

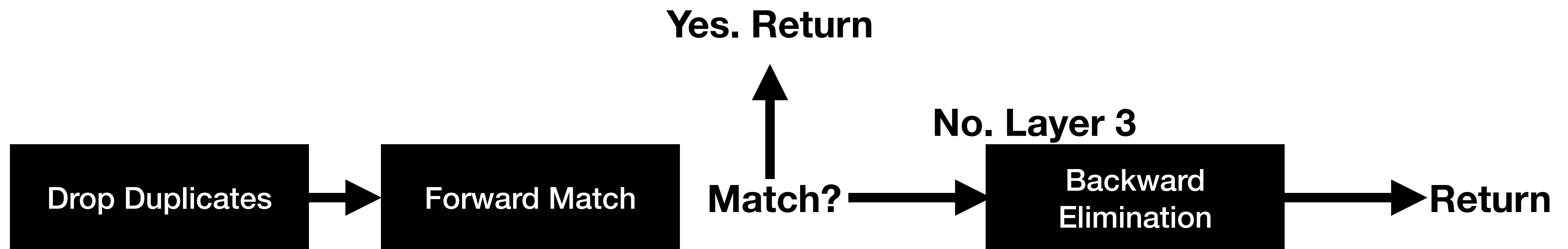
Algorithm

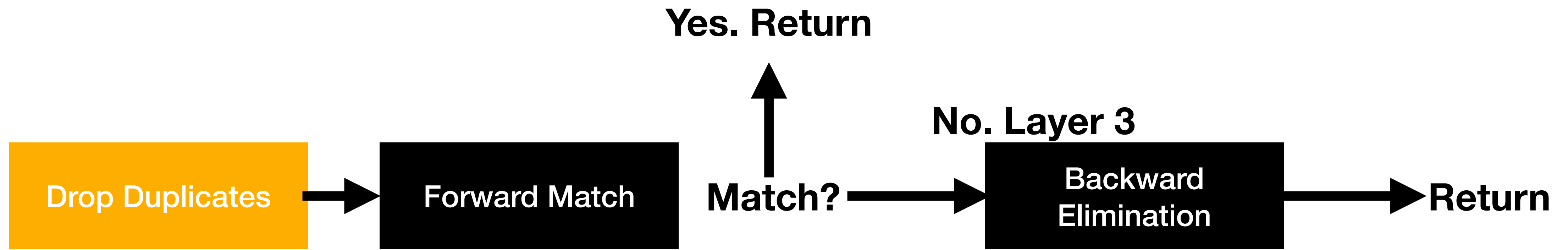
Reconciliation Account

2021/01/27

Three Layers

- Drop Duplicates
- Forward Match
- Backward Elimination





Layer 1: Drop Duplicates

- **Single Drop**
- Merge Drop
- Reversed Merge

The simple duplicate values will be dropped in this step.

Internal Credit	Match!	External Debit
100		99
40		1
70		40
...		...

Layer 1: Drop Duplicates; Pseudo Code

- **Single Drop**
- Merge Drop
- Reversed Merge

```
while True:
    update = False
    for amount_x in internal_account:
        for amount_y in external_account:
            If amount_x == amount_y:
                internal_account = internal_account.exclude(amount_x)
                external_account = external_account.exclude(amount_y)
                update = True
                break
        If update:
            break
    If not update:
        break
```

Layer 1: Drop Duplicates

- Single Drop
- **Merge Drop**
- Reversed Merge Drop

This step will create a temp column to store the accumulate sum in the first column.

Internal Credit	Accumulate Sum (temp)
100	100
40	X
70	170
...	...

External Debit	Accumulate Sum (temp)
99	99
1	100
40	X
...	...

Layer 1: Drop Duplicates

- Single Drop
- **Merge Drop**
- Reversed Merge Drop

This step will create a temp column to store the accumulate sum in the first column.

Internal Credit	Accumulate Sum (temp)
100	100
40	X
70	170
...	...

External Debit	Accumulate Sum (temp)
99	99
1	100
40	X
...	...

Layer 1: Drop Duplicates

- Single Drop
- **Merge Drop**
- Reversed Merge Drop

Then start matching from the top. If the same value is reached, drop all the values above.

Internal Credit	Accumulate Sum (temp)
100	100
40	X
70	170
...	...

Match!

External Debit	Accumulate Sum (temp)
99	99
1	100
40	X
...	...

Layer 1: Drop Duplicates

- Single Drop
- **Merge Drop**
- Reversed Merge Drop

Then start matching from the top. If the same value is reached, drop all the values above.

