

State-space PTA

Kimpson¹, O’Leary, Melatos, Evans, others, etc. ^{★†}

¹Royal Astronomical Society, Burlington House, Piccadilly, London W1J 0BQ, UK

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ABSTRACT

This is an abstract

Key words: editorials, notices – miscellaneous

1 INTRODUCTION

- Introduce PTAs generally.
- Types of astrophysical source to be detected with PTAs.
- Why we want to do parameter estimation
- Advantages of doing this with state-space approach

2 MODEL

The intrinsic pulsar frequency f evolves as (See Vargas and Melatos)

$$\frac{df}{dt} = -\gamma[f - f_{\text{EM}}(t)] + \dot{f}_{\text{EM}} + \xi(t) \quad (1)$$

where f_{EM} is the solution of the electromagnetic spindown equation, γ a proportionality constant, and $\xi(t)$ a white noise process that satisfies,

$$\langle \xi(t)\xi(t') \rangle = \sigma^2 \delta(t - t') \quad (2)$$

Over the timescales that we are interested in, we can express the EM spindown simply as

$$f_{\text{EM}}(t) = f_{\text{EM}}(0) + \dot{f}_{\text{EM}}(0)t \quad (3)$$

Completely, the frequency evolution is then

$$\frac{df}{dt} = -\gamma[f - f_{\text{EM}}(0) - \dot{f}_{\text{EM}}(0)t] + \dot{f}_{\text{EM}}(0) + \xi(t) \quad (4)$$

This intrinsic frequency can be related to a measured frequency as

$$f_M = fg(\bar{\theta}, t) + N_M \quad (5)$$

where $g(\theta, t)$ ("measurement function") is a function of some parameters, $\bar{\theta}$ and time t , whilst N_M is a Gaussian measurement noise that satisfies

$$\langle N_M(t)N_M(t') \rangle = \Sigma^2 \delta(t - t') \quad (6)$$

Explicitly, it can be shown that the measurement function is,

$$g(\bar{\theta}, t) = 1 - \frac{1}{2} \frac{h_{ij}(t)q(t)^i q(t)^j}{1 + \bar{n} \cdot \bar{q}(t)} [1 - e^{i\Omega(1 + \bar{n} \cdot \bar{q}(t))d}] \quad (7)$$

There are a few terms to unpack and define in this equation. $q(t)$ is the vector that connects the Earth and the pulsar, n the vector that connects the Earth and the GW source, Ω the (constant) angular frequency of the GW and d the Earth-pulsar distance. $h_{ij}(t)$ is the metric perturbation due to the GW = $g_{ij} - \eta_{ij}$. It is given by the familiar plane-wave relation:

$$h_{ij}(t) = H_{ij} e^{i(\bar{k} \cdot \bar{x} - \Omega t + \Phi_0)} \quad (8)$$

where \bar{k} is the GW 3-wavevector = $\Omega \bar{n}$ and $x = -\bar{q}t$ (photon propagates from pulsar to Earth), and for phase offset (GW phase at Earth when $t = 0$) Φ_0 . We can also write this as,

$$h_{ij}(t) = H_{ij} e^{(-i\Omega t(1 + \bar{n} \cdot \bar{q}) + \Phi_0)} \quad (9)$$

The amplitude tensor H_{ij} is

$$H_{ij} = h_+ e_{ij}^+(\bar{n}, \psi) + h_\times e_{ij}^\times(\bar{n}, \psi) \quad (10)$$

where h_+, \times are the constant amplitudes of the gravitational plane wave, and $e_{ij}^{+, \times}(\bar{n}, \psi)$ are the polarisation tensors which are uniquely defined by the principal axes of the GW. ψ is the polarisation angle of the GW.

Going forward, for now we will take $q(t) = q$ i.e. the pulsar locations are constant with respect to the Earth. This may have already been "done" during the barycentring when pulsar TOAs are generated, in which case q is the vector from the SSB to the pulsar

Lets review and categorise all the parameters:

$$\bar{\theta} = \bar{\theta}_{\text{PSR}} \cup \bar{\theta}_{\text{GW}} \cup \bar{\theta}_{\text{noise}} \quad (11)$$

$$\bar{\theta}_{\text{PSR}} = [\gamma, f_{\text{EM}}(0), \dot{f}_{\text{EM}}(0), d] \quad (12)$$

$$\bar{\theta}_{\text{GW}} = [h_+, h_\times, \delta, \alpha, \psi, \Omega, \Phi_0] \quad (13)$$

$$\bar{\theta}_{\text{noise}} = [\sigma, \Sigma] \quad (14)$$

We can also express the measurement equation generally as,

$$g(\bar{\theta}, t) = 1 - A \cos(-\Omega t(1 + n \cdot q) + \Phi_0) \quad (15)$$

★ Contact e-mail: mn@ras.ac.uk

† Present address: Science magazine, AAAS Science International, 82-88 Hills Road, Cambridge CB2 1LQ, UK

2 *Kimpson*

where

$$A = \frac{1}{2} \frac{H_{ij} q^i q^j}{1 + \vec{n} \cdot \vec{q}} [1 - e^{i\Omega(1 + \vec{n} \cdot \vec{q})d}] \quad (16)$$

[Vargas & Melatos \(2023\)](#)

3 PTA PULSARS

3.1 References

REFERENCES

Vargas A., Melatos A., 2023, TBD, 1, 1

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