

# Learning Context Effects in Triadic Closure

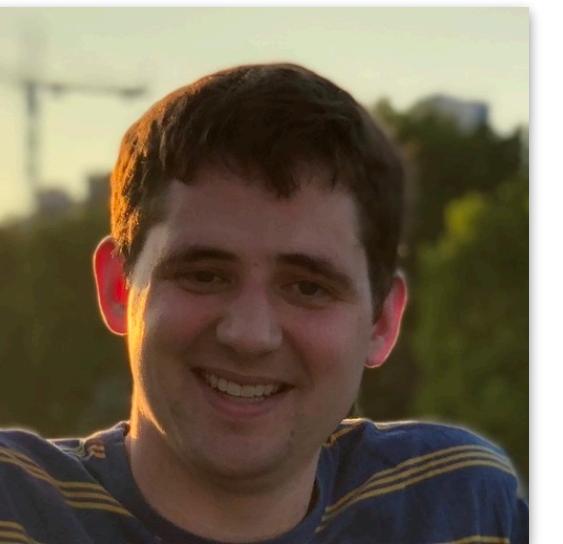
Kiran Tomlinson

SINM 2020

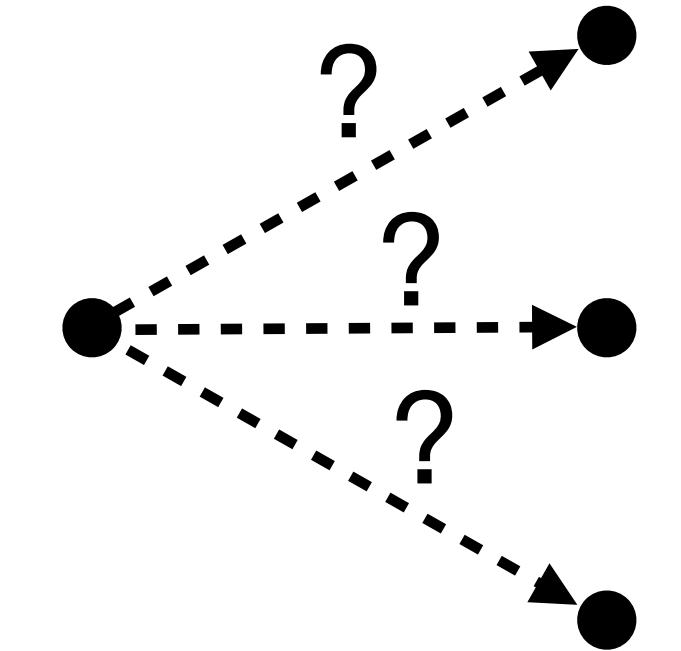
research with Austin R. Benson



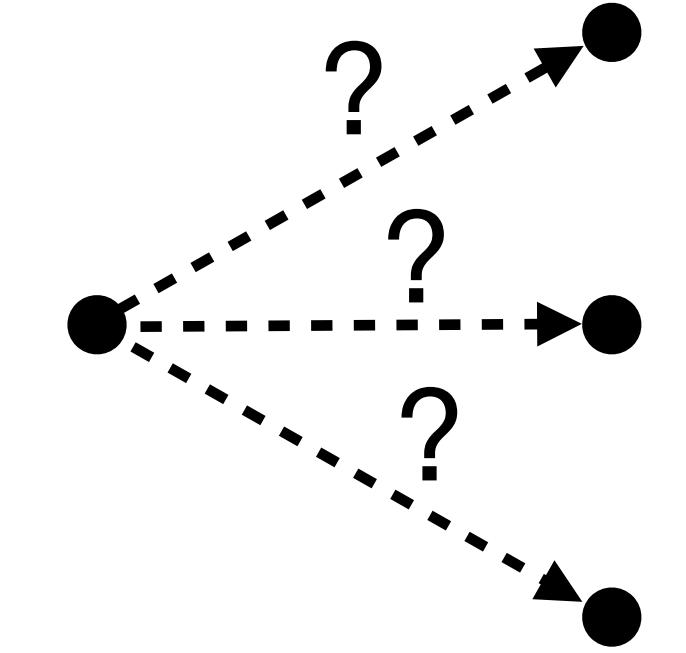
Slides: [bit.ly/lcl-slides](https://bit.ly/lcl-slides)  
Preprint: [bit.ly/lcl-paper](https://bit.ly/lcl-paper)  
Code: [bit.ly/lcl-code](https://bit.ly/lcl-code)  
Data: [bit.ly/lcl-data](https://bit.ly/lcl-data)



# What factors drive edge formation?

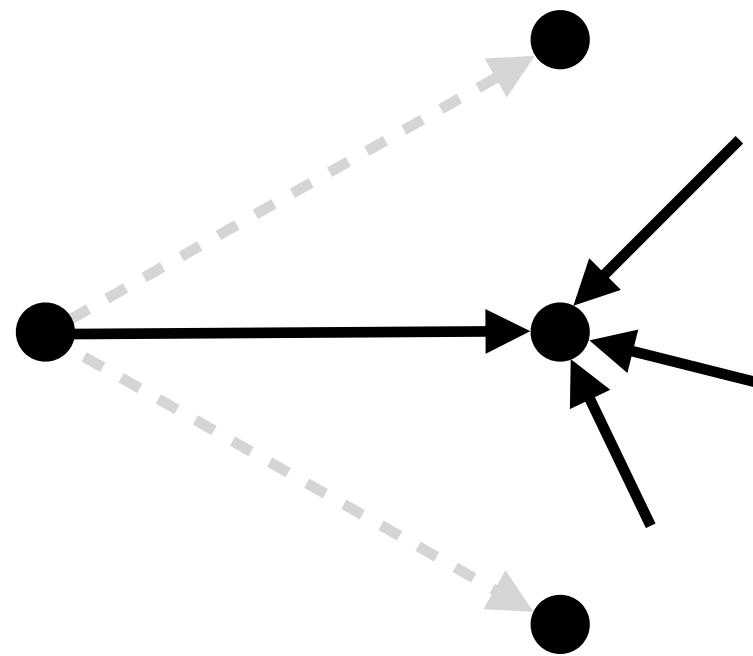


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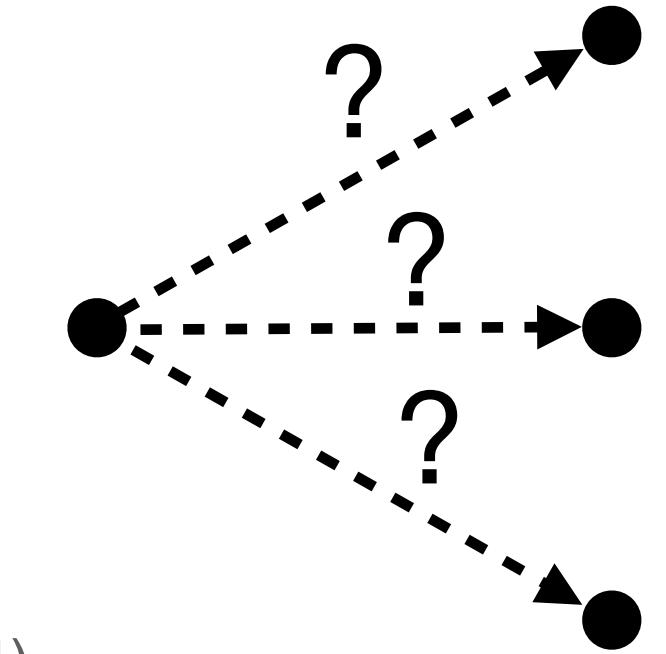


Preferential attachment

(Barabási & Albert, *Science* 1999)

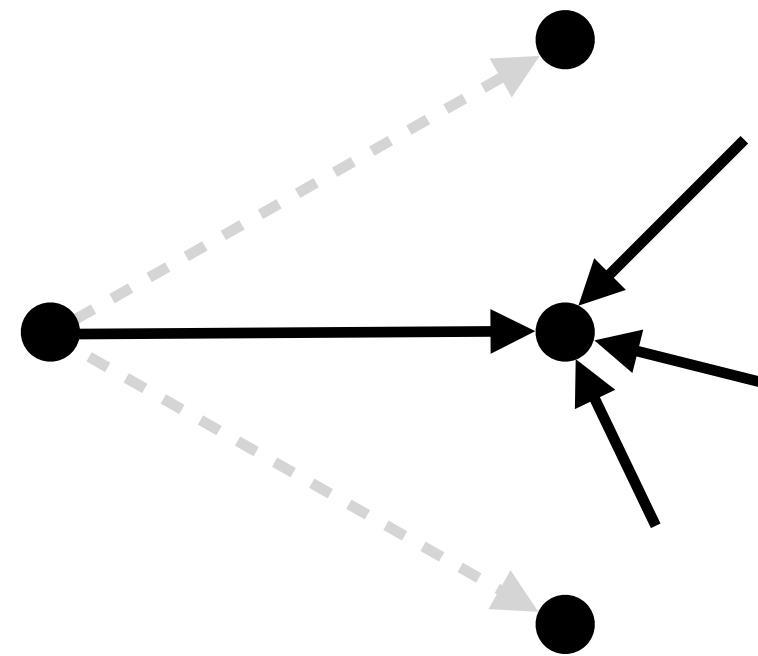


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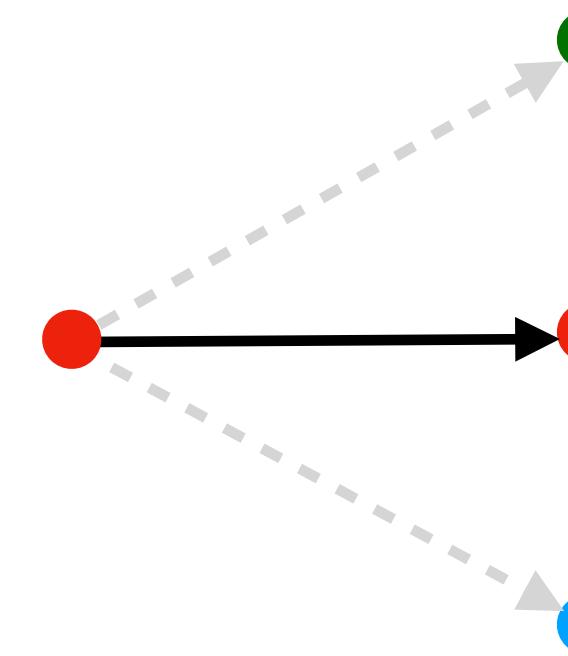
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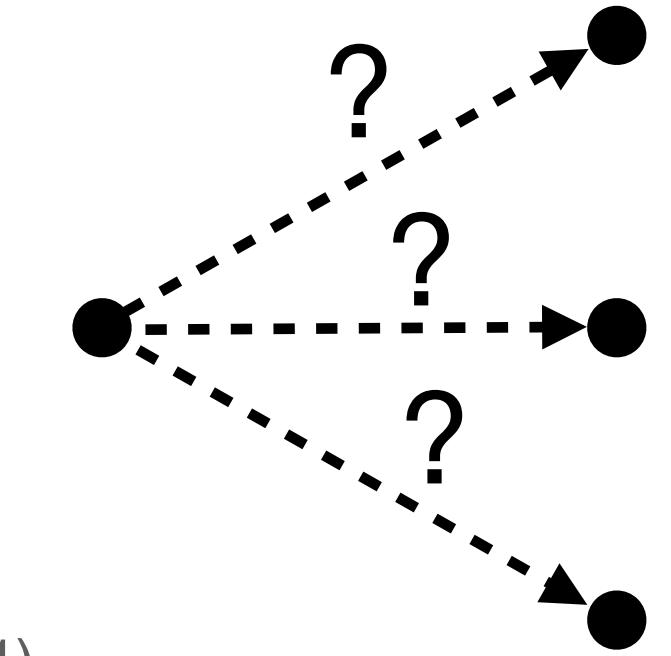


## Homophily

(McPherson et al., *Annual Review of Sociology* 2001)  
(Papadopoulos et al., *Nature* 2012)

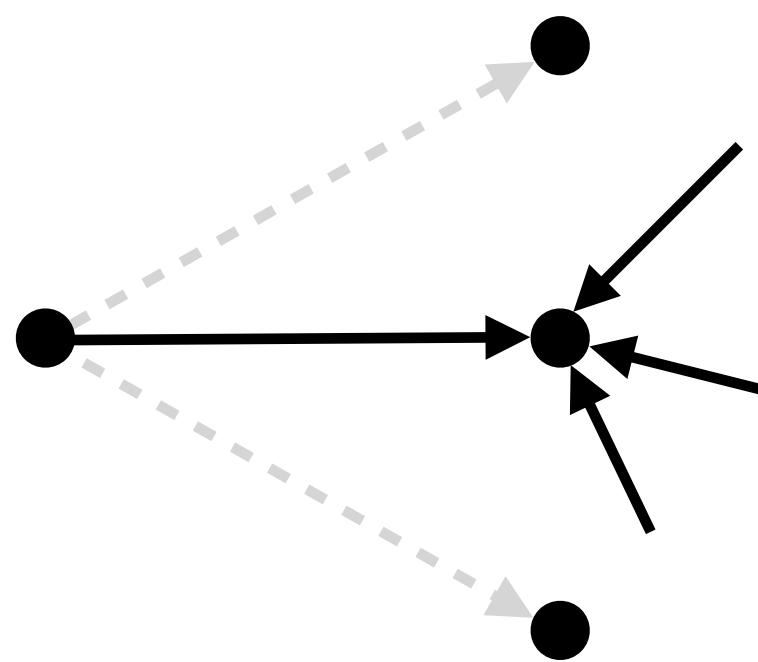


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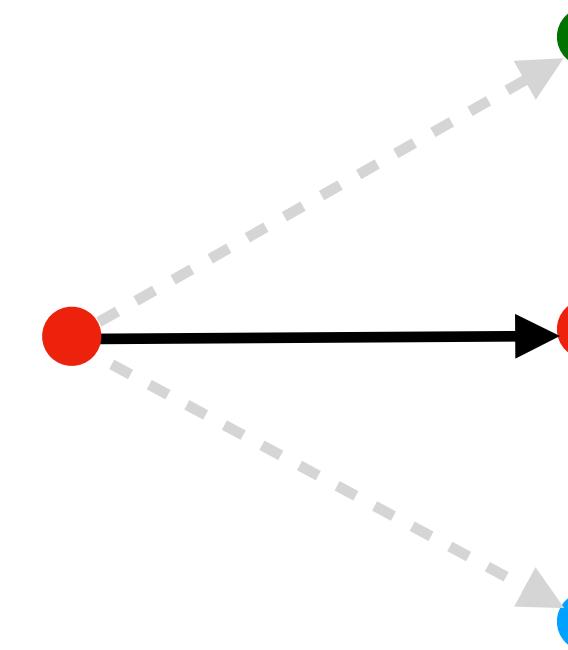
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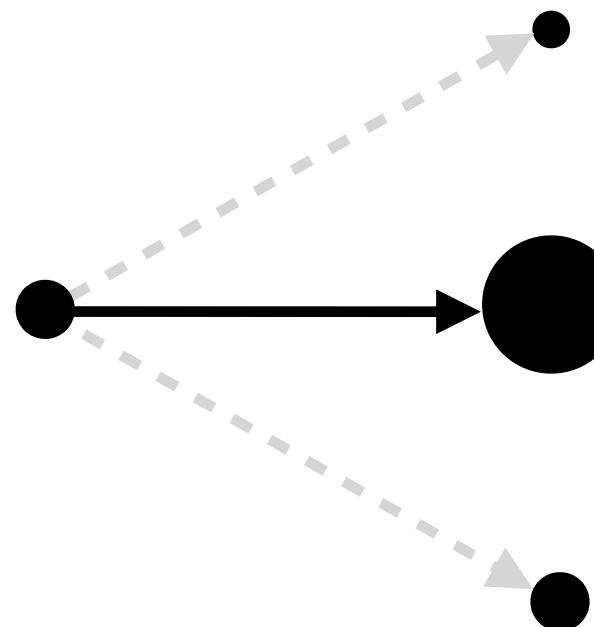
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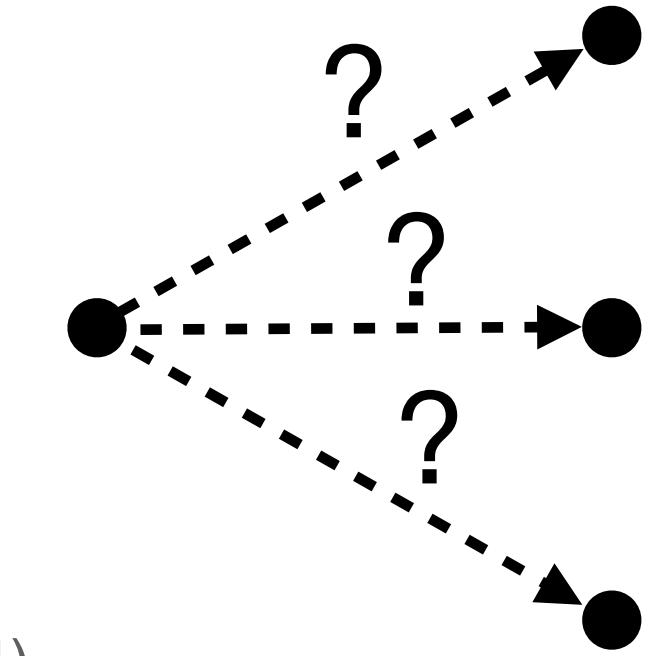


