Discretisation of PID equation:

PN 31/07/2013

$$\varepsilon(t) = \tilde{\omega} - \omega(t)$$

$$V(t) = K_P \left(\varepsilon(t) + \frac{1}{T_I} \int_0^t \varepsilon(\tau)\, d\tau + T_D \frac{d}{dt} \varepsilon(t) \right)$$

Discretise above equation to run in code. Firstly let:

$$\int_0^t \varepsilon(\tau)\, d\tau = \sum_{i=1}^{N} \frac{1}{2} (\varepsilon_i + e_{i-1}) \Delta t$$

$$\frac{d}{dt} \varepsilon(t) = \frac{\varepsilon_i - e_{i-1}}{\Delta t}$$

Then

$$\varepsilon(t) = \tilde{\omega} - \omega(t)$$

$$V_i = K_P \left(\varepsilon_i + \frac{1}{T_I} \sum_{j=1}^{i} \frac{1}{2} \left(\varepsilon_j + e_{j-1} \right) \Delta t_i + T_D \frac{\varepsilon_i - e_{i-1}}{\Delta t_i} \right)$$

Writing out more simply for use in loop:

$$V_i = P_i + I_i + D_i$$

$$P_i = K_P \varepsilon_i$$
$$I_i = I_{i-1} + K_I \left(\varepsilon_i + e_{i-1} \right) \Delta t_i$$
$$D_i = K_D \frac{\varepsilon_i - e_{i-1}}{\Delta t_i}$$

$$K_I = \frac{K_P}{2T_I}$$
$$K_D = K_P T_D$$

Note that in the present application of a motor speed controller ε has units of s^{-1} while V_i is a control voltage, thus K_p must implicitly contain the conversion factor between motor speed and volts having the unit V/RPM .

<u>“Velocity” PID variant (inappropriate for motor control):</u>

$$\frac{d}{dt}V_i = \frac{d}{dt}P_i + \frac{d}{dt}I_i + \frac{d}{dt}D_i$$

To first order:

$$\frac{V_i - V_{i-1}}{\Delta t_i} = \frac{P_i - P_{i-1}}{\Delta t_i} + \frac{I_i - I_{i-1}}{\Delta t_i} + \frac{D_i - D_{i-1}}{\Delta t_i}$$

Cancel dti and solve for Vi:

$$V_i = V_{i-1} + P_i - P_{i-1} + I_i - I_{i-1} + D_i - D_{i-1}$$

Coefficients as above.