Improved Hot-Wire Drift Corrections and Routine – Paul Nathan 06/02/09

Abstract

This paper contains a derivation of corrections to the calibration coefficients of a hot wire operating in constant temperature (CT) mode. These corrections allow for variations in ambient temperature over time, as well as a change of the heat transfer characteristics of the wire due to degradation or contamination over the duration of the measurements. The basic strategy involves obtaining a set of calibration coefficients at the beginning and end of the experiment, correcting the latter calibration coefficients for ambient temperature drift, interpolating the anemometer voltage in time, obtaining a new set of coefficients from a mock calibration at this time, and then correcting the new coefficients to correspond to the ambient temperature at that time. Tabulated thermo-physical property data from up-to-date sources are used to formulate empirical relationships between the relevant properties and temperature. These are then directly substituted into the steady-state heat transfer equation for a hot-wire, and no simplifying approximations are made.

Introduction

The corrections that can be found in the established literature take the form of either re-scaling of the anemometer voltage with a temperature correction parameter (Bearman (1969), Kanevče & Oka (1976)) or re-scaling of the King's law calibration coefficients (which themselves contain temperature dependent quantities) with simple empirical relationships (Collis & Williams (1959), Comte-Bellot (1976), Bruun (1995) and references therein). The Bearman correction is perhaps a product of the circumstances at the time, where it was far simpler to correct the anemometer voltage using analogue electronics than to store and process digital data. Despite this, the Bearman and Kanevče & Oka corrections contain some significant assumptions – namely that the thermal conductivity, Prandtl number and a particular combination of thermal conductivity, density and dynamic viscosity do not change with temperature

$$k(T)\left(\frac{\rho(T)}{\mu(T)}\right)^{0.5} \approx const. \tag{1}$$

The Collis & Williams (hereon referred to as C&W) correction does allow for the variation of thermal conductivity with temperature but still makes the assumption in (1). Investigation into the validity of this assumption revealed that it is only true if the exponent is 0.5 because the function is dominated by a turning point at the film temperatures typically achieved by a hot wire operating in air. However, in reality the exponent is more like 0.4. With this value the turning point is absent, and there is a definite positive correlation with temperature. It is the objective hereon to derive a set of corrections that do not make this assumption, and it will be seen that the resulting corrections actually turn out to be more elegant in form.

Derivation

Starting from the steady-state energy balance for a hot-wire, and the definition of the Nusselt number $\mathrm{Nu} = hd/k$

$$I^2 R_w = h\pi dl\left(T_w - T_a\right) = \pi lk\left(T_w - T_a\right)\mathrm{Nu} \tag{2}$$

Using Ohms law to convert $I^2 R_w$ into $E_w^{\,2}/R_w$ and then using the basic relationship between wire voltage and bridge voltage (for a non temperature-compensated bridge, as shown below)

$$E_w = E \frac{R_w}{R_1 + R_L + R_w} \tag{3}$$

and then solving (2) for the Nusselt number gives

$$Nu = \frac{E^2 R_w}{\pi l \left(R_1 + R_L + R_w \right)^2 k \left(T_w - T_a \right)} \tag{4}$$

In CT mode, R_w and T_w are constant. Also, for a given probe, l is constant. Lumping the constant terms together allows (4) to be written as

$$Nu = C_1 \frac{E^2}{k \left(T_w - T_a \right)} \tag{5}$$

$$\text{where } C_1 = \frac{R_w}{\pi l \left(R_1 + R_L + R_w \right)^2}$$

King's 1914 law with a generic exponent along with the empirical Nusselt number temperature loading factor of C&W that accounts for a temperature jump between the surface of the wire and the gas adjacent to it shall be used as the empirical relationship for a hot-wire (the jump is caused by molecular slip since the Knudsen number around a typical 5μm diameter hot-wire in operation is on the order of 0.1 which is significantly close to the continuum assumption limit)

$$Nu \left(\frac{T_f}{T_a} \right)^{-0.17} = A + B Re^n \tag{6}$$

where $T_f = \frac{1}{2}\left(T_w + T_a \right)$ is the so-called 'film temperature' at which the thermo-physical properties are evaluated. (4) can be substituted into (6) to give

$$E^2 = k \left(T_w - T_a \right) \left(\frac{T_w + T_a}{2 T_a} \right)^{0.17} \left(A' + B' \left(\frac{Ud}{\nu} \right)^n \right) \tag{7}$$

$$\text{where } A' = \frac{A}{C1} \qquad B' = \frac{B}{C1}$$

Now, from the empirical relationship proposed by Kramers (1946) for incompressible, forced convection, $Nu = 0.42 Pr^{0.26} + 0.57 Pr^{0.33} Re^{0.5}$ (with properties evaluated at the film temperature), it is evident that A and B contain the Prandtl number. Indeed this is a function of temperature and should be included as a variable but the functional form of the empirical relationship has $n = 0.5$ and so the factors and exponents involving the Prandtl number cannot be applied to the general case. It will be seen later that around the typical film temperatures, the Prandtl number is very weakly correlated with temperature compared to the other quantities such as thermal conductivity and kinematic viscosity. Thus, the Prandtl number alone is hereon assumed constant.

The mean wire temperature, T_w, appearing in (7) can be obtained from the definition of resistivity and the assumption that a small change in resistivity about some reference value (subscript c for 'cold') is proportional to a small change in temperature

$$R = \frac{\rho L}{A}$$

$$\Delta\left(\frac{\rho}{\rho_c}\right) = \Delta\left(\frac{R}{R_c}\right) = \alpha_c \Delta T$$

$$\frac{dR}{dT} = \alpha_c R_c \tag{8}$$

$$R_w = R_c\left(1 + \alpha_c\left(T_w - T_c\right)\right)$$

$$\therefore T_w = \frac{\Omega}{\alpha_c} + T_c \text{ where } \Omega = \frac{R_w}{R_c} - 1$$

The quantity Ω is defined as the 'overheat ratio'. Bearman and Comte-Bellot defined the overheat ratio using the ratio of temperatures rather than resistances. Since the linear relationship in (8) does not intersect the origin at absolute zero, the ratio of resistances is *not* equal to the ratio of temperatures, but proportioned by a constant $\alpha_c T_c$. Sometimes the overheat ratio is defined as just the ratio of the resistances without the decrement, so care should be taken when interpreting this number.

