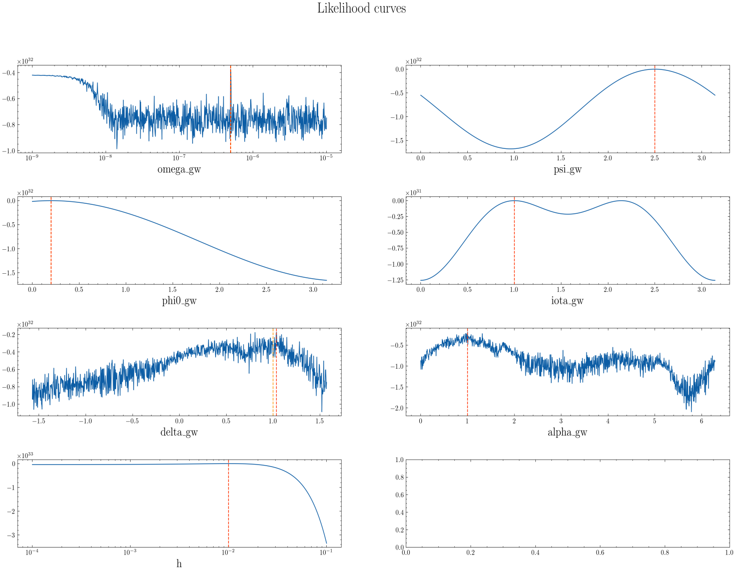
EE/GW meeting, May 19, 2023

Last time: Problems at small h, nasty likelihood curves

All likelihood curves for $\bar{\theta}_{\mathrm{GW}}$:



Drop PSR terms from model?

$$f_{\mathrm{measured}} = f_{\mathrm{emitted}}\, g(\tau; \bar{\theta}) \tag{1}$$

with

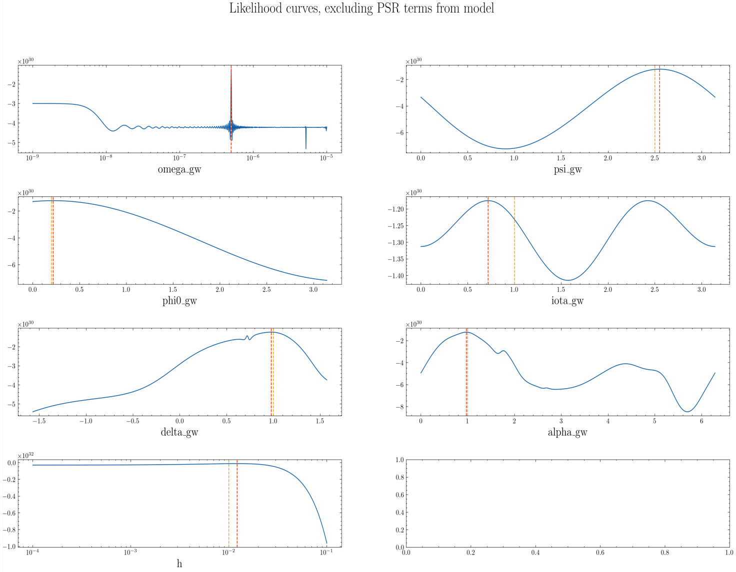
$$g(\tau; \bar{\theta}) = 1 - \frac{1}{2} \frac{H_{ij} q^i q^j}{(1 + \bar{n} \cdot \bar{q})} \left[\cos(-\Omega\tau + \Phi_0) - \cos(-\Omega\tau + \Phi_0 + \Omega(1 + \bar{n} \cdot \bar{q}) d)\right] \tag{2}$$

Generate the fake data using the full expression for $g(\tau; \bar{\theta})$, but use $g'(\tau; \bar{\theta})$ in the Kalman measurement model

$$g'(\tau; \bar{\theta}) = 1 - \frac{1}{2} \frac{H_{ij} q^i q^j}{(1 + \bar{n} \cdot \bar{q})} \left[\cos(-\Omega\tau + \Phi_0)\right] \tag{3}$$

Drop PSR terms from model?

