

Adaptive cancellation of mains power interference in continuous gravitational wave searches with a hidden Markov model

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Continuous gravitational wave searches with terrestrial, long-baseline interferometers are hampered by long-lived, narrowband features in the power spectral density of the detector noise, known as lines. Candidate GW signals which overlap spectrally with known lines are typically vetoed. Here we demonstrate a line subtraction method based on adaptive noise cancellation, using a recursive least squares algorithm, a common approach in electrical engineering applications, such as processing such as audio and biomedical signals signal processing. We validate the line subtraction method by combining it with a hidden Markov model, a standard continuous wave search tool, to detect a synthetic continuous wave signal with an unknown and randomly wandering frequency which overlaps with the strong mains power line at 60 Hz in the Laser Interferometer Gravitational Wave Observatory. We show that the synthetic continuous wave signal can be successfully recovered once the 60 Hz line has been subtracted. The performance of the line subtraction method with respect to the characteristics of the 60 Hz line and the control parameters of the method is explored and quantified recursive least squares algorithm is quantified in terms of receiver operating characteristic curves.

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I. INTRODUCTION

Instrumental noise artifacts in gravitational wave (GW) searches with terrestrial, long-baseline interferometers are classified according to their duration and spectral properties. Short-lived, non-stationary, recurrent noise events such as optomechanical glitches typically last for seconds and exhibit distinctive spectral signatures, e.g. they can be chirp-like [1–5]. Long-lived, quasi-stationary, broadband noise sources include seismic disturbances at low frequencies, test mass thermal fluctuations at intermediate frequencies, and photon shot noise at high frequencies [6–10]. Long-lived narrowband spectral artifacts — termed instrumental lines — are caused by electrical subsystems (e.g. mains power, clocks, oscillators), mechanical subsystems (e.g. test mass and beam-splitter violin modes) and calibration processes [11], although sometimes the origin of a specific feature is unknown. Instrumental lines are disruptive, especially for continuous wave (CW) searches where the target astrophysical signal is quasi-monochromatic and resembles the noise artifact spectrally. Many above-threshold candidates discovered in published CW searches are vetoed because they coincide with known instrumental lines [12–14], for instance, in data from Observing Run 3 (O3) with the Laser Interferometer Gravitational Wave Observatory (LIGO), Virgo and the Kamioka Gravitational Wave Detector (KAGRA) [e.g. 15–17]

Several techniques have been implemented by the LIGO-Virgo-KAGRA (LVK) Collaboration to identify, characterize and suppress instrumental noise artifacts [18, 19]. Some techniques identify and veto an artifact (or gate data segments) based on its time-frequency signature [20–22]. Other techniques perform offline

noise subtraction with reference to auxiliary data from physical environmental monitors (PEMs) [23–26]. PEMs can be used to witness correlated noise and generate a reference signal directly or elucidate and quantify multichannel couplings [27, 28]. Additionally some techniques are based on machine learning [29–32]. In CW searches specifically, the distinctive amplitude and frequency (Doppler) modulations associated with the Earth’s rotation and revolution respectively can be exploited to discriminate between terrestrial noise artifacts and astrophysical signals [33, 34].

In most of the situations above, the practical effect of an instrumental line is to excise the relevant part of the observing band from a CW search. That is, if an above-threshold above-threshold CW search candidate coincides with a known instrumental line, the candidate is vetoed under current practice without further analysis, such as e.g. without comparing the expected strength of the noise line with the measured strength of the candidate¹. In this paper we take a first step towards lifting the above limitation. We introduce an adaptive noise cancellation (ANC) scheme based on an adaptive recursive least squares (ARLS) method which suppresses narrowband noise proportional to a known PEM reference signal. We then apply the ANC scheme to a CW search algorithm based on a hidden Markov model (HMM), which detects and tracks quasi-monochromatic GW signals with wandering frequency and has been tested and validated thoroughly in multiple LVK searches [12–14, 35]. We demonstrate

¹ A regularly updated log of narrowband instrumental lines in the LVK detector is maintained at dcc.ligo.org/LIGO-T2100200/public for public reference.

with synthetic data that the ANC scheme and HMM algorithm together can successfully detect a GW signal lying under the 60 Hz mains power line, if the signal exceeds a well-defined minimum amplitude. The approach extends naturally to other instrumental lines, a topic for future work.

The paper is organized as follows. In Section II we outline a mathematical model for the 60 Hz mains power line and the PEM reference signal, and justify the assumptions of the model by reference to data from the LIGO Livingston interferometer. In Section III we introduce ANC formulated as an ARLS method. In Section IV we validate the ANC scheme on synthetic GW strain data, in conjunction with an HMM Viterbi algorithm, and demonstrate the successful recovery of a frequency-wandering CW signal. In Section V we quantify the performance of the HMM Viterbi algorithm and ANC scheme as a function of the key parameters of the mains power interference and ANC filter. Concluding remarks are given in Section VI.

II. POWER GRID MAINS POWER INTERFERENCE

The goal of this paper is to detect a quasi-monochromatic GW signal in a data stream contaminated by two kinds of noise: additive Gaussian noise which is fundamental and irreducible, and additive non-Gaussian interference from a long-lived narrow spectral feature which can be filtered out in principle given an accurately measured reference signal. For this work we consider the spectral line at 60 Hz that results from the North American alternating current [power grid mains power](#) as the additive non-Gaussian interference, as it is a long-standing impediment to many CW searches with the LIGO interferometers. In Section II A we briefly review the 60 Hz LIGO interference line before proceeding in Section II B to specify the assumed mathematical forms of the interference and reference signals. In Section II C we justify the assumptions of Section II B by analysing differential arm length (DARM channel) and environmental ([power grid mains power](#) monitoring) data from the LIGO interferometers.

A. LIGO 60 Hz Interference

LIGO data contains multiple long-duration narrow lines in addition to [the usual](#) Gaussian noise. These lines are shown in Figure 1 where we plot the amplitude spectral density of the GW strain channel `*:DCS-CALIB_STRAIN_C01.AR` for both LIGO-Hanford (H1, orange curve) and LIGO-Livingston (L1, blue curve). The small amplitude [random](#) fluctuations are the result of Gaussian noise whereas the large amplitude spikes are the long-duration lines. The provision of mains

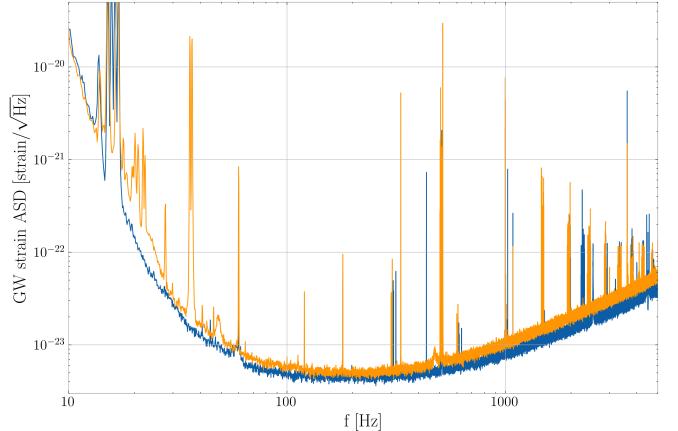


FIG. 1. Sensitivity plot for LIGO-Hanford (orange) and LIGO-Livingston (blue) for a snapshot of data (10.0 minutes) from O3 (channel `*:DCS-CALIB_STRAIN_C01.AR`, see Ref. [36, 37]). The vertical axis plots the amplitude spectral density of the detector noise in units of $\text{Hz}^{-1/2}$, while the horizontal axis plots the observation frequency in units of Hz. The spectral line at 60 Hz is clearly visible as a spike rising three decades, along with multiple other instrumental lines at other frequencies.

power electricity in North America via an alternating current ([AC](#)) with frequency 60 Hz leads to a [line in the power spectral density of the LIGO noise](#) distinct line in Figure 1. The mains power couples to the GW strain channel, because the performance of the high-sensitivity electronic components within LIGO varies with respect to the input power voltage. Additionally, the magnetic fields that arise from the AC mains supply couple to magnets on optical components.

Some spectral lines are static, but the 60 Hz line wanders in time, due to variations in the load in the North American power grid. The wandering is displayed in the cascade plot in Figure 2, where the amplitude spectral density of the output signal of the LIGO-Hanford PEM channel `H1:PEM-CS_MAINSMON_EBAY_1_DQ` is plotted for 210 5-min samples stacked vertically. The peak of the amplitude spectral density varies from 59.969 Hz to 60.053 Hz, across the 210 samples with an average value of 60.004 Hz. Its full width half maximum varies from 5.294 mHz to 29.377 mHz, with an average value of 10.749 mHz. For a full review of LIGO spectral artifacts, including the 60 Hz line, we refer the reader to [11].