## Fitness

(Bianconi & Barabási, *Europhysics Letters* 2001)  
(Caldarelli et al., *Physical Review Letters* 2002)

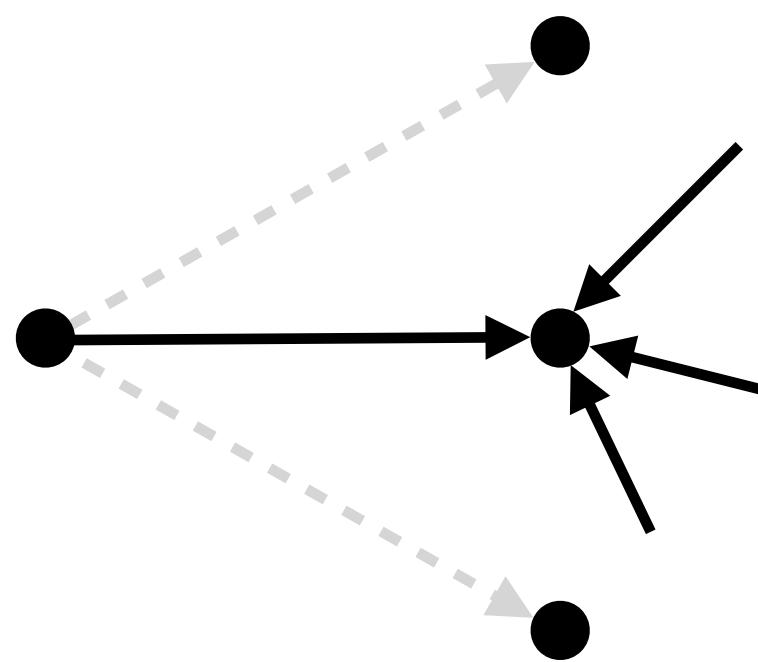


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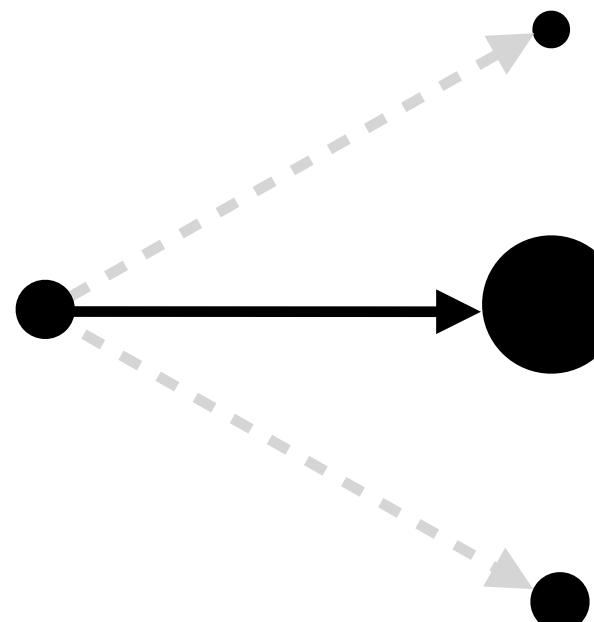
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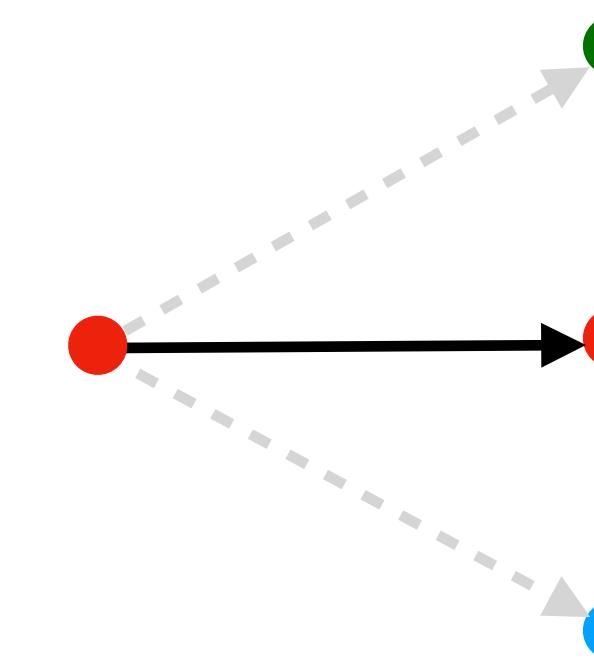
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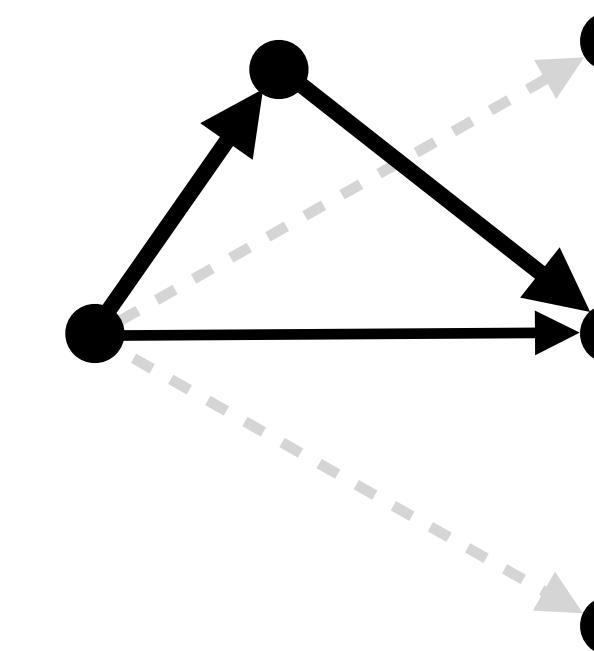
## Homophily

(McPherson et al., *Annual Review of Sociology* 2001)  
(Papadopoulos et al., *Nature* 2012)



## Triadic closure

(Rapoport, *Bulletin of Mathematical Biophysics* 1953)  
(Jin et al., *Physical Review E* 2001)



# “Choosing to grow a graph”

(Overgoor et al., *SINM* '19 & *WWW* '19)

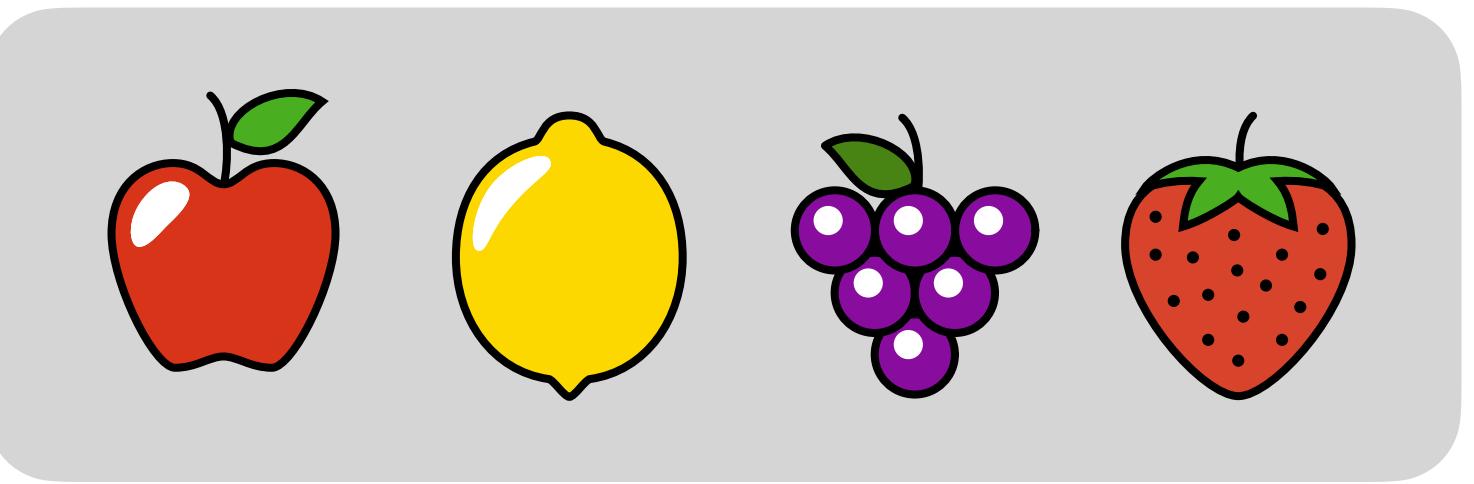
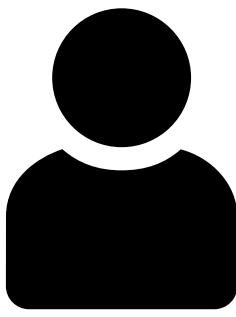
(Gupta & Porter, *arXiv* 2020)

# “Choosing to grow a graph”

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(Gupta & Porter, *arXiv* 2020)

Traditional discrete choice:



*chooser*

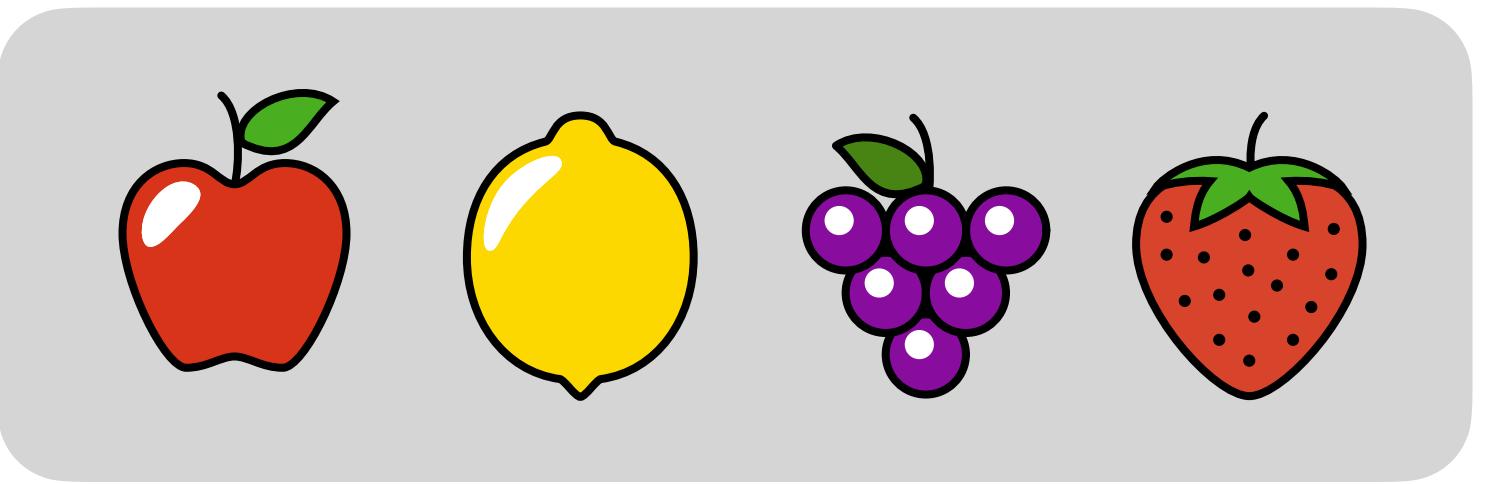
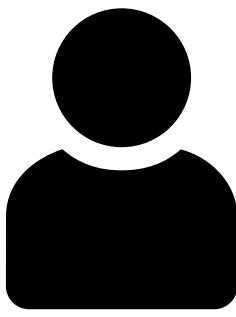
*choice set*

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*chooser*

*choice set*

(under-explored in sociology)

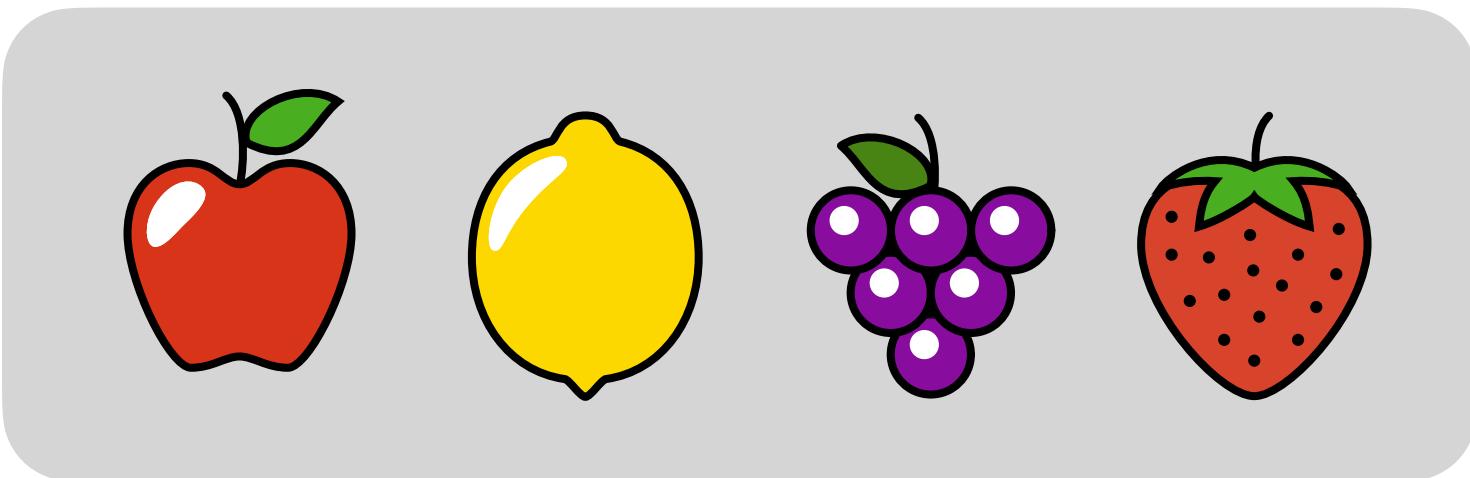
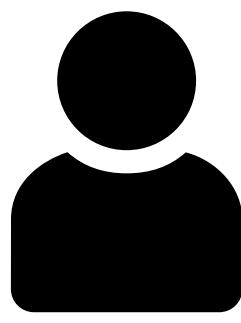
(Bruch & Feinberg, *Annual Review of Sociology* 2017)

