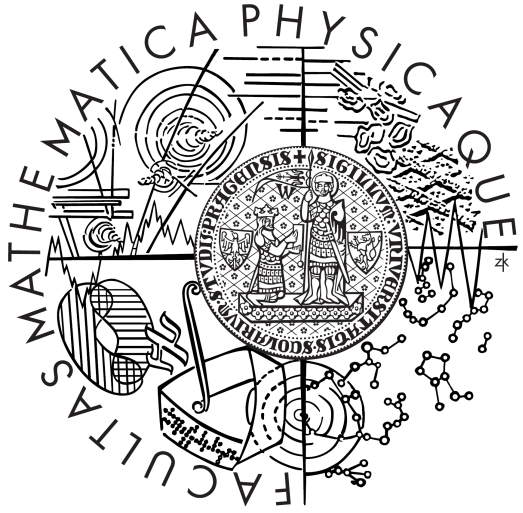
Charles University in Prague
Faculty of Mathematics and Physics

MASTER THESIS



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Typed Functional Genetic Programming

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Study programme: Theoretical Computer Science
Specialization: Non-Procedural Programming and Artificial Intelligence

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Dedication.

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Contents

Introduction

1. Definitions

Let us first say some basic definitions.

1.1 Lambda term

Let V be set of *variable names*.
Let C be set of *constant names*.
Then Λ is set of λ-*terms* inductively defined as follows:

$$x \in V \Rightarrow x \in \Lambda$$
$$c \in C \Rightarrow c \in \Lambda$$
$$M, N \in \Lambda \Rightarrow (MN) \in \Lambda$$
$$x \in V, M \in \Lambda \Rightarrow (\lambda x.M) \in \Lambda$$

TODO

- TALK ABOUT "parenthesis" conventions.

1.2 Type

Let A be set of *atomic type names*.
Then $\mathbb{T}$ is set of *types* inductively defined as follows:

$$\alpha \in A \Rightarrow \alpha \in \mathbb{T}$$
$$\sigma, \tau \in \mathbb{T} \Rightarrow (\sigma \to \tau) \in \mathbb{T}$$

TODO

- TALK ABOUT $\tau_1 \to \cdots \to \tau_n \to \alpha$
- TALK ABOUT arrow/parenthesis conventions.

1.3 Statement of a form $M : \sigma$

Let Λ be set of λ-*terms*.
Let $\mathbb{T}$ be set of *types*.
A *statement* $M : \sigma$ is a pair $(M, \sigma) \in \Lambda \times \mathbb{T}$.
$M : \sigma$ is vocalized as *"M has type σ"*.[1]
The type σ is the *predicate* and the term M is the *subject* of the statement.

[1] $M : \sigma$ can be also imagined as $M \in \sigma$

1.4 Context

Let $\Gamma \in \mathfrak{P}(\Lambda \times \mathbb{T})$. ($\Gamma$ is a set of *statements* of a form $M : \sigma$.)
Then Γ is *context* if it obeys following conditions[2]:

$$\forall (x, \sigma) \in \Gamma : x \in V \cup C$$
$$\forall s_1, s_2 \in \Gamma : s_1 \neq s_2 \Rightarrow \pi_1(s_1) \neq \pi_1(s_2)$$

In other words context is a set of statements with distinct variables or constants as subjects.

TODO: TALK ABOUT Context represents library/building blocks.

1.5 Statement of a form $\Gamma \vdash M : \sigma$

By writing $\Gamma \vdash M : \sigma$ we say *statement* $M : \sigma$ *is derivable from context* Γ .

We construct valid statements of form $\Gamma \vdash M : \sigma$ by using inference rules.