Internal Credit	Accumulate Sum (temp)
100	100
40	X
70	170
...	...

External Debit	Accumulate Sum (temp)
99	99
1	100
40	X
...	...

Layer 1: Drop Duplicates; Pseudo Code

- Single Drop
- **Merge Drop**
- Reversed Merge Drop

```
internal_account.add_col(cumsum)
external_account.add_col(cumsum)
while True:
    update = False
    for i_idx in internal_account:
        for e_idx in external_account:
            If internal_account[i_idx, cumsum] == external_account[e_idx, cumsum]:
                internal_account = internal_account[i_idx + 1:, :]
                external_account = internal_account[e_idx + 1:, :]
                update = True
                break
        If update:
            break
    If not update:
        break
```

Layer 1: Drop Duplicates

- Single Drop
- Merge Drop
- **Reversed Merge Drop**

This step will reverse the data frame and conduct merge drop again.

Internal Credit	Accumulate Sum (temp)
39	39
1	40
15	55
...	...

External Debit	Accumulate Sum (temp)
20	20
20	40
40	80
...	...

Layer 1: Drop Duplicates

- Single Drop
- Merge Drop
- **Reversed Merge Drop**

This step will reverse the data frame and conduct merge drop again.

Internal Credit	Accumulate Sum (temp)
39	39
1	40
15	55
...	...

External Debit	Accumulate Sum (temp)
20	20
20	40
40	80
...	...

Layer 1: Drop Duplicates

- Single Drop
- Merge Drop
- **Reversed Merge Drop**

This step will reverse the data frame and conduct merge drop again.

Internal Credit	Accumulate Sum (temp)
39	39
1	40
15	55
...	...

External Debit	Accumulate Sum (temp)
20	20
20	40
40	80
...	...

Layer 1: Drop Duplicates

- Single Drop
- Merge Drop
- **Reversed Merge Drop**

This step will reverse the data frame and conduct merge drop again.

Internal Credit	Accumulate Sum (temp)
39	39
1	40
15	55
...	...

External Debit	Accumulate Sum (temp)
20	20
20	40
40	80
...	...

Layer 1: Drop Duplicates

- Single Drop
- Merge Drop
- **Reversed Merge Drop**

This step will reverse the data frame and conduct merge drop again.

Internal Credit	Accumulate Sum (temp)
39	39
1	40
15	55
...	...

Match!

External Debit	Accumulate Sum (temp)
20	20
20	40
40	80
...	...

Layer 1: Drop Duplicates

- Single Drop
- Merge Drop
- **Reversed Merge Drop**

This step will reverse the data frame and conduct merge drop again.

Internal Credit	Accumulate Sum (temp)
39	39
1	40
15	55
...	...

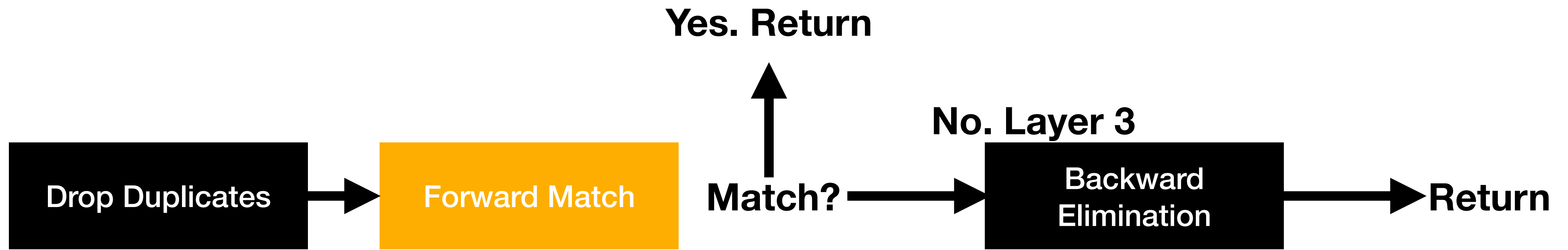
External Debit	Accumulate Sum (temp)
20	20
20	40
40	80
...	...

Layer 1: Drop Duplicates; Pseudo Code

- Single Drop
- Merge Drop

- **Reversed Merge Drop**

```
internal_account.reverse()
external_account.reverse()
internal_account.add_col(cumsum)
external_account.add_col(cumsum)
while True:
    update = False
    for i_idx in internal_account:
        for e_idx in external_account:
            If internal_account[i_idx, cumsum] == external_account[e_idx, cumsum]:
                internal_account = internal_account[i_idx + 1:, :]
                external_account = internal_account[e_idx + 1:, :]
                update = True
                break
        If update:
            break
    If not update:
        break
```



Layer 2: Forward Match

A helper function will be used to determine the optimal selection of the combination of two array.

