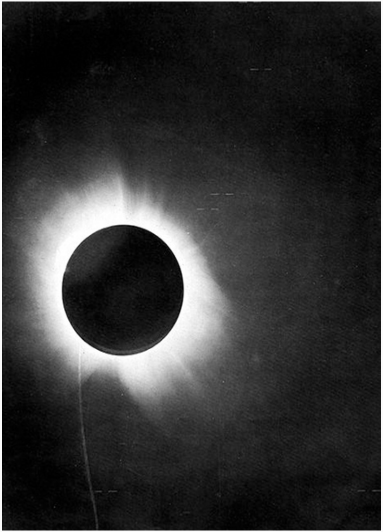
Pulsar Timing in Extreme Mass Ratio Binaries

A General Relativistic approach

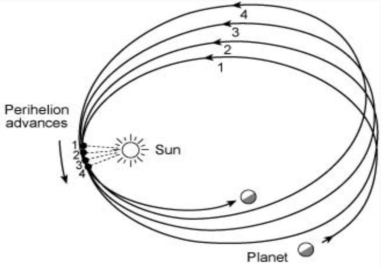
Tom Kimpson. w/ Kinwah Wu, Silvia Zane
MSSL, January 31, 2019

Introduction

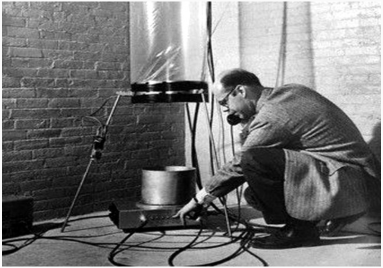
Success of GR: Classical Tests



Light Deflection



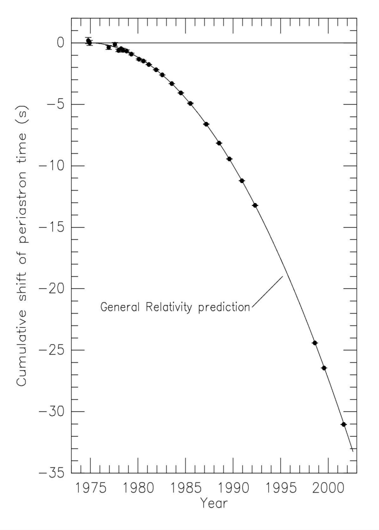
Mercury perihelion precession



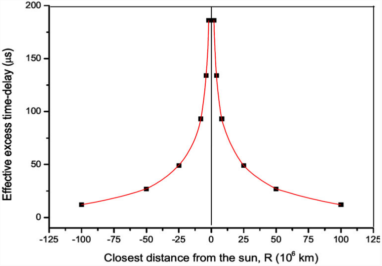
Gravitational Redshift

Credits, clockwise: *Dyson et al. 1919, Lang 2010, Harvard Univ. 2000*

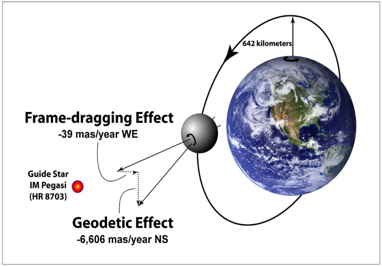
Success of GR: Modern Tests



Binary Pulsar



Shapiro Delay



Geodetic precession

Credits, clockwise: *Weisberg et al. 2010* , *Schaff J. 2012, Everitt et al. 2011*

