2/4 c)

$$M = \left\{ \frac{2n-1}{n+4} \,\middle|\, n \in \mathbb{N} \right\}$$

$n+4 \neq 0 \Rightarrow n \neq (-4) \leftarrow$ nemožné jelikož $n \in \mathbb{N}$

$$\frac{2n-1}{n+4} = \frac{2\cdot(n+4)-8-1}{n+4} = \frac{2\cdot(n+4)}{n+4} - \frac{9}{n+4} = 2 - \frac{9}{n+4}$$

pro $n = 1$ (nejmenší možné n)

$\cancel{2 - \frac{9}{n+4}}$

$$2 - \frac{9}{1+4} = 2 - \frac{9}{5} = \frac{1}{5}$$

$$\frac{1}{5} \leq 2 - \frac{9}{n+4} < 2 \Rightarrow \text{množina je } \underline{\text{omezená}}.$$

2/6 - b)

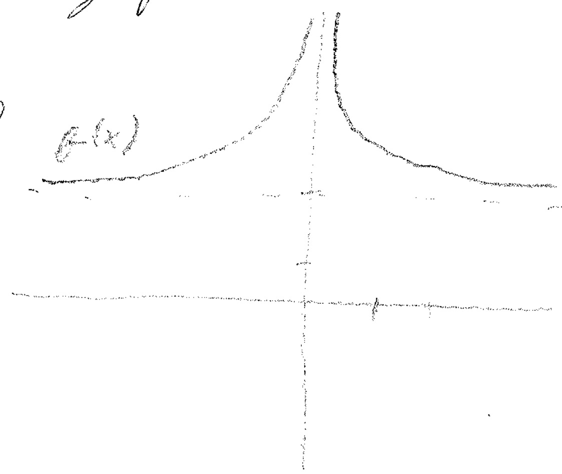
$f(x) = \frac{1}{x^2} + 2$

$D(f) = R - \{0\}$ $\quad x^2 \neq 0 \Rightarrow |x| \neq 0 \Rightarrow x \neq 0$

$H(f) = (\cancel{3}\ 2; \infty)$

fce není prostá

graf:



$f' = \frac{1}{x^2}$

$H(f') = (0; \infty)$

$\lim_{x \to 0} f' = \infty$

$\lim_{x \to \infty} f' = 0$

2/6 - c)

$g(x) = \frac{6x+5}{3x-6}$

$3x - 6 \neq 0$

$3x \neq 6$

$x \neq 2$

$\frac{2 \cdot (3x-6) + 12 + 5}{3x-6} = 2 + \frac{17}{3x-6} \Rightarrow H(g)$

$D(g) = R \setminus \{2\}$

$H(g) = R \setminus \{2\}$

fce je prostá

$x = \frac{6y+5}{3y-6}$

$x(3y-6) = 6y+5$

$3xy - 6x = 6y + 5$

$5 - 6x = y(6-3x)$

$-6x - 5$

$\frac{\cancel{5-6x}}{6-3x} = y$

$\frac{6x+5}{3x-6} = y \Rightarrow y = g^{-1}$

2/9

$f(x) = \frac{2-x^2}{x^2+1}$

$$\frac{2-x^2}{x^2+1} = \frac{2-(x^2+1)+1}{x^2+1} = -1 + \frac{3}{x^2+1}$$

$x^2 > 0$

$\Downarrow$

$0 < \frac{3}{x^2+1} \leq 3$

$$-1 < -1 + \frac{3}{x^2+1} \leq 2$$

$\Downarrow$

fce je omezená

2/10

$x_1 < x_2$

$$f(x_1) - f(x_2) = \frac{1}{x_1^2} - \frac{1}{x_2^2} = \frac{x_2^2 - x_1^2}{x_1^2 x_2^2}$$

pro $x_1, x_2 \in (-\infty; 0)$

$x_2^2 < x_1^2 \Rightarrow$

$x_2^2 - x_1^2 < 0$

$\Downarrow$

fce je rostoucí

pro $x_1, x_2 \in (0; \infty)$

$x_1^2 < x_2^2 \Rightarrow x_2^2 - x_1^2 > 0$

$\Downarrow$

fce je klesající

3/3 − a)

pro $x_0 = 0$

$$\lim_{x \to 0} f(x) = \frac{2x^2 - 11x + 5}{3x^2 - 14x - 5} = \frac{5}{-5} = \underline{\underline{-1}}$$

pro $x_0 = 5$

$$\lim_{x \to 5} f(x) = \frac{(2x-1)(x-5)}{(3x+1)(x-5)} = \frac{2x-1}{3x+1} = \underline{\underline{\frac{9}{16}}}$$

pro $x_0 = -\infty$

$$\lim_{x \to -\infty} f(x) = \frac{2}{3} \quad \left(\text{dle pravidla pro } \frac{\infty}{\infty}\right)$$

3/3 − b)

pro $x_0 = 1$

$$\lim_{x \to 1} f(x) = \frac{x^3 - x^2 - x + 1}{x^3 - 4x^2 + 5x - 2} = \frac{1-1-1+1}{1-4+5-2} = \frac{0}{0}$$

$$\hookrightarrow \frac{x^2 \cdot (x-1) - (x-1)}{(x-1) \cdot (x^2 - 3x + 2)} = \frac{(x-1)(x^2-1)}{(x-1) \cdot (x^2-3x+2)} = \frac{(x^2-1)}{x^2-3x+2} = \frac{(x+1) \cdot (x-1)}{(x-2) \cdot (x-1)} =$$

$$= \frac{x+1}{x-2} \Rightarrow \lim_{x \to 1} f(x) = \frac{x+1}{x-2} = \frac{1+1}{1-2} = \underline{\underline{-2}}$$

pro $x_0 = 2^+$ a 2^-

$$\frac{x+1}{x-2} = \frac{2+1}{2-2} = \frac{3}{0} \Rightarrow \lim_{x \to 2^+} f(x) = \infty$$

$$\lim_{x \to 2^-} f(x) = (-\infty)$$

3/3 - c)

pro $x_0=0$

$$\frac{3}{1-x^3}+\frac{1}{x-1} = \frac{3}{1}+\frac{1}{-1} = \underline{\underline{2}}$$

pro $x_0=1$

$$\frac{3}{1-1}+\frac{1}{1-1} = \frac{3}{0}+\frac{1}{0}$$

$$\frac{3}{1-x^3}+\frac{1}{x-1} = \frac{3x-3+1-x^3}{(1-x^3)\cdot(x-1)} = \frac{(x-1)\cdot(3-x^2-x-1)}{(1-x^3)\cdot(x-1)} = \frac{-x^2-x+2}{1-x^3} =$$

$$= \frac{-1\cdot(x^2+x-2)}{(x-1)\cdot(x^2+x+1)\cdot -1} = \frac{(x-1)\cdot(x+2)}{(x-1)\cdot(x^2+x+1)} = \frac{x+2}{x^2+x+1} \Rightarrow \frac{1+2}{1+1+1} = \frac{3}{3} = \underline{\underline{1}}$$

pro $x_0=(-\infty)$

$$\frac{3}{1-(-\infty)^3}+\frac{1}{(-\infty)-1} = \frac{3}{\infty}+\frac{1}{-\infty} = 0+0 = \underline{\underline{0}}$$