The 60 Hz line occurs in the neighbourhood of plausible astrophysical signals. For example, mass quadrupole radiation from a rotating mountain on the Crab pulsar would be emitted at twice the star's spin frequency, viz. 30.2 Hz [13, 14].

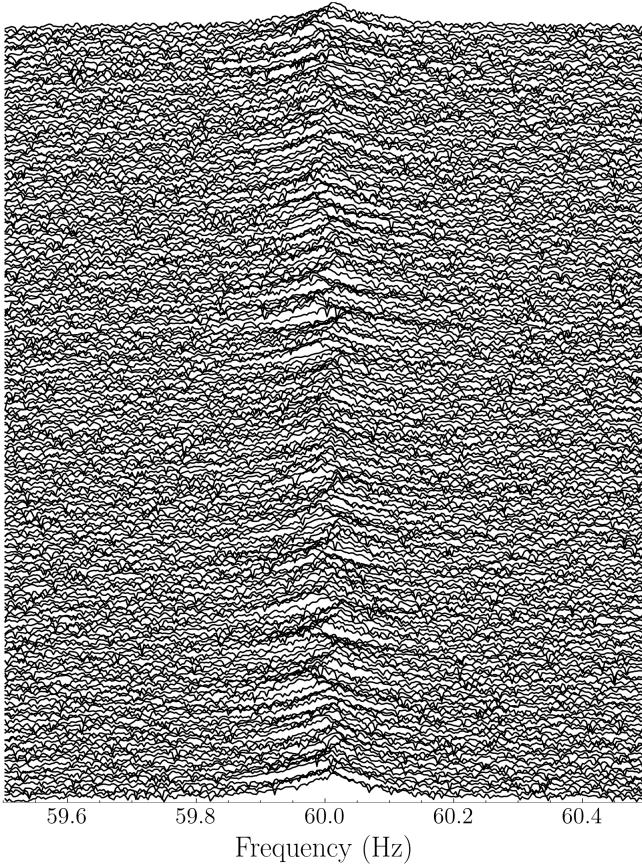


FIG. 2. Cascade plot showing the temporal evolution of the amplitude spectral density (units: $\text{Hz}^{-1/2}$) of the LIGO-Hanford [three-phase](#) mains-power PEM monitor `H1:PEM-CS_MAINSMON_EBAY_1_DQ` (corner station, phase 1) as a function of frequency (units: Hz). Each trace corresponds to 320 s (≈ 5 min) of data (210 traces stacked vertically). The peak of the mains power [60 Hz](#)-line wanders by $\approx \pm 30$ mHz about its time-averaged frequency [60.004 Hz](#).

B. Statement of the problem: signal and noise models

Let $x(t)$ denote the scalar time series output by the GW strain channel of a LIGO-like long baseline interferometer. Suppose that $x(t)$ is sampled at discrete times t_n , with $1 \leq n \leq N$ and uniform sampling interval $\Delta t = t_n - t_{n-1}$. Let $r(t)$ denote the scalar time series output by the environmental reference channel relevant for filtering interference; here $r(t)$ is of one of the three phases of mains power measured at some reference point in the detector. The reference signal is usually sampled less frequently than $x(t)$ at discrete times t_{n_k} , with $1 \leq k \leq K$ and $1 \leq n_k \leq N$. We assume for the sake of convenience that every t_{n_k} coincides with some t_n for all k , but the condition is not essential.

The strain channel is composed of a gravitational wave signal $h(t)$, non-Gaussian interference $c(t)$ (sometimes

called “clutter”) and Gaussian noise $n(t)$ in a linear combination,

$$x(t) = h(t) + c(t) + n(t) . \quad (1)$$

In this paper the gravitational wave signal takes the form predicted by Jaranowski *et al.* [38] for a biaxial rotor, e.g. a neutron star (NS) emitting gravitational waves at multiples of the star’s spin frequency f_\star . The GW signal is quasi-monochromatic, amplitude-modulated by the rotation of the Earth and frequency-modulated by the Earth’s orbital motion. The noise $n(t)$ is white with $\langle n(t_n)n(t_{n'}) \rangle = \sigma_n^2 \delta_{nn'}$. Noise samples $n(t_n)$ are drawn from a Gaussian distribution with zero mean and variance σ_n^2 . The interference $c(t)$ takes a form determined by instrumental processes but is generally a long-lived, narrowband spectral feature. We relate $c(t)$ to the mains voltage $r(t)$ in this paper.

Mains power is characterized by three properties. First, the frequency is maintained at a constant value across the grid to a good approximation by internal grid mechanisms (effectively a phase locked loop), with central frequency $f_{\text{ac}} = 60$ Hz in North America. A slow periodic modulation occurs around f_{ac} with a small amplitude $\Delta f_{\text{ac}} \lesssim 0.5$ Hz and period P , which wanders randomly and uniformly in the range $0 \leq P \leq P_{\text{max}}$. Second, the phase offset $\Theta(t)$ of the voltage $r(t)$ wanders stochastically. We assume that the phase noise is white and Gaussian, with $n_\Theta(t_n)$ drawn from a Gaussian distribution with zero mean and variance σ_Θ^2 , i.e. $\langle n_\Theta(t_n)n_\Theta(t_{n'}) \rangle = \sigma_\Theta^2 \delta_{nn'}$. Third, the voltage amplitude, $A_r(t)$, is random. We assume that samples $A_r(t)$ are distributed uniformly within $[A_{\text{ac}} - \Delta A_{\text{ac}}, A_{\text{ac}} + \Delta A_{\text{ac}}]$. We can then write the reference voltage as,

$$r(t) = A_r(t_n) \cos[2\pi f_{\text{ac}}t + \Theta(t)] + n_r(t_n) , \quad (2)$$

with

$$\Theta(t) = 2\pi\Delta f_{\text{ac}} \cos\left[\frac{2\pi t}{P(t_n)}\right] + n_\Theta(t_n) , \quad (3)$$

for $t_n \leq t \leq t_{n+1}$. That is, at time t_n , random variables $A_r(t_n)$, $P(t_n)$ and $n_\Theta(t_n)$ are drawn from the [distribution-distributions](#) $\mathcal{U}[A_{\text{ac}} - \Delta A_{\text{ac}}, A_{\text{ac}} + \Delta A_{\text{ac}}]$, $\mathcal{U}[0, P_{\text{max}}]$, and $\mathcal{N}[0, \sigma_\Theta^2]$ respectively, [where \$\mathcal{U}\$ denotes uniform, and \$\mathcal{N}\$ denotes normal](#). Equation (2) then steps forward over a time interval Δt ; that is, $r(t)$ is discontinuous from one time step to the next due to random sampling. In Equation (2), $n_r(t_n)$ is the reference signal measurement noise at t_n , assumed to be white and Gaussian with $n_r(t_n)$ drawn from a Gaussian with zero mean and variance σ_r^2 . All the white measurement and process noises are assumed to be independent [for simplicity](#).

Mains power couples into the strain channel in various complicated ways, e.g. through electronic devices,

or inductively through ambient magnetic fields. A central assumption in this work is that the interference in the strain channel is an exact, amplitude-scaled replica of the reference signal up to a delay τ_{delay} which is attributed to spatial propagation between the PEM site and the GW-sensing apparatus. Hence we can express the interference as ,

$$c(t) = A_c(t_n - \tau_{\text{delay}}) \cos \left[2\pi f_{\text{ac}}(t_n - \tau_{\text{delay}}) + \Theta(t_n - \tau_{\text{delay}}) \right], \quad (4)$$

for $t_n \leq t \leq t_{n+1}$. The amplitude A_c is distributed as $\mathcal{U}[A_{\text{ac}} - \Delta A_{\text{ac}}, A_{\text{ac}} + \Delta A_{\text{ac}}]$. For this work we consider $0 \leq \tau_{\text{delay}} \leq 10\Delta t$, but wider or narrower ranges are possible and straightforward to implement. The assumption that $c(t)$ is a scaled replica of $r(t)$ is tested in Section II C.

C. Coherence between $x(t)$ and $r(t)$

A key assumption in Section II B is that the 60 Hz interference recorded in the reference PEM channel is also present in the strain channel. That is, the mains voltage recorded by the PEM is imprinted onto the strain channel up to a proportionality constant and time delay. In order to test this assumption we calculate the coherence between the strain channel and the PEM channel.

