Perturbed Universes and Inflationary Models

MSci. Thesis

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2 Abstract

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$$P(k) = A\left(\frac{k}{k_0}\right)^{n_s-1} \tag{1}$$

$$n_s - 1 = \frac{\log(P(k))}{\log(k)} \tag{2}$$

4.9 Spectral Index

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6 Matching Conditions

6.1 Outline

The matching condition is a simple analytical method that can be used to approximate the spectral index of an inflationary model to a good degree of accuracy so long as $a(\eta)$ is given. This can be achieved by assuming the growing modes that freeze out and exit the horizon are proportional to the scale factor and some function of k. The functional dependence on k can then be calculated by matching the modes at horizon crossing. At horizon crossing the following equation is valid.

$$\left[k^2 - \frac{a''}{a}\right] = 0 \tag{3}$$

$$\frac{1}{\sqrt{k}} \approx a(\eta)F(k) \tag{4}$$

Furthermore

$$P(k) = k^3\zeta^2 = k^3\frac{v^2}{a^2} = k^3F(k)^2 \tag{5}$$

6.2 Constant w

$$a \approx \eta^{\frac{2}{3w+1}} \tag{6}$$

$$n_s(w) = 4 \pm \frac{3(1-w)}{3w+1} \tag{7}$$

6.3 Jacobi Elliptic Function Solutions

The matching condition method can be used to derive an expression for $a(\eta)$ that gives scale invariance. Equation 4 can be expressed purely in terms of the scale factor and its derivatives with respect to conformal time using equation 3 to give the following.

$$a(\eta) \approx \frac{1}{F\left(\sqrt{\frac{a''}{a}}\right)\left(\frac{a''}{a}\right)^{\frac{1}{4}}} \tag{8}$$

From equations 2 and 5 it can be seen that $F(k) \propto k^{-\frac{3}{2}}$ is required for exact scale invariance. If this condition is imposed in equation 8 a second order differential equation can be found for the the scale factor $a(\eta)$.

$$a''(\eta) \propto a(\eta)^3 \tag{9}$$

It is immediately clear to see that $a \propto \frac{1}{|\eta|}$ is a solution.

7 Hamiltonian Jacobi

7.1 Outline

The evolution of scalar fields is described by the Klein [1] Gordon Equation

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \tag{10}$$

$$H^2 = \frac{8\pi}{3m_{pl}^2}[\frac{1}{2}\dot{\phi}^2 + V(\phi)] \tag{11}$$

8 Generalised Equation of State

8.1 $n_s = 1$ (Intermediate Inflation)

$$H''H - 2(H')^2 = 0 \tag{12}$$

$$H(\phi) = \frac{\alpha}{\beta + \phi} \approx \frac{1}{\phi} \tag{13}$$

8.2 $n_s \neq 1$

$$\left(\frac{H''}{2}\right) - \left(\frac{H'^2}{H}\right) + (n_s - 1)\,\pi H = 0 \tag{14}$$

A trial solution of $H(\phi) = \exp(C\phi)$ can be shown to satisfy equation 14 with $C = 5\sqrt{2}$. This is the same as Power Law Inflation.

8.3 Proof of Slow roll

8.4 Equation of State

$$\rho_\phi = \left(\frac{8\pi\,(n_s - 1)}{9A^2}\right) p_\phi^2 - \left(\frac{n_s + 2}{3}\right) p_\phi \tag{15}$$

8.5 Potential

8.6 Number of e-foldings

9 Conclusion

References

[1] Herbert Goldstein. *Classical mechanics*. Addison-Wesley, Reading, Mass. ; Wokingham, 2nd ed. edition, 1980.