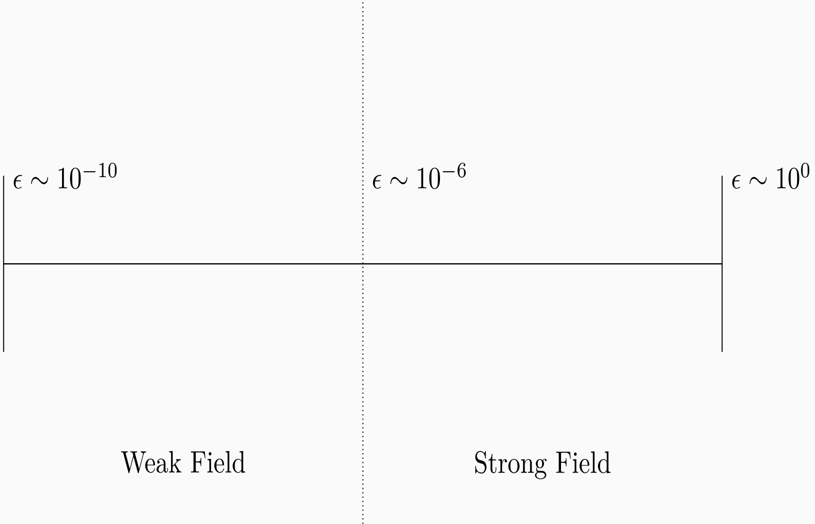
A Problem

GR is incomplete

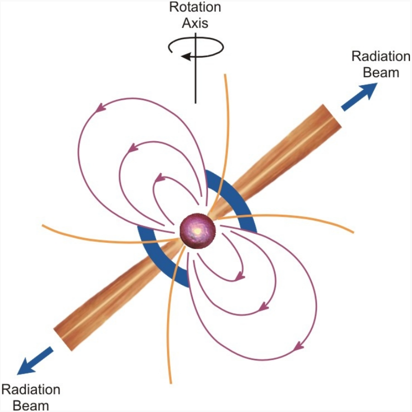
- Field equations = Non-unique
- Singularities + Quantum Gravity

Strong vs. Weak fields

$\epsilon \propto \frac{M}{r}$



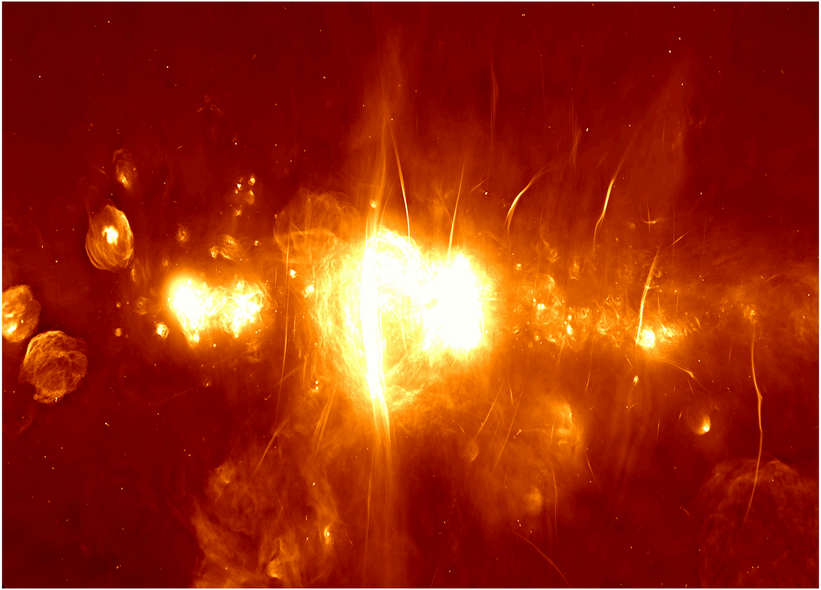
How can we probe strong fields?



- Massive BH = strong gravity
- Pulsars are clocks

  ∴ With pulsars near massive black holes (EMRB)

