CHARACTERISTICS OF ATMOSPHERIC TURBULENCE NEAR THE GROUND

PART I: DEFINITIONS AND GENERAL INFORMATION

1. NOTATION AND UNITS

		SI
$C_{ii}(\tau)$	autocovariance function, $\overline{i(t).\, i(t+\tau)}$ (Section 6.1)	m^2/s^2
$C_{ij}(\underline{r}, \underline{r}'; \tau)$	general cross-covariance function, $\overline{i(\underline{r}; t) j(\underline{r}'; t+\tau)}$ (Section 6.2)	m^2/s^2
$i(\underline{r}; t)$	general value of u, v or w gust component at point $\underline{r}$ and time t where $i = u, v$ or w	m/s
$j(\underline{r}'; t+\tau)$	general value of u, v or w gust component at point $\underline{r}'$ and time $t+\tau$ where $j = u, v$ or w	m/s
^xL_i, ^yL_i, ^zL_i	length scales of turbulence of u, v or w components measured along x, y or z axes respectively	m
n	frequency	Hz
P; $p(i)$	probability and probability density (see Section 4)	
$P_{ij}(\underline{r}, \underline{r}'; n)$	co-spectral function, in-phase component of $S_{ij}(\underline{r}, \underline{r}'; n)$	m^2/s
$Q_{ij}(\underline{r}, \underline{r}'; n)$	quad-spectral function, out-of-phase component of $S_{ij}(\underline{r}, \underline{r}'; n)$	m^2/s
$\underline{r}$; $\underline{r}'$	position vectors denoting points (x, y, z) and (x', y', z') respectively	
$S_{ii}(n)$	power spectral density function (Equation (7.3), Section 7.1)	m^2/s
$S_{ii}(\underline{r}, \underline{r}'; n)$	general cross-spectral density function (Section 7.2)	m^2/s
T	averaging time	s
T_i	time scales of u, v or w components	s
t	time	s
$u(t)$, $v(t)$, $w(t)$	fluctuating components of wind speed along x, y and z axes respectively	m/s

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$\bar{V}_{\tilde{z}}$	mean wind speed at height $\tilde{z}$	m/s
x, y, z	system of rectangular cartesian co-ordinates with x-axis defined in direction of mean wind	
$\tilde{z}$	effective height above ground	m
$\gamma_{ij}^2(\underline{r}, \underline{r}'; n)$	general coherence function (Equation (7.15))	
$\theta_{ij}(\underline{r}, \underline{r}'; n)$	phase shift angle (Equation (7.14))	rad
$\rho_{ii}(\tau)$	autocorrelation function, $C_{ii}(\tau)/\sigma_i^2$	
$\rho_{ij}(\underline{r}, \underline{r}'; \tau)$	general cross-correlation function, $C_{ij}(\underline{r}, \underline{r}'; \tau)/(\sigma_i\sigma_j')$	
σ_i	standard derivation of u, v or w component, $[\overline{i(t)^2}]^{½}$	m/s
τ	incremental time lag	s
ω	circular frequency, $2\pi n$	rad/s

Suffixes

i, j	refers to u, v or w component
A prime ($'$)	denotes a value at the point (x', y', z').
A bar ($^{-}$)	denotes a value averaged over time T.

The values in round brackets ( ) after a functional symbol denote the dependent variables; e.g. $S_{ij}(\underline{r}, \underline{r}'; n) = S_{uu}(z, z'; n)$ is the frequency (n) dependent cross-spectral density of the u component of turbulence measured at points separated vertically.

2. INTRODUCTION

The purpose of this Item is to provide background information concerning a series of Items giving data on the properties of turbulence in the atmospheric boundary layer. Because atmospheric turbulence is continually changing in time and space it is necessary to describe its properties in statistical terms. This Item provides a definition of the parameters used and, where possible, an explanation of their physical significance. Further, more detailed, information on the mathematical aspects of statistical analysis can be found in standard text books on the subject such as References 2 and 5.

No attempt is made in this Item to explain in detail how the data are applied in practice since this will depend on the nature of the problem, of which there are many types. Reference 3 provides information on how the data can be applied to particular cases of wind loading on ground-based structures and References 4 and 6 discuss appropriate means of applying the data to the study of aircraft response to atmospheric turbulence.

3. BACKGROUND

3.1 Origin of Turbulence in the Earth's Boundary Layer

At a considerable distance above the earth's surface wind is created, in the first instance, by differential heating of the atmosphere producing pressure gradients which are subsequently modified by the rotation of the earth.

As with the flow of any fluid over a surface, a boundary layer is formed over the earth's surface, called the atmospheric boundary layer, in which the wind speed decreases from a maximum value at the top of the layer to zero at the earth's surface. This reduction in velocity is due to both the frictional drag of the surfaces and the drag of all bodies protruding into the air flow (such as trees, mountains and buildings). These retarding forces are transmitted through the layer by shear forces (Reynolds stresses) and by the exchange of momentum due to the vertical movement of air. The process of momentum exchange between layers is the mechanism leading to the generation, and decay, of eddies termed turbulence. The resulting mixing of the air produces, along all three orthogonal axes, fluctuations in wind speed, commonly called gusts, which vary in size in both time and space.