The temperature coefficient of resistivity at the calibration reference temperature, α_c (with units $^\circ K^{-1}$), can be obtained from an empirical fit through tabulated data, here taken from the CRC Handbook of Chemistry & Physics (2001). The data shown below is for pure tungsten.
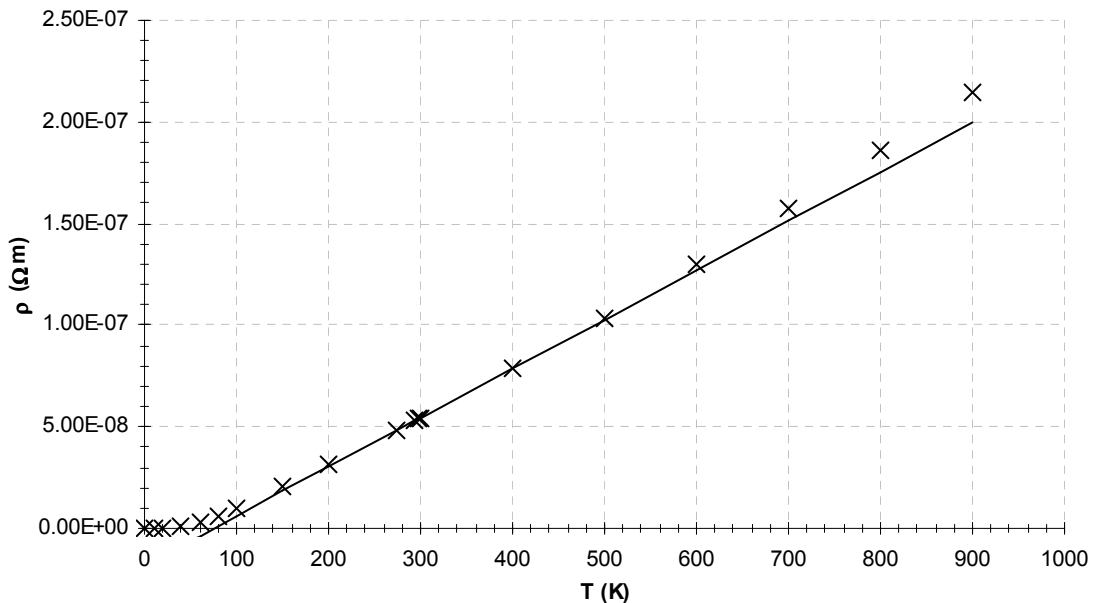

Figure 1: Variation of resistivity with temperature for pure tungsten

Over the range 200 – 500K, the resistivity has an almost constant slope. Choosing the reference temperature T_c to be 298K, the value of α_c is 0.00448346 to 6 s. f. and is constant only over the specified range, which is adequate for hot wire anemometry. A similar procedure for platinum yields 0.00355043. Thus for a given overheat ratio a platinum wire will attain a higher mean temperature than a tungsten wire. If a different reference temperature is employed, the coefficient may be exactly adjusted from its definition as follows

$$\alpha_{c0} = \frac{1}{R_{c0}}\frac{dR}{dT} \qquad \alpha_{c1} = \frac{1}{R_{c1}}\frac{dR}{dT}$$

$$\frac{\alpha_{c1}}{\alpha_{c0}} = \frac{R_{c0}}{R_{c1}} = \frac{\rho_{c0}}{\rho_{c1}} \tag{9}$$

$$\alpha_{c1} = \alpha_{c0}\frac{R_{c0}}{R_{c1}} = \alpha_{c0}\frac{\rho_{c0}}{\rho_{c1}}$$

The thermo-physical property functions shall take the form suggested by C&W since they are both simple and accurate over the required range. The typical film temperature of a hot wire ranges from about 365 – 400K for overheat ratios ranging from 0.6 – 0.9 respectively, with the corresponding mean wire temperatures ranging from about 430 – 500K. Therefore a reference temperature of 400K shall be used for the properties, and they will have empirical functions fitted in the range 350 – 500K. The functions are as follows, where subscripts i and j represent the experimental conditions at times i and j respectively

$$\frac{k}{k_{ref}} = a_1\left(\frac{T}{T_{ref}}\right)^a \qquad \therefore \frac{k_i}{k_j} = \frac{k_i}{k_{ref}}\cdot\frac{k_{ref}}{k_j} = \left(\frac{T_i}{T_j}\right)^a$$

$$\frac{\mu}{\mu_{ref}} = a_2\left(\frac{T}{T_{ref}}\right)^b \qquad \therefore \frac{\mu_i}{\mu_j} = \frac{\mu_i}{\mu_{ref}}\cdot\frac{\mu_{ref}}{\mu_j} = \left(\frac{T_i}{T_j}\right)^b$$

$$\frac{\rho}{\rho_{ref}} = a_3\left(\frac{T}{T_{ref}}\right)^c \qquad \therefore \frac{\rho_i}{\rho_j} = \frac{\rho_i}{\rho_{ref}}\cdot\frac{\rho_{ref}}{\rho_j} = \left(\frac{T_i}{T_j}\right)^c \tag{10}$$

$$\text{similarly } \frac{v_i}{v_j} = \frac{\mu_i}{\mu_j}\cdot\frac{\rho_j}{\rho_i} = \left(\frac{T_i}{T_j}\right)^b\left(\frac{T_i}{T_j}\right)^{-c} = \left(\frac{T_i}{T_j}\right)^{b-c}$$