1.6 Inference rule

Basically speaking, inference rules are used for deriving statements of a form $\Gamma \vdash M : \sigma$ from yet derived statements of such a form. Those inference rules are written in the following form:

$$\frac{\Gamma_1 \vdash M_1 : \sigma_1 \qquad \Gamma_2 \vdash M_2 : \sigma_2 \quad \cdots \quad \Gamma_n \vdash M_n : \sigma_n}{\Gamma_{n+1} \vdash M_{n+1} : \sigma_{n+1}}$$

Suppose we have yet derived statements $\Gamma_1 \vdash M_1 : \sigma_1, \Gamma_2 \vdash M_2 : \sigma_2, \ldots, \Gamma_n \vdash M_n : \sigma_n$. It allows as to use the inference rule to derive statement $\Gamma_{n+1} \vdash M_{n+1} : \sigma_{n+1}$.

For deriving statements including types of a form $(\sigma \to \tau)$ are essential those two inference rules:

$$\frac{\Gamma \vdash M : \sigma \to \tau \qquad \Gamma \vdash N : \sigma}{\Gamma \vdash (MN) : \tau}$$

$$\frac{\Gamma \cup \{(x, \sigma)\} \vdash M : \tau}{\Gamma \vdash (\lambda x.M) : \sigma \to \tau}$$

This kind of inference rules allows us to derive new statements from yet derived statements, but what if we do not have any statement yet? For this purpose we have other kinds of inference rules such as *axiom* inference rule:

$$\frac{(x, \sigma) \in \Gamma}{\Gamma \vdash x : \sigma}$$

Let us consider an example statement of a form $\Gamma \vdash M : \sigma$:

$$\{\} \vdash (\lambda f.(\lambda x.(fx))) : (\sigma \to \tau) \to (\sigma \to \tau)$$

This statement is derived as follows:

[2] The π_1 corresponds to the projection of the first component of the Cartesian product.

$$\dfrac{\dfrac{\dfrac{(f,\sigma\to\tau)\in\{(f,\sigma\to\tau),(x,\sigma)\}}{\{(f,\sigma\to\tau),(x,\sigma)\}\vdash f:\sigma\to\tau}\qquad\dfrac{(x,\sigma)\in\{(f,\sigma\to\tau),(x,\sigma)\}}{\{(f,\sigma\to\tau),(x,\sigma)\}\vdash x:\sigma}}{\{(f,\sigma\to\tau),(x,\sigma)\}\vdash (fx):\tau}}{\{(f,\sigma\to\tau)\}\vdash(\lambda x.(fx)):\sigma\to\tau}}{\{\}\vdash(\lambda f.(\lambda x.(fx))):(\sigma\to\tau)\to(\sigma\to\tau)}$$

1.7 Term generating grammar

Inference rules are good for deriving statements of a form $\Gamma \vdash M : \sigma$, but our goal is slightly different; we would like to generate many λ-terms M for a given type σ and context Γ.

Our approach will be to take each inference rule and transform it to a rule of term generating grammar. With this term generating grammar it will be much easier to reason about generating λ-terms.

It won't be a grammar in classical sense because we will be operating with infinite sets of nonterminal symbols and rules. [3]

Let $Non = Type \times Context$ be our *nonterminal* set. So for every $i \in Non$ is $i = (\sigma_i, \Gamma_i)$.

Let's consider each relevant inference rule and its corresponding grammar rule.

First inference rule is *implication elimination* also known as *modus ponens*:

$$\frac{\Gamma \vdash M : \sigma \to \tau \qquad \Gamma \vdash N : \sigma}{\Gamma \vdash (MN) : \tau}$$

For every $\sigma, \tau \in \mathbb{T}$ and for every *context* $\Gamma \in \mathfrak{P}(\Lambda \times \mathbb{T})$ there is a grammar rule of a form[4]:

$$(\tau, \Gamma) \longmapsto \Big((\sigma \to \tau, \Gamma) \ \_ \ (\sigma, \Gamma) \Big)$$

Second inference rule is *implication introduction*:

$$\frac{\Gamma \cup \{(x,\sigma)\} \vdash M : \tau}{\Gamma \vdash (\lambda x.M) : \sigma \to \tau}$$

$\forall \sigma, \tau \in \mathbb{T}$ $\forall$*context* $\Gamma \in \mathfrak{P}(\Lambda \times \mathbb{T})$ $\forall x \in V$ such that there is no $(x, \rho) \in \Gamma$ there is a grammar rule:

$$(\sigma \to \tau, \Gamma) \longmapsto \Big(\ \lambda \ \mathrm{x} \ . \ (\tau, \Gamma \cup \{(x,\sigma)\}) \ \Big)$$

[3] TODO : mention terminal symbols - situation around variables and their construction with ' symbol.