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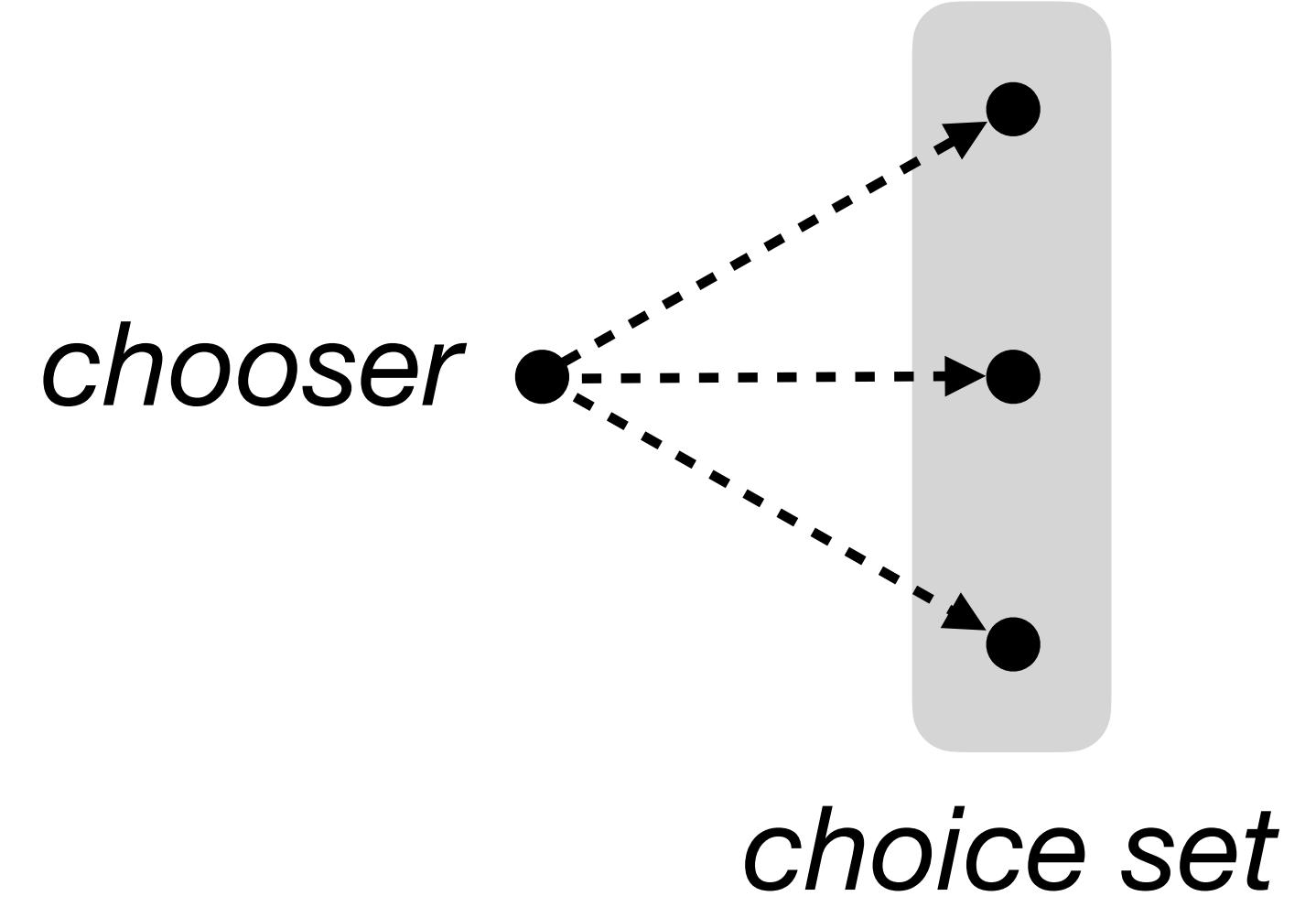
*choice set*

(under-explored in sociology)

(Bruch & Feinberg, *Annual Review of Sociology* 2017)



in network growth

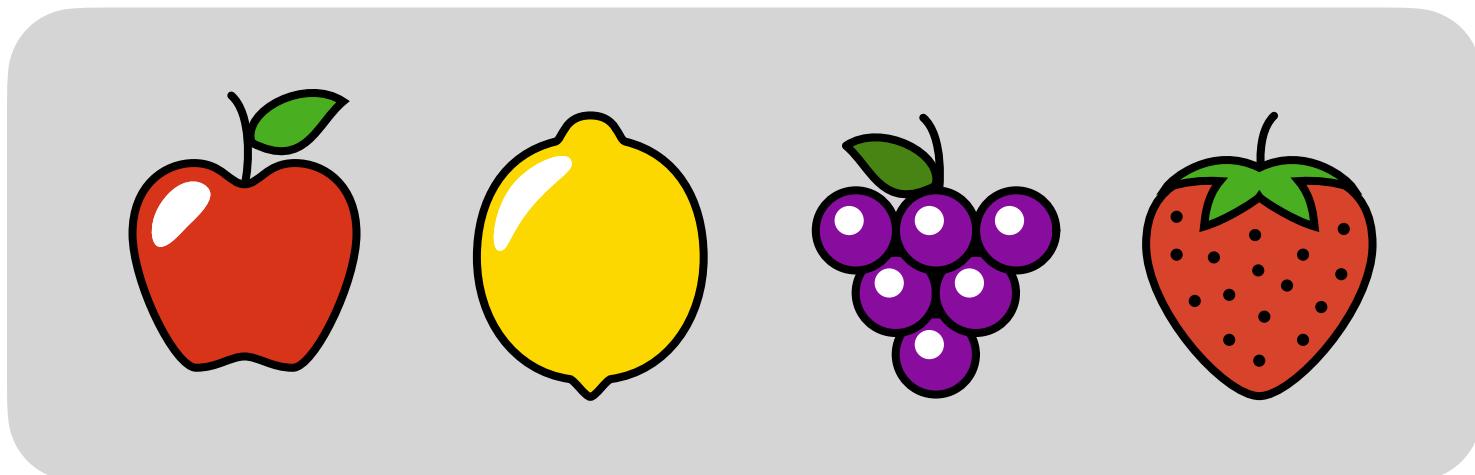
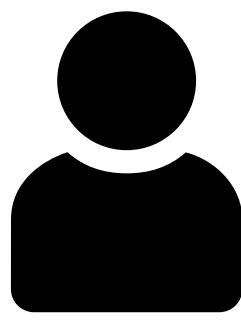


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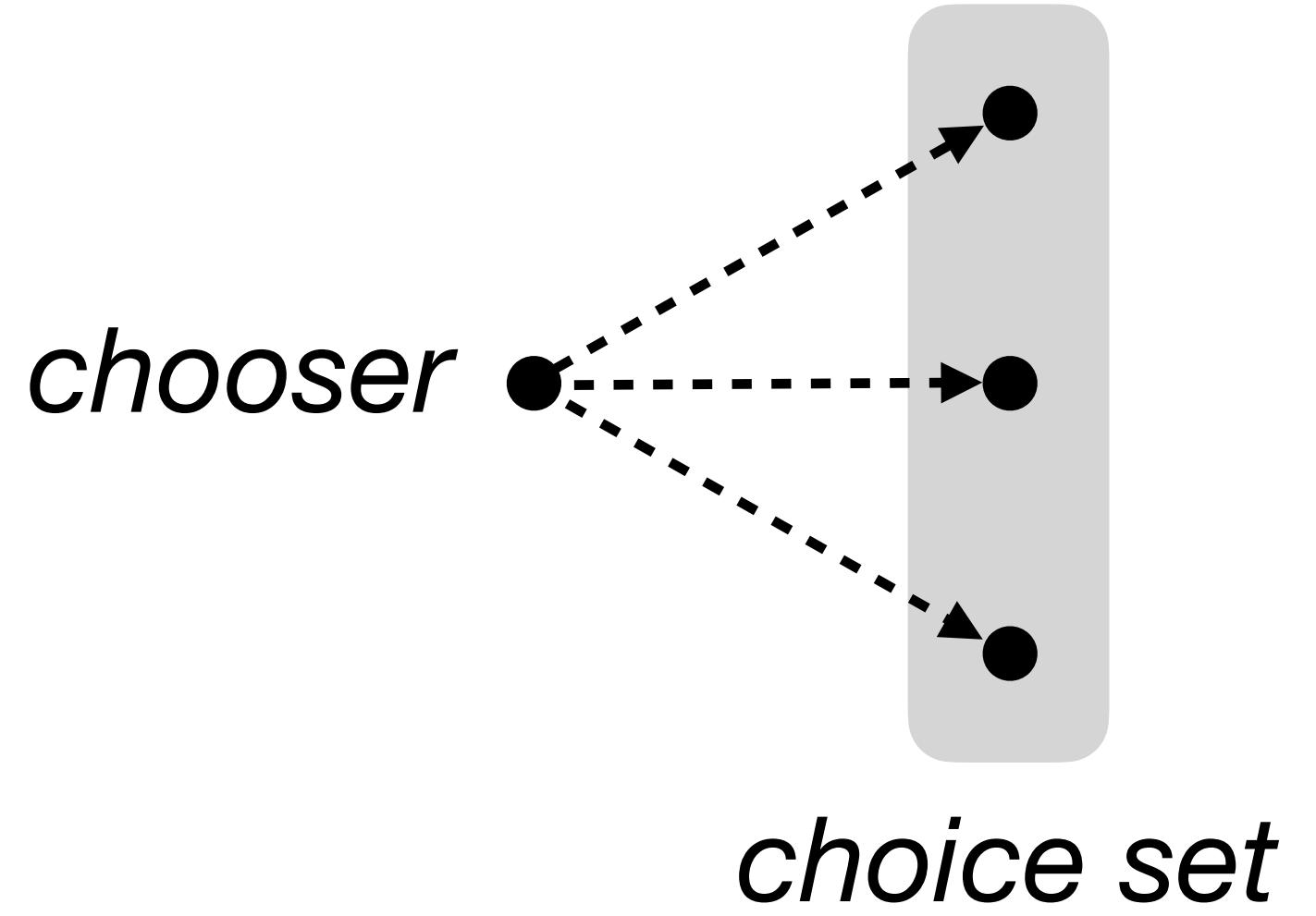
## Key usage

Timestamped edges

→ meaningful choice sets



in network growth



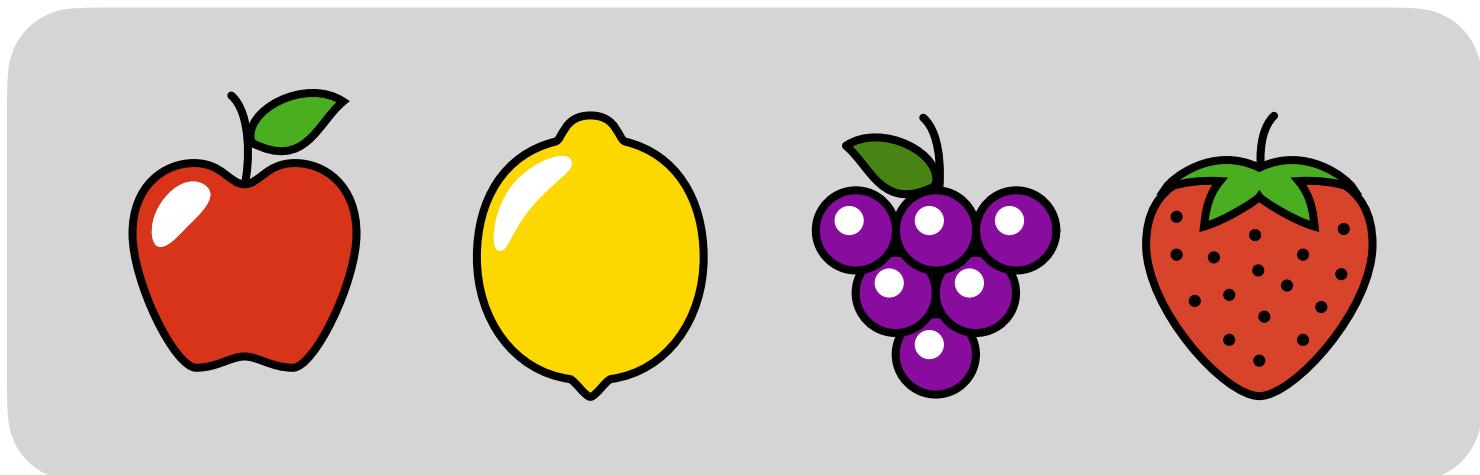
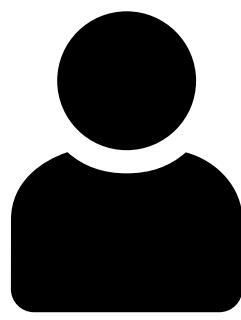
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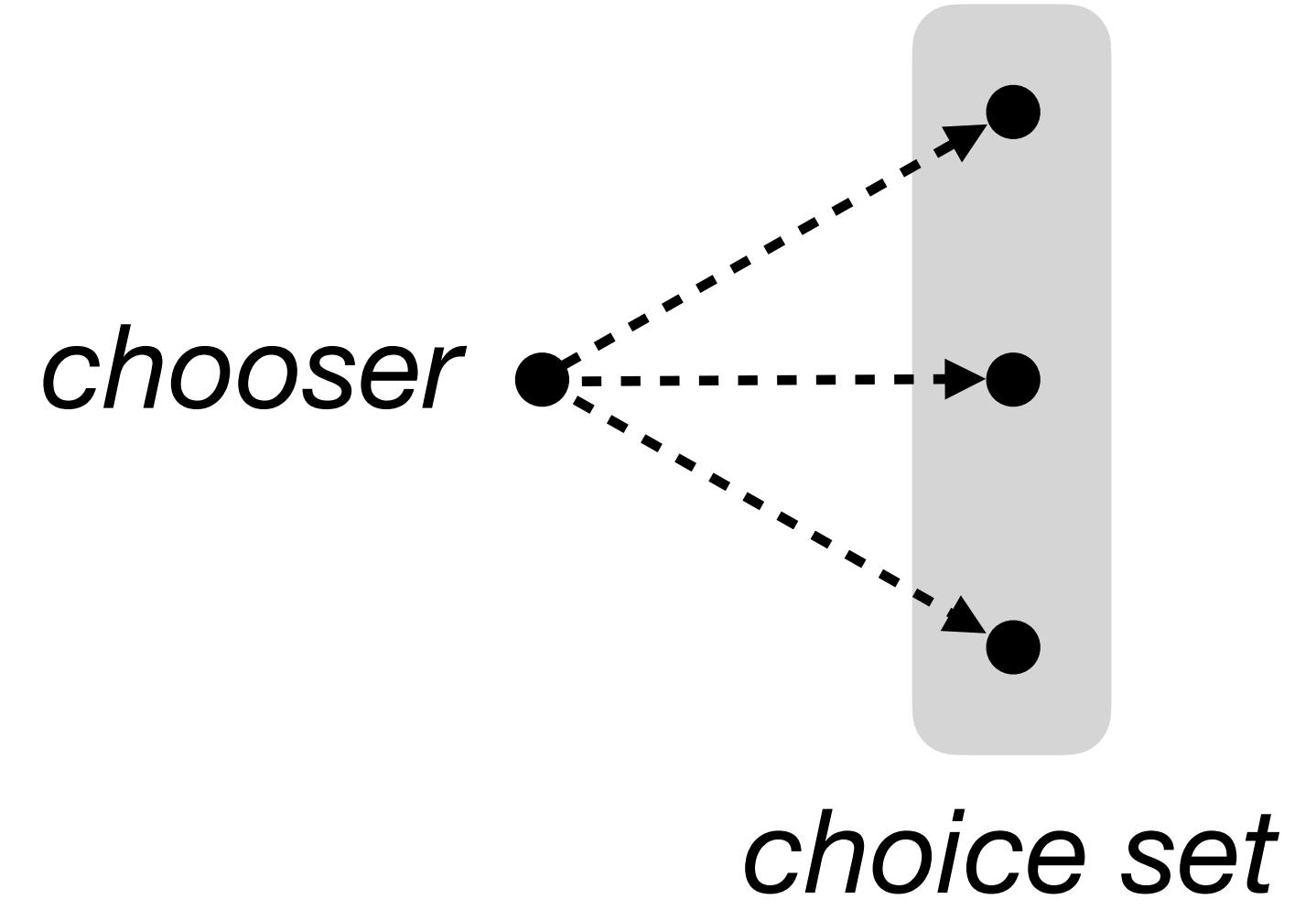
*choice set*