Although it is possible to take into account the relative humidity of the air for the density equation, this is perhaps pedantic overkill and the air shall be assumed dry and to obey the ideal gas law. This immediately gives $a_3 = 1$ and $c = -1$. The remaining unknown exponents can be found from examination of the relevant property data in Rogers & Mayhew (1995).
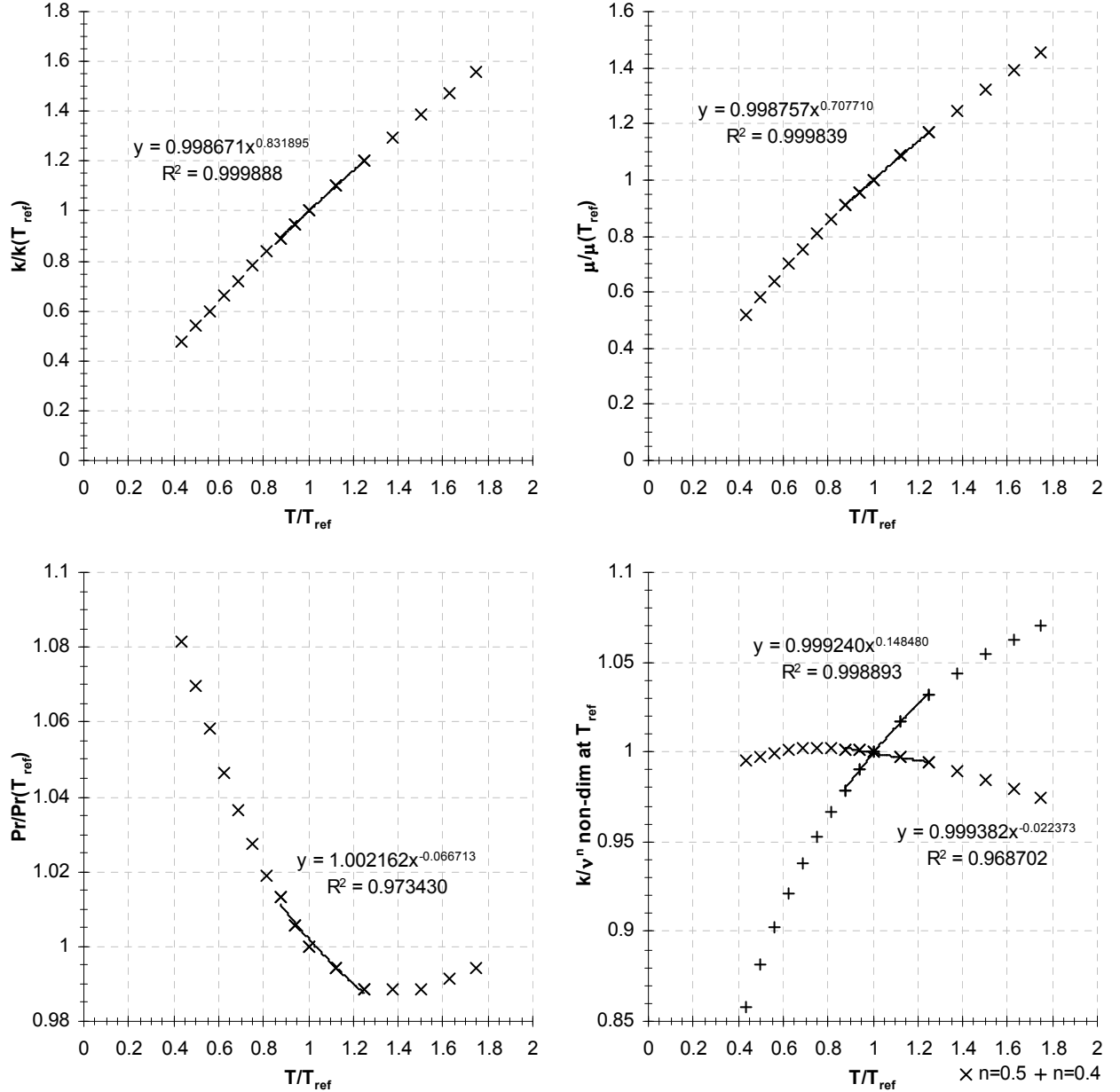
Figure 2: Temperature variation of air thermal conductivity, kinematic viscosity, Prandtl number and eqn. (1) non-dimensionalised by the appropriate reference values. Data fit in 350 – 500K range also shown

The breakdown of the assumption of eqn. (1) for $n \neq 0.5$ is clearly shown in the bottom right graph of Figure 2. Also, the weak correlation of Prandtl number with temperature in the operating range is evident in the bottom left graph, which reveals the close proximity of a turning point. The top two graphs and their least-squares power law fit in the specified range provide the following values for the exponents in (10)

$$a = 0.831895 \qquad b = 0.707710$$
$$a_1 \approx a_2 \approx 1$$

(11)

The value for a is within the range given by Bruun of $0.80 - 0.86$ but b is slightly below ($0.76 - 0.90$), while C&W give 0.8 and 0.76 respectively. In the most general case where n is allowed to vary between calibrations, the expression for the dynamic viscosity ratio in (10) is insufficient and must include a constant factor as follows

$$\left(\frac{v_i}{v_{ref}}\right)^{n_i}\left(\frac{v_{ref}}{v_j}\right)^{n_j} = \frac{\left(\frac{a_2}{a_3}\right)^{n_i}}{\left(\frac{a_2}{a_3}\right)^{n_j}}\left(\frac{T_i}{T_{ref}}\right)^{n_i(1+b)}\left(\frac{T_{ref}}{T_j}\right)^{n_j(1+b)}$$

$$\frac{v_i^{n_i}}{v_j^{n_j}} = \left(\frac{T_i^{n_i}}{T_j^{n_j}}\right)^{1+b}\left(\frac{v_{ref}}{T_{ref}^{1+b}}\right)^{n_i-n_j} = C_3^{n_i-n_j}\left(\frac{T_i^{n_i}}{T_j^{n_j}}\right)^{1+b} \tag{12}$$

where $C_3 = 9.32925\times10^{-10}$

There exists an ambiguity in the definition of the Nusselt number as to what temperature to evaluate the thermal conductivity for a given heat transfer coefficient. Since there is a strong temperature gradient normal to the wire, should the air properties be evaluated right at the surface of the wire or some distance away from it? Historically this has been resolved by evaluating the properties at the film temperature $T_f = \frac{1}{2}(T_w + T_a)$ as mentioned earlier (c.f. Bruun p. 213 and references therein). The physical property temperature dependencies can now be written in terms of the film temperature (N.B. all temperatures must be in kelvin)

$$\frac{k_i}{k_j} = \left(\frac{T_w + T_{ai}}{T_w + T_{aj}}\right)^{0.831895}$$

$$\frac{\mu_i}{\mu_j} = \left(\frac{T_w + T_{ai}}{T_w + T_{aj}}\right)^{0.707710}$$

$$\frac{\rho_i}{\rho_j} = \left(\frac{T_w + T_{ai}}{T_w + T_{aj}}\right)^{-1} \tag{13}$$

$$\frac{v_i^{n_i}}{v_j^{n_j}} = C_4^{n_i-n_j}\left(\frac{\left(T_w + T_{ai}\right)^{n_i}}{\left(T_w + T_{aj}\right)^{n_j}}\right)^{1.70771}$$

where $C_4 = \dfrac{C_3}{2^{1.70771}} = 2.85611\times10^{-10}$

Now, from an experimental calibration the velocity-voltage relationship is of the form

$$E^2 = A'' + B''U^n \tag{14}$$

where the numerical values for A'', B'' and n are obtained from some sort of non-linear regression (Gauss-Newton method, say) on the experimental records of E and U. Linking (14) to the theoretical relationship (7) reveals that

$$A'' = k(T_w - T_a)\left(\frac{T_w + T_a}{2T_a}\right)^{0.17} A'$$

$$B'' = k(T_w - T_a)\left(\frac{T_w + T_a}{2T_a}\right)^{0.17}\left(\frac{d}{v}\right)^n B' = \frac{A''}{A'}\left(\frac{d}{v}\right)^n B' \tag{15}$$

Thus only A'' and B'' depend upon the ambient temperature while n does not and is a fixed constant of the system. It will only vary with degradation or contamination of the hot-wire.