3.2 Significance of Turbulence

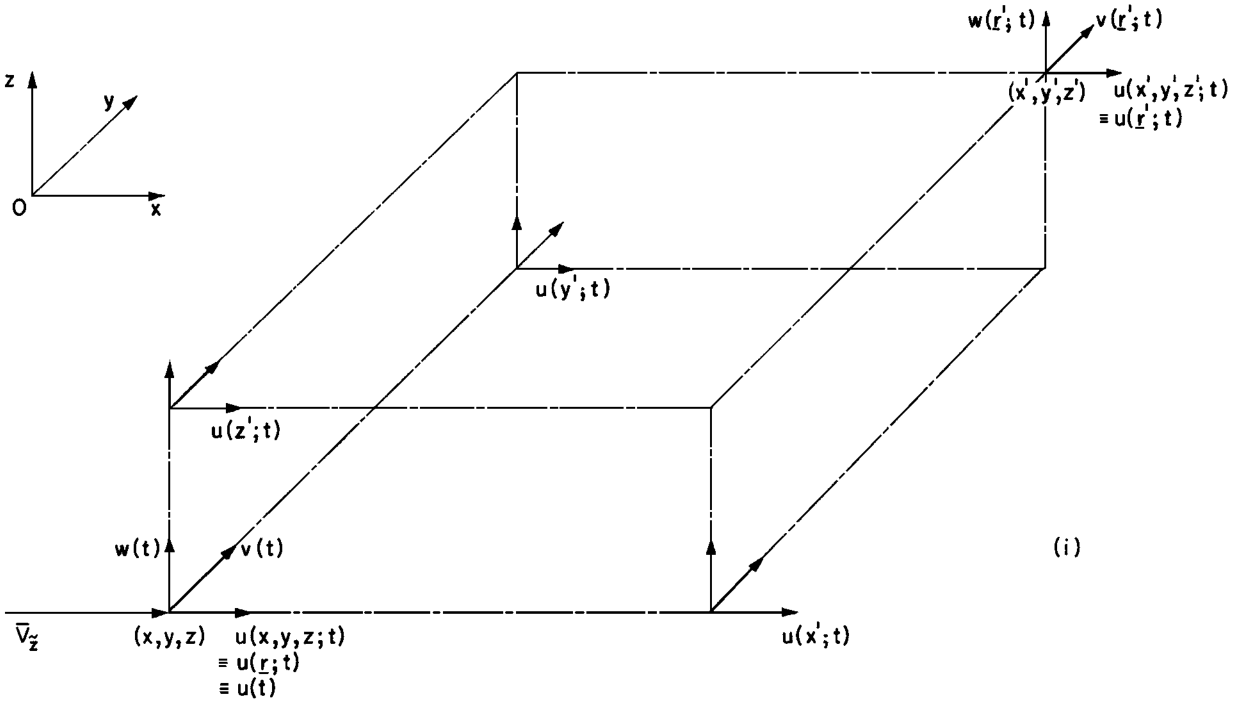
The necessity to consider the turbulent nature of the atmosphere arises for two separate reasons.

(i) The points of separation and reattachment of the flow on bluff bodies are affected to a large extent by the turbulence in the approaching air stream and consequently the pressure in the wake of the body is also affected. Thus even the quasi-static forces and moments acting on a body can be appreciably influenced by the turbulence in the approaching air stream.

(ii) Because of the variations of velocity in turbulence, dynamic forces, in addition to the quasi-static ones, are exerted on a body immersed in a turbulent flow. These fluctuating forces gain additional importance if the body is able to respond dynamically to the excitation. In the case of large structures in the atmosphere, it is not enough to know how the fluctuating velocities vary with time; the way they vary in space is also important. This means that a complete statistical description of the atmospheric wind is required.

3.3 The Turbulence Field

Consider the turbulence field as defined by Sketch 3.1. The flow is moving with a mean velocity $\overline{V}$ along the x-axis and $u(t)$, $v(t)$ and $w(t)$ are the instantaneous longitudinal, lateral and vertical components of fluctuating velocities.



Sketch 3.1

The mean of any gust component is zero since there is only mean motion in the x-direction. The flow velocity at any point in space is thus a vector quantity which has a magnitude and direction which both change rapidly with time. This vector quantity has a magnitude

$$\left[\{\overline{V} + u(t)\}^2 + v^2(t) + w^2(t)\right]^{1/2} \tag{3.1}$$

where

$$\overline{V} = \frac{1}{T}\int_0^T V(t)\,\mathrm{d}t\,. \tag{3.2}$$

The interval T is defined as the averaging time and the value of $\overline{V}$ defined by Equation (3.2) is called the value of wind speed averaged over time T. Ideally, T should be chosen large enough so that the same value of $\overline{V}$ would be calculated for any value of T larger than that chosen. In measurements in the atmosphere, due to the development or decay of weather systems or the passage of fronts, this ideal situation is rarely achieved. Many calculations of a quasi-static nature are performed using wind speeds averaged over short periods of the order of seconds. Data to assist these calculations are presented in ESDU 82026.

The three gust components $u(t)$, $v(t)$ and $w(t)$ at a given point are random functions of time (t) and also vary in space as illustrated in Sketch 3.1. However, there is usually some interrelation between measurements of a gust component at a point taken at different time intervals and also between gust components measured at two points in space. The interrelation, or correlation, decreases as the time lag or separation distance between measurements increases.

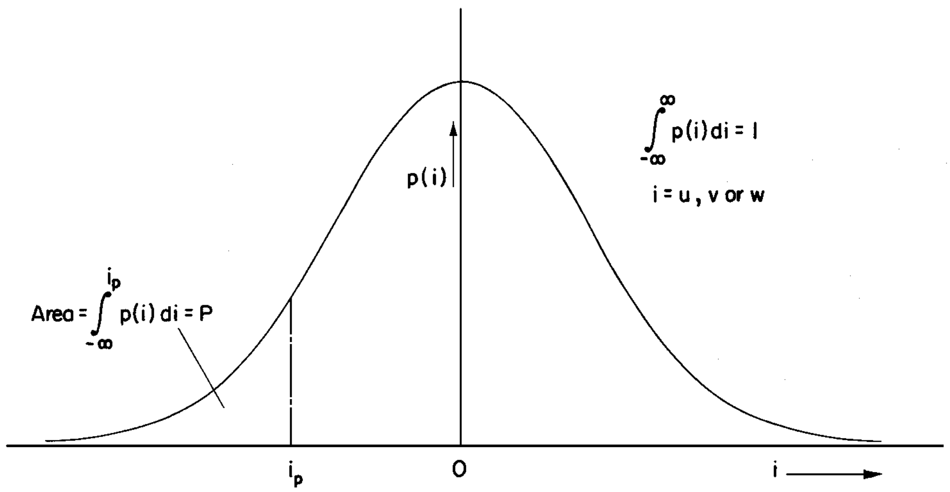
It is for these reasons that in calculating, for example, the response of large structures in the wind it is necessary to take into account the correlation between gust components at difference points on the structure to allow for the non-uniform action of gusts in both a time and spatial sense. For ground-based structures the properties of the longitudinal component are the most important but the lateral component does contribute to lateral forces and the vertical component is important in the context of lift forces on bridge decks. For aircraft the properties of the longitudinal and vertical components are of particular importance.

Two kinds of correlations can be made. In the first, measurements of the components of velocity are made at a single point and these can be related to themselves or one another, either simultaneously or at different times. In the second kind, measurements are made at two different locations (two-point measurements) and these again can be related with or without time delays.

In both kinds, to present a full quantification of the phenomena, data have to be presented in the three domains of amplitude, time and frequency. Special names are allocated to each quantity and the relationships between these quantities are outlined graphically in Figure 1. In particular, information about the amplitude of the velocity components is discussed in terms of probability density distribution in Section 4 and turbulence intensity in Section 5; information about the time domain is presented as correlation functions in Section 6 and about the frequency domain in Section 7.