(under-explored in sociology)

(Bruch & Feinberg, *Annual Review of Sociology* 2017)



in network growth



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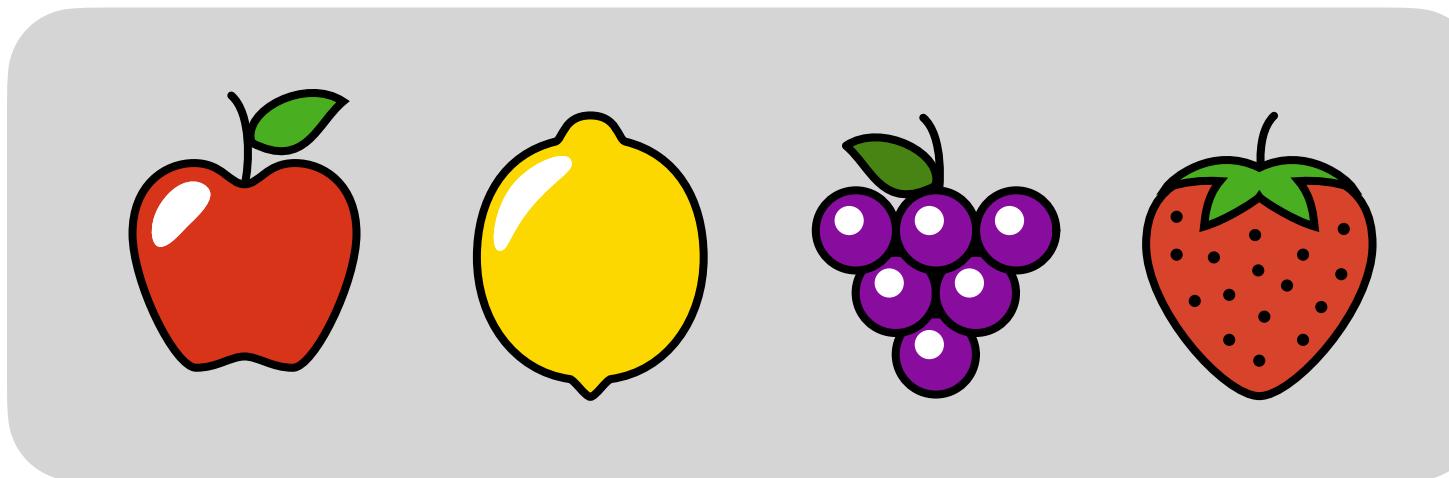
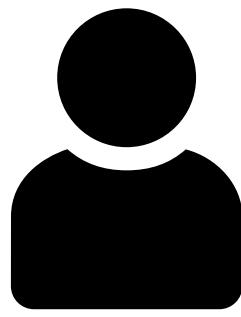
Infer relative importance of edge formation mechanisms from data

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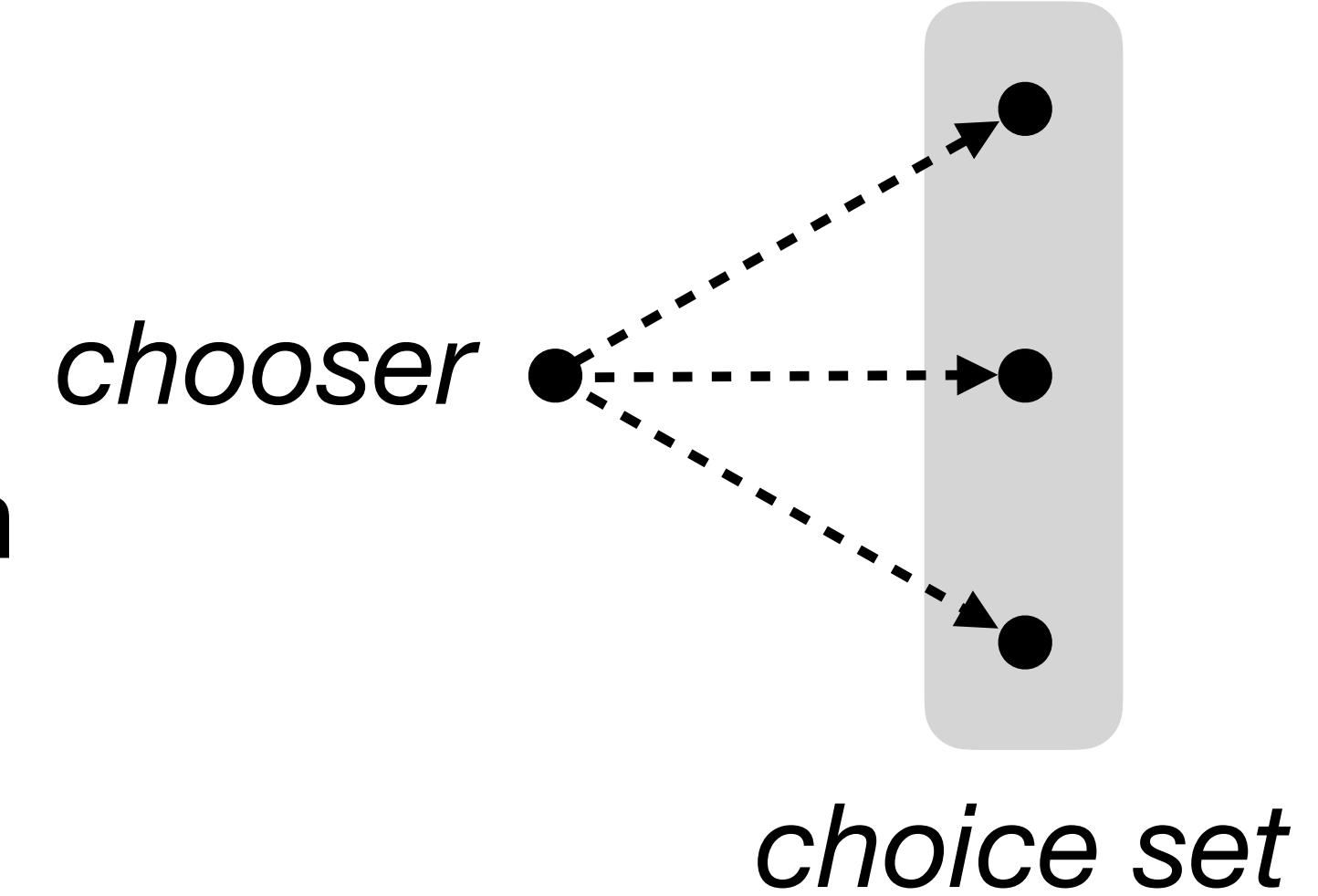
Timestamped edges

→ meaningful choice sets

Infer relative importance of edge formation mechanisms from data



in network growth



$$\Pr(i, C) = \frac{\exp(\theta^T x_i)}{\sum_{j \in C} \exp(\theta^T x_j)}$$

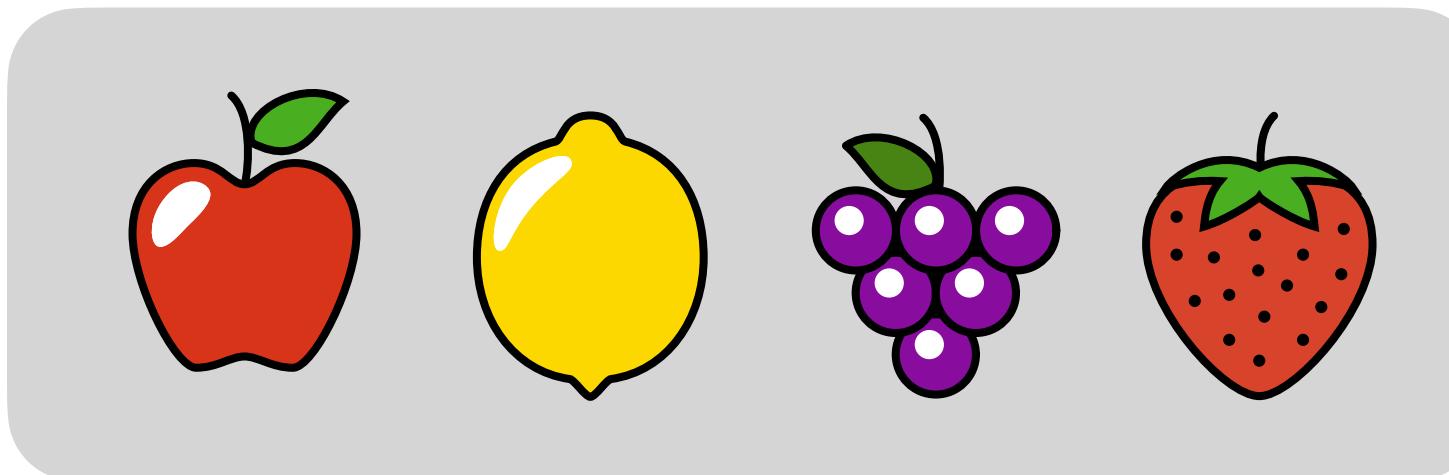
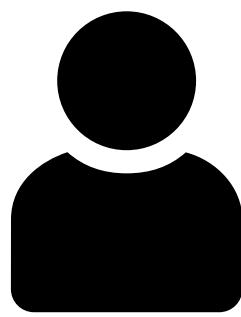
Multinomial logit  
(MNL) (McFadden, 1973)

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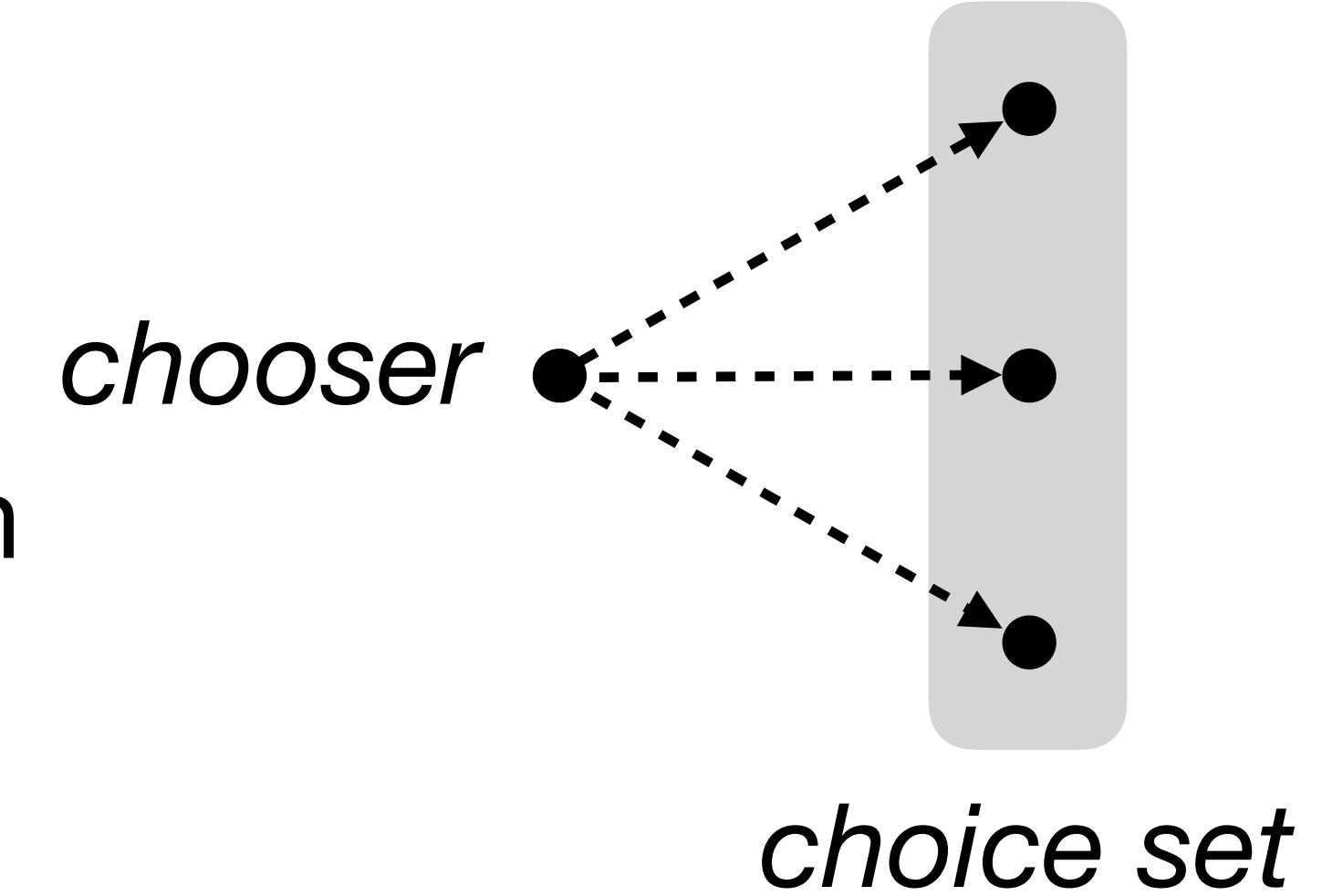
Timestamped edges