The coherence between two time series $x(t)$ and $r(t)$ is given by

$$C_{xr}(f) = \frac{|P_{xr}(f)|^2}{P_{xx}(f)P_{rr}(f)} \quad (5)$$

where f is the frequency, Fourier frequency and P_{xr} is the (cross) power spectral density

$$P_{xr}(f) = \int_{-\infty}^{\infty} R_{xr}(\tau) e^{-2\pi i f \tau} d\tau \quad (6)$$

for cross-correlation,

$$R_{xr}(\tau) = \int_{-\infty}^{\infty} x(t)r(t+\tau) dt. \quad (7)$$

$P_{xx}(f)$ and $P_{rr}(f)$ are defined analogously to Equation (6). $R_{xx}(f)$ and $0 \leq C_{xr}(f) \leq 1$. $R_{rr}(f)$ are defined analogously to Equation (7). In Equation (5) $0 \leq C_{xr}(f) \leq 1$. $C_{xr}(f) = 0$ indicates that the two signals are completely unrelated, whilst $C_{xr}(f) = 1$ indicates that the two signals have an ideal, noiseless, linear relationship i.e., i.e. the convolution $x(t) = g(t) * r(t)$ for impulse response function $g(t)$. If The intermediate situation $0 < C_{xr}(f) < 1$ then this indicates that the relationship between $x(t)$ and $r(t)$ is non-linear, either

due to measurement noise or else contributions to $x(t)$ from additional signals. Heuristically, for linear systems, $C_{xr}(f)$ can be understood as the fraction of the power in $x(t)$ that is produced by $r(t)$, at Fourier frequency f .

We use data for the strain and PEM channels from the first part of the third LIGO observing run, O3a [36]. These data are obtained via the LIGO Data Grid ² using the GWPY package [39]. In O3a there are nine independent PEM channels at both LIGO-Livingston and LIGO-Hanford that directly measure the mains voltage ³. In this paper we consider just the LIGO-Livingston data, as they suffice to make the point. Specifically, there are 3 PEM mains voltage monitors in the electronics bay in each of the X-arm end station (EX), the Y-arm end station (EY) and the corner station (CS). Each of the three PEMs at each station measures a separate component of three-phase mains power. The strain data are sampled at 16384 Hz and downsampled to 1024 Hz to match the sampling rate of the PEM channels.

Figure 3 displays $C_{xr}(f)$ between relating the high latency, calibrated strain channel L1:DCS-CALIB_STRAIN_C01_AR and each of the nine reference PEM channels over a 10 minute interval. For all channels there is a clear spike in the $C_{xr}(f)$ at 60 Hz, with a mean amplitude = 0.85 (unitless).

It is important to note that whilst $C_{xr}(f = 60\text{Hz})$ between $x(t)$ and $r(t)$ is strong, it is also time variable also varies with time. Figure 4 displays the coherence spectrogram, i.e. the time-varying coherence, between the strain channel and the PEM channel L1:PEM-CS_MAINSMON_EBAY_1_DQ (c.f. top panel, Figure 3) over a 1-hour 1-hr time interval (c.f. the 10 minute interval of Figure 3). $C_{xy}(f) C_{xr}(f)$ is calculated in 10-s blocks; that is, every pixel in Figure 4 is 10 s wide horizontally. We use a Fourier transform window of length 0.5 s, with each window overlapping by 0.25 s (c.f. Welch's method, Ref. [40]). Similar results are observed to The results resemble Figure 3; there is a strong coherence (mean value = 0.73) spectral feature at 60 Hz spectral feature with high coherence ($C_{xr}(f = 60\text{Hz}) = 0.73$ on average) which persists across the full 1-hour 1-hr time interval. However, whilst the feature is persistent, $C_{xr}(f = 60\text{Hz})$ is not constant across the entire interval. Instead it changes in time, with a variance = 0.018. These variations are evident visible in the changing colours of the spectrogram at 60 Hz, leading to an apparent “patchiness” of the line. Additional features in the $C_{xr}(f)$ spectrum are seen near 0.2 kHz and 0.3 kHz, corresponding to instrumental lines recorded by

² <https://computing.docs.ligo.org/guide/computing-centres/lhg/>

³ <https://git.ligo.org/detchar/ligo-channel-lists/-/blob/master/03/L1-03-pem.ini>

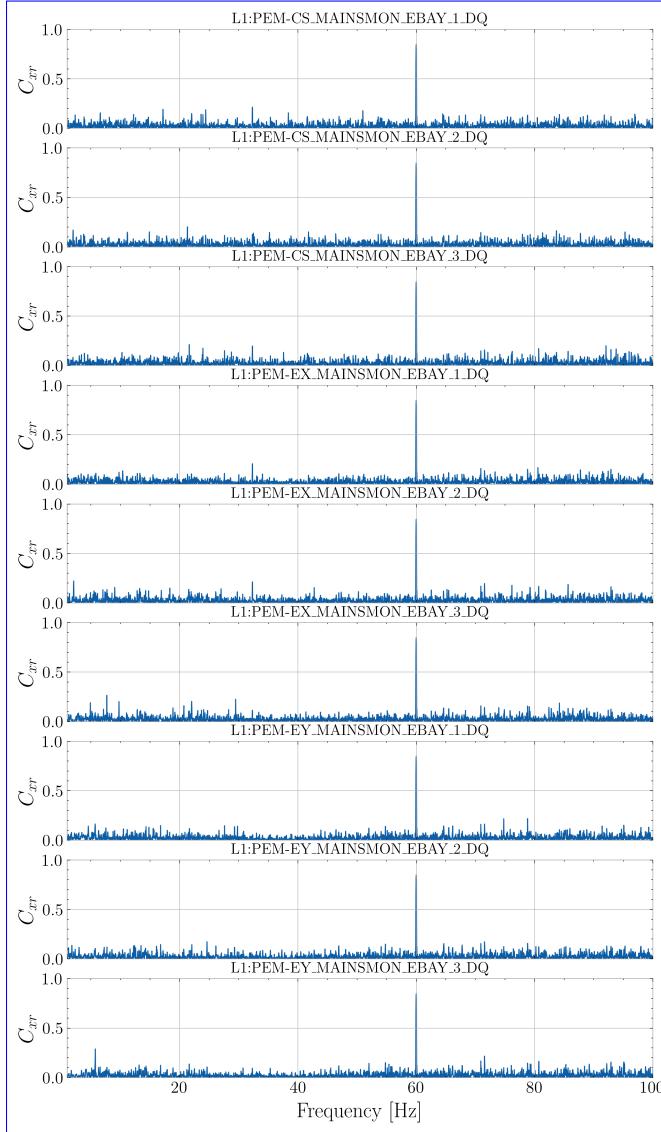


FIG. 3. Coherence $C_{xy}C_{xx}$, defined by Equation (5), between relating the LIGO-Livingston strain channel L1:DCS-CALIB_STRAIN_C01_AR and the nine three-phase mains-power PEM channels at the LIGO-Livingston end stations and corner station during a 10 minute observation interval. Clear features at 60 Hz are present in every channel panel, with amplitudes > 0.8 (unitless).

the PEM which imprint onto the strain channel and are unrelated to the mains power. Similar results are obtained for the other PEM channels shown in Figure 3. Figures 3 and 4 justify in part the assumptions in Section II B.

III. ANC SCHEME

Adaptive noise cancellation (ANC) ANC is a method for recovering an estimate of an estimating

an underlying signal corrupted or obscured by some additive noise, which is itself estimated independently [41]. In contrast to other common optimal filtering methods (e.g. Wiener, Kalman) ANC requires no *a priori* knowledge model of either the signal or the noise. Instead, ANC makes use of a reference input, which is correlated in some unknown way with the noise in the primary signal. The reference is filtered and subtracted from the primary data so as to recover the underlying signal. For our purposes, the primary timeseries data is the gravitational-wave strain channel, $x(t)$, described by Equation (1), and the reference is a PEM recording a mains voltage, $r(t)$, described by Equation (2). The objective of ANC is to remove the clutter $c(t)$ in Equation (1) with the aid of $r(t)$ whilst leaving $h(t)$ i.e. the signal of interest, intact.