4. PROBABILITY DENSITY DISTRIBUTION

A probability density can be obtained in the following way. The instantaneous value $i(t)$ of a fluctuating velocity is recorded a large number of times over a period realising a large number of individual readings of differing values (i). These readings (N_0 in total) are organised in ascending order and a series of 'cells' then obtained which contain a number (N) of readings in a narrow band of values between i and $i + \delta i$. In practice the probability density $p(i)$ is then given by (for small δi) the ratio $N/(N_0 \delta i)$. A probability density function for all values of i can then be defined as in Sketch 4.1.



Sketch 4.1 Probability density function

The probability (P) that any value i will be less than i_p is given by

$$P = \int_{-\infty}^{i_p} p(i)\,\mathrm{d}i\,, \tag{4.1}$$

i.e., P is the proportion of all values having values not greater than i_p. The quantity $(1-P)$ is the probability that the value i will be greater than i_p. The probability density function p therefore demonstrates the likelihood of a given amplitude of the signal.

The term ‘percentile’ is often used to define the value of i below which the given percentage $(100P)$ of the data lies.

If the integral

$$\int_{-\infty}^{+\infty} i.p(i)\mathrm{d}i$$

is evaluated, it will equal the mean value $\bar{i}$ which is zero. Thus, the first moment of area about the $i = 0$ axis will be zero. The second moment of area about the $i = 0$ axis gives the variance (standard deviation squared)

$$\sigma_i^2 = \int_{-\infty}^{\infty} i^2.p(i)\mathrm{d}i\,. \tag{4.2}$$

Since the first two moments yield useful quantities, it is conventional in statistical analysis to proceed further and consider the k^{th} moment where

$$m_k = \int_{-\infty}^{\infty} i^k.p(i)\mathrm{d}i\,. \tag{4.3}$$

In practice only the next two moments are used; the third moment divided by σ_i^3 to produce a dimensionless quantity is called the ‘skewness’ and the fourth moment (divided by σ_i^4) is called the ‘kurtosis’. These are often used to characterise departures of real turbulence properties from those given by the ‘normal’ or Gaussian probability density function (see Item No. 85020) for which $m_3/\sigma_i^3 = 0$ and $m_4/\sigma_i^4 = 3$.

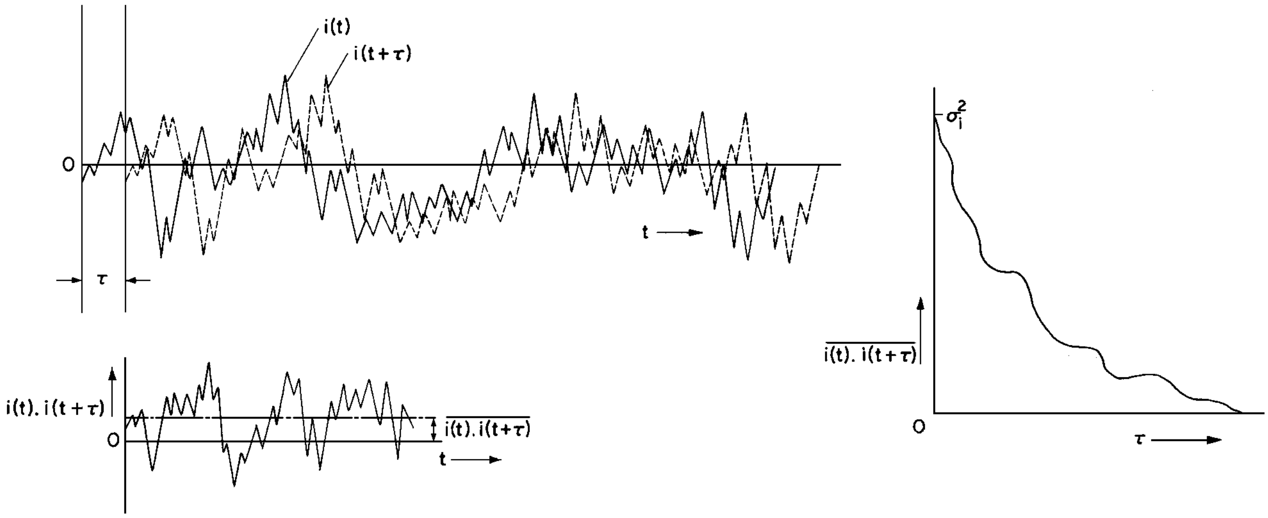
5. TURBULENCE INTENSITY

Turbulence intensity is a measure of the magnitude of turbulence fluctuations. It is defined as the ratio of the standard deviation of the instantaneous fluctuating velocity component to the mean wind speed averaged over, say, 1 hour, i.e., $\sigma_u/\bar{V}_{\tilde{z}}$, $\sigma_v/\bar{V}_{\tilde{z}}$, or $\sigma_w/\bar{V}_{\tilde{z}}$.

6. COVARIANCE AND CORRELATION FUNCTIONS

A covariance function is the mean product of fluctuating velocity components measured at one or more points in space either simultaneously or with a time lag between them. In particular, covariance functions may be formed from measurements at (i) a single point or (ii) at two points in space. In the first case the function provides information on the extent of eddies or gusts in a time sense. In the second case the function yields information about the relationship between turbulent characteristics in different regions of a turbulent flow field in both a spatial and a time sense.

6.1 Correlations of Measurements at One Point



Sketch 6.1 Autocovariance function

A covariance function is formed by the mean product of two fluctuating velocity components measured at the same point but at times t and $(t+\tau)$, i.e.,

$$C_{ij}(\tau) = \overline{i(t).j(t+\tau)} = \lim T \to \infty \frac{1}{T}\int_0^T i(t).j(t+\tau)\mathrm{d}t \tag{6.1}$$

where $i, j = u, v$ or w. There are three velocity components giving 9 functions in all but the most important of these are the autocovariance functions given by

$$C_{uu}(\tau) = \overline{u(t).u(t+\tau)}, \quad C_{vv}(\tau) = \overline{v(t).v(t+\tau)}, \quad C_{ww}(\tau) = \overline{w(t).w(t+\tau)}, \tag{6.2a}$$

and the cross-covariance function given by

$$C_{uw}(\tau) = \overline{u(t).w(t+\tau)}. \tag{6.2b}$$

When the time lag, τ, is zero then the autocovariance functions reduce to the variances, denoted by σ_u^2, σ_v^2 and σ_w^2. The fourth function reduces to $\overline{u(t)w(t)}$ which gives the Reynolds stress $(= -\overline{u(t)w(t)}$. air density). It is usual to 'normalise' the covariance functions by dividing by the appropriate standard deviations of the constituent velocity components to form correlation functions, i.e.,

$$\rho_{ij}(\tau) = C_{ij}(\tau)/(\sigma_i\sigma_j), \tag{6.3}$$

so that, for example,

$$\rho_{uu}(\tau) = \overline{u(t).u(t+\tau)}/\sigma_u^2,$$

and $\rho_{uu}(\tau)$, $\rho_{vv}(\tau)$, and $\rho_{ww}(\tau)$ and called autocorrelation functions.