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in network growth



*choice set*

$$\Pr(i, C) = \frac{\exp(\theta^T x_i)}{\sum_{j \in C} \exp(\theta^T x_j)}$$

↑  
node

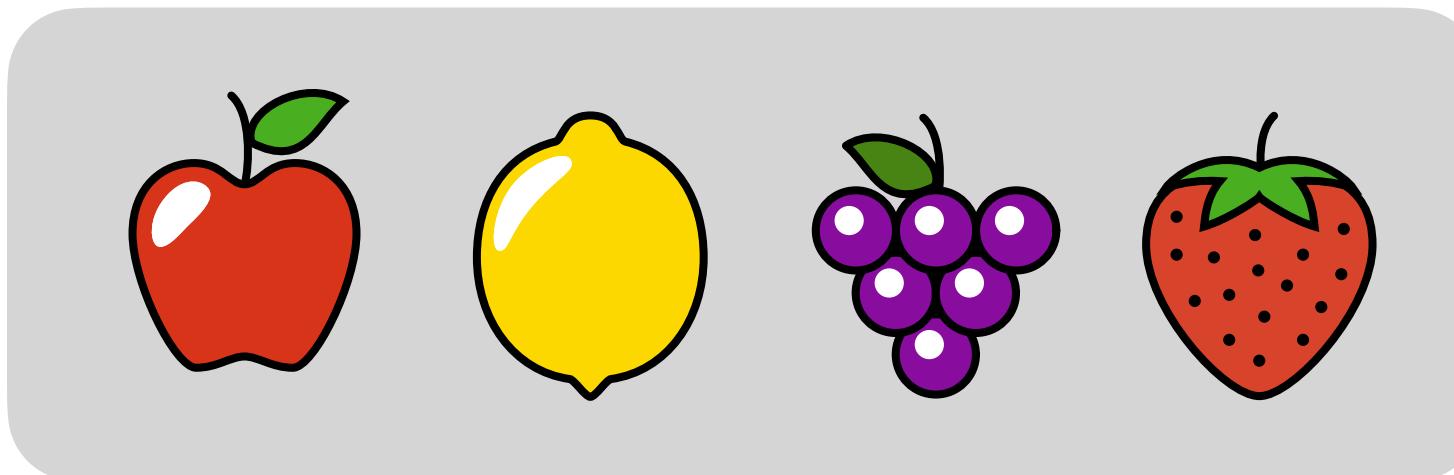
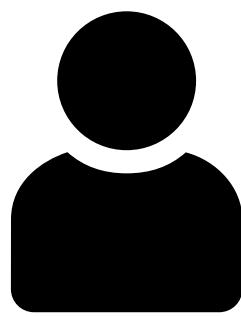
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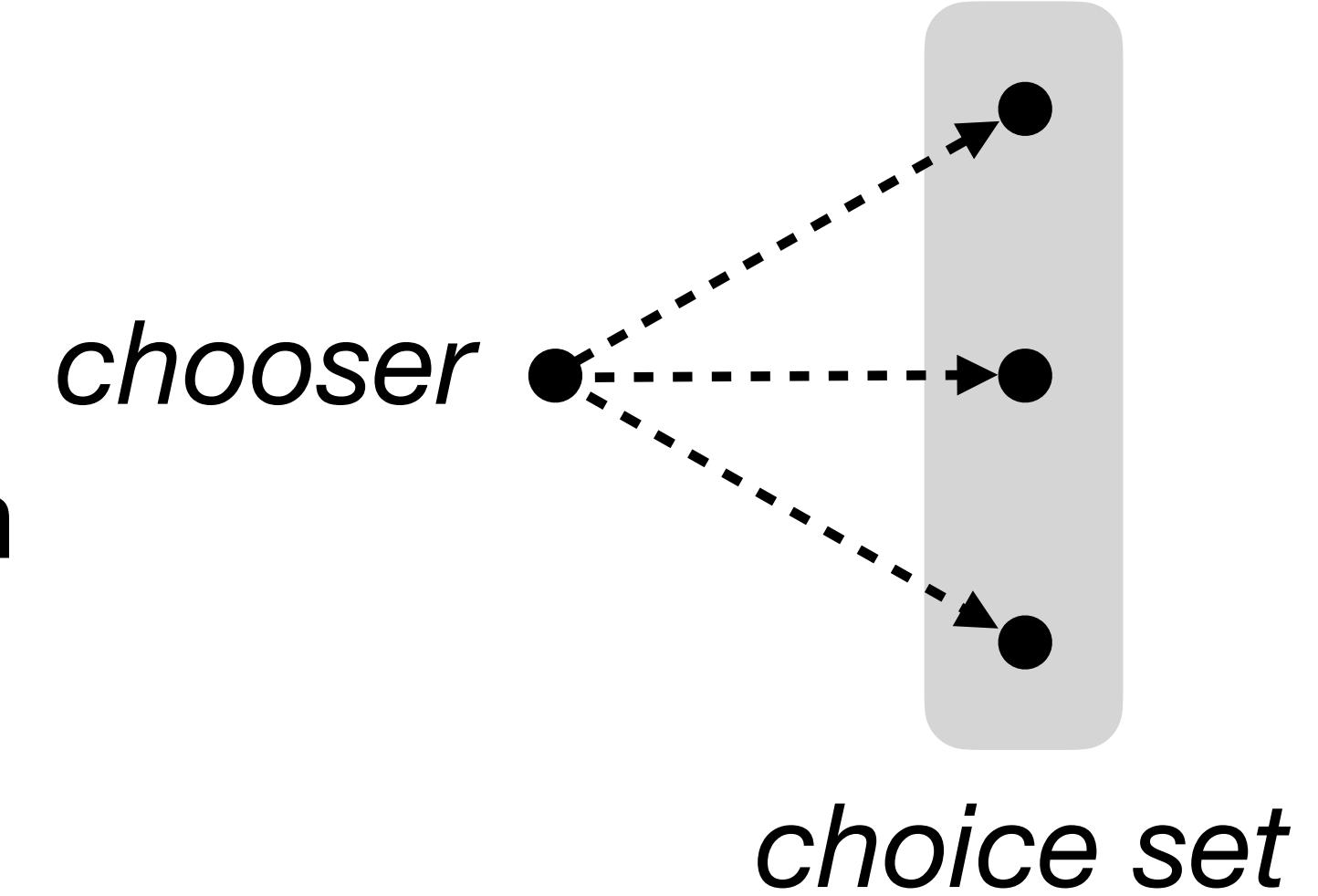
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in network growth



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node                      choice set

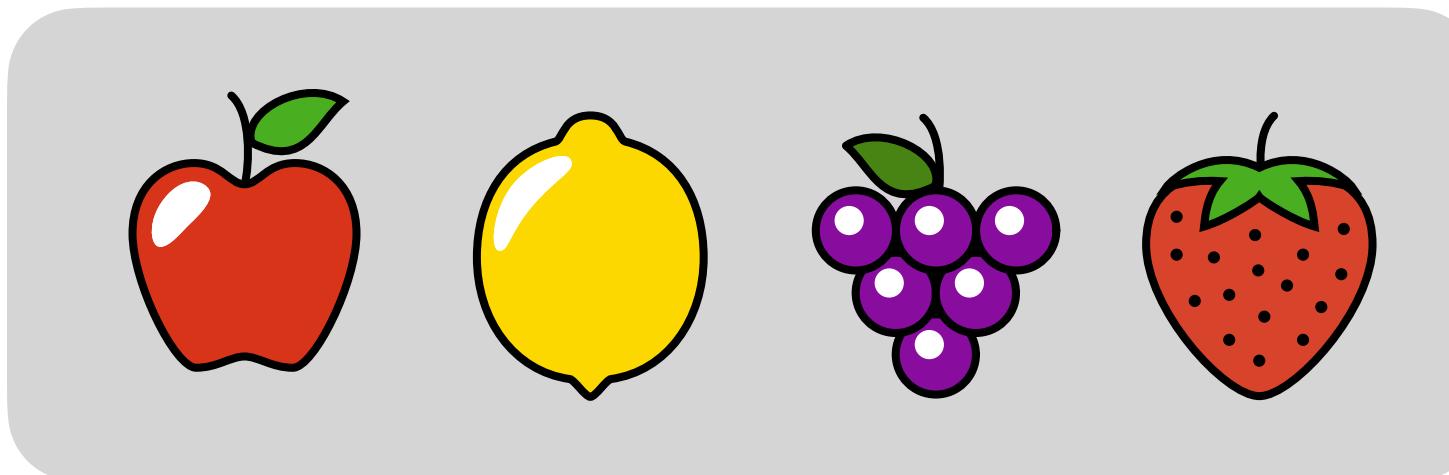
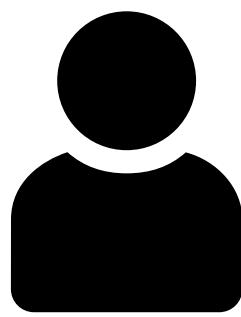
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## Key usage

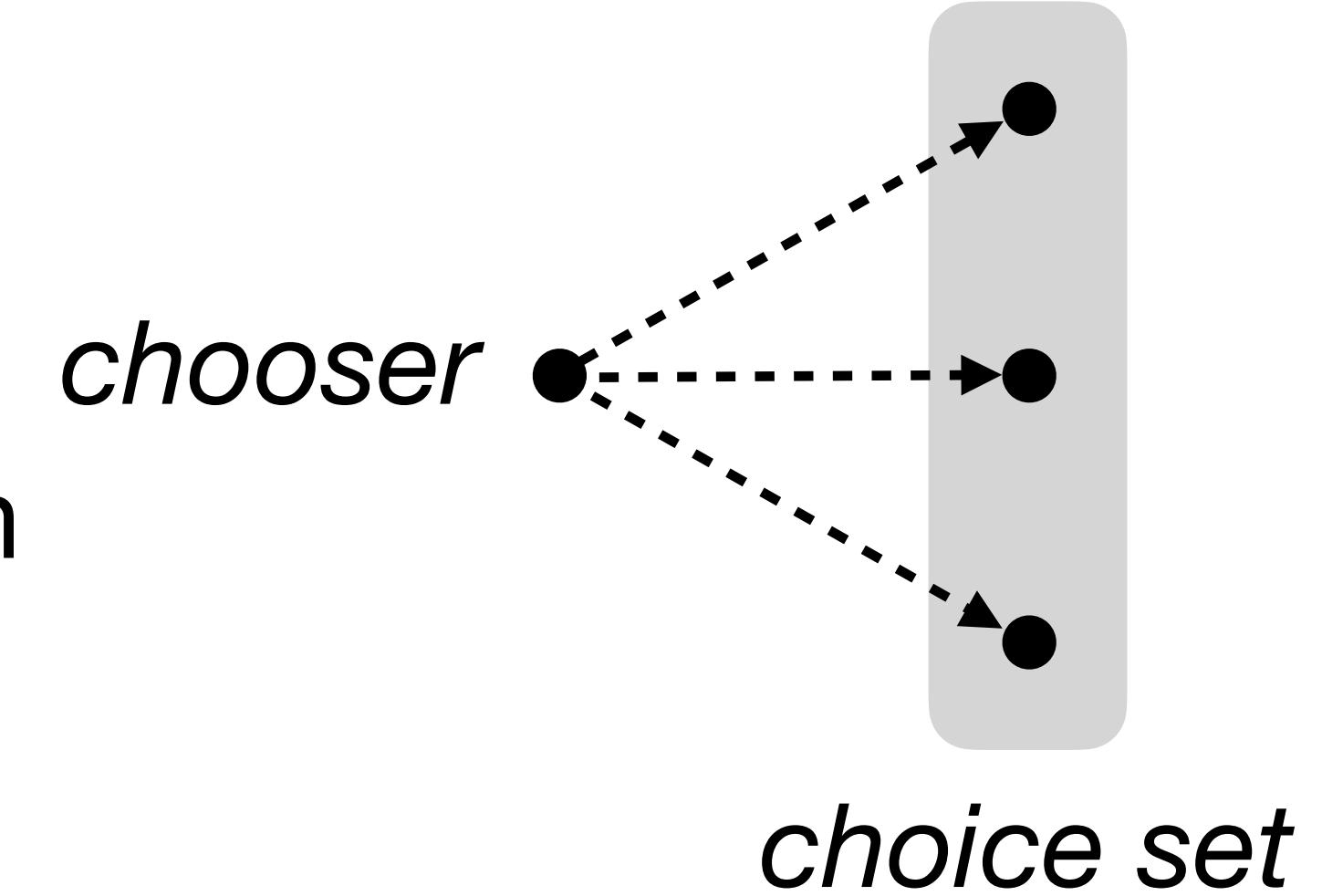
Timestamped edges

→ meaningful choice sets

Infer relative importance of edge formation mechanisms from data



in network growth



preferences

$$\Pr(i, C) = \frac{\exp(\theta^T x_i)}{\sum_{j \in C} \exp(\theta^T x_j)}$$

node                      choice set

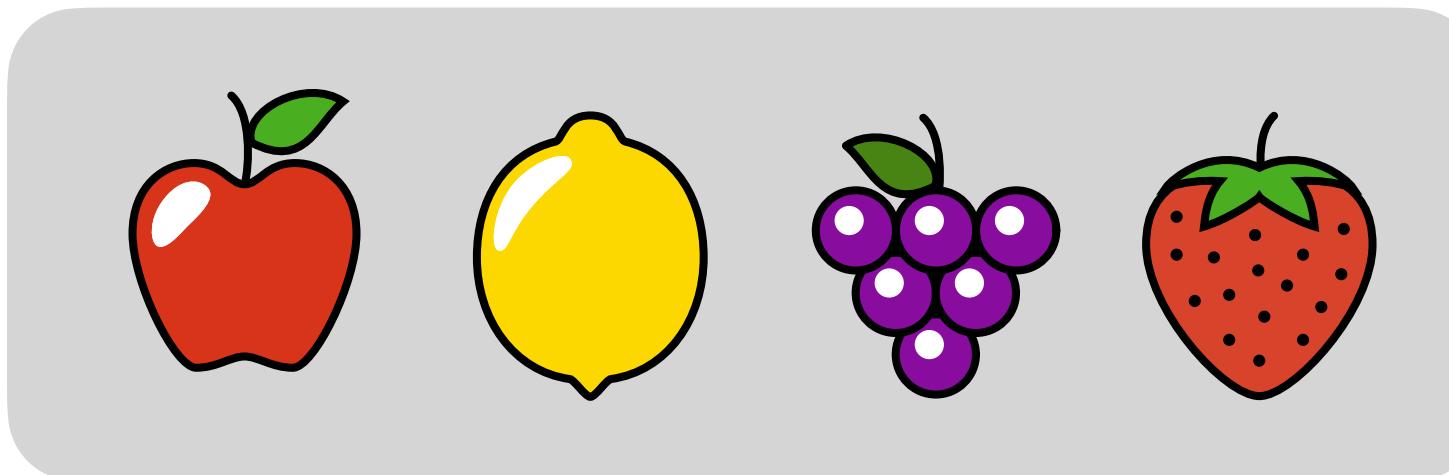
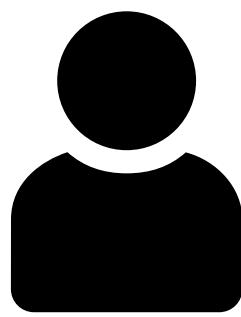
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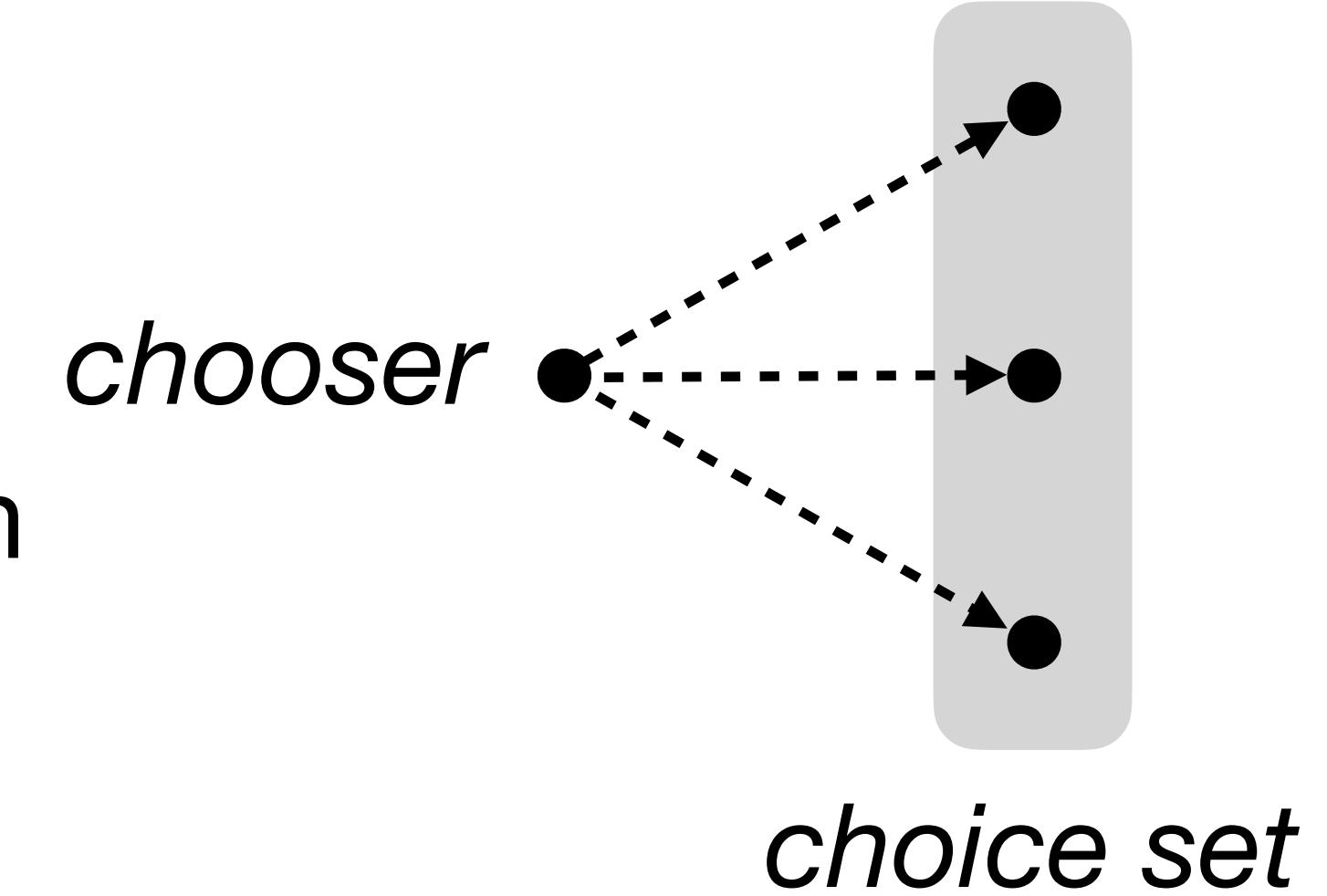
Timestamped edges