[4] Terminal symbols for parenthesis and normally *space* now $\_$ (for *function application* operator) are visually highlighted.

Third inference rule is *axiom*:

$$\frac{(x,\sigma) \in \Gamma}{\Gamma \vdash x : \sigma}$$

$\forall \sigma \in \mathbb{T}\ \forall context\ \Gamma \in \mathfrak{P}(\Lambda \times \mathbb{T})\ \forall x \in V \cup C$ such that $(x,\sigma) \in \Gamma$ there is a grammar rule:

$$(\sigma, \Gamma) \longmapsto \mathrm{x}$$

We will demonstrate λ-term generation on example. Again on $(\lambda f.(\lambda x.(fx)))$. We would like to generate λ-term of a type $(\sigma \to \tau) \to (\sigma \to \tau)$ with $\Gamma = \{\}$.

$$\begin{aligned}
&((\sigma \to \tau) \to (\sigma \to \tau), \{\}) \\
\longmapsto &\Big(\lambda \mathrm{f}.(\sigma \to \tau, \{(f, \sigma \to \tau)\}) \Big) \\
\longmapsto &\Big(\lambda \mathrm{f}.\ \Big(\lambda \mathrm{x}.\ (\tau, \{(f, \sigma \to \tau), (x, \sigma)\}) \Big) \Big) \\
\longmapsto &\Big(\lambda \mathrm{f}.\ \Big(\lambda \mathrm{x}.\ \Big((\sigma \to \tau, \{(f, \sigma \to \tau), (x, \sigma)\}) __ (\sigma, \{(f, \sigma \to \tau), (x, \sigma)\})\Big) \Big) \Big) \\
\longmapsto &\Big(\lambda \mathrm{f}.\ \Big(\lambda \mathrm{x}.\ \Big(\mathrm{f} __ (\sigma, \{(f, \sigma \to \tau), (x, \sigma)\})\Big) \Big) \Big) \\
\longmapsto &\Big(\lambda \mathrm{f}.\ \Big(\lambda \mathrm{x}.\ \Big(\mathrm{f} __ \mathrm{x}\Big) \Big) \Big)
\end{aligned}$$

1.7.1 "Barendregt-like" inference and grammar rules

Inference rule 1:

$$\frac{\Gamma \cup \{(x_1, \tau_1), \ldots, (x_n, \tau_n)\} \vdash M : \alpha}{\Gamma \vdash (\lambda x_1 \ldots x_n.M) : \tau_1 \to \cdots \to \tau_n \to \alpha}$$

... there is a grammar rule:

$$(\tau_1 \to \cdots \to \tau_n \to \alpha, \Gamma) \longmapsto \Big(\lambda x_1 \ldots x_n.\ (\alpha, \Gamma \cup \{(x_1, \tau_1), \ldots, (x_n, \tau_n)\}) \Big)$$

Inference rule 2:

$$\frac{(f, \rho_1 \to \cdots \to \rho_m \to \alpha) \in \Gamma \qquad \Gamma \vdash M_1 : \rho_1 \quad \cdots \quad \Gamma \vdash M_m : \rho_m}{\Gamma \vdash (f M_1 \ldots M_m) : \alpha}$$

... there is a grammar rule:

$$(\alpha, \Gamma) \longmapsto \Big(\mathrm{f} \ \_\_ (\rho_1, \Gamma) \_\_ \ \ldots \ \_\_ (\rho_m, \Gamma) \Big)$$

TODO

- SHOW correctness of those inference rules by composing them of $E^{\rightarrow}$, $I^{\rightarrow}$ and *axiom*.
- SHOW more examples of inference rules transformed into grammar rules.
- DESCRIBE general algorithm for this transformation.
- TALK ABOUT $\tau_1 \rightarrow \cdots \rightarrow \tau_n \rightarrow \alpha$
- TALK ABOUT $\beta\eta$-normal form which is generated by this method.