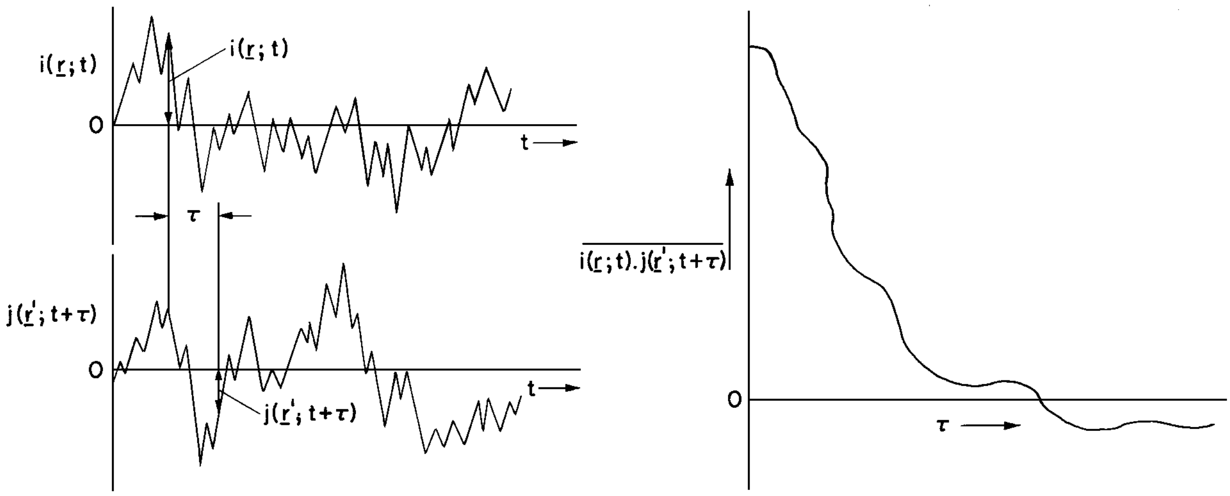
For small values of τ, $\rho_{uu}(\tau)$ for example is close to unity since the variation of the two values of the velocity component with time will be in phase. As the interval τ is increased the value of $\rho_{uu}(\tau)$ will decrease until for large values of τ it will tend to zero since there will be a decreasing relationship, or correlation, between the two values particularly when they are separated by long periods. In general, it can be stated that when τ is much larger than a typical time scale of turbulence (see Section 6.3) then the correlation between the gust components will be insignificant.

6.2 Correlations of Measurements at Two Points

A spatial covariance function is formed by multiplying together pairs of values of velocity components measured simultaneously at two separated points and taking the time-averaged value, e.g.

$$C_{uu}(x' - x) = \overline{u(x, y, z).u(x', y, z)} .$$

As the distance $x' - x$ is increased the value of the function $C_{uu}(x' - x)$ decreases and tends to zero for large values of $x' - x$ since there will be little or no correlation between velocity components measured at two points separated by a large distance.



Sketch 6.2 General cross-covariance function

In addition, spatial correlations can be evaluated between one component at a point (x, y, z) and another component at a different point (x', y', z') measured at some time interval (τ) later. This general cross-covariance function (Sketch 6.2) is expressed in the form

$$C_{ij}(\underline{r}, \underline{r}'; \tau) = \overline{i(x, y, z; t).j(x', y', z'; t + \tau)} \text{ (for } i, j = u, v \text{ or } w) \tag{6.4}$$

where for notational convenience $\underline{r}$ and $\underline{r}'$ denote the position vectors of the two points in question. (Note that negative values of the covariance and cross-variance functions can occur because the fluctuating velocity components forming the correlations can be of opposite sign so that their product, and mean product, can become negative.)

Again, the cross-covariance functions are usually normalised by dividing by the standard deviations of the constituent components to form cross-correlation functions, i.e.,

$$\rho_{ij}(\underline{r}, \underline{r}'; \tau) = \frac{C_{ij}(\underline{r}, \underline{r}'; \tau)}{\sigma_i \sigma_j} \tag{6.5}$$

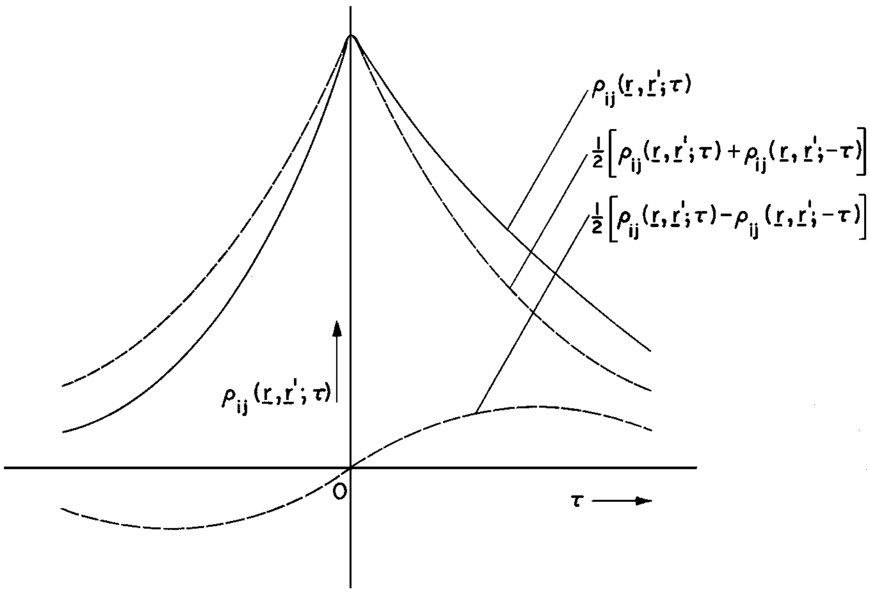
so that, for example, for the two points (x, y, z) and (x, y, z') separated vertically

$$\rho_{uu}(z, z'; \tau) = \overline{u(x, y, z; t).u(x, y, z'; t+\tau)}/(\sigma_u \ \sigma'_u).$$

Whilst the autocorrelations are symmetrical functions of τ, i.e., $\rho_{ii}(\tau) = \rho_{ii}(-\tau)$, the cross correlations are, in general, non-symmetrical functions of τ. In this case it is convenient to re-write the cross correlations in the form

$$\rho_{ij}(\underline{r}, \underline{r}'; \tau) = \frac{1}{2}\left[\rho_{ij}(\underline{r}, \underline{r}'; \tau) + \rho_{ij}(\underline{r}, \underline{r}'; -\tau)\right] + \frac{1}{2}\left[\rho_{ij}(\underline{r}, \underline{r}'; \tau) - \rho_{ij}(\underline{r}, \underline{r}'; -\tau)\right] \tag{6.6}$$

so that the first half of the right-hand side of Equation (6.6) is a symmetrical function of τ and the second half is an antisymmetrical function of τ (see Sketch 6.3). This feature is used in determining the in-phase and out-of-phase parts of the corresponding cross-spectra as discussed in Section 7.2.