All likelihood curves for $\bar{\theta}_{\mathrm{GW}}$:

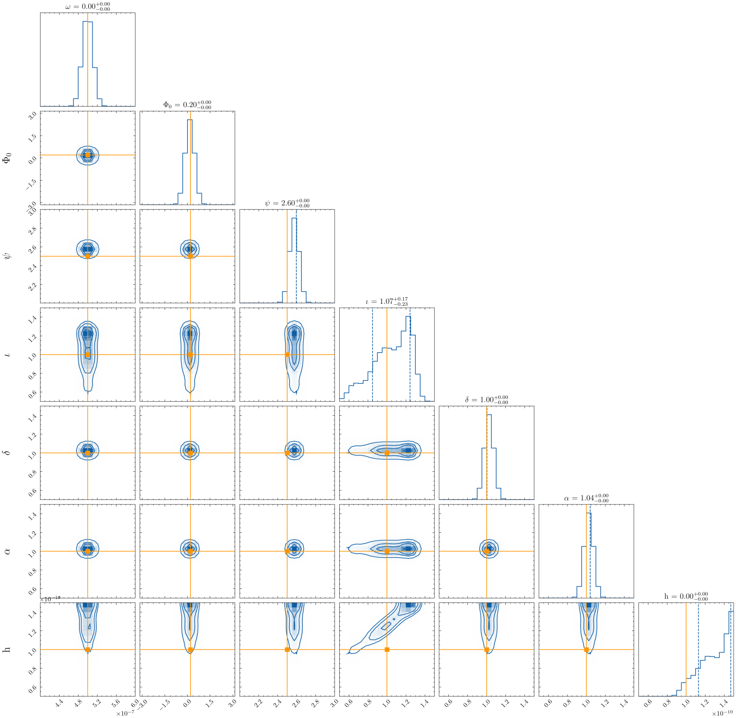


Full NS search for all parameters

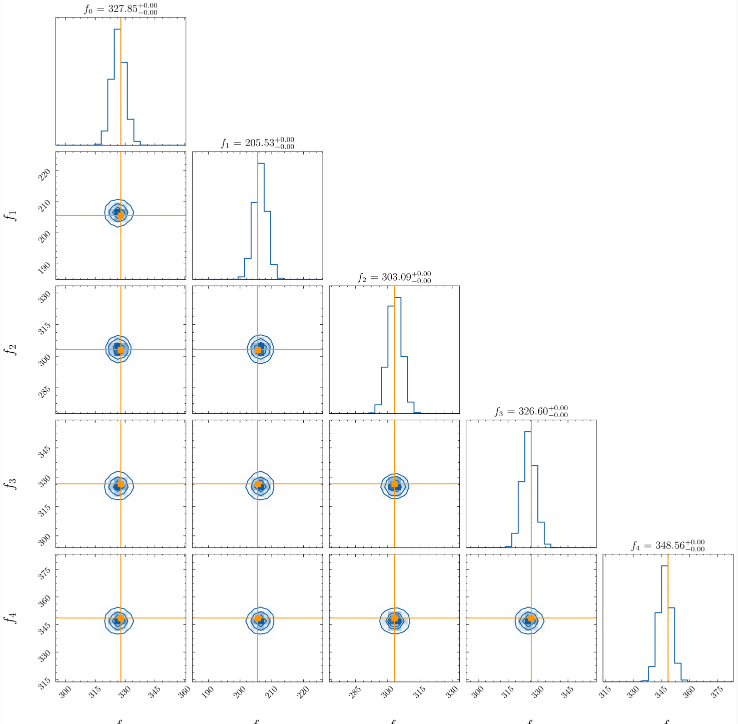
$\bar{\theta} = [\bar{\theta}_{\mathrm{GW}},\, f,\, (t=0)\, \dot{f}(t=0)]$

with $h = 10^{-10}$, $\sigma_m = 10^{-11}$

Note: pulsar distance is no longer part of model. γ doesn't matter



$\omega = 0.00^{+0.00}_{-0.00}$
$\Phi_0 = 0.20^{+0.00}_{-0.00}$
$\psi = 2.60^{+0.00}_{-0.00}$
$\iota = 1.07^{+0.17}_{-0.23}$
$\delta = 1.00^{+0.00}_{-0.00}$
$\alpha = 1.04^{+0.00}_{-0.00}$
h = $0.00^{+0.00}_{-0.00}$
Φ_0
ψ
ι
δ
α
h
$\times 10^{-7}$
$\times 10^{-10}$