A. Filter

The first step in implementing ANC is to convert $r(t)$ into an estimate of the clutter, $\hat{c}(t)$. The clutter estimate is subtracted from the primary signal in the time domain, defining a residual

$$e(t) = x(t) - \hat{c}(t) . \quad (8)$$

In the limit $\hat{c}(t) \rightarrow c(t)$, one obtains $e(t) \rightarrow h(t) + n(t)$ and recovers a noise cancelled timeseries. The clutter estimate is modelled by a finite duration impulse response (FIR) filter,

$$\hat{c}_k = \mathbf{w}^\top \mathbf{u}_k . \quad (9)$$

In Equation (9), $\hat{c}_k = \hat{c}(t_{n_k})$ is the clutter estimate at the n_k -th time step, \mathbf{u}_k is the tap-input vector composed of M running samples of the reference signal arranged backwards in time,

$$\mathbf{u}_k = [r_k, r_{k-1}, \dots, r_{k-M+1}] , \quad (10)$$

and \mathbf{w} is the tap-weight vector,

$$\mathbf{w} = [w_1, w_2, \dots, w_M] . \quad (11)$$

The goal of ANC is to determine the optimal tap weights \mathbf{w}_{opt} , that minimise the mean square error cost function,

$$\mathbf{w}_{\text{opt}} = \arg \min \sum_{k=1}^K \lambda^{K-k-1} |e_k|^2 , \quad (12)$$

where $0 \leq \lambda \leq 1$ is a “forgetting factor” which gives exponentially less weight to older samples and can be freely chosen. The choice of λ in this paper is discussed in Section III B. The minimization problem described by Equation (12) can be solved in many ways. Here we implement an ARLS algorithm. The algorithm is outlined in Section III B.

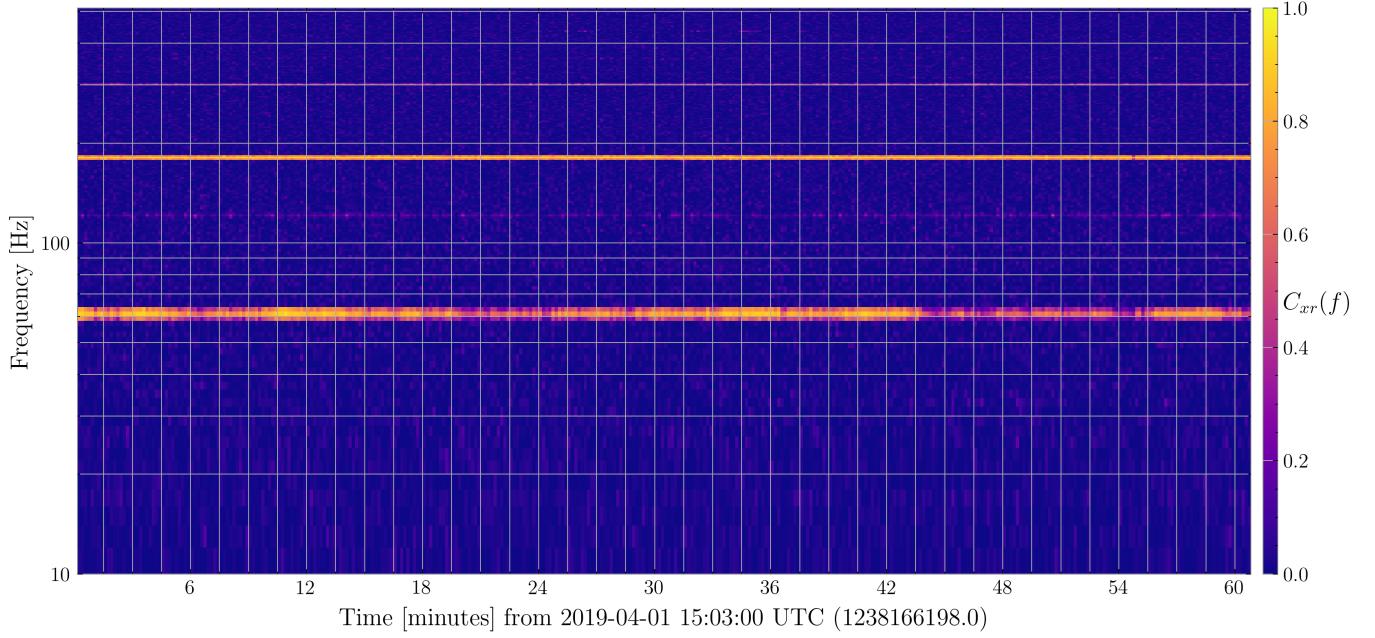


FIG. 4. Coherence spectrogram between the strain channel L1:DCS-CALIB_STRAIN_C01_AR and the PEM channel L1:PEM-CS_MAINSMON_EBAY_1.DQ covering a 1 hour period of O3 data. The ~~x-axis~~^{horizontal axis} is the time in minutes from the start of O3a, ~~separated~~^{divided} into ~~10s~~^{10 - s} bins. The ~~y-axis~~^{vertical axis} is the frequency in Hz. The pixel colour denotes the coherence $C_{xy}C_{xr}$, defined Equation (5), in that time-frequency bin; see colour bar at right (dimensionless). A strong High coherence is observed at 60 Hz due to the ~~main~~^{mains} power interference. Additional coherence features ~~can also be observed~~ are visible at 0.2 kHz and 0.3 kHz due to instrumental lines of a different provenance, c.f. Figure 1.

B. ARLS algorithm

As its name implies, the ARLS algorithm is adaptive, in the sense that the weights \mathbf{w} are continually updated based on new data. It is also recursive, in the sense that \mathbf{w} are ~~is~~^{updated} iteratively as new data ~~arrives~~^{arrive}, rather than reprocessing all previous data. Abbreviated pseudocode is presented below for the sake of reproducibility and the reader's convenience. The reader is referred to standard signal processing textbooks, e.g. [42] for more information.

ARLS proceeds as follows:

1. Initialise the tap weights $\mathbf{w} = \mathbf{0}$. Initialise a covariance matrix $\mathbf{P} = \langle \mathbf{w}\mathbf{w}^T \rangle = \delta^{-1}\mathbf{I}$ of rank M for regularisation parameter $0 < \delta \ll 1$ and an $M \times M$ identity matrix \mathbf{I} where the superscript T symbolises transposition.
2. For $1 \leq k \leq K$:
 - (a) Estimate the clutter \hat{c}_k from Equation (9)
 - (b) Calculate the residual e_k from Equation (8)
 - (c) Calculate the gain vector \mathbf{g}_k

$$\mathbf{g}_k = \frac{\mathbf{P}\mathbf{u}_k}{\lambda + \mathbf{u}_k^T \mathbf{P} \mathbf{u}_k} \quad (13)$$

(d) Update the tap weights

$$\mathbf{w}_k = \mathbf{w}_{k-1} + e_k \mathbf{g}_k \quad (14)$$

(e) Update the covariance matrix

$$\mathbf{P}_k = \mathbf{P}_{k-1} \lambda^{-1} - \mathbf{g}_k \mathbf{u}_k^T \lambda^{-1} \mathbf{P}_{k-1} \quad (15)$$

The pseudocode is ~~also illustrated~~^{depicted} in Figure 5 via ~~as~~^a block diagram. At time t the filter receives the reference signal $r(t)$ and produces an estimate of the clutter $\hat{c}(t)$, given the current tap weights \mathbf{w} . By comparing $\hat{c}(t)$ with the strain data $x(t)$ a residual ~~is~~^{is calculated} $e(t)$ is calculated. The residual is then used in conjunction with $r(t)$ to update the weights and the algorithm continues iteratively.

ARLS has three free parameters: the order parameter M , the forgetting factor λ , and the regularisation parameter δ . Larger values of M ~~corresponds to an increased model~~^{increases the model's} complexity. Whilst this generally increases the accuracy of the resulting estimates, it also increases the computational overhead and hence the latency between receiving the input data and updating the weights. In Sec. IV we trial a selection of M values for synthetic GW data. Regarding the forgetting factor, $\lambda = 1$ corresponds to infinite memory, ~~causing~~^{whereupon} the filter becomes a ““growing window” RLS [43, 44].

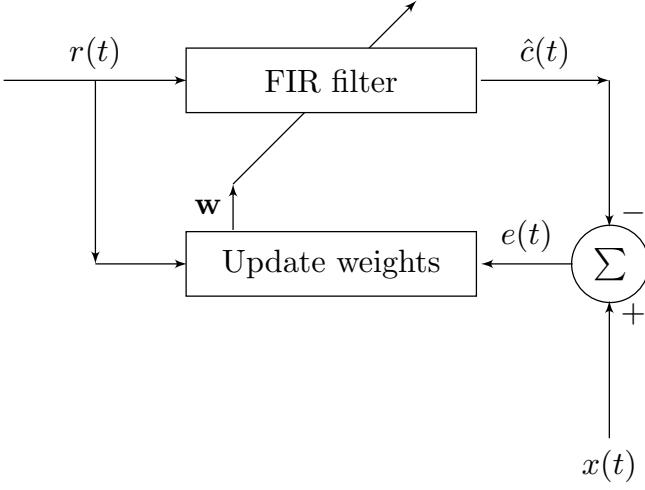


FIG. 5. Block diagram of the ARLS method described in Section III B. The reference signal $r(t)$ is passed to an FIR filter to construct an estimate $\hat{c}(t)$ of the clutter. Subtracting $\hat{c}(t)$ from the primary signal $x(t)$ provides a residual $e(t)$ which controls the process to update to update the weights w of the FIR filter. The method proceeds iteratively. The diagonal line across the FIR filter block denotes that the parameters of the block (i.e. w) are being updated, but are not part of the signal processed by the block.