Currently, the number is set to 1000 per array so that the maximum operation will not exceed $1000 * 1000 + 1000$.

Internal Credit
1000
20
70
...

External Debit
20
1
40
...

Layer 2: Forward Match; Pseudo Code

A helper function will be used to determine the optimal selection of the combination of two array.

Currently, the number is set to 1000 per array so that the maximum operation will not exceed $1000 * 1000 + 1000$.

The mathematical formula for optimization is:

$$\max(x) \text{ subject to } \sum_{i=1}^x \binom{n}{i} < 1000$$

Where x is the optimal number of choose; n is the total length of the input array.

```
def helper_func(array):  
    num = len(array)  
    counter = 0  
    limit = 1000  
  
    for n in num:  
        counter += combination(num, n)  
        If counter > limit:  
            return max(n - 1, 1)
```

Layer 2: Forward Match

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

The precedence goes:

comb. with X trans. > comb. with Y trans;
where $\text{len}(X) < \text{len}(Y)$.

(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

Internal Credit
1000
20
70
...

External Debit
20
1
40
...

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

The precedence goes:

comb. with X trans. > comb. with Y trans;
where len(X) < len(Y).

(1) comb. with X trans. > (2) comb. with Y trans;
where len(X) == len(Y), (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

The precedence goes:

comb. with X trans. > comb. with Y trans;
where $\text{len}(X) < \text{len}(Y)$.

(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

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comb. with X trans. > comb. with Y trans;
where len(X) < len(Y).

(1) comb. with X trans. > (2) comb. with Y trans;
where len(X) == len(Y), (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

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where $\text{len}(X) < \text{len}(Y)$.

(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

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where $\text{len}(X) < \text{len}(Y)$.

(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

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where len(X) < len(Y).

(1) comb. with X trans. > (2) comb. with Y trans;
where len(X) == len(Y), (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

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where len(X) < len(Y).

(1) comb. with X trans. > (2) comb. with Y trans;
where len(X) == len(Y), (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

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where $\text{len}(X) < \text{len}(Y)$.

(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

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(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

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where len(X) < len(Y).

(1) comb. with X trans. > (2) comb. with Y trans;
where len(X) == len(Y), (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

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(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

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where $\text{len}(X) < \text{len}(Y)$.

(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

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comb. with X trans. > comb. with Y trans;
where $\text{len}(X) < \text{len}(Y)$.

(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

The precedence goes:

comb. with X trans. > comb. with Y trans;
where $\text{len}(X) < \text{len}(Y)$.

(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

The precedence goes:

comb. with X trans. > comb. with Y trans;
where $\text{len}(X) < \text{len}(Y)$.

(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

The precedence goes:

comb. with X trans. > comb. with Y trans;
where $\text{len}(X) < \text{len}(Y)$.

(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

The precedence goes:

comb. with X trans. > comb. with Y trans;
where $\text{len}(X) < \text{len}(Y)$.

(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

The precedence goes:

comb. with X trans. > comb. with Y trans;
where len(X) < len(Y).

(1) comb. with X trans. > (2) comb. with Y trans;
where len(X) == len(Y), (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

The precedence goes:

comb. with X trans. > comb. with Y trans;
where $\text{len}(X) < \text{len}(Y)$.

(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

The precedence goes:

comb. with X trans. > comb. with Y trans;
where $\text{len}(X) < \text{len}(Y)$.

(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

The precedence goes:

comb. with X trans. > comb. with Y trans;
where $\text{len}(X) < \text{len}(Y)$.

(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

The precedence goes:

comb. with X trans. > comb. with Y trans;
where len(X) < len(Y).

(1) comb. with X trans. > (2) comb. with Y trans;
where len(X) == len(Y), (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

After layer 1, we only need to handle the remaining data frame.

In this layer, combination of the transactions will be calculated; the match combination will be returned.

The precedence goes:

comb. with X trans. > comb. with Y trans;
where $\text{len}(X) < \text{len}(Y)$.

(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

Internal Credit	External Debit
1000	20
20	1
70	40
...	...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

When a combination match, the program will record the instance and continue executing to see if there is a more concise combination. After the completion of iteration, the most concise combination will be returned.

The precedence goes:

comb. with X trans. > comb. with Y trans;
where $\text{len}(X) < \text{len}(Y)$.

(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

Internal Credit	Match!	External Debit
1000		20
20		1
70		40
...		...

