3/4-c) $D(f) = (-\infty; -\frac{1}{\sqrt{2}}) \cup (\frac{1}{\sqrt{2}}; \infty)$ $2x^2 - 1 \geq 0$ $x \neq 0$

$2x^2 \geq 1$

$x^2 \geq \frac{1}{2}$

$|x| \geq \frac{1}{\sqrt{2}}$

pro $x_0 = 0$

f není v okolí 0 definována

pro $x_0 = \pm\infty$

$$\lim_{x\to\pm\infty} \frac{1+\sqrt{2x^2-1}}{x} = \left\langle\!\left\langle \frac{\infty}{\pm\infty} \right\rangle\!\right\rangle = \lim_{x\to\pm\infty} \frac{1+\sqrt{x^2(2-\frac{1}{x^2})}}{x} = \lim_{x\to\pm\infty} \frac{1+|x|\sqrt{2-\frac{1}{x^2}}}{x} =$$

$$= \lim_{x\to\pm\infty} \left(\underbrace{\frac{1}{x}}_{\to 0} + \frac{\sqrt{2-\underbrace{\frac{1}{x^2}}_{\to 0}}}{\underbrace{\operatorname{sgn}(x)}_{\to \pm 1}} \right) = \left\langle\!\left\langle 0 + \frac{\sqrt{2-0}}{\pm 1} \right\rangle\!\right\rangle = \pm\sqrt{2}$$

4/1-b)

$$\lim_{x\to\infty} \frac{4^{2x}+5^x}{4^{2x+1}-5^{x+1}} = \lim_{x\to\infty} \frac{4^{2x}+5^x}{4\cdot 4^{2x}-5\cdot 5^x} = \lim_{x\to\infty} \frac{1+\underbrace{\left(\frac{5}{16}\right)^x}_{\to 0}}{4\cdot 1-5\cdot \underbrace{\left(\frac{5}{16}\right)^x}_{\to 0}} = \frac{1}{4}$$

4/2-a)

$$\lim_{x\to\infty} \left(2+\frac{1}{x}\right)^x = \left\langle\!\left\langle \left(2+\frac{1}{\infty}\right)^\infty \right\rangle\!\right\rangle \Rightarrow f(x) > 2^x \Rightarrow \lim_{x\to\infty} \left(2+\frac{1}{x}\right)^x = \infty$$

4/4 -a)

pro $x_0 = 0$

$$\lim_{x\to 0} \frac{x^2-1}{\sin(x+1)} = \frac{0-1}{\sin(0+1)} = \frac{-1}{\sin(1)}$$

pro $x_0 = -1$

$$\lim_{x\to -1} \frac{x^2-1}{\sin(x+1)} = \left(\left(\frac{0}{0}\right)\right) = \lim_{x\to -1} \frac{(x+1)\cdot(x-1)}{\sin(x+1)} = \lim_{x\to -1} (x-1)\frac{(x+1)}{\sin(x+1)} = \lim_{x\to -1} (x-1)\frac{1}{\frac{\sin(x+1)}{x+1}} =$$

$$= -2\cdot\frac{1}{\frac{\sin 0}{0}} = \frac{-2}{1} = -2$$

4/6-c)

$$f(x) = \left(\ln \frac{2x+1}{x}\right)^{\cos(\pi x)}$$

$x \neq 0$

$\frac{2x+1}{x} > 0$

$\frac{x\cdot(2+\frac{1}{x})}{x} > 0$

$2+\frac{1}{x} > 0$

$\frac{1}{x} > -2$

$x < 0$ $\quad$ $x > 0$

$1 < -2x$ $\quad$ $1 > -2x$

$-\frac{1}{2} < x$ $\quad$ $-\frac{1}{2} > x$

$x \in (-\infty; -\frac{1}{2}) \cup (0; \infty)$

9/6 - c)