Sketch 6.3 Symmetrical and antisymmetrical parts of cross-correlation function

In general the zero lag $(\tau = 0)$ correlations $\rho_{uu}(x'-x)$, $\rho_{vv}(y'-y)$ and $\rho_{ww}(z'-z)$ are called longitudinal correlation functions since the direction of the velocity components being correlated at two points is parallel to the line joining the two points. Conversely, the correlation functions $\rho_{uu}(y'-y)$, $\rho_{uu}(z'-z)$, $\rho_{vv}(x'-x)$, $\rho_{vv}(z'-z)$, $\rho_{ww}(x'-x)$ and $\rho_{ww}(y'-y)$ are called lateral correlations. In isotropic* turbulence all the longitudinal correlation functions are the same and so are the lateral correlation functions. However, in the Earth's boundary layer, particularly near the ground, the condition of true isotropy does not exist and in general all the correlation functions are different.

* For isotropic turbulence the statistical properties do not change when the reference coordinate axes are rotated, i.e. the properties are independent of direction in the turbulence field.

6.3 Length and Time Scales of Turbulence

The length scales of turbulence are useful quantities because they are a comparative measure of the average size of a gust in appropriate directions. Similarly, the time scales characterise the average duration of the effect of a gust at a point. These scales are obtained by integrating the appropriate spatial or autocorrelation functions over the complete range of the appropriate spatial or time variable.

There are nine length scales defined by (for $i = u$, v and w)

$$\left.\begin{aligned} {}^{x}L_i &= \int_0^{\infty} \rho_{ii}(x'-x)\,\mathrm{d}(x'-x)\,, \\ {}^{y}L_i &= \int_0^{\infty} \rho_{ii}(y'-y)\,\mathrm{d}(y'-y)\,, \\ {}^{z}L_i &= \int_0^{\infty} \rho_{ii}(z'-z)\,\mathrm{d}(z'-z)\,, \end{aligned}\right\} \tag{6.7}$$

and three time scales defined by

$$T_i = \int_0^{\infty} \rho_{ii}(\tau)\,\mathrm{d}\tau\,. \tag{6.8}$$

The length scales ${}^{x}L_u$, ${}^{y}L_v$ and ${}^{z}L_w$ are called longitudinal scales because they are derived from the longitudinal correlation functions (see Section 6.2) and the remaining scales are called lateral scales after the lateral correlation functions from which they are derived.

6.4 Taylor's Hypothesis

Taylor's Hypothesis implies that provided $\overline{V}_{\tilde{z}}$ is much greater than $u(t)$ then the turbulence field can be considered to be frozen in space and convected past a point with velocity $\overline{V}_{\tilde{z}}$. Thus the variation of $u(t)$ with time when the turbulence field is viewed from a stationary point is the same as the variation observed from the point moving with velocity $\overline{V}_{\tilde{z}}$ across the 'frozen' field in the x-direction.

In general, Taylor's Hypothesis can be applied to atmospheric turbulence except at very low frequencies (large wavelengths). Data from Reference 1 suggest that it can be applied at least for wavelengths ($\overline{V}_{\tilde{z}}/n$) less than about 300m.

Assuming Taylor's Hypothesis applies, spatial separations in the along-wind direction can be related to an equivalent time lag so that, for example,

$$\rho_{uu}(x'-x) = \rho_{uu}(\overline{V}_{\tilde{z}}\tau) \text{ and } {}^{x}L_i = T_i\overline{V}_{\tilde{z}}\,. \tag{6.9}$$

7. SPECTRAL DENSITY FUNCTIONS

It is often more convenient to work in the frequency domain rather than the time domain. In this context spectral functions of atmospheric turbulence are appropriate because they provide information on the frequency distribution of the kinetic energy of the various fluctuating velocity components. Used in conjunction with certain transfer functions (mechanical and aerodynamic admittance – see Reference 3) they provide information about the dynamic loading on, and response of, building and aircraft structures in the atmospheric wind.

Fourier analysis enables the amplitudes of a time-varying process to be determined as a function of frequency. Thus power spectral density functions and the corresponding covariance functions (Section 6) can be expressed as Fourier transforms of each other (see Sections 7.1 and 7.2).

The one-dimensional power spectral density functions $S_{ii}(n)$ (the Fourier transform of the autocovariance functions $C_{ii}(\tau)$) provide information on the energy input from different frequency ranges at a single point. Such information is useful in the calculation of wind-loading on, or in response of, structures that are small compared with the scale of turbulence. However, many structures are large enough for appreciable variation of the gust properties to take place over the structure in both the vertical and cross-wind directions. In this case a knowledge of the cross-spectral density functions $S_{ij}(\underline{r}, \underline{r}'; n)$ (the Fourier transform of the cross-covariance functions $C_{ij}(\underline{r}, \underline{r}'; \tau)$) is essential. This gives information about the spatial variation of gust energies at individual frequencies. It is important in, for example, the calculation of the dynamic wind loading due to horizontal gusts along the span of a large bridge or up the face of a high-rise building, or in the calculation of the rolling response due to variations in vertical gust velocities across the wing span of a large aircraft.