The corrections to A'' and B'' will have the form

$$A''_i = A''_j \cdot \frac{A''_i}{A''_j} \qquad B''_i = B''_j \cdot \frac{B''_i}{B''_j} \tag{16}$$

This quotient form naturally lends itself to the use of the property functions given in (13). Firstly, the correction quotient for A'' after substituting the thermal conductivity equation in (13) into (15)

$$\frac{A''_i}{A''_j} = \left(\frac{T_w + T_{ai}}{T_w + T_{aj}}\right)^a \left(\frac{T_w - T_{ai}}{T_w - T_{aj}}\right) \left(\frac{T_w + T_{ai}}{T_w + T_{aj}}\right)^{0.17} \left(\frac{T_{aj}}{T_{ai}}\right)^{0.17} \tag{17}$$

By a fortuitous combination of empirical exponents, $a + 0.17 \approx 1$ and so the above simplifies to

$$\frac{A''_i}{A''_j} = \left(\frac{T_w^2 - T_{ai}^2}{T_w^2 - T_{aj}^2}\right) \left(\frac{T_{aj}}{T_{ai}}\right)^{0.17} \tag{18}$$

Allowing n to vary in the most general case, the correction quotient for B'' is

$$\begin{aligned}
\frac{B''_i}{B''_j} &= \frac{A''_i}{A''_j} \cdot \frac{v_j^{n_j}}{v_i^{n_i}} \cdot d^{n_i - n_j} \\
&= \frac{A''_i}{A''_j} \left(\frac{C_4}{d}\right)^{n_j - n_i} \left(\frac{(T_w + T_{aj})^{n_j}}{(T_w + T_{ai})^{n_i}}\right)^{1.70771}
\end{aligned} \tag{19}$$

If one wishes to keep n fixed at the value obtained from the first calibration, then the correction quotient for B'' simplifies to

$$\frac{B''_i}{B''_j} = \frac{A''_i}{A''_j} \left(\frac{T_w + T_{aj}}{T_w + T_{ai}}\right)^{1.70771n} \tag{20}$$

and the wire diameter no longer features in the correction.

Comparison with existing corrections

The present corrections can now be compared to the existing corrections mentioned previously. For quick reference they are given below, re-expressed from their original form as corrections to the calibration coefficients

Bearman:

$$\frac{A''_i}{A''_j} = \frac{B''_i}{B''_j} = \left(1 + \frac{1}{2}\left(\frac{T_{aj} - T_{ai}}{T_w - T_{ai}}\right)\right)^2 \tag{21}$$

Kanevče & Oka:

$$\frac{A''_i}{A''_j} = \frac{B''_i}{B''_j} = \left(\frac{T_w - T_{ai}}{T_w - T_{aj}}\right) \tag{22}$$

C&W:

$$\frac{A''_i}{A''_j} \text{ same as (17)}$$

$$\frac{B''_i}{B''_j} = \left(\frac{T_w - T_{ai}}{T_w - T_{aj}}\right)\left[\left(\frac{T_w + T_{ai}}{T_w + T_{aj}}\right)\left(\frac{T_{aj}}{T_{ai}}\right)\right]^{0.17} \tag{23}$$

Comte-Bellot (hereon C-B) applied to the C&W correction quotients above:

$$\frac{A''_i}{A''_j} \to \frac{A''_i}{A''_j} \times \left[\frac{1 + 0.49\left(\dfrac{T_w}{T_{ai}} - 1\right)}{1 + 0.49\left(\dfrac{T_w}{T_{aj}} - 1\right)}\right]$$

$$\frac{B''_i}{B''_j} \to \frac{B''_i}{B''_j} \times \left[\frac{1 + 0.12\left(\dfrac{T_w}{T_{ai}} - 1\right)}{1 + 0.12\left(\dfrac{T_w}{T_{aj}} - 1\right)}\right] \tag{24}$$

The graph below shows all these corrections applied over a drift range of $\pm 10K$ for the case of $\Omega = 0.8$ and $n = 0.4$.
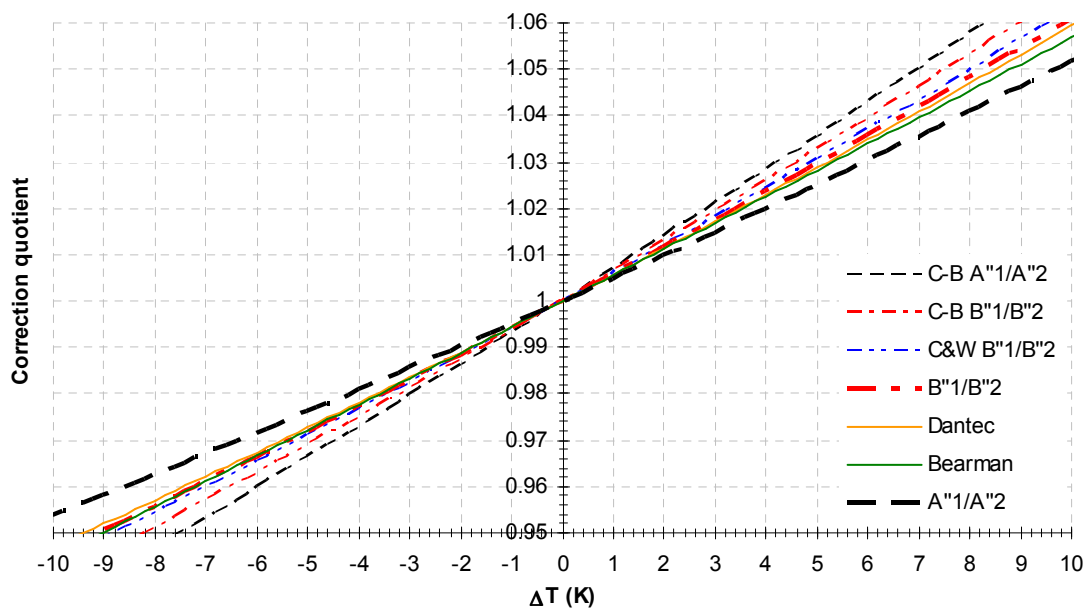