Hunting Grounds: Galactic centre



Credit: *SARAO*

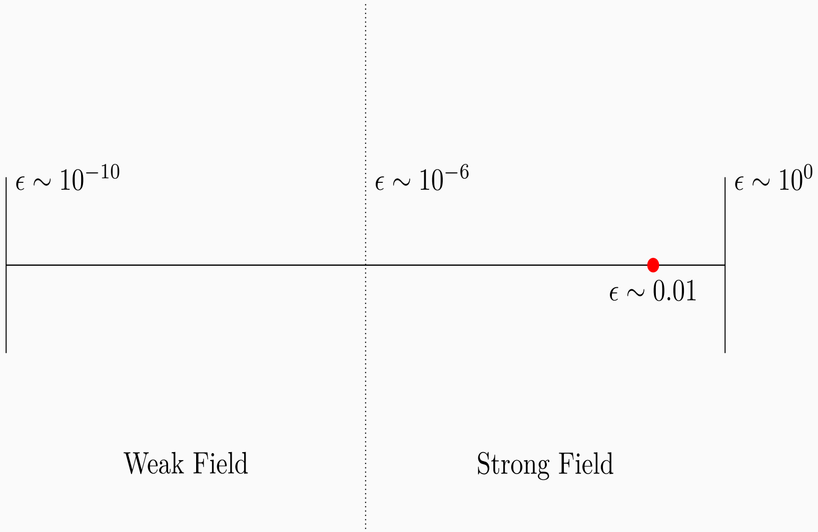
Hunting Grounds: Globular clusters

Credit: *NASA & ESA*

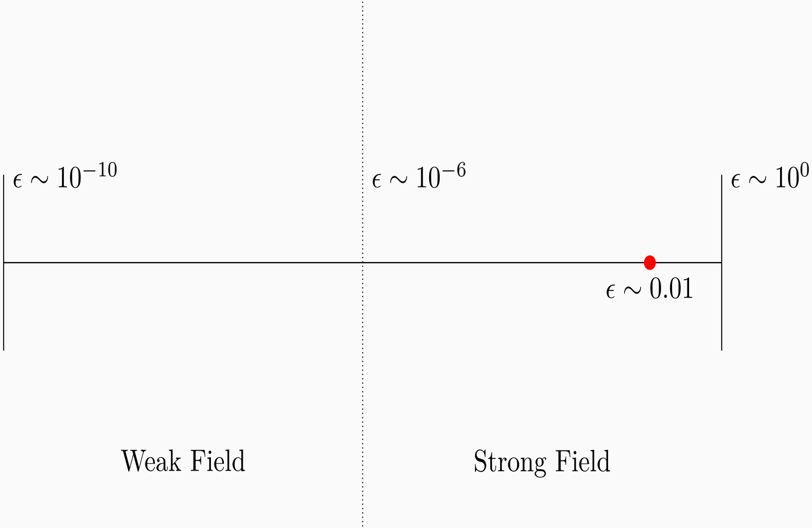
Detection Prospects

- $\lesssim 10^4$ PSR at $\sim$1 pc (*Wharton et al. 2012, ApJ 753:108, Rajwade et al. 2017, MNRAS 471:730*)
- Closest semi-major axis $\lesssim$ 100 AU
- Closest pericentre distance $\sim$ 2 AU (*Zhang et al. 2014, ApJ 784:106*)

Detection Prospects



Detection Prospects



No such PSR-EMRB yet detected!

Scientific prospects

- Strong field GR tests
- Alternative Gravitational theories

Scientific prospects

3 important parameters:

$$M, \chi, Q$$

Fundamental Physics

- No Hair Theorem ($Q = -\chi^2$)
- Cosmic Censorship Conjecture ($\chi \leq M^2$)

Astrophysics

- Astrophysical BH = Kerr solution?
- Constrain low end of $M - \sigma$ relation / Existence of IMBH

This Work

Goal: Use the next-generation radio telescopes to time a pulsar in orbit around a massive black hole at the Galactic centre

Require theoretical basis for calculation of the $t - \nu$ behaviour.

This Work: Why?

- Prospects
- Detection
- Modelling

This Work: How?

Behaviour of light + Orbital Dynamics = Timing signal ($t - \nu$)

Ray Tracing

Ray Tracing in Vacuum

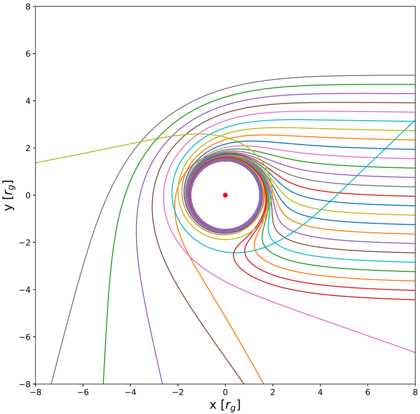
Vacuum Hamiltonian

$$H = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu$$

$$\dot{x}^\mu = \frac{dH}{dp_\mu}; \dot{p}_\mu = -\frac{dH}{dx^\mu}$$

Constants

- Energy
- Angular Momentum
- Mass
- Carter Constant



Ray Tracing in Kerr plasma

Vacuum

$$H = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu$$

Plasma

$$H = \frac{1}{2}(g^{\mu\nu} p_\mu p_\nu + \omega_p^2)$$

Ray Tracing in Kerr plasma

Vacuum

$$H = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu$$

Plasma

$$H = \frac{1}{2}(g^{\mu\nu} p_\mu p_\nu + \omega_p^2)$$

Homogeneous

$\omega_p^2 = const.$

This is OK

Inhomogeneous

$\omega_p^2 = \omega_p^2(r, \theta)$

This is a problem

Plasma frequency = mass

Massive particle in vacuum:

$$p^\mu p_\mu + m$$

Massless particle in plasma:

$$p^\mu p_\mu + \omega_p^2$$

Losing Carter

Constants

- Energy
- Angular Momentum
- Mass
- Carter Constant

Must define

$$\omega_p^2 = \frac{f(r) + g(\theta)}{\Sigma(r, \theta)}$$

$\omega_p^2 \rightarrow$ dispersion

Orbital Dynamics

- Textbook GR: point particles.
- Real pulsars $\neq$ point particles!

