# State-space PTA

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Last updated 2020 June 10; in original form 2013 September 5

**ABSTRACT**
This is an abstract

**Key words:** editorials, notices – miscellaneous

## 1 INTRODUCTION

- Introduce PTAs generally.
- Types of astrophysical source to be detected with PTAs.
- Why we want to do parameter estimation
- Advantages of doing this with state-space approach

## 2 MODEL

The intrinsic pulsar frequency $f$ evolves as (See Vargas and Melatos)

$$\frac{df}{dt} = -\gamma[f - f_{\rm EM}(t)] + \dot{f}_{\rm EM} + \xi(t) \tag{1}$$

where $f_{\rm EM}$ is the solution of the electromagnetic spindown equation, $\gamma$ a proporionality constant, and $\xi(t)$ a white noise process that satisfies,

$$\langle \xi(t)\xi(t') \rangle = \sigma^2 \delta(t-t') \tag{2}$$

Over the timescales that we are interested in, we can express the EM spindown simply as

$$f_{\rm EM}(t) = f_{\rm EM}(0) + \dot{f}_{\rm EM}(0)t \tag{3}$$

Completely, the frequency evolution is then

$$\frac{df}{dt} = -\gamma[f - f_{\rm EM}(0) - \dot{f}_{\rm EM}(0)t] + \dot{f}_{\rm EM}(0) + \xi(t) \tag{4}$$

This intrinsic frequency can be related to a measured frequency as

$$f_M = f g(\bar{\theta}, t) + N_M \tag{5}$$

where $g(\theta, t)$ ("measurement function")is a function of some parameters, $\bar{\theta}$ and time $t$, whilst $N_M$ is a Gaussian measurement noise that satisfies

$$\langle N_M(t) N_M(t') \rangle = \Sigma^2 \delta(t-t') \tag{6}$$

Explicitly, it can be shown that the measurement function is,

$$g(\bar{\theta}, t) = 1 - \frac{1}{2}\frac{h_{ij}(t) q(t)^i q(t)^j}{1 + \bar{n}\cdot\bar{q}(t)}[1 - e^{i\Omega(1+\bar{n}\cdot\bar{q}(t))d}] \tag{7}$$

There are a few terms to unpack and define in this equation. $q(t)$ is the vector that connects the Earth and the pulsar, $n$ the vector that connects the Earth and the GW source, $\Omega$ the (constant) angular frequency of the GW and $d$ the Earth-pulsar distance. $h_{ij}(t)$ is the metric perturbation due to the GW = $g_{ij} - \eta_{ij}$. It is given by

$$h_{ij}(t) = H_{ij} e^{(i(\Omega(\bar{n}\cdot\bar{q}-t)+\Phi_0))} \tag{8}$$

for phase offset (GW phase at Earth when $t = 0$) $\Phi_0$. The amplitude tensor $H_{ij}$ is

$$H_{ij} = h_+ e^+_{ij}(\bar{n}, \psi) + h_\times e^\times_{ij}(\bar{n}, \psi) \tag{9}$$

where $h_{+,\times}$ are the constant amplitudes of the gravitational plane wave, and $e^{+,\times}_{ij}(\bar{n}, \psi)$ are the polarisation tensors which are uniquely defined by the principal axes of the GW. $\psi$ is the polarisation angle of the GW.

Going forward, for now we will take $q(t) = q$ i..e. the pulsar locations are constant with respect to the Earth. This may have already been "done" during the barycentreing when pulsar TOAs are generated, in which case $q$ is the vector from the SSB to the pulsar

Lets review and categorise all the parameters:

$$\bar{\theta} = \bar{\theta}_{\rm PSR} + \bar{\theta}_{\rm GW} + \bar{\theta}_{\rm noise} \tag{10}$$

$$\bar{\theta}_{\rm PSR} = [\gamma, f_{\rm EM}(0), \dot{f}_{\rm EM}(0), d] \tag{11}$$

$$\bar{\theta}_{\rm GW} = [h_+, h_\times, \delta, \alpha, \psi, \Omega, \Phi_0] \tag{12}$$

$$\bar{\theta}_{\rm noise} = [\sigma, \Sigma] \tag{13}$$

We can also express the measurement equation generally as,

$$g(\bar{\theta}, t) = 1 - A\cos(\Omega(n\cdot q - t) + \Phi_0) \tag{14}$$

where

$$A = \frac{1}{2}\frac{H_{ij} q^i q^j}{1 + \bar{n}\cdot\bar{q}}[1 - e^{i\Omega(1+\bar{n}\cdot\bar{q})d}] \tag{15}$$

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### 2.1 References

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This paper has been typeset from a TEX/LATEX file prepared by the author.