Figure 3: Comparison of present corrections with various others. For ease of identification, legend order corresponds to curve order when viewed from uppermost curve on the positive x side through to the lowermost curve. $\Omega = 0.8$, $n = 0.4$

The present corrections are very similar to the C&W corrections for this particular case, however they have the advantage that the B'' correction can alter with n. Decreasing n causes the B'' correction curve to effectively rotate clockwise towards the A'' correction curve (and vice-versa). This is because a smaller n implies a reduced sensitivity to the kinematic viscosity term in (20). Increasing the overheat ratio causes all the curves to effectively rotate clockwise about unity and converge. As expected this is because a larger overheat reduces the ambient temperature sensitivity of the hot wire. It should be noted that in the (unphysical) limit of infinite overheat ratio, those corrections involving the C&W Nusselt number temperature loading factor as well as the C-B corrections do not asymptote to unity as would be expected (infinite overheat implies zero sensitivity to ambient temperature) but retain the $\left(T_{aj}/T_{ai}\right)^{0.17}$ term (while the C-B corrections retain an extra $\left(T_{aj}/T_{ai}\right)$ multiple as well). Hence the factor is technically unphysical, but its confirmation by Bradbury & Castro (1972) suggests that it is valid in the realistic overheat ratio operating range, where for tungsten an overheat ratio above 1.3 would begin to cause oxidation for very long wires, and about 1.0 for millimetre length wires (the actual temperature distribution is a hyperbolic cosine type function (c.f. Bruun p. 24) , and so the temperature at the centre of the wire is always above the mean).

Expounding of correction procedure

Before continuing it is perhaps a good idea to clarify exactly what Figure 3 is showing. The corrections solve the following problem: A calibration has been carried out at time i and at time j, by what do the calibration coefficients at time j need to be multiplied in order to correct them back to those of time i, assuming that all the drift is caused by ambient temperature variation? For example, if the ambient temperature at time j is higher than that at time i, it is necessary to scale up the coefficients at time j to account for the increased heat transfer between the wire and the air (which manifests itself as a increased bridge voltage) at the lower ambient temperature that existed a time i. For this reason the corrections are greater than unity for a positive ΔT. The opposite also applies, of course.

Furthermore, consider the following problem: A calibration was carried out at time i only and measurements are being made at some later time j where the ambient temperature has changed. By what do these calibration coefficients need to be multiplied in order for them to return the true flow speed for a given bridge voltage, despite the ambient temperature change? This problem is solved simply by swapping i and j in the correction quotient equations ((18) and (19)) and applying them as appropriate to the calibration coefficients obtained at time i. This sort of correction shall be called a 'Type 1a' correction. It has the following schematic
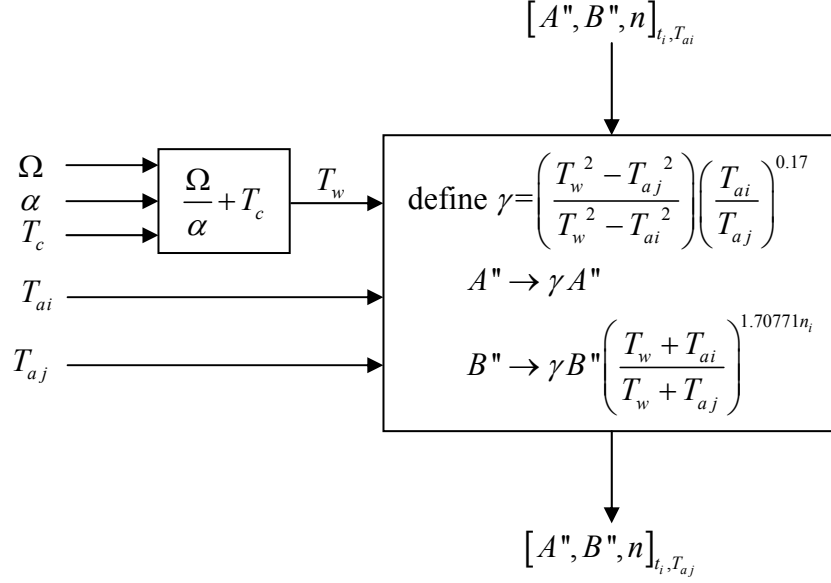
$$[A'',B'',n]_{t_i,T_{ai}}$$

Figure 4: Schematic for a Type 1a correction – calibration at time *i*, measurement at time *j*

For practical simplicity the hot-wire and cable 'cold' resistances are measured at the same ambient conditions as at calibration, and the bridge resistance calculated likewise. This enables T_c in (8) to equal T_{ai} for this particular case. The Type 1a correction is an open ended method, and there is no way to check whether the corrections are still within sensible bounds after a long elapsed time. By having a calibration at the start and end of a set of measurements, the drift correction can simply comprise of an interpolation in time between the two calibrations. The simplest procedure would be to linearly interpolate each of the coefficients. However there is nothing to suggest that the coefficients should each vary independently and in a linear fashion in time. A more robust procedure, hereby called a 'Type 2' correction , involves the following steps:

1. Correct A'' and B'' from calibration 2 back to the temperature at calibration 1 using a Type 1b correction (defined in Figure 5)

2. Form an array of E^2 from the velocity array of calibration 1 and the temperature-corrected coefficients from calibration 2 using (14)

3. Linearly interpolate between this E^2 array and the original array from calibration 1 in time according to the current time fraction, creating a new interpolated E^2 array

4. Send this interpolated E^2 array along with the velocity array of calibration 1 into a non-linear regression routine to obtain a new set of calibration coefficients for this particular time (effectively, carry out a mock calibration)

5. Perform a Type 1c correction (defined in Figure 6Figure 5) on these new coefficients to correct them to the temperature that exists at the particular time in question

