Formula for the Roots of the General Cubic Equation

$$y = y_0 + a_3\left(x - x_0\right)^3 + a_2\left(x - x_0\right)^2 + a_1\left(x - x_0\right)$$

$$\begin{aligned} a &= a_3 \\ b &= a_2 \\ c &= a_1 \\ d &= y_0 - y \end{aligned}$$

$$\begin{aligned} D_0 &= b^2 - 3ac \\ D_1 &= 2b^3 - 9abc + 27a^2 d \\ D_2 &= D_1^{\,2} - 4D_0^{\,3} \end{aligned}$$

$$\begin{aligned} C &= \sqrt[3]{\frac{D_1 \pm \sqrt{D_2}}{2}} \\ x_{0,1,2} &= x_0 - \frac{1}{3a}\left(b + \kappa^n C + \frac{D_0}{\kappa^n C}\right) \\ \kappa &= \frac{-1 + i\sqrt{3}}{2} \\ n &= 0,1,2 \end{aligned}$$

In the context of monotone cubic spline segments for hot-wire calibration, only the following two cases are relevant

$$\text{if } D_2 \geq 0$$

$$\begin{aligned} C &= \sqrt[3]{\frac{D_1 + \sqrt{D_2}}{2}} \\ x &= x_0 - \frac{1}{3a}\left(b + C + \frac{D_0}{C}\right) \end{aligned}$$

$$\text{else}$$

$$\begin{aligned} C &= \sqrt[3]{\frac{D_1 - i\sqrt{\left|D_2\right|}}{2}} \\ x &= x_0 - \frac{1}{3a}\left(b + \kappa^n C + \frac{D_0}{\kappa^n C}\right) \\ \kappa &= \frac{-1 + i\sqrt{3}}{2} \\ n &= 1 \end{aligned}$$

The n = 2 root is not used in the present context.

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14-OCT-2024