In this work we use $\lambda = 0.9999$ as a default. Different values of λ are investigated in Section V B. The regularisation parameter δ must be “suitably chosen” chosen judiciously to balance competing priorities [45, 46]. A large δ leads to rapid convergence of the ARLS algorithm, but with correspondingly larger variance in the parameter estimates before convergence. Conversely, a smaller δ leads to slower convergence and smaller variance in the parameter estimates. General guidelines for choosing δ are not readily available. Instead δ is typically empirically selected for the particular problem. In this work we found $\delta = 100$ to be a reasonable choice, since we have a large given the uncertainty in our initial estimate of the weights. We refer the reader to Chapter 9 of Ref. [42] for a full review of adaptive least squares estimation in the context of linear filtering.

IV. VALIDATION TESTS WITH AN HMM

We test the ANC scheme in Section III by combining it with a HMM to search for a CW signal with randomly wandering frequency, which overlaps the mains power line. Many other validation tests are possible, of course, and no individual test represents the final word on the efficacy of the scheme. Nonetheless, HMM-based searches are a key element of the data analysis program of the LVK Collaboration, generating a substantial corpus of published results [47–52]. In the context of this

paper, validations tests involving a HMM are especially powerful, because they present the ANC scheme with an additional challenge: to subtract the mains power line well enough to reveal detect an underlying CW signal, which is itself “noisy” (in the sense that its frequency wanders) rather than monochromatic. That is, the ANC-HMM combination is tasked to not only remove the stochastic 60-Hz interference but also distinguish it from the process noise in the CW signal’s frequency. If the ANC succeeds in this complicated task, one may be confident that it also succeeds in the easier task of subtracting the mains power line to reveal detecting an underlying CW signal whose frequency is constant.

In what follows, we employ the HMM scheme introduced by Suvorova *et al.* [35], which has been thoroughly tested through multiple LVK searches [12–14]. We do not cover any details of the HMM scheme in this work and refer the reader to Suvorova *et al.* [35] for further information. A synthetic CW signal $h(t)$, whose frequency executes an unbiased random walk, is injected into synthetic Gaussian noise $n(t)$ typical of the LIGO detectors. A Clutter $c(t)$ proportional to a simulated 60-Hz mains power line $e(t)$ is superposed, whose properties mimic the real system electrical supply (see Section II A). The challenge is to recover the $h(t)$ by passing the data $x(t)$ through the ANC filter and feeding the output into the HMM tracker. In Section IV A we describe how we create a synthetic time series $x(t)$. In Section IV B we present a representative example and compare the HMM’s ability to detect and track $h(t)$ before and after applying ANC.

A. Synthetic data

In the tests that follow, we work with representative synthetic data, generated under controlled conditions, rather than directly with the LIGO data. We can generate synthetic data for the constituents of Equation (1)–, namely $h(t)$, $c(t)$, and $n(t)$, by applying the recipe in Section II B as follows.

Starting with $h(t)$ we assume that the source is isolated (i.e. not in a binary) and that the GW signal is quasimonochromatic (i.e. the frequency varies slowly over many wave cycles if at all). We also place the source at a fixed sky position and neglect for simplicity the rotation and revolution of the Earth. The latter corrections are included readily in searches with real data through the maximum likelihood \mathcal{F} -statistic [38]. Under these assumptions the GW model reduces to

$$h(t) = h_0 \sin[2\pi\phi_{\text{gw}}(t)], \quad (16)$$

where h_0 is the constant amplitude, $\phi_{\text{gw}}(t)$ is a random phase,

$$\phi_{\text{gw}}(t) = \int_0^t ds f_{\text{gw}}(s), \quad (17)$$

and the GW frequency $f_{\text{gw}}(t)$ evolves stochastically and piecewise linearly from one time step to the next according to

$$f_{\text{gw}}(t_{n+1}) = f_{\text{gw}}(t_n) + \epsilon_n \quad (18)$$

where ϵ_n is a zero-mean Gaussian random variable with variance σ_f^2 viz.

$$\epsilon_n = \mathcal{N}(0, \Delta t \sigma_f^2). \quad (19)$$

Hence the synthetic GW signal $h(t)$ is completely described by the parameters h_0 , σ_f , and $f_{\text{gw}}(t=0)$

The clutter and reference signals evolve according to Equations (2) and (4) respectively. For this initial study we take the amplitudes $A_r(t) = a_r$ and $A_c(t) = a_c$ to be constants. Similarly, $P(t) = P$, and $\hat{P}(t) = \hat{P}$ to be constant. Different values for P values are explored in Section V A. Under these assumptions, Equations (2) and (4) reduce to

$$r(t) = a_r \cos \left[2\pi f_{\text{ac}} t + 2\pi \Delta f_{\text{ac}} \cos \left(\frac{2\pi t}{P} \right) + n_{\Theta}(t_n) \right] + n_r(t_n), \quad (20)$$

$$c(t) = a_c \cos \left\{ 2\pi f_{\text{ac}}(t_n - \tau_{\text{delay}}) + 2\pi \Delta f_{\text{ac}} \cos \left[\frac{2\pi(t_n - \tau_{\text{delay}})}{P} \right] + n_{\Theta}(t_n - \tau_{\text{delay}}) \right\} \quad (21)$$

$$c(t) = a_c \cos \left\{ 2\pi f_{\text{ac}}(t_n - \tau_{\text{delay}}) + 2\pi \Delta f_{\text{ac}} \cos \left[\frac{2\pi(t_n - \tau_{\text{delay}})}{P} \right] + n_{\Theta}(t_n - \tau_{\text{delay}}) \right\}$$

The synthetic reference and clutter data are fully described by the parameters $a_r, a_c, f_{\text{ac}}, \Delta f_{\text{ac}}, P, \sigma_{\Theta}^2, a_r, a_c, P, f_{\text{ac}}, \Delta f_{\text{ac}}, \sigma_{\Theta}^2, \sigma_r^2$.

The 11 free parameters are summarised in Table I, along with their chosen injected values. There are three amplitude parameters h, a_r, a_c , with $h \ll a_r, a_c$, h_0, a_r, a_c with $h \ll a_r, a_c$. There are four noise parameters $\sigma_{\Theta}^2, \sigma_r^2, \sigma_{\Theta}^2$ and σ_r^2 , $\sigma_{\Theta}, \sigma_r, \sigma_{\Theta}$ and σ_r . In this paper σ_{Θ}^2 and σ_r^2 , σ_{Θ} and σ_r have fixed, constant values, whilst different values for σ_{Θ}^2 and σ_r^2 , σ_{Θ} and σ_r are investigated in Section IV B and Section V A respectively. There are three additional parameters that specify the modulation of the central frequency: f_{ac} , Δf_{ac} and P . In this paper f_{ac} is fixed at 60 Hz. Different values of Δf_{ac} and P are investigated in Section V A. Finally there is the initial GW frequency, $f_{\text{gw}}(t=0)$.

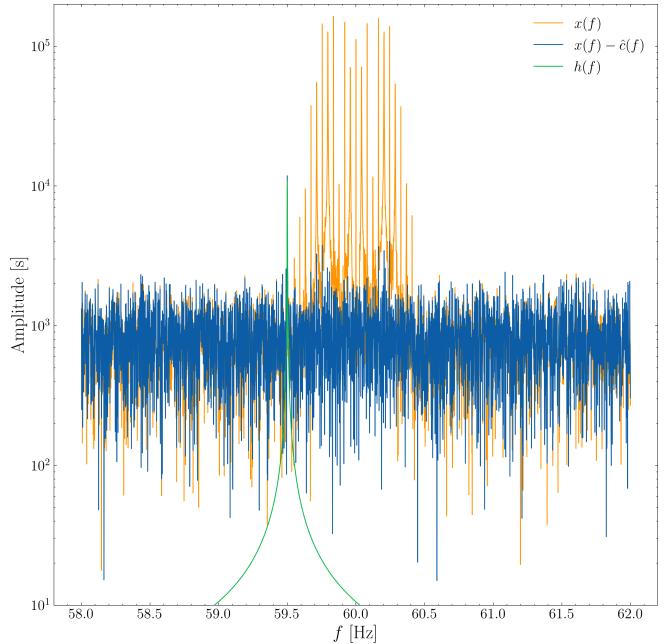


FIG. 6. Spectrum (i.e. modulus of the Fourier transform) of the data $x(t)$ (orange curve), GW signal $h(t)$ (green curve) and decluttered output of the ANC filter $x(t) - \hat{e}(t)$ (blue curve) for a system with parameters described in Table I. ANC suppresses by ≈ 20 dB the orange spikes near 60 Hz, which corresponds to mains power interference, by ≈ 20 dB.