6. Use these new coefficients in (14) solved for U to get the corrected velocity from the recorded bridge voltage

7. Repeat steps 1 through 6 for every measurement point.

Step 1 is necessary to eliminate any ambiguity regarding the temperature that represents the interpolated values. The Type 1b and 1c corrections are described below. The Type 1c correction can be viewed as a reversed Type 1b correction:
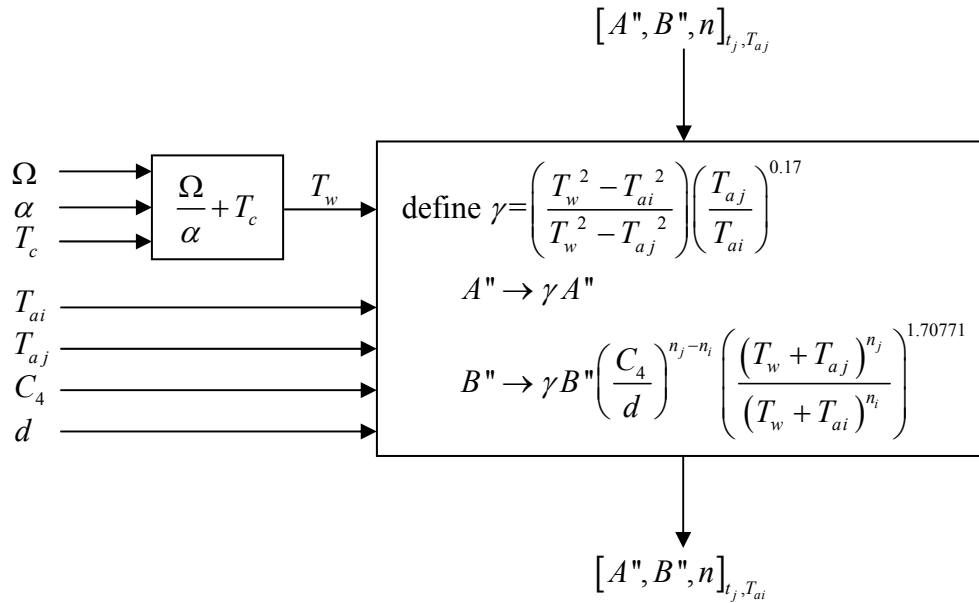
$$[A'',B'',n]_{t_j,T_{aj}}$$

$$\Omega, \alpha, T_c \longrightarrow \boxed{\frac{\Omega}{\alpha}+T_c} \xrightarrow{T_w}$$

$$\text{define } \gamma = \left(\frac{T_w^2 - T_{ai}^2}{T_w^2 - T_{aj}^2}\right)\left(\frac{T_{aj}}{T_{ai}}\right)^{0.17}$$

$$A'' \rightarrow \gamma A''$$

$$B'' \rightarrow \gamma B''\left(\frac{C_4}{d}\right)^{n_j - n_i}\left(\frac{(T_w + T_{aj})^{n_j}}{(T_w + T_{ai})^{n_i}}\right)^{1.70771}$$

$$[A'',B'',n]_{t_j,T_{ai}}$$

Figure 5: Schematic for a Type 1b correction – correct calibration constants at time *j* back to conditions at time *i*

$$[A'',B'',n]_{t_k,T_{ai}}$$

$$\Omega, \alpha, T_c \longrightarrow \boxed{\frac{\Omega}{\alpha}+T_c} \xrightarrow{T_w}$$

$$\text{define } \gamma = \left(\frac{T_w^2 - T_{ak}^2}{T_w^2 - T_{ai}^2}\right)\left(\frac{T_{ai}}{T_{ak}}\right)^{0.17}$$

$$A'' \rightarrow \gamma A''$$

$$B'' \rightarrow \gamma B''\left(\frac{C_4}{d}\right)^{n_i - n_k}\left(\frac{(T_w + T_{ai})^{n_i}}{(T_w + T_{ak})^{n_k}}\right)^{1.70771}$$
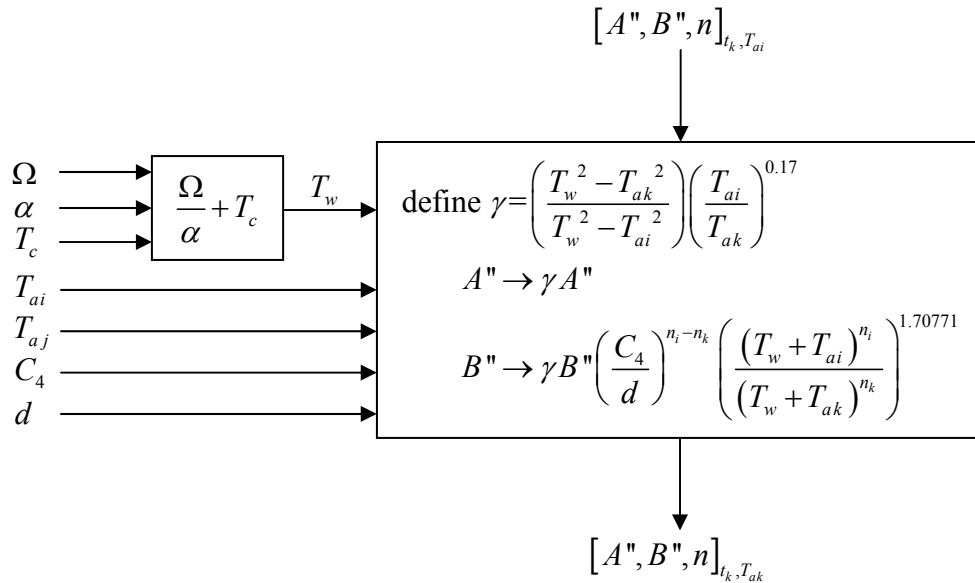
$$[A'',B'',n]_{t_k,T_{ak}}$$

Figure 6: Schematic for a Type 1c correction – correct intermediate calibration constants at time *k*, but with conditions at time *i*, to conditions at time *k*