Assume that today's imbalance for Internal Credit & External Debit is **1030**

Layer 2: Forward Match

Right Red Square - Left Red Square

When a combination match, the program will record the instance and continue executing to see if there is a more concise combination. After the completion of iteration, the most concise combination will be returned.

The precedence goes:

comb. with X trans. > comb. with Y trans;
where $\text{len}(X) < \text{len}(Y)$.

(1) comb. with X trans. > (2) comb. with Y trans;
where $\text{len}(X) == \text{len}(Y)$, (i) means the time order of the match.

For example, the yellow square pair will be returned instead of the red square pair.

Internal Credit	Return Yellow	External Debit
10		5
7		15
30		40
...		...

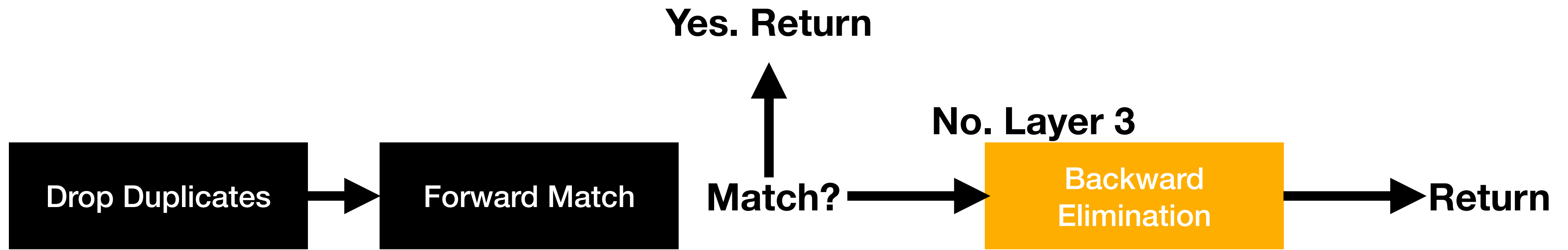
Assume that today's imbalance for Internal Credit & External Debit is **10**

Layer 2: Forward Match; Pseudo Code

```
n_1 = helper_func(internal_account)
n_2 = helper_func(external_account)
internal_combs = get_combination(array=internal_account, choose_to=n_1)
external_combs = get_combination(array=external_account, choose_to=n_2)
Candidates = []

for int_comb in internal_combs:
    for ex_comb in external_combs:
        If internal_account.select(int_comb).sum() - external_account.select(ex_comb).sum() == target:
            candidates.append((int_comb, ex_comb))

shortest = inf
for cand in candidates:
    total_len = len(cand[0]) + len(cand[1])
    If total_len < shortest:
        shortest = total_len
        match_pair = cand
return internal_account.select(cand[0]), external_account.select(cand[1])
```



Layer 3: Backward Elimination

A helper function will be used to determine the optimal selection of the combination of two array.

Currently, the number is set to 1000 per array so that the maximum operation will not exceed $1000 * 1000 + 1000$.

Internal Credit
1000
20
70
...

External Debit
20
1
40
...

Layer 2: Forward Match; Pseudo Code

A helper function will be used to determine the optimal selection of the combination of two array.

Currently, the number is set to 1000 per array so that the maximum operation will not exceed $1000 * 1000 + 1000$.

The mathematical formula for optimization is:

$$\max(x) \text{ subject to } \sum_{i=0}^x \binom{n}{t} < 1000 \quad t = n - i$$

Where x is the optimal number of choose; n is the total length of the input array.

```
def helper_func(array):  
    num = len(array)  
    counter = 0  
    limit = 1000  
  
    for n in num:  
        counter += combination(num, n)  
        If counter > limit:  
            return max(n - 1, 1)
```

Layer 3: Backward Elimination

If the layer 2 cannot find the match transactions, layer 3 will be executed.

In this layer, combination of the transactions will be calculated; the matched combination will be returned.

The precedence will be same as layer 2.

The only difference is that in this layer. The number of choose is started from n , $n-1$, $n-2$, etc., where n is the length of the input array.

If the length of the matched array is shorter than the original array. The unmatched parts will be drop and the array will be updated.

Internal Credit	External Debit
1000	20
20	1
70	30
...	...

Assume that today's imbalance for Internal Credit & External Debit is **37.5**

Layer 3: Backward Elimination

If the layer 2 cannot find the match transactions, layer 3 will be executed.

In this layer, combination of the transactions will be calculated; the matched combination will be returned.

The precedence will be same as layer 2.

The only difference is that in this layer. The number of choose is started from n , $n-1$, $n-2$, etc., where n is the length of the input array.

