Perturbed Universes and Inflationary Models

MSci. Thesis

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$$P(k) = A\left(\frac{k}{k_0}\right)^{n_s-1} \tag{1}$$

$$n_s - 1 = \frac{\log(P(k))}{\log(k)} \tag{2}$$

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5.1 Outline

$$\left[k^2 - \frac{a''}{a}\right] = 0 \tag{3}$$

$$\frac{1}{\sqrt{k}} \approx a(\eta)F(k) \tag{4}$$

$$P(k) = k^3\frac{v^2}{a^2} = k^3F(k)^2 \tag{5}$$

5.2 Constant w

$$a \approx \eta^{\frac{2}{3w+1}} \tag{6}$$

$$n_s(w) = 4 \pm \frac{3(1-w)}{3w+1} \tag{7}$$

5.3 Jacobi Elliptic Function Solutions

The matching condition method can be used to derive an expression for $a(\eta)$ that gives scale invariance. Equation 4 can be expressed purely in terms of the scale factor and its derivatives with respect to conformal time using equation 3 to give the following.

$$a(\eta) \approx \frac{1}{F\left(\sqrt{\frac{a''}{a}}\right)\left(\frac{a''}{a}\right)^{\frac{1}{4}}} \tag{8}$$

From equations 2 and 5 it can be seen that $F(k) \propto k^{-\frac{3}{2}}$ is required for exact scale invariance. If this condition is imposed in equation 8 a second order differential equation can be found for the the scale factor $a(\eta)$.

$$a''(\eta) \propto a(\eta)^3 \tag{9}$$

It is immediately clear to see that $a \propto \frac{1}{|\eta|}$ is a solution.

6 Hamiltonian Jacobi

6.1 Outline

The evolution of scalar fields is described by the Klein [1] Gordon Equation

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \tag{10}$$

$$H^2 = \frac{8\pi}{3m_{pl}^2}[\frac{1}{2}\dot{\phi}^2 + V(\phi)] \tag{11}$$

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7 Conclusion

References

[1] Herbert Goldstein. *Classical mechanics*. Addison-Wesley, Reading, Mass. ; Wokingham, 2nd ed. edition, 1980. ID: 44IMP$_ALMA_DS$2141267570001591.