A schematic for the Type 2 correction is shown below, with braces {} denoting arrays. Also, the recorded bridge voltage, E_{rec} is usually modified by a gain and offset of the form $E_{rec} = \text{Gain}(E + \text{Offset})$, and this first needs to be removed to recover the true voltage, E.
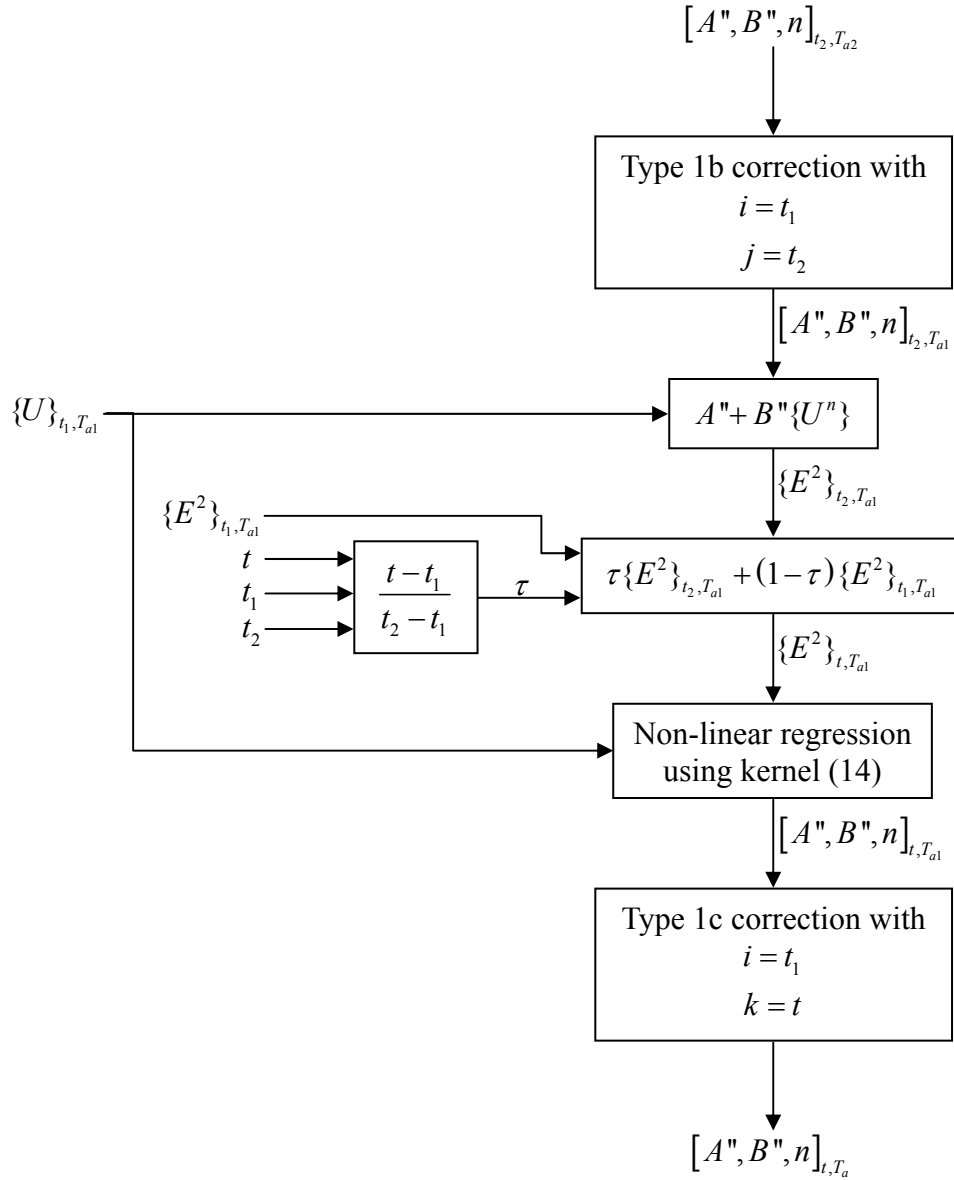
$$\left[A'',B'',n\right]_{t_2,T_{a2}}$$

Type 1b correction with
$$i = t_1$$
$$j = t_2$$

$$\left[A'',B'',n\right]_{t_2,T_{a1}}$$

$\{U\}_{t_1,T_{a1}}$

$$A'' + B''\{U^n\}$$

$$\{E^2\}_{t_2,T_{a1}}$$

$\{E^2\}_{t_1,T_{a1}}$

t
t_1
t_2

$$\frac{t-t_1}{t_2-t_1}$$

τ

$$\tau\{E^2\}_{t_2,T_{a1}} + (1-\tau)\{E^2\}_{t_1,T_{a1}}$$

$$\{E^2\}_{t,T_{a1}}$$

Non-linear regression using kernel (14)

$$\left[A'',B'',n\right]_{t,T_{a1}}$$

Type 1c correction with
$$i = t_1$$
$$k = t$$

$$\left[A'',B'',n\right]_{t,T_a}$$

Figure 7: Schematic for a Type 2 correction

Experimental results

The Type 2 correction can be validated by applying to some real calibration data. The following test case consists of 2 calibrations taken 3hrs apart in the freestream. Each set contains 10 speed/voltage pairs ranging from 2m/s to 11m/s, with each point acquired at a 20kHz sampling rate (after passing through a 9.2kHz low-pass filter) and averaged over 1min. The bridge voltage gain and offset were 40 and -1.64 respectively. The freestream turbulence intensity was 0.16%, and the air was taken in through 5micron filters. The anemometer bridge was manufactured at Newcastle University, Australia. The hot-wire employed was an Auspex (USA) 2.5micron diameter, 0.5mm length tungsten wire. The overheat ratio was set at a modest 0.65 to prolong wire life. The ambient temperature was on average 24.27ºC during the first calibration and on average 24.6ºC during the second calibration.

