(e)Everyone besides Sam has taken at least two different math classes.

Excercise 1.10.10

Wrong because the ambiguity of the word “Besides”.

Next time, clarify the use of the term first.

$\cancel{\forall x \exists y \exists z((y \neq z) \rightarrow (T(x,y) \land T(x,z)))}$

(f)Sam has taken exactly two math classes.

$\cancel{\exists x \exists y(((x \neq y) \rightarrow (T(Sam,x) \land T(Sam,y))) \rightarrow \forall z(((z \neq y) \land (z \neq x)) \rightarrow \neg T(Sam,z)))}$

Question 12:
Exercise 1.8.2: Applying De Morgan's law for quantified statements to English statements.
In the following question, the domain of discourse is a set of male patients in a clinical study. Define the following predicates:
P(x): x was given the placebo
D(x): x was given the medication
M(x): x had migraines

Translate each statement into a logical expression. Then negate the expression by adding a negation operation to the beginning of the expression. Apply De Morgan's law until each negation operation applies directly to a predicate and then translate the logical expression back into English.

Sample question: Some patient was given the placebo and the medication.
$\exists x\ (P(x) \land D(x))$
Negation: $\neg\exists x\ (P(x) \land D(x))$
Applying De Morgan's law: $\forall x\ (\neg P(x) \lor \neg D(x))$
English: Every patient was either not given the placebo or not given the medication (or both).

(b)Every patient was given the medication or the placebo or both.
$\forall x\big(D(x) \lor P(x)\big)$
Negation: $\neg\forall x\big(D(x) \lor P(x)\big)$
Applying De Morgan's law: $\exists x\big(\neg D(x) \land \neg P(x)\big)$
English: Some patient was neither given the medication nor given the placebo.

(c)There is a patient who took the medication and had migraines.
$\exists x\big(D(x) \land M(x)\big)$
Negation: $\neg\exists x\big(D(x) \land M(x)\big)$
Applying De Morgan's law: $\forall x\big(\neg D(x) \lor \neg M(x)\big)$
English: Every patient was either not given the medication or not have migraines.

(d)Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, $p \rightarrow q \equiv \neg p \lor q$.)
$\forall x\big(P(x) \rightarrow M(x)\big)$
$\forall x\big(\neg P(x) \lor M(x)\big)$
Negation: $\neg\forall x\big(\neg P(x) \lor M(x)\big)$
Applying De Morgan's law: $\exists x\big(P(x) \land \neg M(x)\big)$
English: Some patient was given the placebo and not have migraines.

(e)There is a patient who had migraines and was given the placebo.
$\exists x\big(M(x) \land P(x)\big)$
Negation: $\neg\exists x\big(M(x) \land P(x)\big)$
Applying De Morgan's law: $\forall x\big(\neg M(x) \lor \neg P(x)\big)$
English: Every patient was either not given the placebo or not have migraines.

Exercise 1.9.4: De Morgan's law and nested quantifiers.
Write the negation of each of the following logical expressions so that all negations immediately precede predicates. In some cases, it may be necessary to apply one or more laws of propositional logic.

(c) $\exists x \, \forall y \, (P(x, y) \to Q(x, y))$
$\forall x \, \exists y \, (P(x, y) \land \neg Q(x, y))$

(d) $\exists x \, \forall y \, (P(x, y) \leftrightarrow P(y, x))$
$\cancel{\forall x \exists y(\neg P(x, y) \leftrightarrow P(y, x))}$

~(((P(x, y) -> P(y, x)) ^ (P(y, x) -> P(x, y)))
= ~((~P(x, y) v P(y, x)) ^ (~P(y, x) v P(x, y)))
= ~(~P(x, y) v P(y, x)) v ~(~P(y, x) v P(x, y))
= (P(x, y) ^ ~P(y, x)) v (P(y, x) ^ ~P(x, y))

(e) $\exists x \, \exists y \, P(x, y) \land \forall x \, \forall y \, Q(x, y)$
$\forall x \forall y(\neg P(x, y)) \lor \exists x \exists y(\neg Q(y, x))$