B. Representative worked examples

To orient the reader, we start with two representative examples where the frequency wandering is (i) ‘low’, with $\sigma_f(N\Delta t)^{1/2}/f_{\text{ac}} = 0.047$, and (ii) ‘high’, with $\sigma_f(N\Delta t)^{1/2}/f_{\text{ac}} = 0.15$. In both cases the effective SNR $h_0/\sigma_n^2 = 0.025$ signal-to-noise ratio equals $h_0/\sigma_n = 0.025$. All other parameters are specified in Table I. For clarity of illustration/exposition, we work with a single PEM in this section and extend to multiple PEMs in Section V B.

We start by checking visually that the ANC filter removes most of the excess power from the 60 Hz interference. In Figure 6 we plot the modulus of the Fourier transform of the synthetic data $x(t)$, the underlying GW signal $h(t)$, and the decluttered output of the ANC filter, $e(t) = x(t) - \hat{e}(t)$, for Fourier frequencies from 58 Hz to 62 Hz for case (i). Before filtering, the spectrum of $x(t)$ has multiple modes about $f_{\text{ac}} = 60$ Hz peaks near $f = f_{\text{ac}} = 60$ Hz, visible as the orange spikes in Figure 6. After filtering, the spectrum of $e(t)$ is flatter, as indicated by the blue curve in Figure 6, with the exception of a clear feature at 59.5 Hz coincident with the GW injection at $f_{\text{gw}}(t=0)$ (green curve). In rough terms, the ANC scheme achieves 20 dB of suppression in case (i). Similar results (not plotted)

| | | Section IV | Section V |
|---------------------------------|--|---|---|
| h_0 | GW strain amplitude | 0.025 | 0.022 |
| a_r | Mains voltage amplitude at PEM | $\mathcal{U}(10 - 0.01, 10 + 0.01)$ | $\mathcal{U}(10 - 0.01, 10 + 0.01)$ |
| a_c | Interference voltage amplitude | $\mathcal{U}(1.2 - 0.001, 1.2 + 0.001)$ | $\mathcal{U}(1.2 - 0.001, 1.2 + 0.001)$ |
| $\sigma_n^2 \sigma_r$ | Strain channel measurement noise | 1 | 1 |
| $\sigma_r^2 \sigma_L$ | PEM voltage measurement noise | $10^{-4} 10^{-2}$ | $10^{-4} 10^{-2}$ |
| $\sigma_\theta^2 \sigma_\Theta$ | PEM voltage phase noise | $10^{-2} 10^{-1}$ | $\{10^{-3}, 10^{-2}, 10^{-1}\} \{10^{-3/2}, 10^{-1}, 10^{-1/2}\}$ |
| $\sigma_f^2 \sigma_{fL}$ | GW frequency noise | $\{0.01, 0.1\} \{0.1, \sqrt{0.1}\} s^{1/2}$ | $0.001 60 0.04 s^{1/2}$ |
| $f_{gw}(t=0)$ | Initial GW frequency | 59.5 Hz | 59.9 Hz |
| f_{ac} | Central interference frequency | 60 Hz | 60 Hz |
| Δf_{ac} | Amplitude of interference frequency modulation | 1 Hz | {0.5, 1.0, 1.5} Hz |
| P | Period of interference frequency modulation | 10^2 s | $\{10^1, 10^2, 10^3\}$ s |

TABLE I. Parameters, their physical meaning, and the injected values used to generate synthetic data. The injected values are listed for validating the ANC scheme in Section IV, and quantifying the performance of the scheme as a function of the properties of the mains power interference in Section V. a_r and a_c are randomly drawn from separate uniform distributions, denoted by \mathcal{U} . The braces notation (e.g. $\{0.01, 0.1\}$) indicates that multiple injected values have been trialled. The quoted injected values have been of the three amplitude parameters and the four noise parameters and the are scaled such that $\sigma_n^2 = 1$.

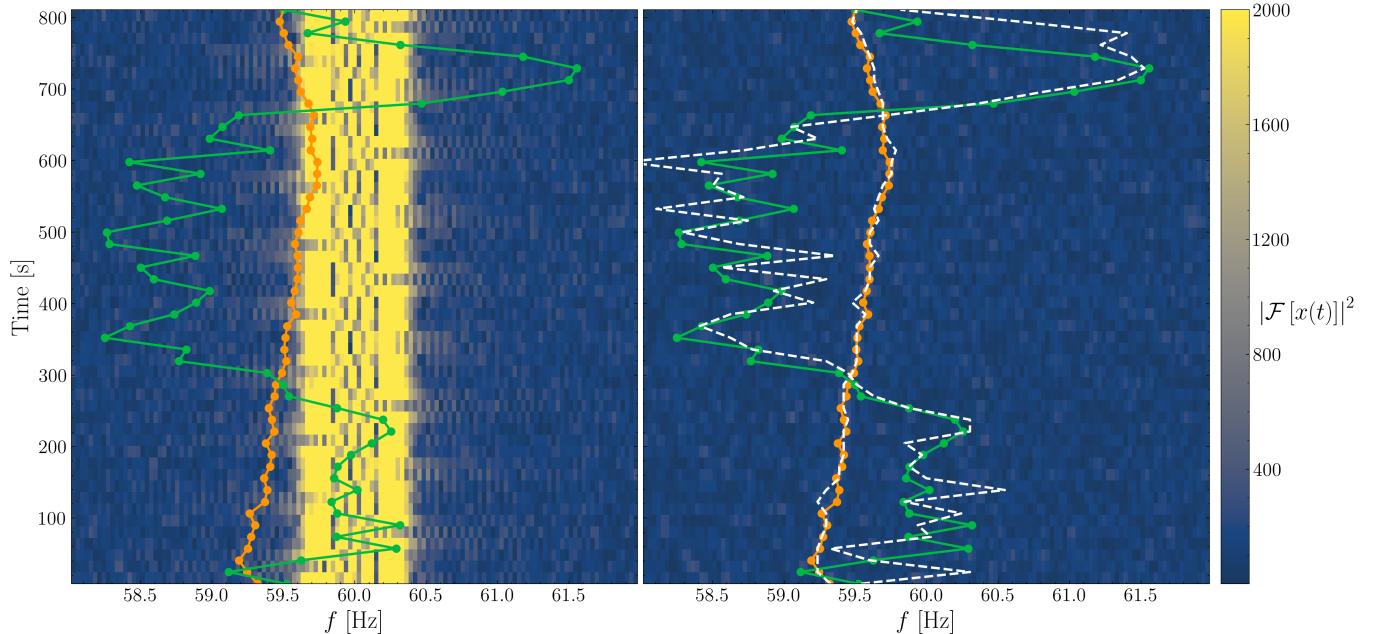


FIG. 7. HMM tracking of a CW signal with spin wandering in the presence of 60 Hz mains power interference before (left panel) and after (right panel) ANC. Colored contours (arbitrary units; see color bar at right) show the spectrogram of the data, i.e. the squared modulus of the Fourier transform of $x(t)$. The 60 Hz line is visible as a vertical yellow band in the left panel; it cannot be discerned in the right panel. Injected signals: lower spin wandering, with $\sigma_f(N\Delta t)^{1/2}/f_{ac} = 0.047$ (solid orange curve), and $h_0/\sigma_n = 0.025$, higher spin wandering, with $\sigma_f(N\Delta t)^{1/2}/f_{ac} = 0.15$ (solid green curve) and $h_0/\sigma_n = 0.025$. In both cases $h_0/\sigma_n^2 = 0.025$. Recovered signals: dashed white curves. Neither signal can be detected before ANC, so the dashed white curves appear in the right panel only.

are obtained for case (ii).

