4/3 - b)

$$f(x) = \operatorname{tg} x - \frac{1}{\cos x} = \frac{\sin x - 1}{\cos x}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{\sin x - 1}{\cos x} = \left\langle\!\left\langle \frac{1-1}{0} \right\rangle\!\right\rangle = \left\langle\!\left\langle \frac{0}{0} \right\rangle\!\right\rangle = \lim_{x \to \frac{\pi}{2}} \frac{(\sin x - 1)\cdot(\sin x + 1)}{\cos x \cdot (\sin x + 1)} =$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\sin^2 x - 1}{\cos x(\sin x + 1)} = \lim_{x \to \frac{\pi}{2}} \frac{\frac{1-\cos 2x}{2} - 1}{\cos x(\sin x + 1)} =$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\frac{1 - \cos 2x - 2}{2}}{\cos x(\sin x + 1)} = \lim_{x \to \frac{\pi}{2}} \frac{-\frac{\cos 2x + 1}{2}}{\cos x(\sin x + 1)} = \lim_{x \to \frac{\pi}{2}} \frac{\frac{-\cos^2 x + \sin^2 x - 1}{2}}{\cos x(\sin x + 1)} =$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\frac{-\cos^2 x + \sin^2 x - \sin^2 x - \cos^2 x}{2}}{\cos x(\sin x + 1)} = \lim_{x \to \frac{\pi}{2}} \frac{\frac{-2\cos^2 x}{2}}{\cos x(\sin x + 1)} = \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{\sin x + 1} = \frac{0}{1+1} = \underline{\underline{0}}$$

$$\lim_{x \to \frac{\pi}{4}} \frac{-\cos x}{\sin x + 1} = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} + 1} = \frac{-\frac{1}{\sqrt{2}}}{\frac{\sqrt{2}+1}{\sqrt{2}}} = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}+1} = \frac{-1}{\sqrt{2}+1} = \frac{-1(\sqrt{2}-1)}{2-1} = \underline{\underline{1-\sqrt{2}}}$$

5/1)

$$f'(-1) = \lim_{x\to -1} \frac{f(x)-f(-1)}{x+1} = \lim_{x\to -1} \frac{\left(x+(x+1)\arcsin\sqrt{\frac{x}{x-1}}\right) - \left(-1+0\cdot\arcsin\sqrt{\frac{-1}{-2}}\right)}{x+1} =$$

$$= \lim_{x\to -1} \frac{1+x+(x+1)\arcsin\sqrt{\frac{x}{x-1}}}{x+1} = \lim_{x\to -1} \frac{(x+1)\cdot\left(1+\arcsin\sqrt{\frac{x}{x-1}}\right)}{x+1} =$$

$$= \lim_{x\to -1} 1+\arcsin\sqrt{\frac{x}{x-1}} = 1+\arcsin\sqrt{\frac{1}{2}} = 1+\frac{\pi}{4} = \frac{5\pi}{4}$$

5/3-b)

$$\cancel{f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}} =$$

n...

$$q: y = \frac{-x}{2}+4 \Rightarrow \frac{-1}{2} = k_q = k_n \Rightarrow k_t = \frac{-1}{k_n} = 2$$

$x \neq \pm 1$

$$f'(x) = \left(\frac{2x}{1-x^2}\right)' = 2\cdot\left(\frac{(x)'\cdot(1-x^2) - x\cdot(1-x^2)'}{(1-x^2)^2}\right) = 2\frac{1\cdot(1-x^2)-x\cdot(-2x)}{(1-x^2)^2} =$$

$$= 2\cdot\frac{1-x^2+2x^2}{(1-x^2)^2} = \frac{2\cdot(x^2+1)}{(1-x^2)^2}$$

$$\frac{2\cdot(x^2+1)}{(1-x^2)^2} = 2$$

$$2\cdot(x^2+1) = 2\cdot(1-x^2)^2$$

$$(x^2+1) = (1-x^2)^2$$

$$x^2+1 = 1-2x^2+x^4$$

$$0 = -3x^2+x^4$$

$$0 = x^2\cdot(x^2-3)$$

$x^2=0 \Rightarrow |x|=0 \Rightarrow \underline{x=0}$

$x^2-3=0 \Rightarrow |x|=\sqrt{3} \Rightarrow x_2=\sqrt{3},\ x_3=-\sqrt{3}$

$$f(0) = \frac{0}{1-0} = 0$$

$$f(\sqrt{3}) = \frac{2\sqrt{3}}{1-3} = -\sqrt{3}$$

$$f(-\sqrt{3}) = \frac{2\cdot-\sqrt{3}}{1-3} = \sqrt{3}$$

$$\boxed{\begin{array}{l} n_1: y = \frac{-x}{2} \\ n_2: y = -\sqrt{3} - \frac{x}{2} \\ n_3: y = \sqrt{3} - \frac{x}{2} \end{array}}$$

5/3-b - l'H)

$$\lim_{x\to\frac{\pi}{2}} \frac{\sin x - 1}{\cos x} = \left\langle\!\left\langle \frac{0}{0} \right\rangle\!\right\rangle \overset{l'H}{=} \lim_{x\to\frac{\pi}{2}} \frac{\cos x}{-\sin x} = \frac{0}{-1} = \underline{\underline{0}}$$