7.1 One-dimensional Power Spectral Density

Each fluctuating velocity component can be regarded as being compounded of oscillations of cosine and sine form of varying amplitude and frequency, and in the general case can be represented by the sum of a Fourier cosine and sine series. A power spectral density function, e.g. $S_{ii}(n)$ (see Sketch 7.1), may be defined so that the total energy, or variance, associated with each gust component over the frequency range $0 \le n < \infty$ can be represented by

$$\sigma_i^2 = \int_0^\infty S_{ii}(n)\,\mathrm{d}n \text{ (for } i = u, v \text{ or } w). \tag{7.1}$$

The quantity $S_{ii}(n).\delta n$ is a measure of the energy associated with that component over the narrow frequency band between n and $n + \delta n$. In practice, one way of obtaining a power spectral density $S_{ii}(n)$ at frequency n is as follows. A signal $i(t)$ is put through a narrow band-pass filter so that only those parts of the signal $i(t)$ corresponding to a frequency bandwidth of δn centred about frequency n remain; the average mean-square of the filtered signal $i(t; n; \delta n)$ is then given by

$$\frac{1}{T_0}\int_0^{T_0} i^2(t; n, \delta n)\,\mathrm{d}t$$

and the spectral power density at frequency n is defined as

$$S_{ii}(n) = \lim_{\substack{T_0 \to \infty \\ \delta n \to 0}} \frac{1}{T_0 \delta n} \int_0^{T_0} i^2(t; n, \delta n) \mathrm{d}t . \tag{7.2}$$

The power spectral density function can also be obtained from the autocovariance function (and vice versa) using a Fourier transform, i.e.

$$S_{ii}(n) = 2 \int_{-\infty}^{\infty} C_{ii}(\tau) e^{-(\sqrt{-1}\, 2\pi n \tau)} \mathrm{d}\tau \tag{7.3}$$

and
$$C_{ii}(\tau) = \int_0^{\infty} S_{ii}(n) e^{(\sqrt{-1}\, 2\pi n \tau)} \mathrm{d}n \tag{7.4}$$

where, for a stationary process*, $S_{ii}(n)$ is defined as a 'single-sided' function (because it contains only positive frequencies in the range $0 \le n < \infty$) and is twice the magnitude of the equivalent symmetrical 'double-sided' function $S_{ii}^{\star}(n)$ (containing frequencies in the range $-\infty < n < \infty$) for which $S_{ii}^{\star}(n) = S_{ii}^{\star}(-n)$. In practice it is values of $S_{ii}(n)$ that are derived.

The assumption of stationarity means that the autocovariance functions are also real and even functions of τ (i.e. $C_{ii}(\tau) = C_{ii}(-\tau)$) so that Equations (7.3) and (7.4) reduce to

$$S_{ii}(n) = 4 \int_0^{\infty} C_{ii}(\tau) \cos(2\pi n \tau) \mathrm{d}\tau \tag{7.5}$$

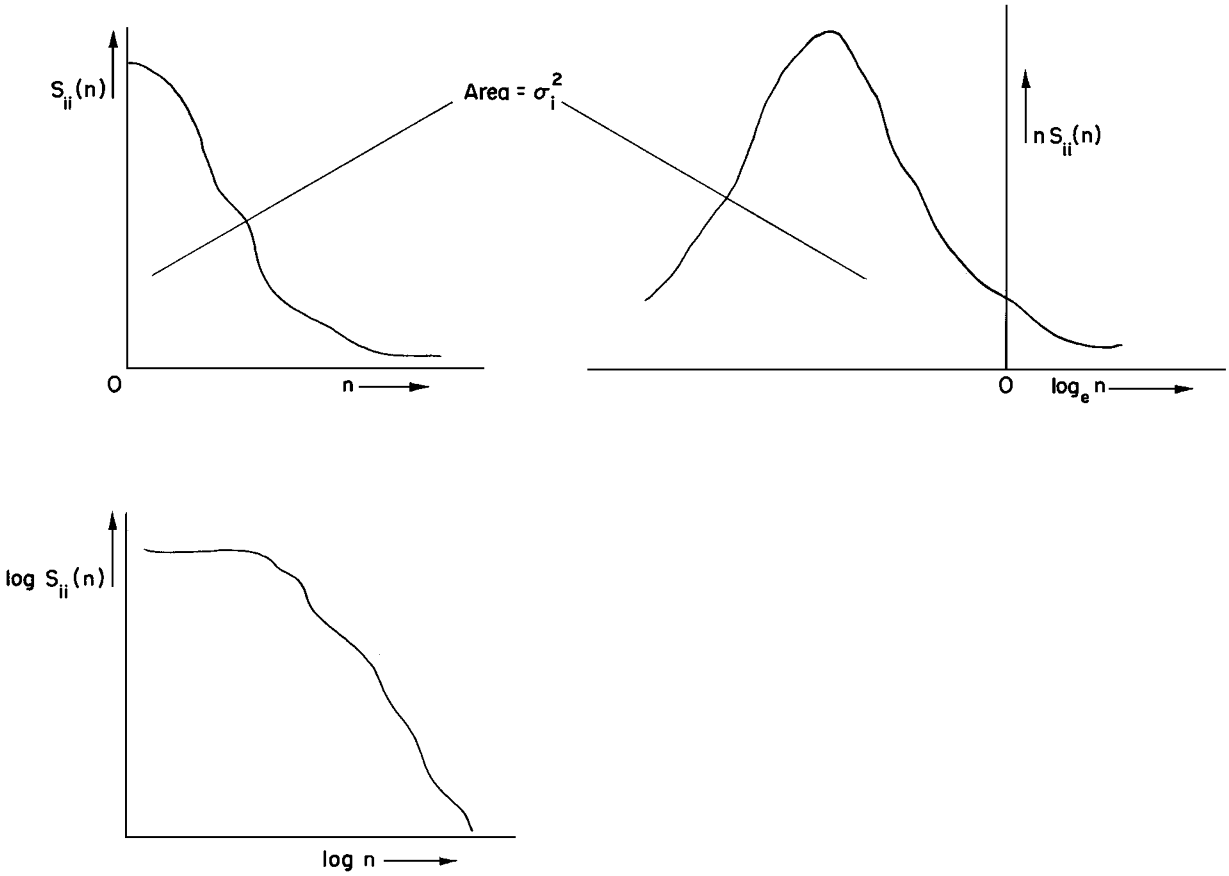
and
$$C_{ii}(\tau) = \int_0^{\infty} S_{ii}(n) \cos(2\pi n \tau) \mathrm{d}n . \tag{7.6}$$

For $\tau = 0$,
$$C_{ii}(\tau) = \int_0^{\infty} S_{ii}(n) \mathrm{d}n = \sigma_i^2 . \tag{7.7}$$

Thus, if power spectra are presented in the form $S_{ii}(n)$ plotted against n the area under the curve is σ_i^2. Power spectra are often given in the normalised form $nS_{ii}(n)/\sigma_i^2$ plotted against $\log_e n$ or $\log_e (n/\bar{V}_{\tilde{z}})$ when the area under the curve is unity, i.e.

$$\frac{1}{\sigma_i^2} \int_{-\infty}^{\infty} n S_{ii}(n) \mathrm{d}(\log_e n) = 1 . \tag{7.8}$$

* A stationary process is defined as one in which the statistical properties of the fluctuating velocity components at a given point are independent of time.