If the length of the matched array is shorter than the original array. The unmatched parts will be drop and the array will be updated.

Internal Credit	External Debit
1000	20
20	1
70	30
...	...

Assume that today's imbalance for Internal Credit & External Debit is **37.5**

Layer 3: Backward Elimination

If the layer 2 cannot find the match transactions, layer 3 will be executed.

In this layer, combination of the transactions will be calculated; the matched combination will be returned.

The precedence will be same as layer 2.

The only difference is that in this layer. The number of choose is started from n , $n-1$, $n-2$, etc., where n is the length of the input array.

If the length of the matched array is shorter than the original array. The unmatched parts will be drop and the array will be updated.

Internal Credit	Match!	External Debit
1000		20
20		1
70		30
...		...

Assume that today's imbalance for Internal Credit & External Debit is **37.5**

Layer 3: Backward Elimination

If the length of the matched array is shorter than the original array, as the example, the unmatched parts will be drop and the array will be updated.

The same operation will be executed in the updated array until there is nothing to be updated (the length of the matched array is equal to the length of the original array)

Internal Credit	Match!	External Debit
1000		20
20		1
70		30
...		...

Assume that today's imbalance for Internal Credit & External Debit is **37.5**

Layer 3: Backward Elimination

If the length of the matched array is shorter than the original array, as the example, the unmatched parts will be drop and the array will be updated.

The same operation will be executed in the updated array until there is nothing to be updated (the length of the matched array is equal to the length of the original array)

Internal Credit	Update!	External Debit
1000		1
20		23
65		78
...		...

Assume that today's imbalance for Internal Credit & External Debit is **37.5**

Layer 3: Backward Elimination

A helper function will be used to determine the optimal selection of the combination of two array.

Currently, the number is set to 1000 per array so that the maximum operation will not exceed $1000 * 1000 + 1000$.

Internal Credit
1000
20
65
...

External Debit
1
23
78
...

Layer 3: Backward Elimination

The same operation is executed in the updated array.

Internal Credit	External Debit
1000	1
20	23
65	78
...	...

Assume that today's imbalance for Internal Credit & External Debit is **37.5**

Layer 3: Backward Elimination

The same operation is executed in the updated array.

Internal Credit	External Debit
1000	1
20	23
65	78
...	...

Assume that today's imbalance for Internal Credit & External Debit is **37.5**

Layer 3: Backward Elimination

The same operation is executed in the updated array.

Internal Credit	External Debit
1000	1
20	23
65	78
...	...

Assume that today's imbalance for Internal Credit & External Debit is **37.5**

Layer 3: Backward Elimination

The same operation is executed in the updated array.

Internal Credit	External Debit
1000	1
20	23
65	78
...	...

Assume that today's imbalance for Internal Credit & External Debit is **37.5**

Layer 3: Backward Elimination

The same operation is executed in the updated array.

Eventually, there is no subset of the original array that can match the imbalance amount under the limitation of the helper function. The entire array will be selected, stop updating, and return the result.

Internal Credit	Match!	External Debit
1000		1
20		23
65		78
...		...

Assume that today's imbalance for Internal Credit & External Debit is **37.5**

Layer 3: Backward Elimination

The same operation is executed in the updated array.

Eventually, there is no subset of the original array that can match the imbalance amount under the limitation of the helper function. The entire array will be selected, stop updating, and return the result.

Internal Credit	All is Selected Return!	External Debit
1000		1
20		23
65		78
...		...

Assume that today's imbalance for Internal Credit & External Debit is **37.5**

Layer 3: Backward Elimination; Pseudo Code

while True:

 n_1 = helper_func(internal_account)

 n_2 = helper_func(external_account)

 internal_combs = get_combination(array=internal_account, choose_to=n_1)

 external_combs = get_combination(array=external_account, choose_to=n_2)

 Candidates = []

 for int_comb in internal_combs:

 for ex_comb in external_combs:

 if internal_account.select(int_comb).sum() - external_account.select(ex_comb).sum() == target:

 candidates.append((int_comb, ex_comb))

...to be cont'd in the next page

Layer 3: Backward Elimination; Pseudo Code

while True:

.....cont'd from last page

shortest = inf

for cand in candidates:

total_len = len(cand[0]) + len(cand[1])

If total_len < shortest:

shortest = total_len

match_pair = cand

condition_1 = len(internal_account) == len(internal_account.select(cand[0]))

condition_2 = len(external_account) == len(external_account.select(cand[1]))

if condition_1 & condition_2:

return internal_account, external_account

else:

internal_account, external_account = internal_account.select(cand[0]), external_account.select(cand[1])