In the graphs below, the calibration data are plotted on linearising axes. Both first and second calibrations are presented as dashed lines, the second calibration corrected back onto the first calibration conditions is presented with cross symbols and an arbitrary interpolation to the half way time fraction and ambient temperature of 24.43 ºC is presented with plusses. The top graph shows these results with n allowed to vary, and the bottom graph fixes n to that of the first calibration.
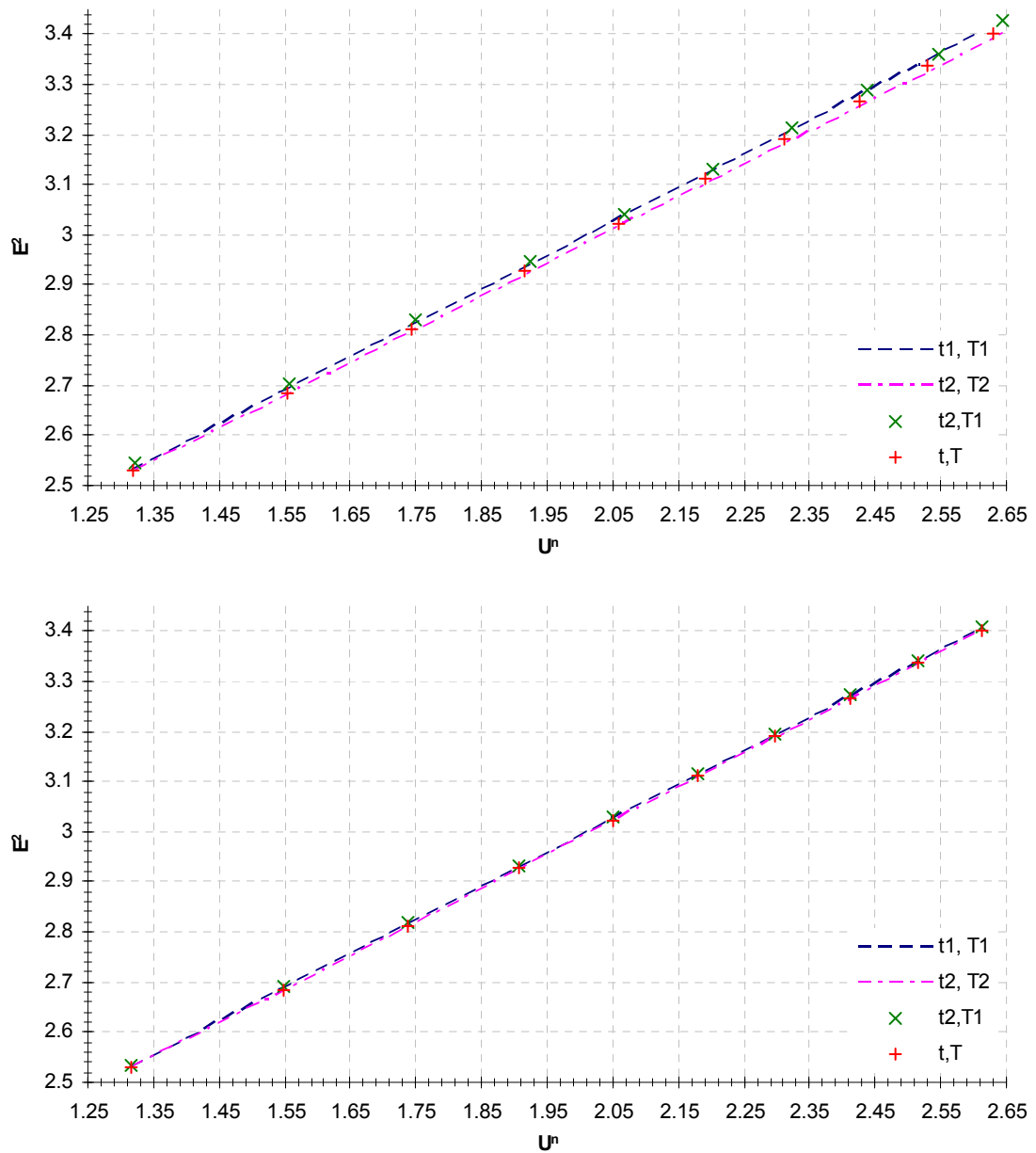
Figure 8: Test case for Type 2 correction validation showing both calibrations and the corrected points. Top graph *n* variable, bottom graph *n* constant

The calibration constants are tabulated below

Table 1: Calibration constants. Left table – *n* variable; right table – *n* fixed

	A"	B"	n		A"	B"	n
t_1, T_{a1}	1.644814	0.673153	0.400286	t_1, T_{a1}	1.644814	0.673153	0.400286
t_2, T_{a2}	1.656703	0.659345	0.405422	t_2, T_{a2}	1.640386	0.673583	0.400286
t_2, T_{a1}	1.660055	0.668410	0.405422	t_2, T_{a1}	1.643705	0.675460	0.400286
t, T_a	1.650865	0.666170	0.402871	t, T_a	1.642650	0.673398	0.400286

Examining the top graph, the corrected points overlap the first calibration line very well. The fact that they do overlap so well indicates that most of the drift is related to ambient temperature and not to wire degradation – the interpolation between calibrations takes care of the drift caused by wire degradation/contamination while the analytical corrections to the constants takes care of ambient temperature effects. Remember, it is not a requirement that the points overlap exactly since they

represent an intermediate stage of the correction. Finally, the midway interpolation points lie between the two calibration lines as they ought to. This latter point demonstrates the correction procedure to be consistent and sensible. Furthermore, inspection of the tabulated values reveals that the calibration constants for the midway case are not simply halfway between the corresponding first and second calibration constants. Admittedly the data is insufficient to demonstrate the robustness of the corrections and ideally a set of calibrations purposely carried out over a large range of ambient temperatures using the same wire and anemometer would be available for testing.

If n is fixed at the value from the first calibration then it appears that the two calibrations almost perfectly coincide and that any corrections are imperceptibly small. At first glance, this might suggest that keeping n fixed is beneficial, but of course this is fallacious since by doing this the second (and any subsequent) calibration constants no longer provide a true least squares fit to the data that minimise the residuals.

The Type 2 correction can be extended to deal with any number of calibrations. For example if three calibrations were carried out during an experiment, the linear interpolation step can be replaced with a quadratic interpolation after correcting the second and third calibration constants back to the ambient temperature condition of the first calibration.

Conclusion

A new hot-wire drift correction procedure has been formulated from first principles, with analytical corrections featuring fewer assumptions than existing correction techniques in the literature. The empirical constants in the analytical corrections were obtained from up-to-date thermo-physical data. The correction proved to be closest to the Collis & Williams correction with the A'' correction being identical but the B'' correction now featuring the kinematic viscosity and the exponent n. If n is allowed to vary between calibrations then the wire diameter also appears in the B'' correction (due to the heat transfer relationship containing the Reynolds number).

Two procedures were developed, called Type 1a and Type 2. The Type 1a procedure was for use with a single calibration made at the start of the experiment and works by correcting the calibration constants for any changes in ambient temperature as the experiment proceeds. This is an open-ended method and cannot correct for wire degradation. The Type 2 procedure was for use with two calibrations, made at the beginning and end of the experiment respectively. This method ensured that all corrections in time are bounded by the two calibration curves and an interpolation between a temperature corrected 'mock' calibration and the first calibration effectively takes into consideration drift caused by events other than ambient temperature change (such as wire degradation or contamination). A subsequent temperature correction brings the new set of calibration constants to the current ambient temperature condition of the measurement in question. The procedure is simple to implement on computer and only additionally requires that the ambient fluid temperature be recorded alongside the anemometer bridge voltage for each measurement point, and of course that the overheat ratio and wire diameter be logged.

The Type 2 correction procedure was verified using an actual set of calibrations taken 3hrs apart in which the temperature varied by 0.3ºC. The results were deemed sensible and consistent. Although the case of keeping n fixed at its value in the first calibration appeared better in terms of having to make much smaller corrections, such a conclusion is erroneous since the second calibration curve is no longer a best fit and has been, in essence, forced to be closer to the first calibration curve, especially on linearising axes.

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