Sketch 7.1 Various presentations of the power spectral density function

7.2 Cross-spectral Density, Coherence

Cross-spectra are derived from Fourier transforms of the cross-covariance functions (Section 6.2), i.e., the single-sided cross-spectral density function for $0 \le n < \infty$ is given by:

$$S_{ij}(\underline{r}, \underline{r}'; n) = 2 \int_{-\infty}^{\infty} C_{ij}(\underline{r}, \underline{r}'; \tau) e^{-(\sqrt{-1}\, 2\pi n \tau)} \mathrm{d}\tau . \tag{7.9}$$

In general, the cross-covariance functions are not symmetrical functions of τ (see Equation (6.6) and Sketch 6.3). Thus when Equation (6.6) is substituted into Equation (7.9) it follows that the cross-spectrum is a complex quantity which can be expressed as the sum of real (in-phase) and imaginary (out-of-phase) components, i.e.

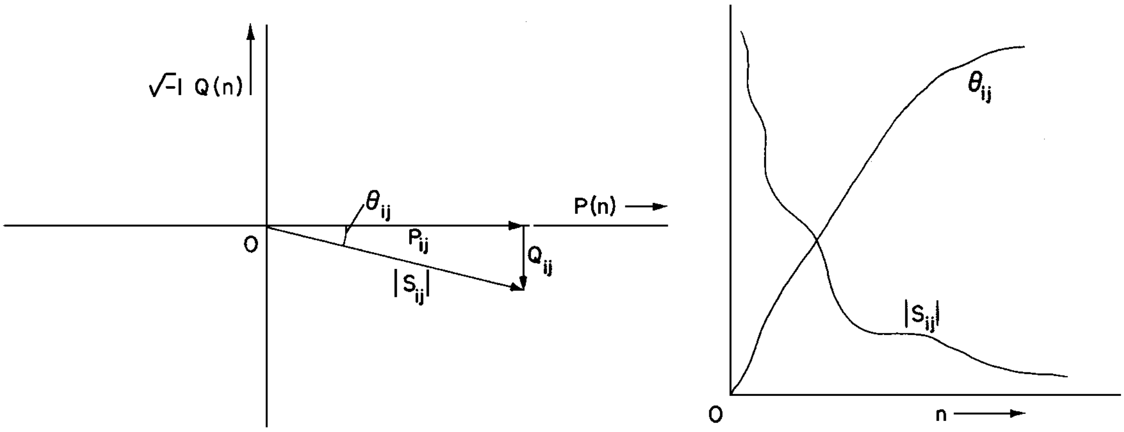
$$S_{ij}(\underline{r}, \underline{r}'; n) = P_{ij}(\underline{r}, \underline{r}'; n) - \sqrt{-1}\, Q_{ij}(\underline{r}, \underline{r}'; n) . \tag{7.10}$$

In Equation (7.10), $\sqrt{-1}$ indicates a 90° phase shift between the components (see Sketch 7.2) which are the Fourier transforms of the symmetrical and anti-symmetrical parts of Equation (6.6), i.e.,

$$P_{ij}(\underline{r}, \underline{r}'; n) = 2 \int_0^\infty [C_{ij}(\underline{r}, \underline{r}'; \tau) + C_{ij}(\underline{r}, \underline{r}'; -\tau)] \cos(2\pi n\tau) d\tau, \tag{7.11}$$

$$Q_{ij}(\underline{r}, \underline{r}'; n) = 2 \int_0^\infty [C_{ij}(\underline{r}, \underline{r}'; \tau) - C_{ij}(\underline{r}, \underline{r}'; \tau)] \sin(2\pi n\tau) d\tau \tag{7.12}$$

The function $P_{ij}(\underline{r}, \underline{r}'; n)$ is called the co-spectral density function and $Q_{ij}(\underline{r}, \underline{r}'; n)$ is called the quad-spectral density function. They are real-valued symmetrical and antisymmetrical functions of n although, as used in Equations (7.11) and (7.12), they are single-sided functions in the range $0 \le n < \infty$.



Sketch 7.2 Component parts of the complex cross-spectral density function

Because the cross-spectral density function is a complex quantity it can be expressed as a function of a modulus and a phase shift, $\theta_{ij}(\underline{r}, \underline{r}'; n)$, so that (see Sketch 7.2)

$$|S_{ij}(\underline{r}, \underline{r}'; n)|^2 = P_{ij}^2(\underline{r}, \underline{r}'; n) + Q_{ij}^2(\underline{r}, \underline{r}'; n) \tag{7.13}$$

and

$$\theta_{ij}(\underline{r}, \underline{r}'; n) = \tan^{-1} \frac{Q_{ij}(\underline{r}, \underline{r}'; n)}{P_{ij}(\underline{r}, \underline{r}'; n)}. \tag{7.14}$$

The normalised cross-spectral density is formed by dividing $|S_{ij}(\underline{r}, \underline{r}'; n)|^2$ by the product of the power spectra of the constituent components at the two points $\underline{r}$ and $\underline{r}'$, i.e.

$$\frac{|S_{ij}(\underline{r}, \underline{r}'; n)|^2}{S_{ii}(n) S'_{jj}(n)} = \gamma_{ij}^2(\underline{r}, \underline{r}'; n) \tag{7.15}$$

and $\gamma_{ij}^2(\underline{r}, \underline{r}'; n)$ is called the coherence.

Physically, a value of $S_{ij}(\underline{r}, \underline{r}'; n)$ at a frequency n can be thought of as being derived from the cross correlation (or mean product) with zero time lag of the identically filtered signals $i(t; n; \delta n)$ and $j(t; n; \delta n)$ at points $\underline{r}$ and $\underline{r}'$ in an analogous way to that described in Section 7.1 for $S_{ii}(n)$. Other values of $S_{ij}(\underline{r}, \underline{r}'; n)$ can then be obtained by repeating this process for other pairs of points $\underline{r}$ and $\underline{r}'$ and for other frequencies. The normalised cross-spectral (or coherence) function for a given frequency can thus be considered to give a measure of the spatial scale of turbulence associated with that frequency, analogous to the length scales of turbulence defined in Section 6.3.

