State-space PTA

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Last updated 2020 June 10; in original form 2013 September 5

ABSTRACT
This is an abstract

Key words: editorials, notices – miscellaneous

1 INTRODUCTION

- Introduce PTAs generally.
- Types of astrophysical source to be detected with PTAs.
- Why we want to do parameter estimation
- Advantages of doing this with state-space approach

2 MODEL

We will take as our model of the intrinsic pulsar frequency f a variation of the phenomenological model of Vargas & Melatos (2023). The frequency evolves according to a Ornstein-Uhlenbeck process (equivalently a Langevin equation) with a time-dependent drift parameter:

$$\frac{df}{dt} = -\gamma[f - f_{\rm EM}(t)] + \dot{f}_{\rm EM} + \xi(t) \tag{1}$$

where $f_{\rm EM}$ is the solution of the electromagnetic spindown equation, γ a proporionality constant, and $\xi(t)$ a white noise process that satisfies,

$$\langle \xi(t)\xi(t')\rangle = \sigma^2 \delta(t-t') \tag{2}$$

Over the timescales that we are interested in, we can express the EM spindown simply as

$$f_{\rm EM}(t) = f_{\rm EM}(0) + \dot{f}_{\rm EM}(0)t \tag{3}$$

Completely, the frequency evolution is then

$$\frac{df}{dt} = -\gamma[f - f_{\rm EM}(0) - \dot{f}_{\rm EM}(0)t] + \dot{f}_{\rm EM}(0) + \xi(t) \tag{4}$$

This intrinsic frequency can be related to a measured frequency as

$$f_M = f g(\bar{\theta}, t) + N_M \tag{5}$$

where $g(\theta, t)$ ("measurement function")is a function of some parameters, $\bar{\theta}$ and time t, whilst N_M is a Gaussian measurement noise that satisfies

$$\langle N_M(t) N_M(t')\rangle = \Sigma^2 \delta(t-t') \tag{6}$$

Explicitly, it can be shown that the measurement function is,

$$g(\bar{\theta}, t) = 1 - \frac{1}{2}\frac{h_{ij}(t)q(t)^i q(t)^j}{1+\bar{n}\cdot\bar{q}(t)}[1 - e^{i\Omega(1+\bar{n}\cdot\bar{q}(t))d}] \tag{7}$$

There are a few terms to unpack and define in this equation. $q(t)$ is the vector from the Earth to the pulsar, n the propagation direction of the GW, Ω the (constant) angular frequency of the GW and d the Earth-pulsar distance. $h_{ij}(t)$ is the metric perturbation due to the GW = $g_{ij} - \eta_{ij}$. It is given by the familiar plane-wave relation:

$$h_{ij}(t) = H_{ij} e^{(i(\bar{k}\cdot\bar{x} - \Omega t + \Phi_0))} \tag{8}$$

where $\bar{k}$ is the GW 3-wavevector = $\Omega\bar{n}$ and $x = -\bar{q}t$ (photon propagates from pulsar to Earth), and for phase offset (GW phase at Earth when $t = 0$) Φ_0. We can also write this as,

$$h_{ij}(t) = H_{ij} e^{(-i\Omega t(1+\bar{n}\cdot\bar{q}) + \Phi_0))} \tag{9}$$

The amplitude tensor H_{ij} is

$$H_{ij} = h_+ e^+_{ij}(\bar{n}, \psi) + h_\times e^\times_{ij}(\bar{n}, \psi) \tag{10}$$

where $h_{+,\times}$ are the constant amplitudes of the gravitational plane wave, and $e^{+;\times}_{ij}(\bar{n}, \psi)$ are the polarisation tensors which are uniquely defined by the principal axes of the GW. ψ is the polarisation angle of the GW.