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Infer relative importance of edge formation mechanisms from data



in network growth



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preferences      node features  
                        ↑  
                        node  
                        ↑  
                        choice set

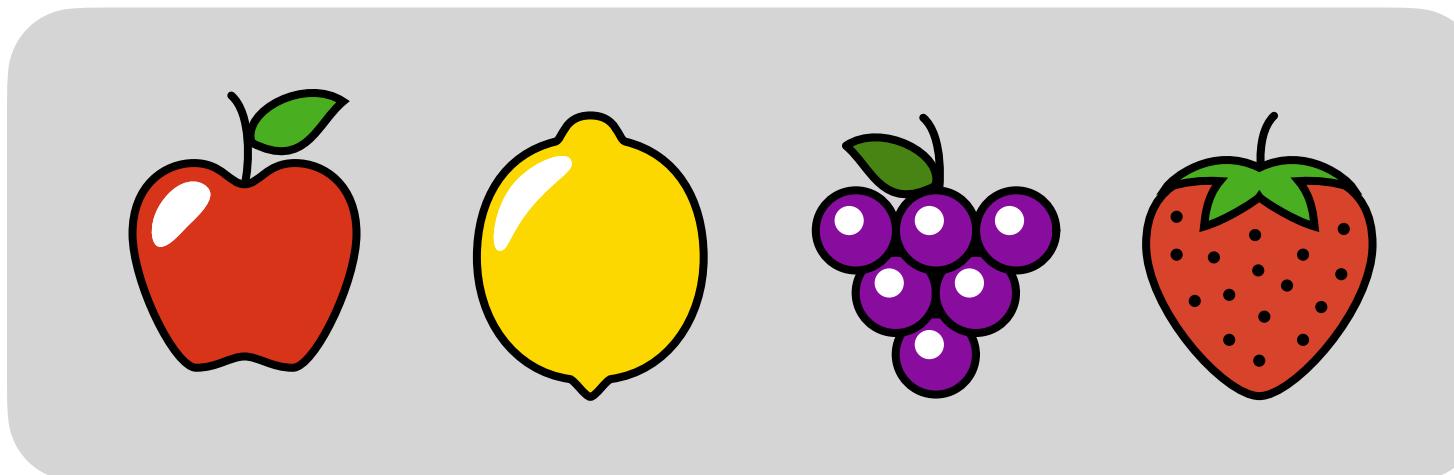
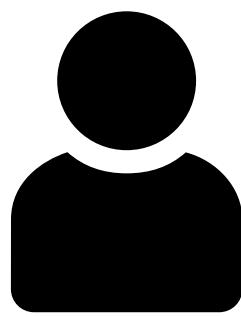
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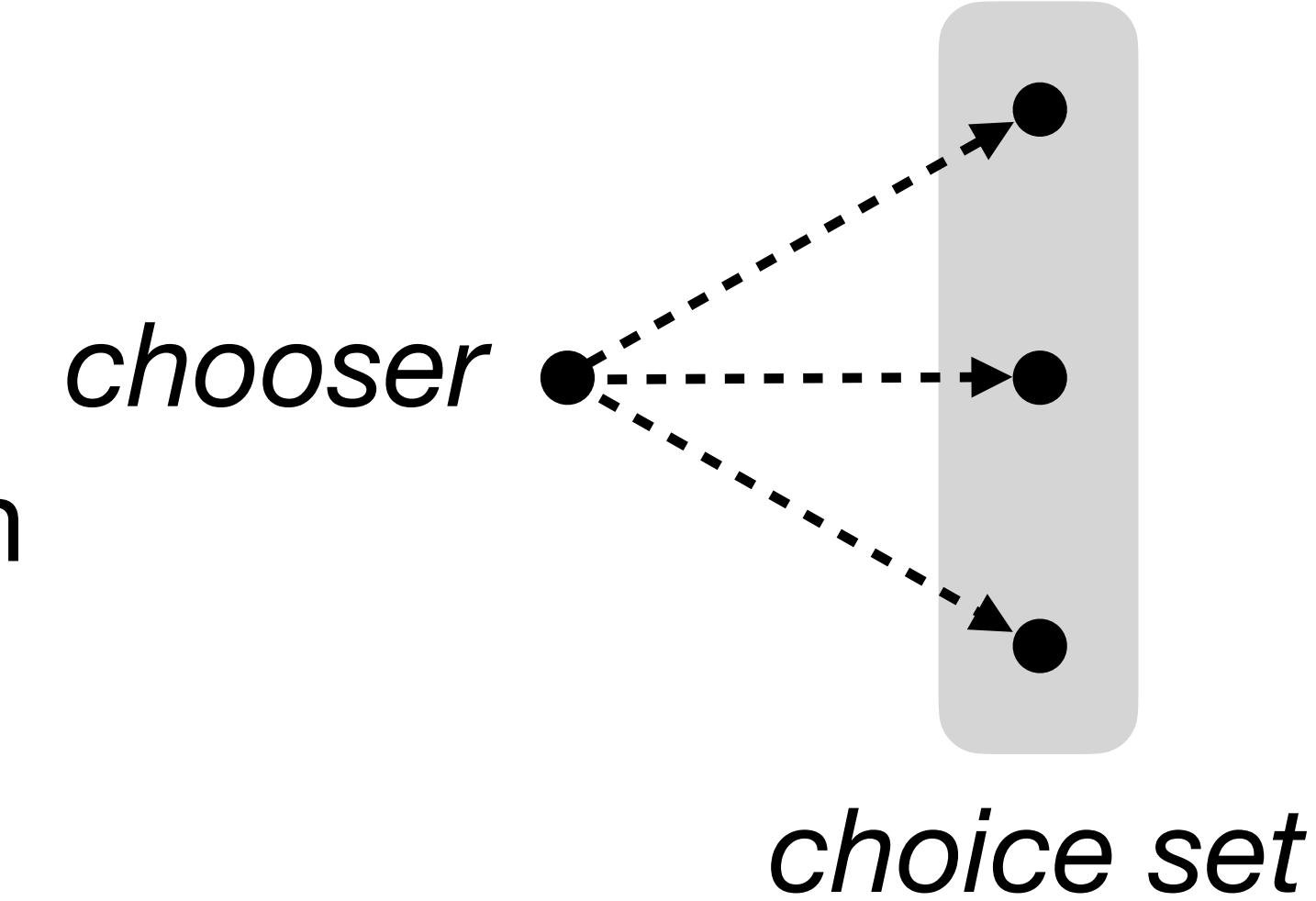
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preferences  
node features  
(similarity, in-degree, fitness...)

node  
choice set

Multinomial logit  
(MNL) (McFadden, 1973)

# The choice set affects preferences

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## *Context effects*

(Huber et al., *Journal of Consumer Research* 1982)

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e.g., *compromise effect*:

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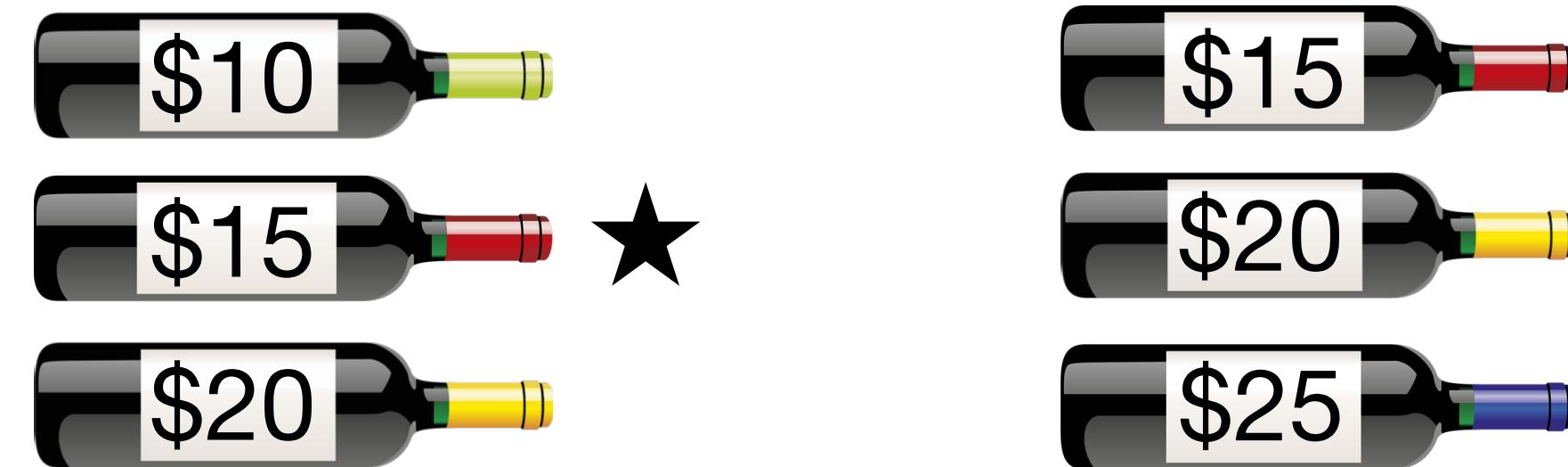
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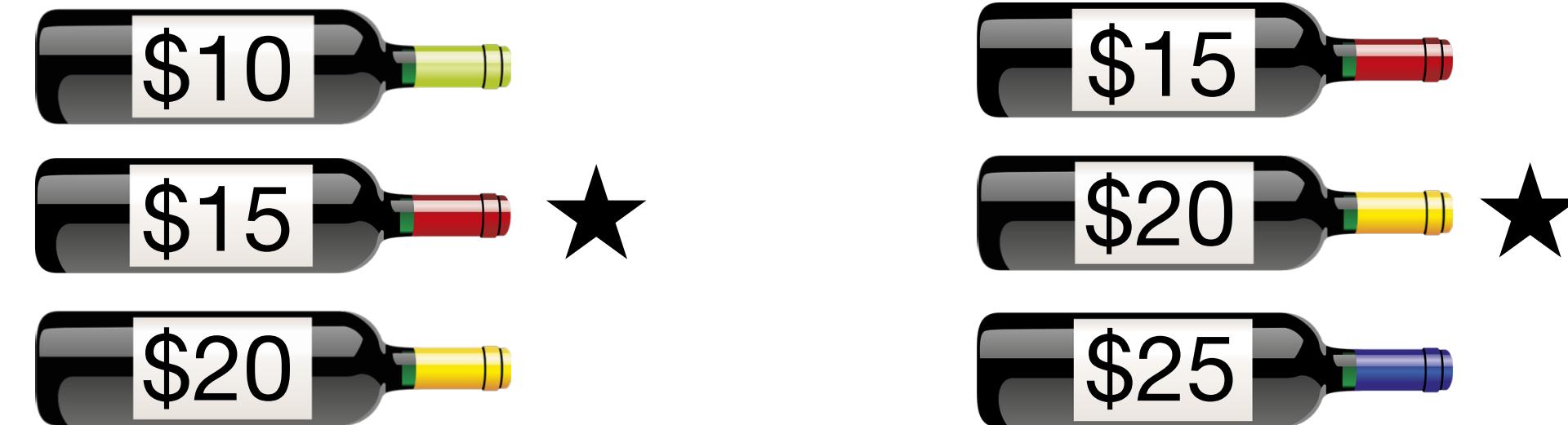
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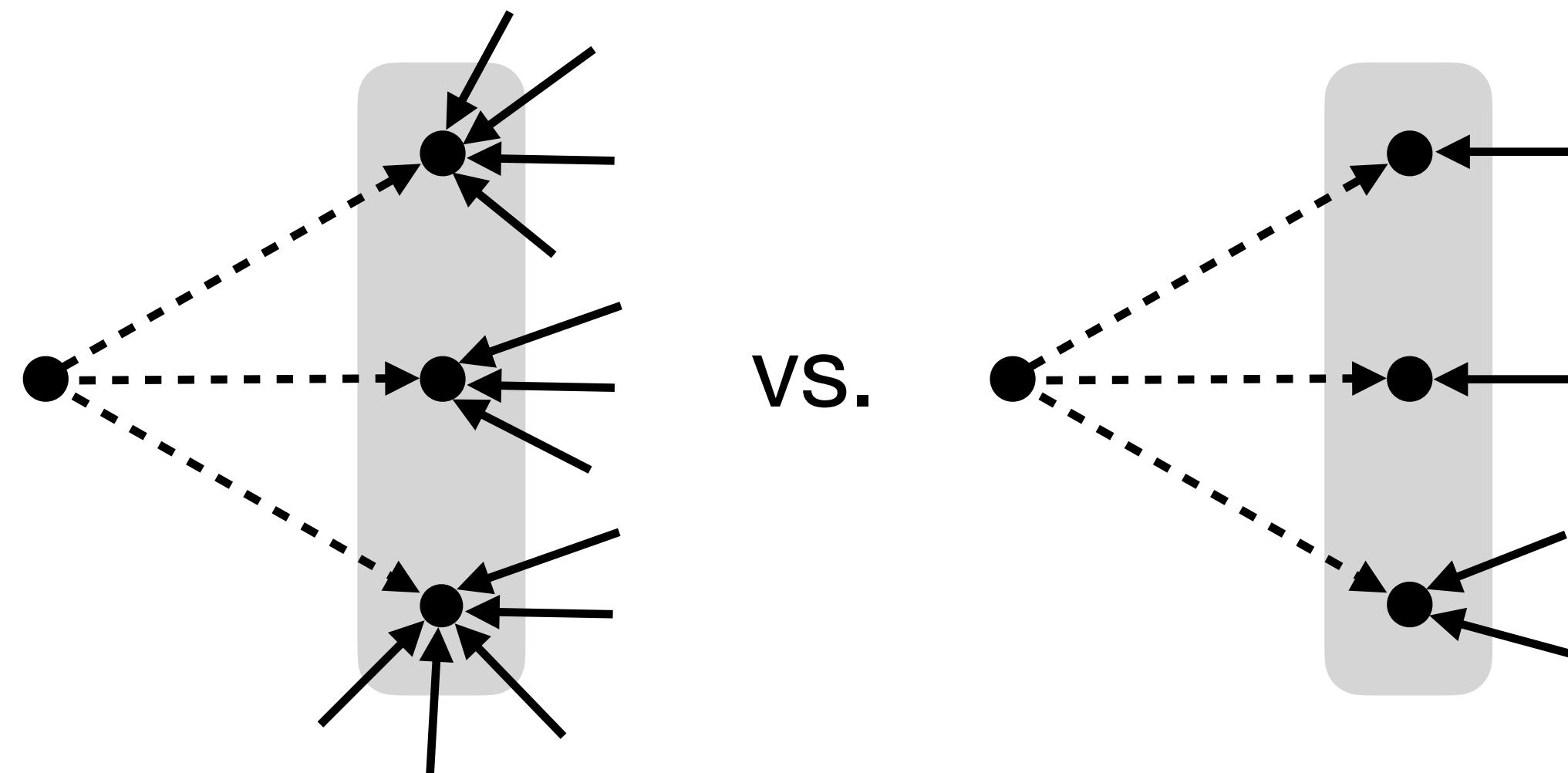
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## In networks

e.g., how do preferences change  
when choosing from a popular group?