$f_0 = 327.85^{+0.00}_{-0.00}$
$f_1 = 205.53^{+0.00}_{-0.00}$
$f_2 = 303.09^{+0.00}_{-0.00}$
$f_3 = 326.60^{+0.00}_{-0.00}$
$f_4 = 348.56^{+0.00}_{-0.00}$
f_1
f_2
f_3
f_4

Next steps:

- Bayes factors vs. strain (currently running)
- Process noise σ_p
- Looser priors on f, $\dot{f}$? Do we need to?
- Can we add PSR terms back in? Do we need to?

Fin.

Backup slides

Sometimes, it is useful to add slides at the end of your presentation to refer to during audience questions.

The best way to do this is to include the `appendixnumberbeamer` package in your preamble and call `\appendix` before your backup slides.

metropolis will automatically turn off slide numbering and progress bars for slides in the appendix.

Where did σ_m value come from?

The measurement noise is

$$\sigma_m = f\frac{\sigma_{\mathrm{TOA}}}{\mathrm{cadence}}$$

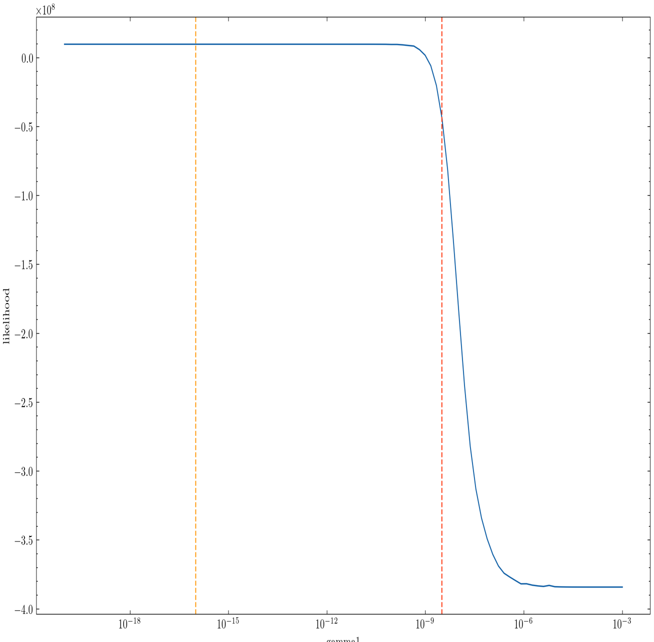
So for a MSP ($f \sim 100 Hz$) observed with a weekly cadence and $\sigma_{\mathrm{TOA}} \sim 1\mu$s:

$$\sigma_m \sim 1.6 \times 10^{-10}$$

The very best pulsars might have $\sigma_{\mathrm{TOA}} \sim 10$ ns:

$$\sigma_m \sim 1.6 \times 10^{-12}$$

γ likelihood



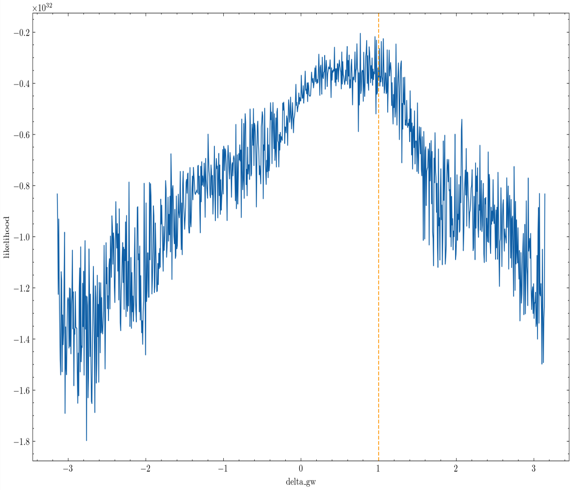
- Why? Very nasty likelihood curves

$$p(\theta|d) \propto p(d|\theta)\pi(\theta)$$

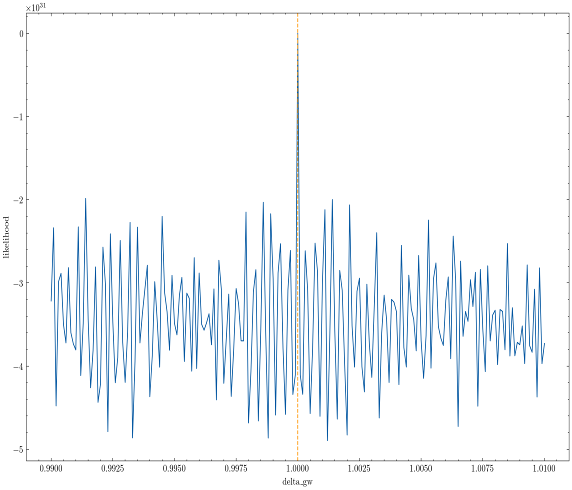
$$p(\theta|d) \propto \mathcal{L}(\theta)\pi(\theta)$$

For uniform prior, posterior shape $\sim$ likelihood shape.

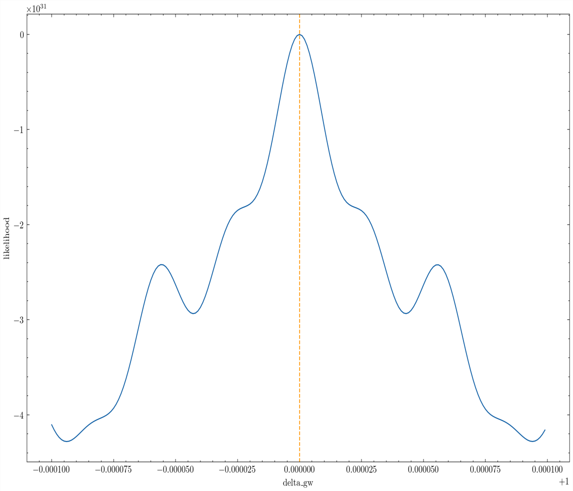
Example: $\mathcal{L}(\delta)$



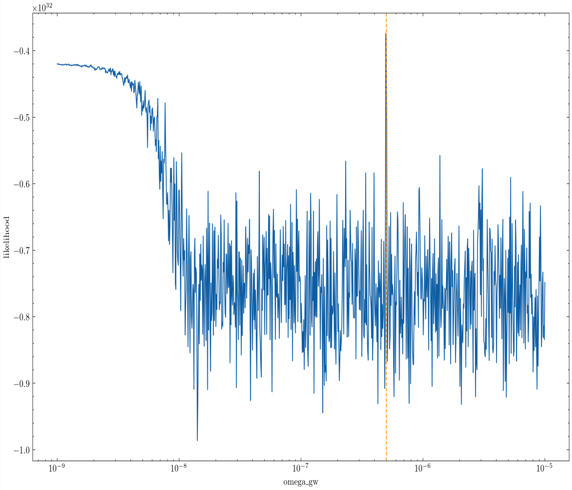
Example: $\mathcal{L}(\delta)$, zoomed 1



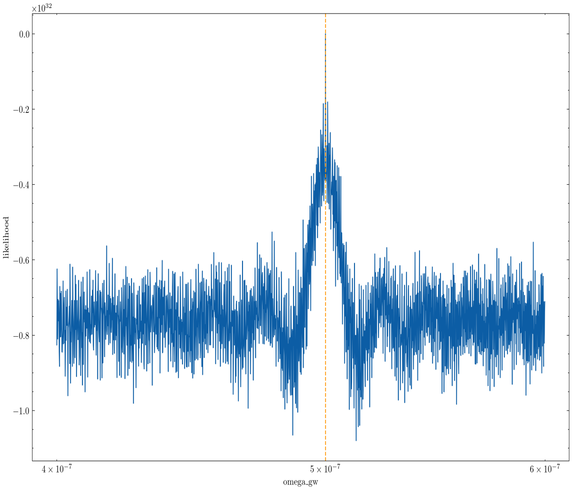
Example: $\mathcal{L}(\delta)$, zoomed 2