Creating the skeleton

$$T^{\mu\nu}{}_{;\nu} = 0$$

Multipole expansion to dipole order:

$$\frac{Dp^{\mu}}{d\tau} = -\frac{1}{2}R^{\mu}_{\nu\alpha\beta}u^{\nu}s^{\alpha\beta}$$

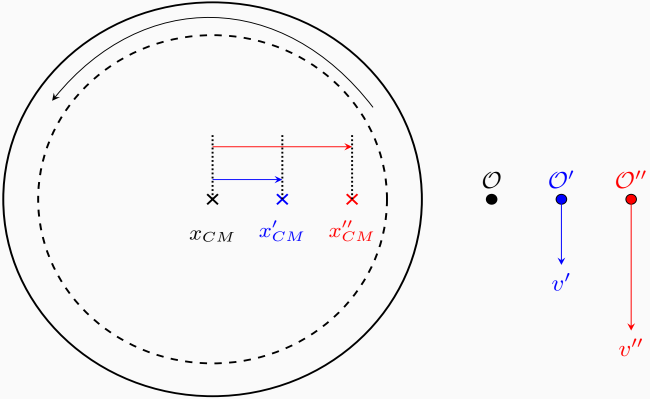
$$\frac{Ds^{\mu\nu}}{d\tau} = p^{\mu}u^{\nu} - p^{\nu}u^{\mu}$$

Equations are indeterminate!

Choosing a centre of mass

Multipole expansion defined w.r.t some reference worldline $z^\alpha(\tau)$.

Centre of mass is observer dependent.



How to choose a reference worldline?

Choosing the observer

$$\frac{Dp^\mu}{d\tau} = -\frac{1}{2} R^\mu_{\nu\alpha\beta} u^\nu s^{\alpha\beta}$$

$$\frac{Ds^{\mu\nu}}{d\tau} = p^\mu u^\nu - p^\nu u^\mu$$

Condition:

$$p_\mu s^{\mu\nu} = 0$$

Choosing the observer

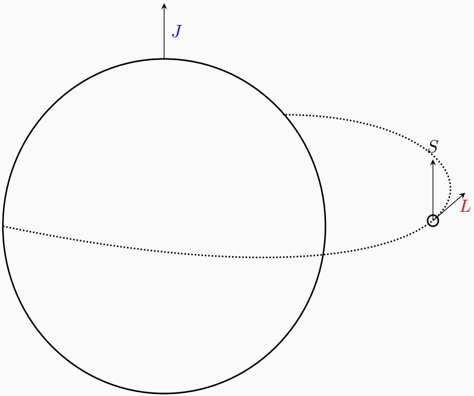
$$\frac{Dp^\mu}{d\tau} = -\frac{1}{2} R^\mu_{\nu\alpha\beta} u^\nu s^{\alpha\beta}$$

$$\frac{Ds^{\mu\nu}}{d\tau} = p^\mu u^\nu - p^\nu u^\mu \qquad \Rightarrow \qquad \frac{dp^\alpha}{d\tau}, \frac{ds^\alpha}{d\tau}, \frac{dx^\alpha}{d\tau}$$

Condition:

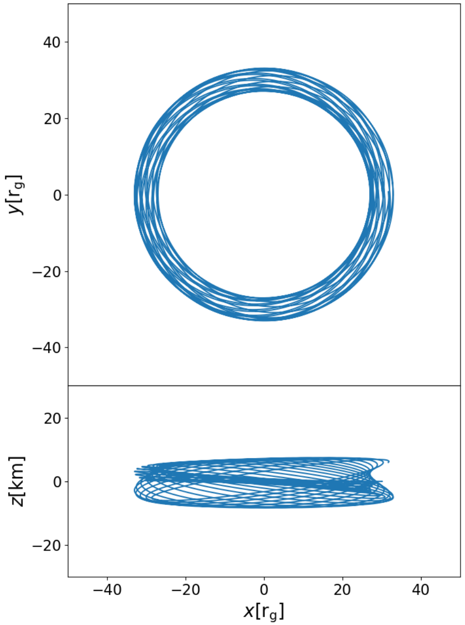
$$p_\mu s^{\mu\nu} = 0$$

Spin couplings

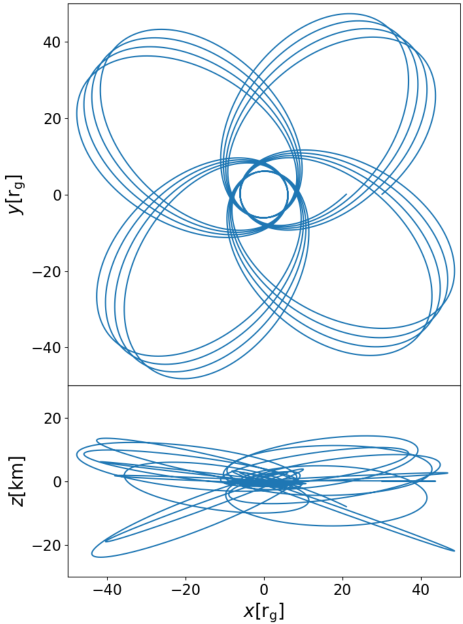


- Spin-spin (S-J)
- Spin-orbit (S-L)
- Spin-curvature

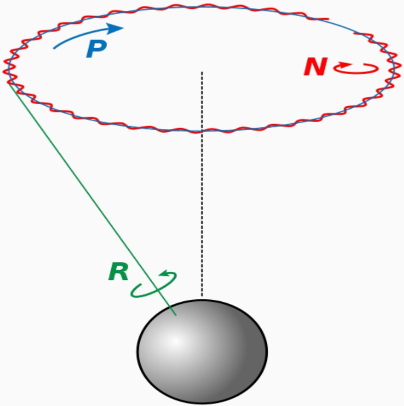
Orbital Dynamics: circular



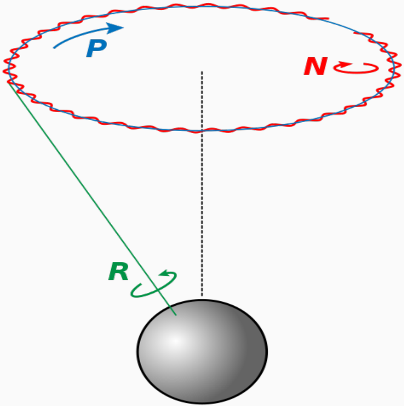
Orbital Dynamics: eccentric

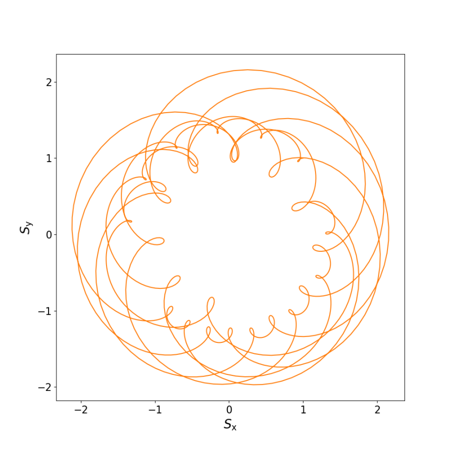


Spin Precession



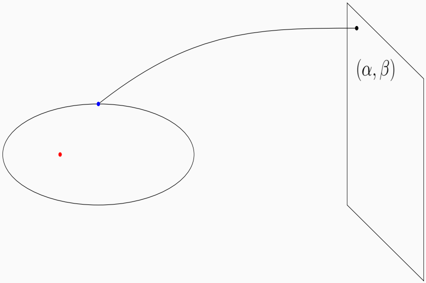
Spin Precession





Timing Signal

Putting it all together...



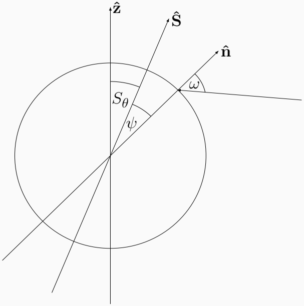
- Ray Tracing ✓
- Orbital Dynamics ✓
- Create timing signal

Optimization problem

$$ds = f(\alpha, \beta)$$

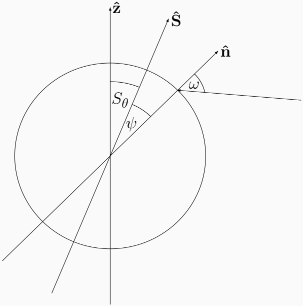
Putting it all together...