7.3 Change of Independent Variable

Sometimes it is convenient to express the spectral density functions using $n/\bar{V}_{\tilde{z}}$ or $\omega(= 2\pi n)$ as the independent variable, i.e. using a spectral density function defined by $\Phi_{ij}(n/\bar{V}_{\tilde{z}})$ or $\Psi_{ij}(\omega)$. These functions are related to each other and in the general case, for $i, j = u, v$ or w,

$$\int_0^\infty S_{ij}(n)\mathrm{d}n = \int_0^\infty \Phi_{ij}(n/\bar{V}_{\tilde{z}})\mathrm{d}(n/\bar{V}_{\tilde{z}}) = \int_0^\infty \Psi_{ij}(\omega)\mathrm{d}\omega \qquad (7.16)$$

so that

$$nS_{ij}(n) = (n/\bar{V}_{\tilde{z}})\Phi_{ij}(n/\bar{V}_{\tilde{z}}) = \omega\Psi_{ij}(\omega) . \qquad (7.17)$$

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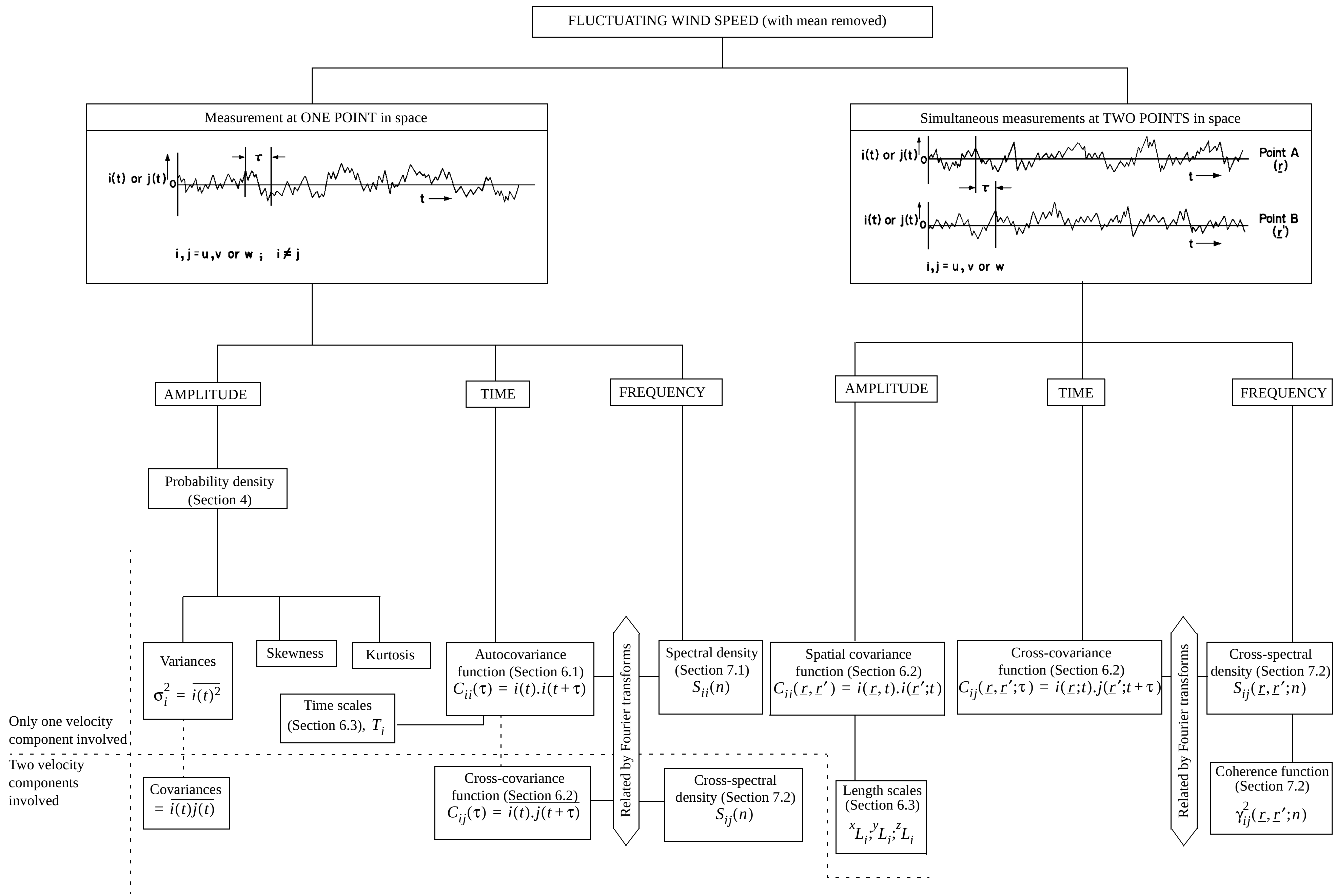

FIGURE 1 SUMMARY OF RELATIONSHIPS

THE PREPARATION OF THIS DATA ITEM

The work on this particular Item was monitored and guided by the following Working Party:

Dr K.J. Eaton	–	Building Research Establishment
Mr R.I. Harris	–	Cranfield Institute of Technology
Mr J.G. Jones	–	Royal Aircraft Establishment
Mr T.V. Lawson	–	University of Bristol
Mr J.F. Woods	–	Royal Aircraft Establishment

on behalf of the Fluid Mechanics Steering Group which first met in 1970 aned now has the following membership:

Chairman		
Mr T.V. Lawson	–	University of Bristol
Members		
Mr K.C. Anthony	–	Ove Arup and Partners
Mr E.C. Firman	–	Central Electricity Research Laboratories
Mr B.H. Fisher	–	Consulting Structural Engineer
Mr L.C. Hickman	–	Kellogg International Corporation
Dr G. Hobson	–	GEC Turbine Generators Ltd
Mr J.R.C. Pedersen	–	Independent
Mr C. Scruton	–	Independent
Mr N.G. Worley	–	Babcock and Wilcox (Operations) Ltd.

The Item was accepted for inclusion in the Aerodynamics Sub-series by the Aerodynamics Committee which first met in 1942 and now has the following membership

Chairman		
Mr E.C. Carter	–	Aircraft Research Association
Vice-Chairman		
Prof. G.M. Lilley	–	University of Southampton
Members		
Prof. L.F. Crabtree	–	University of Bristol
Mr R.L. Dommett	–	Royal Aircraft Establishment
Mr H.C. Garner	–	Royal Aircraft Establishment
Mr J.R.C. Pedersen	–	Independent
Mr J.J. Perrin	–	Aérospatiale, Chatillon
Mr M.R. Pike	–	Rolls-Royce (1971) Ltd, Derby
Mr M.W. Salisbury	–	British Aircraft Corporation Ltd, Weybridge
Mr J.J. Spillman	–	Cranfield Institute of Technology
Mr J. Taylor	–	Hawker Siddeley Aviation Ltd, Woodford
Mr J.W.H. Thomas	–	Hawker Siddeley Aviation Ltd, Hatfield
Mr J. Weir	–	University of Salford

The work on this Item was carried out in the Fluid Mechanics and Physical Properties Group of ESDU. The member of staff who undertook the technical work involved in the initial assessment of the available information and the construction and subsequent development of the Item was

Mr N. Thompson – Group Head