Having established the ability of the ANC scheme to filter out $c(t)$ by calibrating against $r(t)$, we pass the ANC output to the HMM tracker and evaluate its the tracker's performance. The results are shown in Figure 7

for both low and high frequency wandering, for a single realisation of the noise. The figure shows the spectrogram of $x(t)$ as a heat map before (left panel) and after (right panel) ANC filtering. It is clear that ANC suppresses the mains power interference, which is visible as a vertical yellow band in the left panel and is almost absent

from the right panel. The spin wandering of the injected GW source (green solid curve for higher σ_f , orange solid curve for lower σ_f) and the HMM estimate (dashed white curves for both higher and lower σ_f) are superposed onto the spectrogram. The approximate overlap between the solid coloured and dashed white curves confirms that the ANC scheme and HMM tracker detect both injected signals successfully. For lower σ_f , the GW spin frequency wanders close to, but below, the 60 Hz interference line. For higher σ_f , the GW signal crosses the interference line, presenting a more difficult challenge, which is surmounted successfully nevertheless. The time-averaged root-mean-square error in the frequency estimate is 0.038 Hz for low σ_f and 0.47 Hz for high σ_f . In contrast, neither the lower- σ_f nor the higher- σ_f injections are detected by the HMM tracker before ANC in the left panel (which is why there are no dashed white curves in the left panel).

V. ROC CURVES

In this section, we quantify the performance of the HMM tracker, when it analyzes filtered data supplied by the ANC scheme. We do so systematically by computing receiver operating characteristic (ROC) curves as a function of key parameters of the mains power interference (Section V A) and ANC filter (Section V B).

A. Mains power interference parameters

In this section we test how the ANC scheme and HMM tracker perform together as a function of the mains power parameters Δf_{ac} , σ_Θ^2 and P . To this end, we calculate the probability of detecting the CW signal, p_d , as a function of the false alarm probability, p_{fa} , i.e. the ROC curve $p_d(p_{fa})$. We consider three numerical experiments. In the first experiment we vary $\Delta f_{ac} = \{0.5, 1.0, 1.5\}$ Hz whilst holding constant $\sigma_\Theta^2 = 10^{-2}$ and $P = 100$ s. In the second experiment we vary $\sigma_\Theta^2 = \{10^{-3}, 10^{-2}, 10^{-1}\}$ whilst holding constant $\Delta f_{ac} = 1.0$ Hz and $P = 100$ s. In the third experiment we vary $P = \{10, 10^2, 10^3\}$ s whilst holding constant $\Delta f_{ac} = 1.0$ Hz and $\sigma_\Theta^2 = 10^{-2}$. For all experiments $h_0/\sigma_n^2 = 0.022$, $f_{gw}(t=0)$ is fixed to 59.9 we fix $h_0/\sigma_n = 0.022$, $f_{gw}(t=0) = 59.9$ Hz, and $N\Delta t = 800$ s. Note that despite expecting $\Delta f_{ac} < 0.5$ in practice, as discussed in Section II B, in this Section we trial larger values of Δf_{ac} in order to provide set a more difficult test for the ANC scheme and HMM tracker. All other injected parameters are specified in Table I.

Figure 8 displays the ROC curves resulting from the three numerical experiments above — In in the top, middle and bottom panels respectively. For example in the top panel the results of the first experiment are plotted, where we set $\Delta f_{ac} = \{0.5, 1.0, 1.5\}$ Hz (blue,

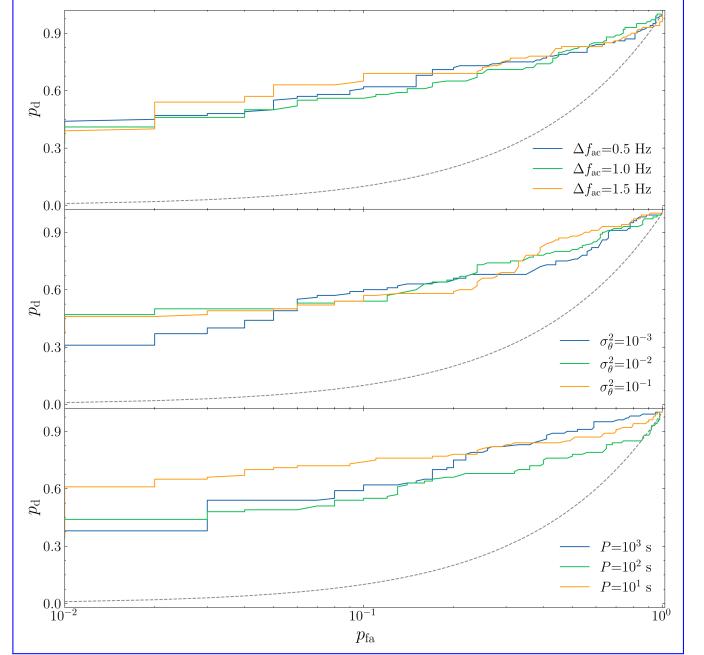


FIG. 8. Detection probability p_d as a function of false alarm probability p_{fa} (i.e. ROC curve) for HMM tracking of a CW signal ($h_0/\sigma_n^2 = 0.022$, $h_0/\sigma_n = 0.022$) in conjunction with the ANC scheme, for different mains power interference parameters. The top panel shows different values of Δf_{ac} , the middle panel and bottom panels display different values of σ_Θ^2 , and the bottom panel different values of P respectively. The grey dashed line curve in all panels denotes corresponds to the performance of a random classifier. For In all panels the curves are broadly overlaid overlapping approximately, illustrating that the performance of the HMM and ANC scheme is resilient insensitive to different the mains power parameters. At $p_{fa} = 0.05$, the mean p_d across the 3 curves is 0.52 (top panel), 0.48 (middle panel) and 0.58 (bottom panel). TK: we could also label as e.g. $\Delta f_{ac}/f_{ac}$ as AM suggests. I personally think it's better to keep the original label Δf_{ac} .

green and orange curves respectively). Also plotted for comparison is the performance of a random classifier (grey dashed line). The performance across different Δf_{ac} , σ_Θ^2 and P values is broadly comparable, with the ROC curves generally overlaid. For all values of Δf_{ac} the ANC scheme and HMM tracker outperform overlapping approximately and outperforming a random classifier (i.e. each of the ROC curves lies above the grey dashed line).

We take $p_d(p_{fa} = 0.05)$ as a scalar metric to quantify the typical performance of the HMM-ANC combination. For in practical applications. In the top panel we obtain $p_d(0.05) = \{0.5, 0.5, 0.57\}$ for $\Delta f_{ac} = \{0.5, 1.0, 1.5\}$ Hz respectively. In the middle panel the results of the second experiment are plotted, where we set $\sigma_\Theta^2 = \{10^{-3}, 10^{-2}, 10^{-1}\}$ (blue, green and orange curves respectively). Again the ANC scheme and HMM tracker

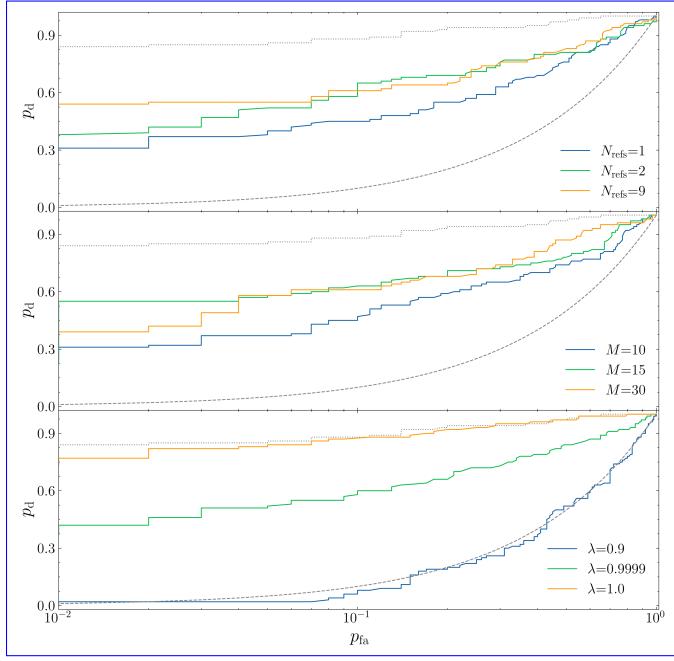


FIG. 9. ROC curves $p_d(p_{fa})$. As Figure 8 but for the HMM/ANC tracking of a CW signal for different ANC filter parameters. The top panel displays, middle and bottom panels display the ROC curves for different number numbers of PEM reference channels, N_{ref} , the middle panel different values of tap weights M , and the bottom panel different values of forgetting factors λ respectively. All panels additionally include the ROC curve for a random classifier (i.e. the lower performance limit, diagonal grey dashed line) and the ROC curve for the case where there is zero interference ROC curve (i.e. the upper performance limit, grey dotted line).

outperform a random classifier, and the results are broadly consistent across different parameter values, with we obtain $p_d(0.05) = (0.44, 0.50, 0.49)$ for $\sigma_\Theta^2 = \{10^{-3}, 10^{-2}, 10^{-1}\}$ respectively. In the bottom panel the results of the third experiment are plotted where we set $P = \{10, 10^2, 10^3\}$ s (blue, green and orange curves respectively). Once again the scheme outperforms a random classifier and is robust to different parameter values, with we obtain $p_d(0.05) = (0.7, 0.49, 0.54)$ for $P = \{10, 10^2, 10^3\}$ s respectively.

