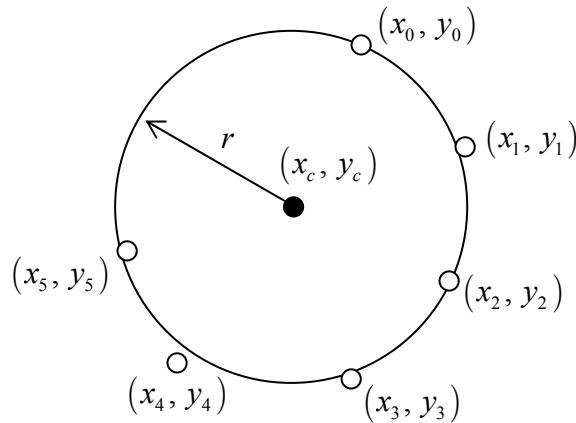


Finding the Best Fit Circle through a Set of Points

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Consider a set of n points on the plane, what circle with radius r and centre (x_c, y_c) best fits these points?



The equation for the circle is given by

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

This may be expanded as follows, with a view to formulating a matrix equation for the three unknowns

$$2x \cdot x_c + 2y \cdot y_c + (r^2 - x_c^2 - y_c^2) = x^2 + y^2$$

In matrix form

$$\underbrace{\begin{pmatrix} x_0 & y_0 & 1 \\ \vdots & \vdots & \vdots \\ x_{n-1} & y_{n-1} & 1 \end{pmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} 2x_c \\ 2y_c \\ r^2 - x_c^2 - y_c^2 \end{pmatrix}}_{\mathbf{X}} = \underbrace{\begin{pmatrix} x_0^2 + y_0^2 \\ \vdots \\ x_{n-1}^2 + y_{n-1}^2 \end{pmatrix}}_{\mathbf{B}}$$

A minimum of three points is required to solve this exactly; otherwise (and more generally) the least-squares solution can be found by first multiplying through by the transpose of the coefficient matrix to turn \mathbf{A} into a 3x3 square matrix, and then solving in the usual manner

$$(\mathbf{A}^T \mathbf{A}) \mathbf{X} = \mathbf{A}^T \mathbf{B}$$

$$\mathbf{X} = (\mathbf{A}^T \mathbf{A})^{-1} (\mathbf{A}^T \mathbf{B})$$

The circle centre coordinates and radius are obtained from the solution vector \mathbf{X}

$$(x_c, y_c) = \left(\frac{X_0}{2}, \frac{X_1}{2} \right)$$
$$r = \sqrt{X_2 + x_c^2 + y_c^2}$$