1.8 Inhabitation tree

Now we will introduce *Inhabitation tree*, structure slightly different from *Inhabitation machine*, which was introduced in [1] by Henk Barendregt. We can think about Inhabitation tree as about unfolded Inhabitation machine. The motivation for using Inhabitation trees is belief that it will help us reason about generation of λ-terms of a given type σ and with a given context Γ.

1.8.1 Definition of Inhabitation tree

Inhabitation tree is a *rooted tree*, possibly infinite. It has two types of nodes:

- Type nodes - containing type $\sigma \in \mathbb{T}$ - aka "OR-node" , Nonterminal-node.
- Symbol nodes - containing "λ-head" (nonempty finite sequence of variable names) or constant name. - aka "AND-node" , Terminal-node.

We construct Inhabitation tree for given type σ and context Γ.
We will define Inhabitation tree by describing its construction for a given (σ, Γ). Notice that it will closely follow the rules from 1.7.1:

- The root of Inhabitation tree for (σ, Γ) is *type node* with σ as type.
- All *type nodes* have as child nodes only *symbol nodes*.
- And all *symbol nodes* have as child nodes only *type nodes*.

Now we will resolve the child nodes of the root node.
There are two cases of σ (recall 1.2):

Atomic type $\sigma = \alpha$ where $\alpha \in A$.

Function type $\sigma = \tau_1 \rightarrow \cdots \rightarrow \tau_n \rightarrow \alpha$ where $n \geq 1, \alpha \in A$.

First case **Atomic type** — i.e., $\sigma = \alpha$ where $\alpha \in A$:
For every $(f, \rho_1 \rightarrow \cdots \rightarrow \rho_m \rightarrow \alpha) \in \Gamma$ where $\alpha \in A$ there is a child *symbol node* of the root containing constant name f. This symbol node containing f has m child subtrees corresponding to Inhabitation trees for $(\rho_1, \Gamma), \ldots, (\rho_n, \Gamma)$.

Compare this case with corresponding grammar rule:

$$(\alpha, \Gamma) \longmapsto \left(\mathrm{f} \ \_ (\rho_1, \Gamma) \ \_ \ \ldots \ \_ (\rho_m, \Gamma) \right)$$

Second case **Function type** — i.e., $\sigma = \tau_1 \to \cdots \to \tau_n \to \alpha$ where $n \geq 1, \alpha \in A$:
For every $i \in \{1, \ldots, n\}$ we create new *variable name* x_i which is not yet included in context Γ as variable or constant name.

There is one and only one child *symbol node* of the root containing "λ-head" $\lambda x_1 \ldots x_n$ which stands for sequence of variable names $(x_1, \ldots, x_n)$. This symbol node containing $\lambda x_1 \ldots x_n$ has one and only one child subtree corresponding to Inhabitation trees for $(\alpha, \Gamma \cup \{(x_1, \tau_1), \ldots, (x_n, \tau_n)\})$.