$$f(x) = \left(\ln\frac{2x+1}{x}\right)^{\cos(\pi x)}$$

$$D(f) = (-\infty; -1) \cup (0; \infty)$$

$\ln\frac{2x+1}{x} > 0$

$\Downarrow$

$$\frac{2x+1}{x} > 1$$

$$2 + \frac{1}{x} > 1$$

$$\frac{1}{x} > -1$$

$x > 0$ $\quad$ $x < 0$

$1 > -x$ $\quad$ $1 < -x$

$-1 < x \;\wedge\; -1 > x \;\wedge\; x \neq 0$

$(0 < x)$

$$\lim_{x\to\pm\infty} \left(\ln\frac{2x+1}{x}\right)^{\cos(\pi x)} =$$

= ~~není definováno~~

neexistuje jelikož $\ll (\ln(2))^{\cos(\pi x)}$

$$\lim_{x\to -1^-} \left(\ln\frac{2x+1}{x}\right)^{\cos(\pi x)} =$$

$$= \lim_{x\to -1^-} \exp\left(\cos(\pi x)\ln\left(\ln\left(\frac{2x+1}{x}\right)\right)\right) = \exp\left(\lim_{x\to -1^-}\cos(\pi x)\lim_{x\to -1^-}\ln\left(\ln\left(\frac{2x+1}{x}\right)\right)\right) =$$

$$= \exp\left(\lim_{x\to -1^-}\cos(\pi x)\ln\left(\lim_{x\to -1^-}\ln\left(\frac{2x+1}{x}\right)\right)\right) = \exp(-1\cdot\ln(0)) = \underline{\underline{\infty}}$$

$$\lim_{x\to -1^-}\ln\left(\frac{2x+1}{x}\right) = \ln\left(\frac{-2+1}{-1}\right) = \ln(1) = \underline{\underline{0}}$$

$$\lim_{x\to 0^+} \exp\left(\lim_{x\to 0^+}\cos(\pi x)\cdot\ln\left(\lim_{x\to 0^+}\ln\left(\frac{2x+1}{x}\right)\right)\right) = \exp(1\cdot\ln(\infty)) = \underline{\underline{\infty}}$$

$$\lim_{x\to 0^+}\ln\left(\frac{2x+1}{x}\right) = \ln\left(\frac{1}{0^+}\right) = \infty$$

4/8-c)

$$\lim_{x\to\infty} \left(\sqrt{x^2+1} - x\right)\cos\sqrt{x^2+1} = \lim_{x\to\infty} \frac{\left(\sqrt{x^2+1}-x\right)\left(\sqrt{x^2+1}+x\right)}{\sqrt{x^2+1}+x}\cos\sqrt{x^2+1} =$$

$$= \lim_{x\to\infty} \frac{x^2+1-x^2}{\sqrt{x^2+1}+x}\cdot\cos\sqrt{x^2+1} = \frac{1}{\underbrace{\sqrt{x^2+1}+x}_{0}}\cdot\cos\sqrt{x^2+1} = \underline{\underline{0}}$$

6/1-f)

$$\lim_{x\to 1} \frac{3\cdot 4^x - 4\cdot 3^x}{5^x - 5\cdot 2^{x-1}} = \left\langle\!\left\langle \frac{12-12}{5-5\cdot 1} \right\rangle\!\right\rangle = \left\langle\!\left\langle \frac{0}{0} \right\rangle\!\right\rangle \overset{L'H}{=} \lim_{x\to 1} \frac{3\cdot 4^x \ln 4 - 4\cdot 3^x \ln 3}{5^x \ln 5 - 5\cdot 2^{x-1}\cdot \ln 2} =$$

$$= \frac{3\cdot 4\cdot \ln 4 - 3\cdot 4\cdot \ln 3}{5\cdot \ln 5 - 5 \ln 2} = \underline{\underline{\frac{12 \ln\left(\frac{4}{3}\right)}{5 \ln\left(\frac{5}{2}\right)}}}$$