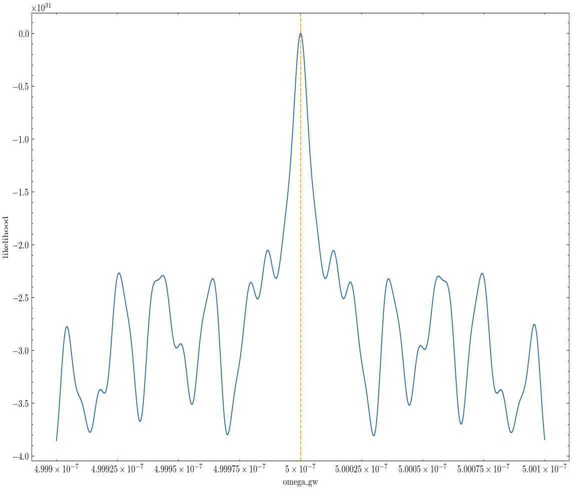
Example: $\mathcal{L}(\omega)$



Example: $\mathcal{L}(\omega)$, zoomed 1



Example: $\mathcal{L}(\omega)$, zoomed 2

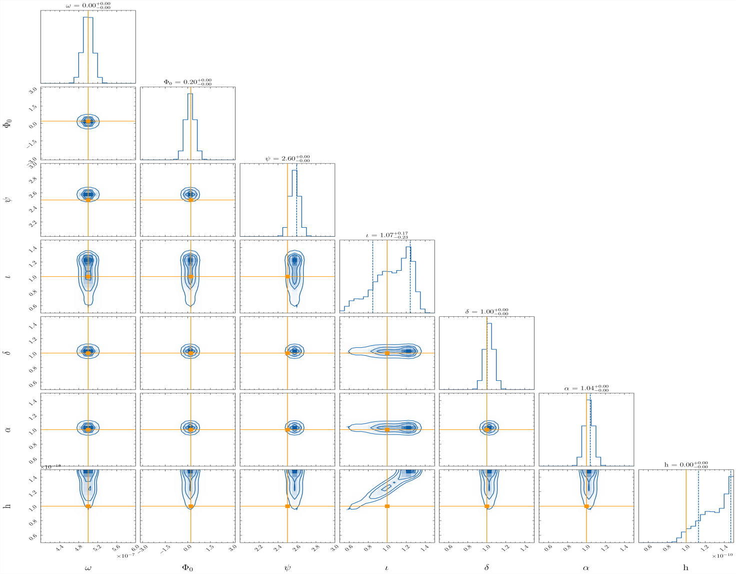


- Problem = overly narrow,biased posteriors
- Cause: noisy, likelihoods which are ”locally Gaussian”. Sampling gets stuck in a local optima and infers posteriors widths based on that optima.

Potential solutions:

1. Better settings for sampler? E.g. more live points (how many?), massively parallel, optimized likelihood evaluations,...
2. Maximum likelihood / loss optimisation?
3. Drop PSR terms from model?
4. MCMC at higher temperatures (trades off evidence..., less efficient)
5. other...?

Option 2: Drop PSR terms from model?



OK, but some biases. c.f. location of the maxima on the previous slide

Lets proceed using this 'Earth terms' model for now.

So far we have been considering inference of only the GW terms $\bar{\theta}_{\rm GW}$. It is important to be able to infer the pulsar terms $\bar{\theta}_{\rm PSR}$.

Lets try and include $f(t=0)$ and $\dot{f}(t=0)$ in our nested sampling.

We have a uniform prior of $\pm 1\%$ about the true value for these two parameters

Earth terms model $\bar{\theta}_{\mathrm{GW}}$ and $f(t=0)$, $\dot{f}(t=0)$