# Finding the Best Fit Circle through a Set of Points

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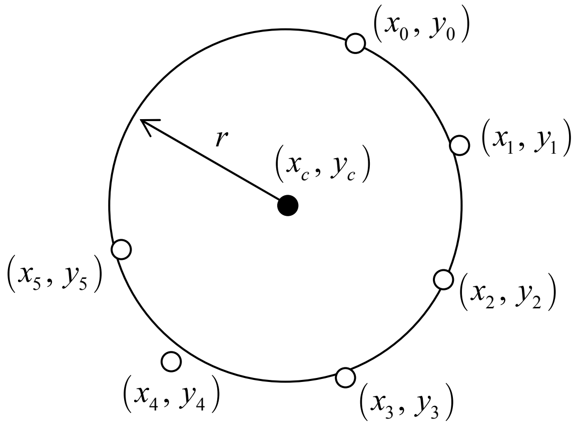
Consider a set of $n$ points on the plane, what circle with radius $r$ and centre $(x_c, y_c)$ best fits these points?




The equation for the circle is given by

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

This may be expanded as follows, with a view to formulating a matrix equation for the three unknowns

$$2x \cdot x_c + 2y \cdot y_c + \left(r^2 - x_c^{\,2} - y_c^{\,2}\right) = x^2 + y^2$$

In matrix form

$$\underbrace{\begin{pmatrix} x_0 & y_0 & 1 \\ \vdots & \vdots & \vdots \\ x_{n-1} & y_{n-1} & 1 \end{pmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} 2x_c \\ 2y_c \\ r^2 - x_c^{\,2} - y_c^{\,2} \end{pmatrix}}_{\mathbf{X}} = \underbrace{\begin{pmatrix} x_0^{\,2} + y_0^{\,2} \\ \vdots \\ x_{n-1}^{\,2} + y_{n-1}^{\,2} \end{pmatrix}}_{\mathbf{B}}$$

A minimum of three points is required to solve this exactly; otherwise (and more generally) the least-squares solution can be found by first multiplying through by the transpose of the coefficient matrix to turn **A** into a 3x3 square matrix, and then solving in the usual manner

$$\left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)\mathbf{X} = \mathbf{A}^{\mathrm{T}}\mathbf{B}$$

$$\mathbf{X} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\left(\mathbf{A}^{\mathrm{T}}\mathbf{B}\right)$$

The circle centre coordinates and radius are obtained from the solution vector **X**

$$(x_c, y_c) = \left(\frac{X_0}{2}, \frac{X_1}{2}\right)$$

$$r = \sqrt{X_3 + x_c^{\,2} + y_c^{\,2}}$$