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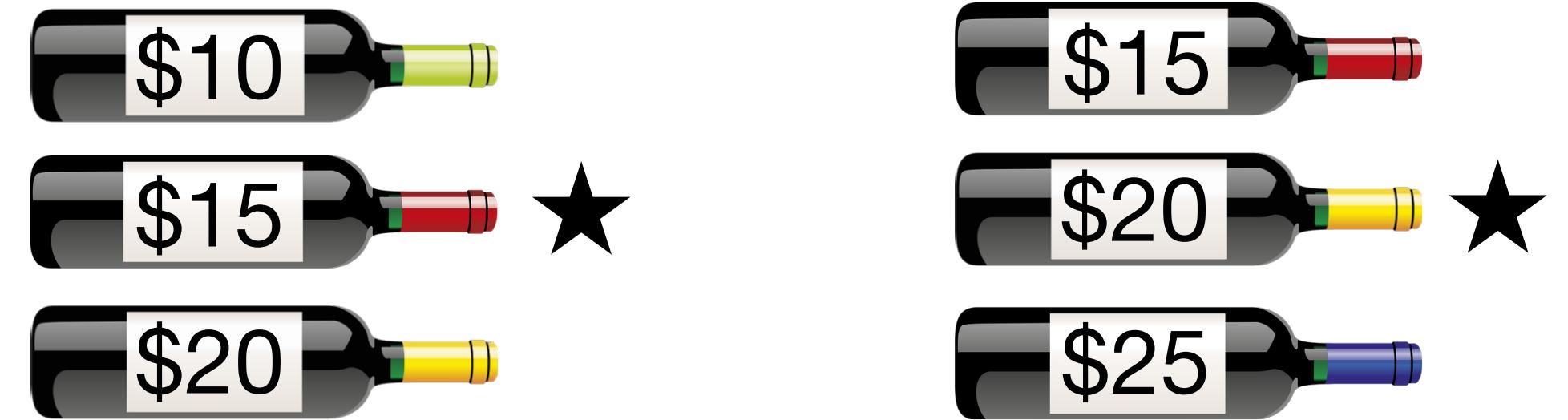
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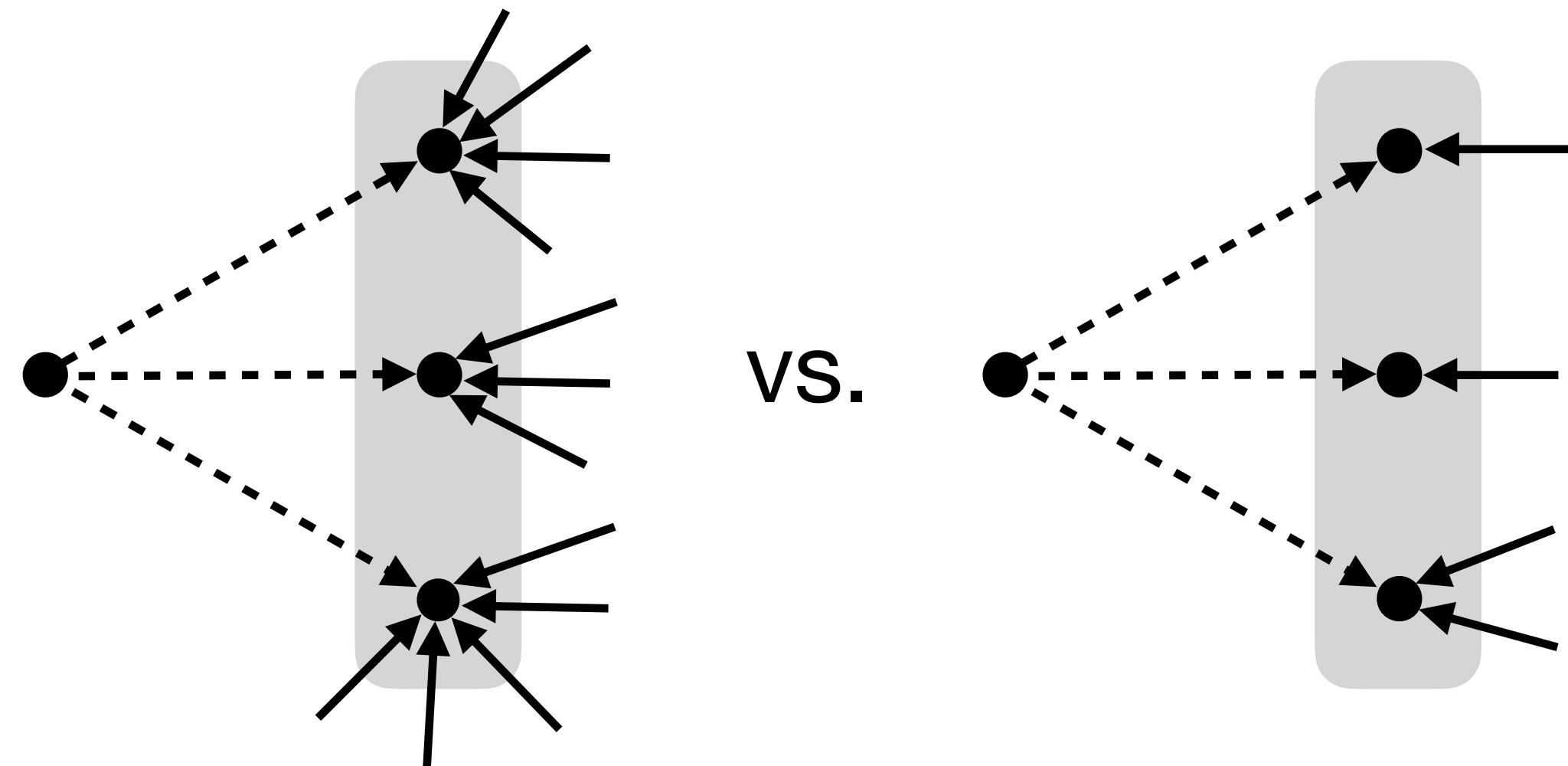
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Our model:

**Linear context logit (LCL)**

$$\Pr(i, C) = \frac{\exp([\theta + Ax_C]^T x_i)}{\sum_{j \in C} \exp([\theta + Ax_C]^T x_j)}$$

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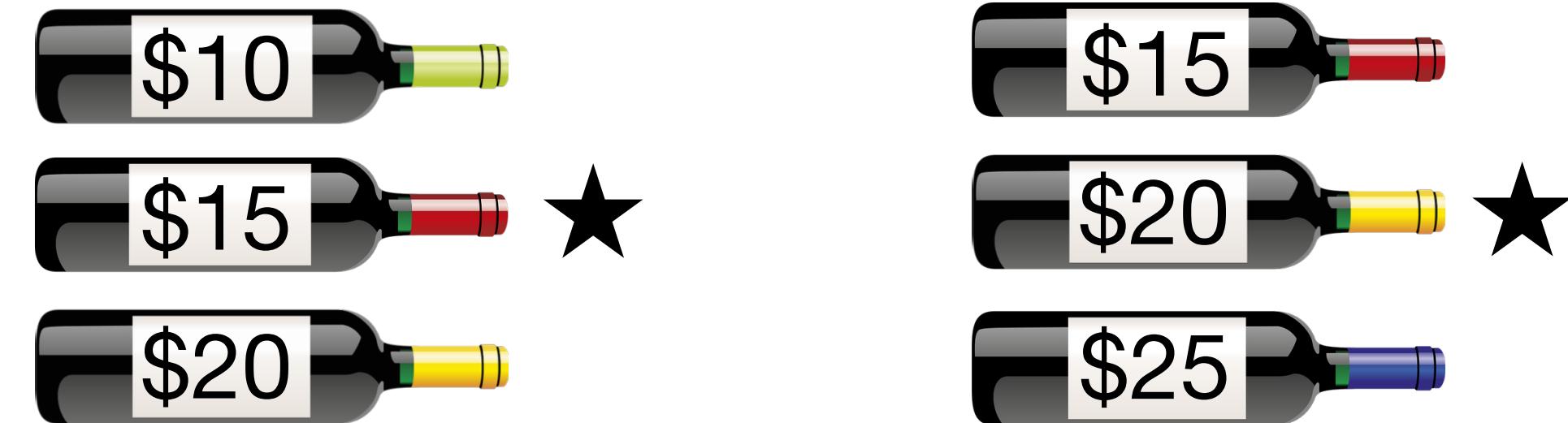
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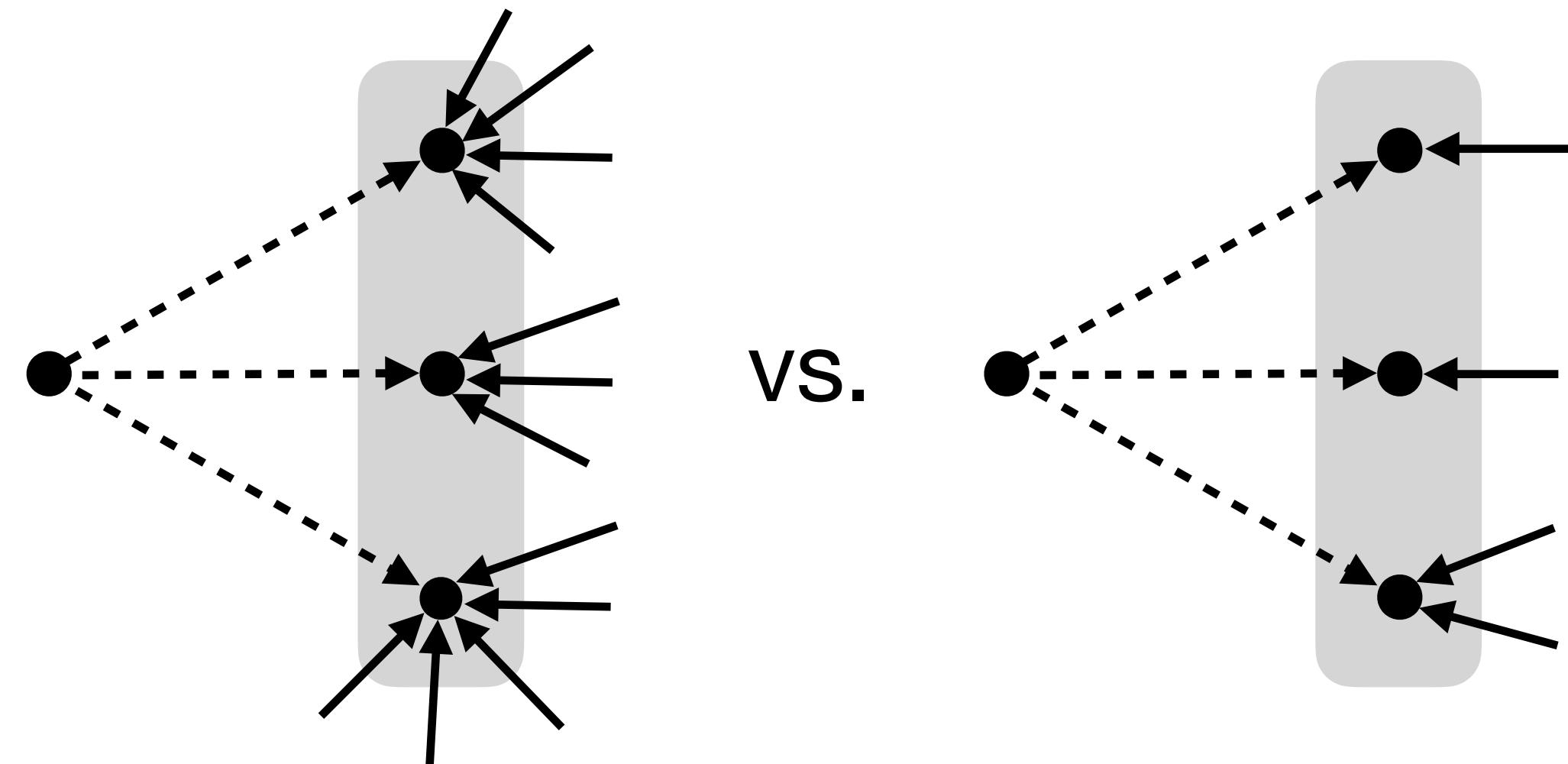
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base preferences