B. Filter ANC filter parameters

We have shown verified in Section V A how that the ANC scheme and HMM tracker are robust to different parameters of insensitive to the mains power interference parameters. In this section we investigate different the performance as a function of the control parameters of the ANC scheme filter itself. Specifically, we investigate three important questions:

1. How does ANC benefit from multiple independent

references?

2. What order ANC filter (M) is required to achieve good interference cancellation?
3. How does ANC performance depend on λ ?

Regarding the first question, the preceding validation tests on synthetic data all assumed that we have

Figure 9 displays the ROC curves for different values of the number of PEM references (N_{ref} , top panel), M (middle panel) and λ (bottom panel). All mains power interference parameters are as specified in Table I. Also plotted for comparison in every panel is the ROC curve of a random classifier (grey dashed diagonal line) and the ROC curve for $c(t) = 0$ (grey dotted line). The zero interference case is included as a performance upper bound. The validation tests in Sections IV B and V A assume a single PEM reference voltage measurement. However, In practice, as discussed in Section II C in practice, there are multiple PEM channels measuring power line interference for LIGO (c.f. Figure 3). Specifically, for O3 there are 9 nine PEM channels which directly measure the mains voltage three-phase voltages at each of the LIGO-Livingston and LIGO-Hanford sites. Multiple PEM channels provided additional independent measurements of the reference voltage; it seems reasonable to suspect that these additional channels may aid the references, which should improve the performance of the ANC filter. Indeed, ANC is commonly used with multiple reference signals in other electrical engineering applications such as noise cancelling headphones [53], communication intelligibility [54, 55] and cardiac monitoring [56]. For the second question, the order of the filter, i.e. the number of taps, is a parameter that can be freely chosen in ARLS. Generally an increased number of taps is expected to improve the performance of the filter due to the increased model complexity. However, this comes at an increased computational cost and also an increased latency. For real time applications tracking the wandering of the GW frequency it is important to minimize both of these variables. In this work we explicitly specify the number of taps; we note that adaptive tap length methods are also available [e.g. 57–59]. Regarding the third question, the use of the forgetting factor λ allows ARLS to handle non-stationary systems, neglecting older data in favour of more recent data. This is useful for cases where the model parameters are time-varying, enabling more rapid convergence and effective tracking. In the case when $\lambda = 1$, ARLS has infinite memory and so exhibits good stability and low misadjustment. However the tracking capability can be reduced for non-stationary systems. Conversely, $\lambda \ll 1$ improves the tracking at the expense of reduced stability and increased misadjustment [60]. In this paper we run ARLS using a specific value for λ , but note that adaptive λ algorithms are also available [61–63].

Figure 9 displays the ROC curves for different values of the number of PEM references (N_{ref} , top panel), M

(middle panel) and λ (bottom panel). All mains power interference parameters are as specified in Table I. Also plotted for comparison in every panel is the ROC curve of a random classifier (grey dashed diagonal line) and the ROC curve when $c(t) = 0$ (grey dotted line). The zero interference case is included to provide a reference on the performance upper limit.

The top panel—Indeed an improvement of this kind is visible in the top pane of Figure 9 which plots the ROC curves for $N_{\text{ref}} = \{1, 2, 9\}$ (blue, green and orange curves respectively). The detection probability $p_{\text{pd}}(0.05) = \{0.38, 0.52, 0.55\}$ for each of the respective N_{ref} . For comparison, in the zero interference case $p_{\text{pd}}(0.05) = 0.85$. It is evident that $N_{\text{ref}} > 1$ are generally an improvement over $N_{\text{ref}} = 1$. However, the improvement of $N_{\text{ref}} = 9$ over $N_{\text{ref}} = 2$ is more modest. This suggests that the inclusion of additional reference PEM channels quickly reaches a point of diminishing returns where the interference is well-captured by a few reference channels, and adding more references beyond this point does not improve the effectiveness.

With respect to the second question concerning the order of the filter, more taps should improve the performance of the filter. The middle panel plots due to the increased model complexity. Indeed, this trend is observed in the middle panel of Figure 9 which plots ROC curves for $M = \{10, 15, 30\}$ (blue, green, and orange curves respectively). The detection probability $p_{\text{pd}}(0.05) = \{0.37, 0.57, 0.58\}$ for each respective M . Analogous to the top panel, increasing M generally increases the detection probability, but the performance quickly saturates such that saturates; the ROC curves for $M = 15$ and $M = 30$ are highly comparable. In the bottom panel we plot comparable. More taps increases the computational costs and latency (i.e. the delay in the signal processing pipeline due to the filter). In this work we explicitly specify the number of taps, while noting that adaptive tap length methods are also available [e.g. 57–59].

With respect to the third question concerning the forgetting factor λ , we find that p_{d} is a strong function of λ at fixed p_{fa} . In the bottom panel of Figure 9, we plot ROC curves for $\lambda = \{1, 2, 9\}$ (blue, green, orange curves respectively), which have respective $p_{\text{pd}}(0.05) = \{0.38, 0.52, 0.55\}$ respectively. A clear hierarchy is evident whereby larger values of λ increase the detection probability. For the synthetic data in this paper $\lambda = 1$ is advantageous for detection since the interference is quasi-stationary (c.f. Equation 21). Consequently, discounting older data with $\lambda < 1$ inhibits the ability of the ANC scheme to filter out the interference. We defer investigation of non-stationary interference (c.f. Equation 4) to a future work. For application to When analysing real astrophysical data λ will need to should be selected optimally e.g. [64]. Algorithms exist to achieve this, as in Ref. [64]. In general one reduces $\lambda = 1$,

neglecting older data in favour of more recent data, to handle non-stationary systems. This is useful for cases where the model parameters are time-varying, enabling more rapid convergence and effective tracking. For $\lambda = 1$, ARLS has infinite memory and so exhibits good stability (i.e. insensitivity to numerical perturbations) and low misadjustment (i.e. w is not erroneously updated away from its optimal value w_{opt}). Conversely, $\lambda < 1$ improves the tracking at the expense of reduced stability and increased misadjustment [60]. In this paper we run ARLS using a specific value for λ , but note that adaptive λ algorithms are also available [61–63].

VI. CONCLUSIONS

In this paper we demonstrate a new line subtraction method based on adaptive noise cancellation ANC for use in continuous gravitational wave searches. We use an adaptive recursive least squares ARLS method in conjunction with an independent, known PEM reference signal to suppress the interference from a long-lived narrow spectral feature. We then search for the continuous wave signal using a HMM Viterbi algorithm, whose frequency wanders stochastically in general, using a standard HMM search pipeline. We test our method the scheme on synthetic data containing modelling the 60 Hz spectral interference line due to interference from the North American power grid. We show how the the ANC and Viterbi algorithm together are able to successfully track the spin-wandering continuous GW signal near the 60 Hz line. The method is shown to be robust mains power supply.

We find that ANC suppresses the 60-Hz interference by approximately 20 dB, allowing the HMM to track continuous-wave signals that overlap with the 60-Hz spectral feature. Quantitatively, we find the method to be insensitive to different mains power interference parameters, whilst larger values of with $p_{\text{d}}(0.05) \sim 0.5$ across a range of values. The method is dependent on the filter parameters N_{ref} , M and λ generally increase the effectiveness of. The detection probability $p_{\text{d}}(0.05)$ generally increases at larger values of these parameters, albeit at a decreasing rate. Specifically, $p_{\text{d}}(0.05) = \{0.55, 0.58, 0.55\}$ for $N_{\text{ref}} = 9$, $M = 30$ and $\lambda = 1$ respectively.

Ref. [11] catalogues long-lived, narrowband interference in the LIGO interferometers from various sources, including electrical subsystems, mechanical subsystems and calibration processes. Some of these instrumental lines are “cleaner” spectrally than mains power interference, while others are not. In the future, it will be interesting to test whether the success achieved by the ANC scheme in this paper extends to other instrumental lines. Such tests involve investment in human and computational resources but, if successful,

~~will increase the proportion of the ANC filter and the probability of detecting the GW signal LIGO band~~

~~accessible to continuous wave searches at frequencies of astrophysical interest, especially at $\lesssim 0.1 \text{ kHz}$.~~

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