Going forward, for now we will take $q(t) = q$ i..e. the pulsar locations are constant with respect to the Earth. This may have already been "done" during the barycentreing when pulsar TOAs are generated, in which case q is the vector from the SSB to the pulsar

Bringing this together, and adopting a trigonometric form of the equations, we can express the measurement equation as

$$g(\bar{\theta}, t) = 1 - A\cos(-\Omega t(1 + n\cdot q) + \Phi_0) \tag{11}$$

where

$$A = \frac{1}{2}\frac{H_{ij}q^i q^j}{1+\bar{n}\cdot\bar{q}}[1 - \cos(\Omega(1+\bar{n}\cdot\bar{q})d)] \tag{12}$$

2.1 Parameters of the model

Lets review and categorise all the parameters the the above model. We can generally separate these into parameters which correspond

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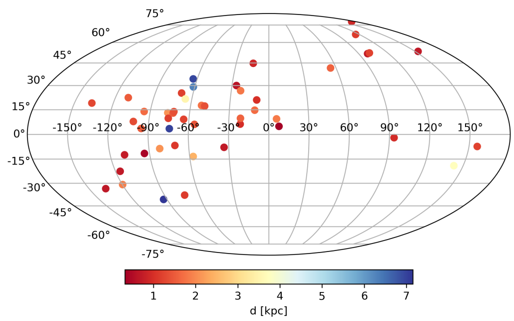

Figure 1. Spatial distribution and distances of NANOGrav pulsars

to the intrinsic frequency evolution of the pulsar, the GW parameters and the noise parameters i.e.

$$\bar{\theta} = \bar{\theta}_{\rm PSR} \cup \bar{\theta}_{\rm GW} \cup \bar{\theta}_{\rm noise} \tag{13}$$

$$\bar{\theta}_{\rm PSR} = [\gamma, f_{\rm EM}(0), \dot{f}_{\rm EM}(0), d] \tag{14}$$

$$\bar{\theta}_{\rm GW} = [h_+, h_\times, \delta, \alpha, \psi, \Omega, \Phi_0] \tag{15}$$

$$\bar{\theta}_{\rm noise} = [\sigma, \Sigma] \tag{16}$$

We can also express the measurement equation generally as,

3 PTA PULSARS

In order to proceed and explore how well this state-space formulation works, we will need to specify a selection of pulsars to make up our PTA. We will take the 47 pulsars that make up the NANOGrav PTA. For each pulsar, we need to specify the complete set of $\bar{\theta}_{\rm PSR}$ as well as σ. The parameters $f_{\rm EM}(0), \dot{f}_{\rm EM}(0), d$ are straightforward to set and can be read directly from the current "present day" best estimates of the pulsar frequency, derivative and distance.

4 METHODS

There are two potential methods we can take

(i) Let the states just be the frequencies $\bar{x} = (f)$. We have a linear equation for both the state evolution and the measurement matrix. Both of these depend on the parameters θ. Use the likelihood output by a standard Kalman filter to run a nested sampler and try to recover θ.

(ii) Let the states be the frequencies and all the unknown parameters $\bar{x} = (f, \bar{\theta}_{\rm GW}, \bar{\theta}_{\rm PSR}, \bar{\theta}_{\rm noise})$. Our state evolution is now linear but our measurement matrix is non-linear - a function of the states(parameters). Use a non-linear estimator such as a EKF/UKF to try to recover all of the states at once.

4.1 Method 1

In the case where the states are just the intrinsic pulsar frequencies, we can write the ODEs in matrix form as

$$d\bar{X} = \bar{A}\bar{X}dt + \bar{N}(t)dt + \bar{\Sigma}d\bar{B}(t) \tag{17}$$

where $\bar{A} = \mathrm{diag}(\gamma_1, \gamma_2, ...)$, $\bar{X} = \mathrm{diag}(f_1, f_2, ...)$, $\bar{N} = \mathrm{diag}(\gamma_1[a_1 + b_1 t] + b_1, ...)$ and we have let $a = f_{EM}(0)$, $b = \dot{f}_{\rm EM}(0)$.

4.2 References

REFERENCES

Vargas A., Melatos A., 2023, TBD, 1, 1

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