4/5-a)

$$f(x) = (1+3\operatorname{tg} x)^{\operatorname{cotg} x} = \exp(\operatorname{cotg} x \cdot \ln(1+3\operatorname{tg} x))$$

$$\lim_{x\to\pi^\pm} f(x) = \lim_{x\to\pi^\pm} \exp(\operatorname{cotg} x \cdot \ln(1+3\operatorname{tg} x)) = \exp\left(\lim_{x\to\pi^\pm} (\operatorname{cotg} x \cdot \ln(1+3\operatorname{tg} x)\right) =$$

$$\langle\!\langle \pm\infty \cdot 0 \rangle\!\rangle = \exp\left(\lim_{x\to\pi^\pm} \frac{\cos x}{\sin x} \cdot \ln(1+3\operatorname{tg} x)\right) = \cancel{\cos x \cdot \ln(1+3\operatorname{tg} x)}$$

$$= \exp\left(\lim_{x\to\pi^\pm} \frac{\cos x \cdot (\ln(1+3\operatorname{tg} x))}{\sin x}\right) = \left\langle\!\left\langle \frac{0}{0} \right\rangle\!\right\rangle \overset{l'H}{=} \cancel{\exp\left(\lim_{x\to\pi^\pm} -\sin x \cdot \ln(1+3\operatorname{tg} x) +\right.}$$

$$\overset{l'H}{=} \exp\left(\lim_{x\to\pi^\pm} \frac{-\sin x \ln(1+3\operatorname{tg} x) + \cos x \cdot \frac{1}{1+3\operatorname{tg} x} \cdot \frac{3}{\cos^2 x}}{\cos x}\right) = \exp\left(\frac{0 \cdot 0 + 1 \cdot \frac{1}{1} \cdot \frac{3}{1}}{1}\right) = \exp(3) =$$

$$= \underline{\underline{e^3}}$$

~~6/1-c)~~

$$f(x) = x \cdot \operatorname{cotg} 2x$$

$$\lim_{x\to0^\pm} x \cdot \operatorname{cotg} 2x = \langle\!\langle 0 \cdot \pm\infty \rangle\!\rangle = \lim_{x\to0^\pm} x \cdot \frac{\operatorname{cotg}^2 x - 1}{2\operatorname{cotg} x} = \lim_{x\to0^\pm} \frac{x \cdot (\operatorname{cotg}^2 x - 1)}{2\operatorname{cotg} x} = \left\langle\!\left\langle \frac{0}{0} \right\rangle\!\right\rangle \overset{l'H}{=}$$

$$\overset{l'H}{=} \lim_{x\to0^\pm} \frac{1 \cdot (\operatorname{cotg}^2 x - 1) + x \cdot 2\operatorname{cotg} x \cdot \frac{-1}{\sin^2 x}}{\frac{-2}{\sin^2 x}} = \lim_{x\to0^\pm} \frac{\operatorname{cotg}^2 x - 1 + \frac{-2x\operatorname{cotg}(x)}{\sin^2 x}}{\frac{-2}{\sin^2 x}} = \lim_{x\to0^\pm} \frac{\frac{\cos^2 x}{\sin^2 x} - 1 + \frac{-2x\operatorname{cotg} x}{\sin^2 x}}{\frac{-2}{\sin^2 x}} =$$

$$= \lim_{x\to0^\pm} \frac{\frac{\cos^2 x - \sin^2 x - 2x\operatorname{cotg} x}{\sin^2 x}}{\frac{-2}{\sin^2 x}} = \lim_{x\to0^\pm} \frac{\cos^2 x - \sin^2 x - 2x\operatorname{cotg} x}{-2}$$

6/1-c)

$$\lim_{x\to 0^{\pm}} x\cdot \cot 2x = \langle\langle 0\cdot \pm\infty \rangle\rangle = \lim_{x\to 0^{\pm}} \frac{x\cos 2x}{\sin 2x} = \left\langle\left\langle \frac{0}{0} \right\rangle\right\rangle \overset{l'H}{=}$$

$$\overset{l'H}{=} \lim_{x\to 0^{\pm}} \frac{1\cdot \cos 2x + -\sin 2x \cdot 2}{2\cos 2x} = \lim_{x\to 0^{\pm}} \frac{\cos 2x - 2\sin 2x}{2\cos 2x} = \frac{1-2\cdot 0}{2\cdot 1} = \frac{1}{2}$$

6/2-d)

$$\lim_{x\to -\infty} \sqrt[3]{e^{x}}\cdot x^{3} = \langle\langle 0\cdot -\infty \rangle\rangle = \lim_{x\to -\infty} \frac{x^{3}}{e^{-\frac{x}{3}}} = \left\langle\left\langle \frac{-\infty}{+\infty} \right\rangle\right\rangle \overset{l'H}{=} \lim \frac{3x^{2}}{e^{-\frac{x}{3}}\cdot -\frac{1}{3}} =$$

$$= \left\langle\left\langle \frac{\infty}{\infty} \right\rangle\right\rangle \overset{l'H}{=} \lim_{x\to -\infty} \frac{6x}{\left(e^{-\frac{x}{3}}\cdot \frac{-1}{3}\right)\cdot \frac{-1}{3} + e^{-\frac{x}{3}}\cdot \frac{-1}{3}} = \left\langle\left\langle \frac{-\infty}{\infty} \right\rangle\right\rangle \overset{l'H}{=} \frac{6}{-\infty} = 0$$