- Pulsar emission $\neq$ isotropic
- Find intersection with radiation point $x^i_{\mathrm{rad}} = R_{\mathrm{PSR}}\hat{\mathsf{n}} + x^i_{\mathrm{pulsar}}$
- $\hat{\mathsf{n}} = \hat{\mathsf{n}}(S_\theta(\tau), \psi)$



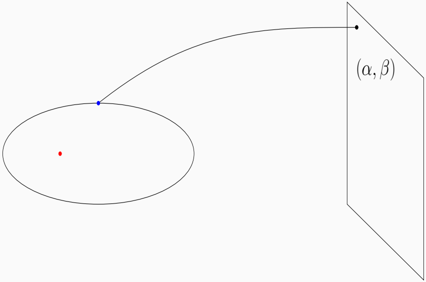
Aberration

- 'Seen' if $\omega < \omega_c$
- Global $\omega \neq$ Local ω
- Transform to coming frame



Effects

Quick Review



- Ray Tracing ✓
- Orbital Dynamics ✓
- Create timing signal ✓

Goal: calculation of the $t - \nu$ behaviour.

Effects to consider

- Gravitational lensing
- Primary/Secondary rays
- Influence of plasma: temporal/spatial dispersion
- Gravitational/Relativistic time dilation
- Orbital Dynamics
- Spin-curvature coupling
- Spin precession
- Relativistic aberration

Photon ToA, pulse profile, intensity, observability

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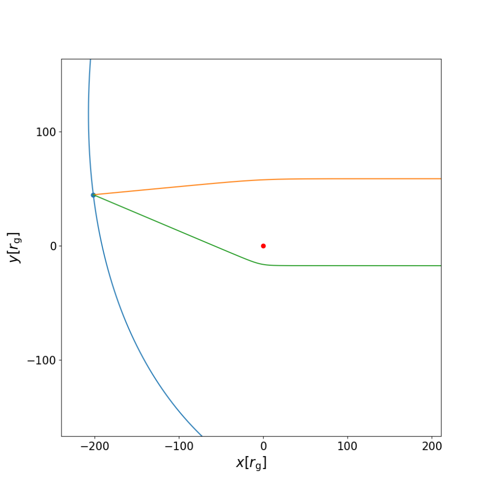
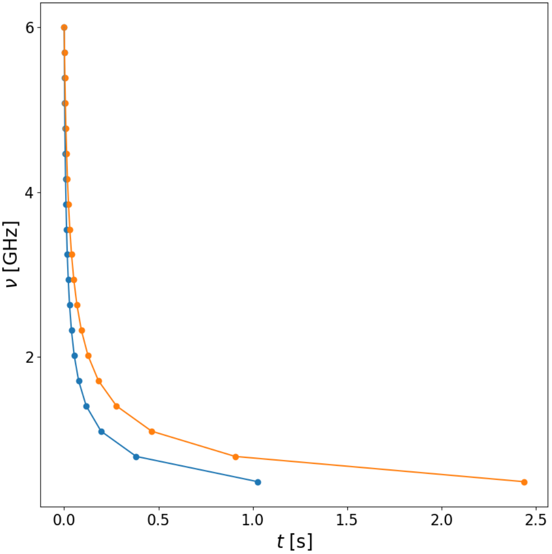
Photon ToA, pulse profile, intensity, observability

Gravitational Bending

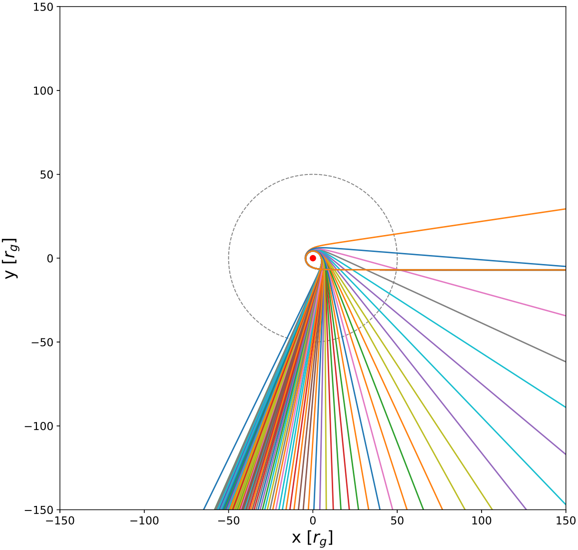


- Deviation from straight lines
- Primary/Secondary Rays

Plasma: temporal dispersion



Plasma: spatial dispersion



Summary

- PSR-EMRB = precision strong-gravity probes
- Require fully relativistic $t-\nu$ model
- Open question: How good are current methods?

Questions?