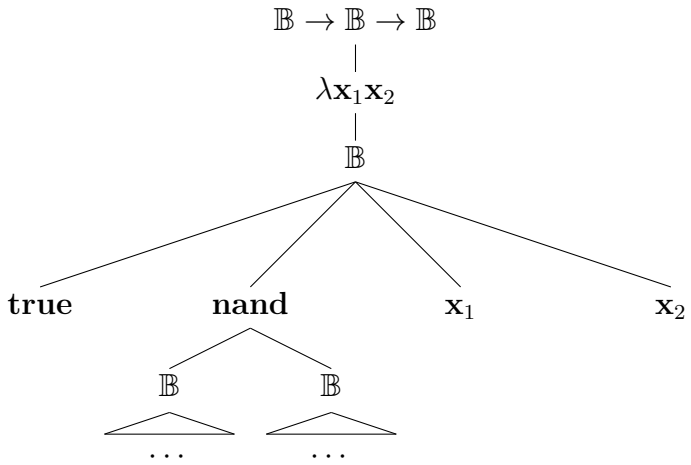
Compare this case with corresponding grammar rule:

$$(\tau_1 \to \cdots \to \tau_n \to \alpha, \Gamma) \longmapsto \left(\lambda x_1 \ldots x_n.\ (\alpha, \Gamma \cup \{(x_1, \tau_1), \ldots, (x_n, \tau_n)\}) \right)$$

Let's consider following (σ, Γ) as a simple example:

$$\begin{aligned} \sigma &= \mathbb{B} \to \mathbb{B} \to \mathbb{B} \\ \Gamma &= \{\ true : \mathbb{B} \\ &\quad ,\ nand : \mathbb{B} \to \mathbb{B} \to \mathbb{B}\ \} \end{aligned}$$

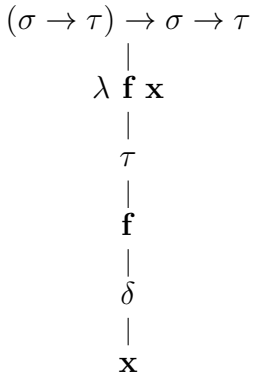
This particular (σ, Γ) results in the following tree:



Second example features our well known example:

$$\begin{aligned} \sigma &= (\sigma \to \tau) \to \sigma \to \tau \\ \Gamma &= \{\} \end{aligned}$$

Which results in following tree:



1.8.2 And-or tree and searching in Inhabitation tree

Let us consider following definition of *And-or tree*[5]:

And-or tree is a rooted tree where each node is labeled as either *and-node* or[6] *or-node*.

By *solving* And-or tree T we mean finding T' subtree of T such that it follows these conditions:

- The root of T' is the root of T.
- Each *and-node* in T' has all the child nodes as in T.
- Each *or-node* in T' has precisely one child node.[7]

Let us now consider following labeling of Inhabitation tree:

- **Type nodes** are labeled as **or-nodes**.
- **Symbol nodes** are labeled as **and-nodes**.

This labeling has following justification:

Selection of exactly one child node in *type node* corresponds to selection of exactly one grammar rule in order to rewrite nonterminal symbol.

Selection of all the child nodes in *symbol node* corresponds to rewriting all the nonterminal symbols in string that is being generated.

TODO: TALK ABOUT : By *solving* And-or tree for Inhabitation tree we obtain that typed tree notation for typed λ-term, which must be firstly INTRODUCED.
TODO: TALK (more?) ABOUT "Barendregt-like" subsection 1.7.1

[5] **TODO**: Mention that on WIKI there is more general definition, but for our purposes is this one sufficient.

[6] xor

[7] **TODO**: MENTION why precisely one and not at least one ..or CHANGE the def.

2. Designed system

2.1 Top level view

2.1.1 Comments about main source files

Eva.hs

2.2 Term generating

2.2.1 A* algorithm

2.3 Crossover

2.3.1 Finding same types

2.3.2 Two basic options

Resolve problems with free variables or avoid variables completely.

2.4 Mutation

2.4.1 Using term generation

3. Problems

In this section will be presented usage of the system in order to solve specific problems.

3.1 Even Parity Problem

3.2 Big Context

3.3 Fly

3.4 Simple Symbolic Regression

3.5 Artificial Ant

3.6 Boolean Alternate

Conclusion

Bibliography

[1] Henk Barendregt, Wil Dekkers, Richard Statman, *Lambda Calculus With Types*. Cambridge University Press, 2010.
`http://www.cs.ru.nl/~henk/book.pdf`