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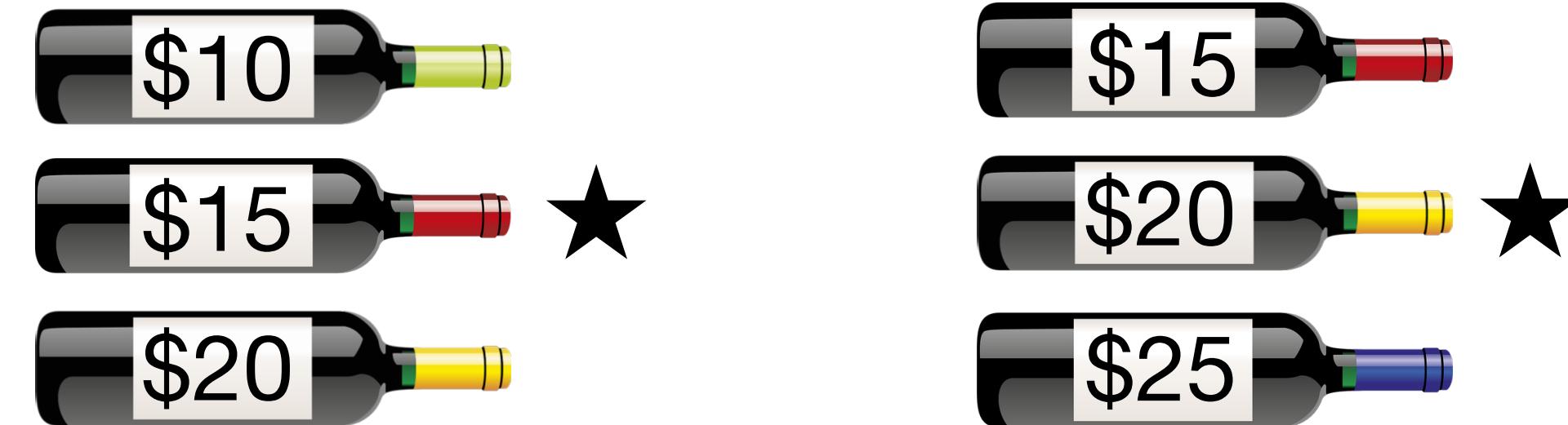
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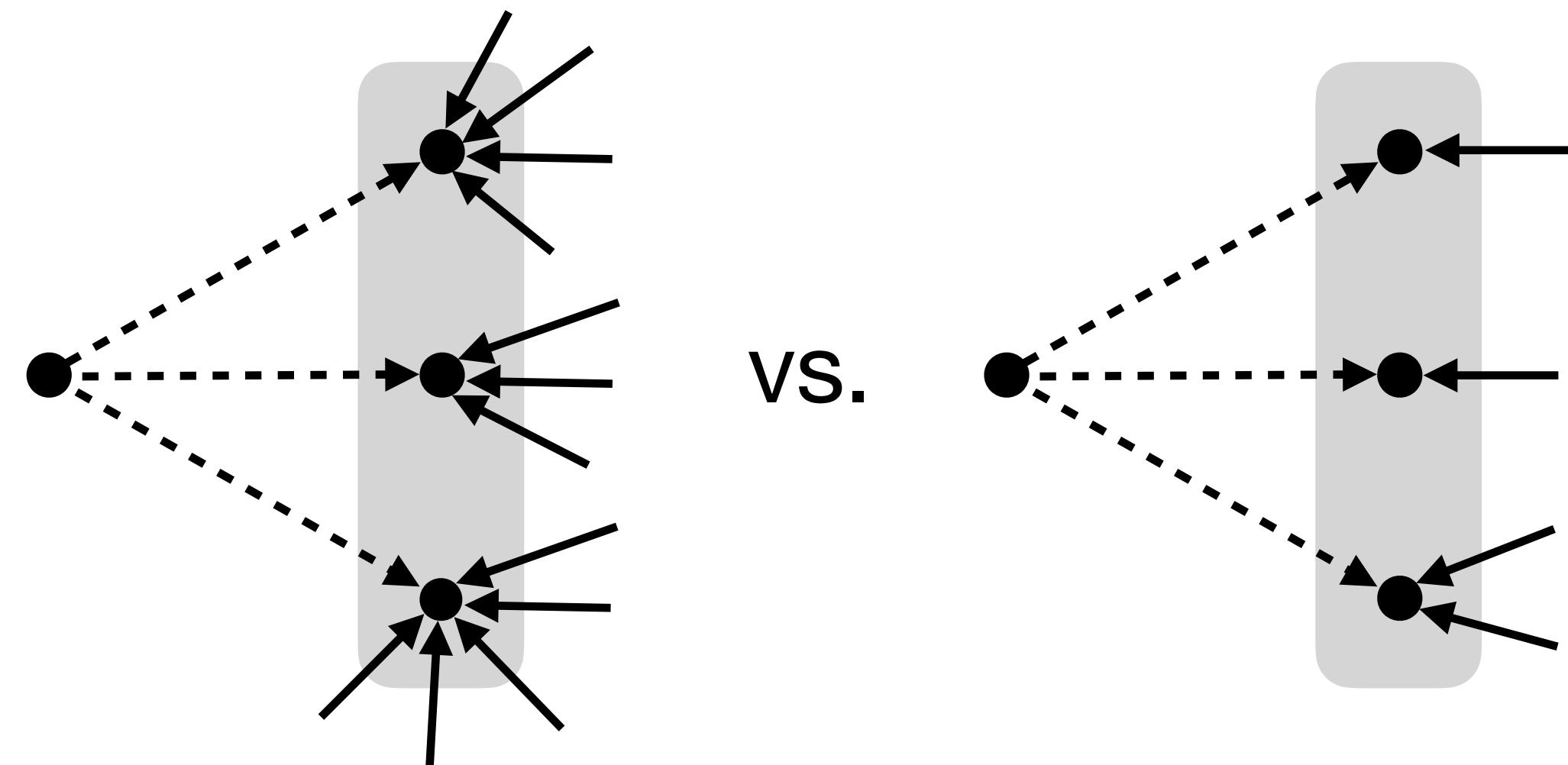
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base preferences      context effect matrix

# The choice set affects preferences

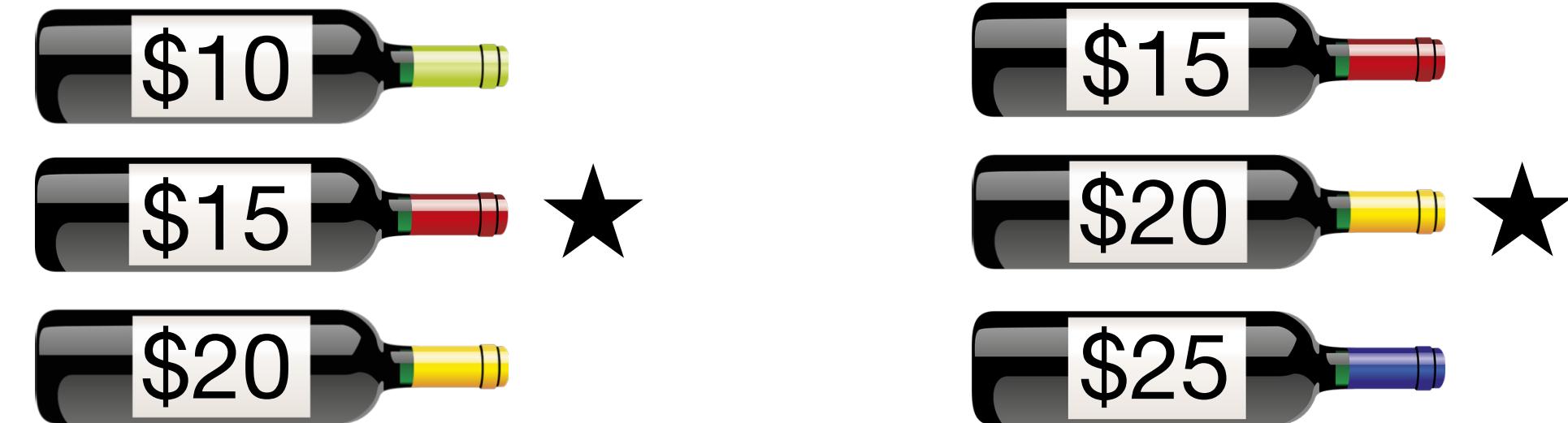
## Context effects

(Huber et al., *Journal of Consumer Research* 1982)

(Simonson & Tversky, *Journal of Marketing Research* 1992)

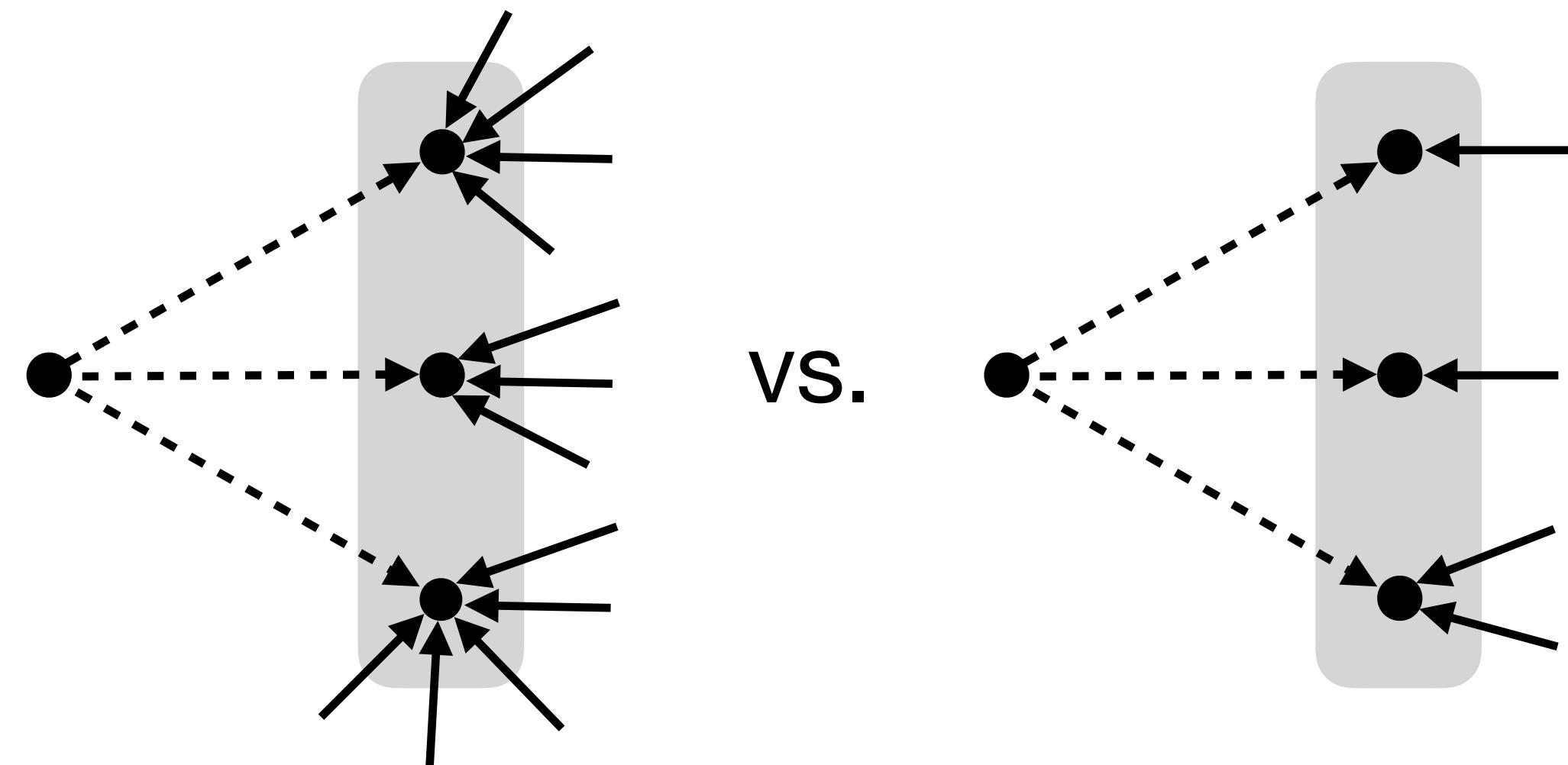
e.g., *compromise effect*:

(Simonson, *Journal of Consumer Research* 1989)



## In networks

e.g., how do preferences change  
when choosing from a popular group?



Our model:

## Linear context logit (LCL)

$$\Pr(i, C) = \frac{\exp([\theta + Ax_C]^T x_i)}{\sum_{j \in C} \exp([\theta + Ax_C]^T x_j)}$$

base preferences      context effect matrix      mean features over choice set

# Choosing to close triangles

Triadic closure offers small choice sets

- tractable inference
- varied choice sets

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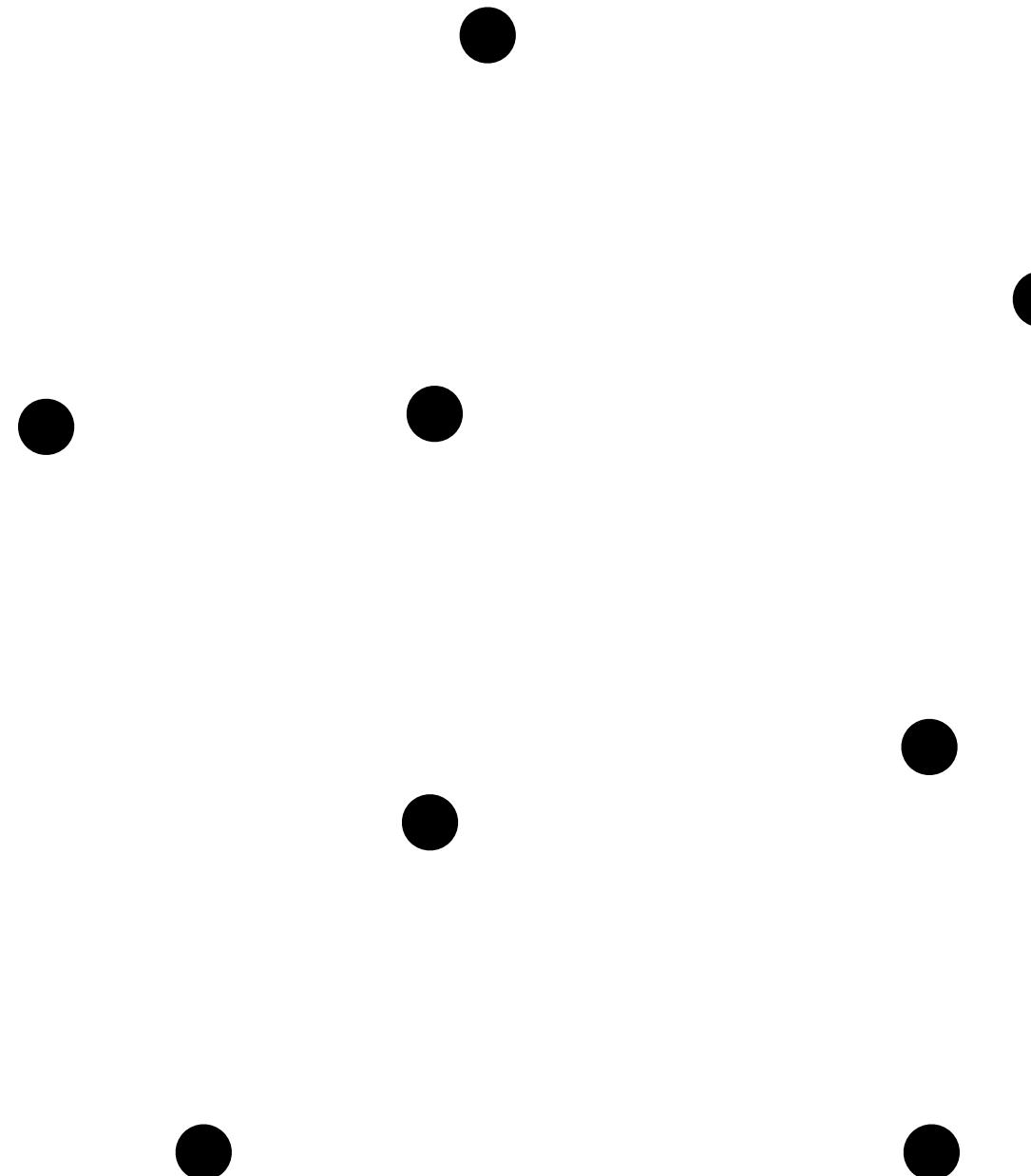
## Our data

Timestamped edges  
(including repeats)

# Choosing to close triangles

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- tractable inference
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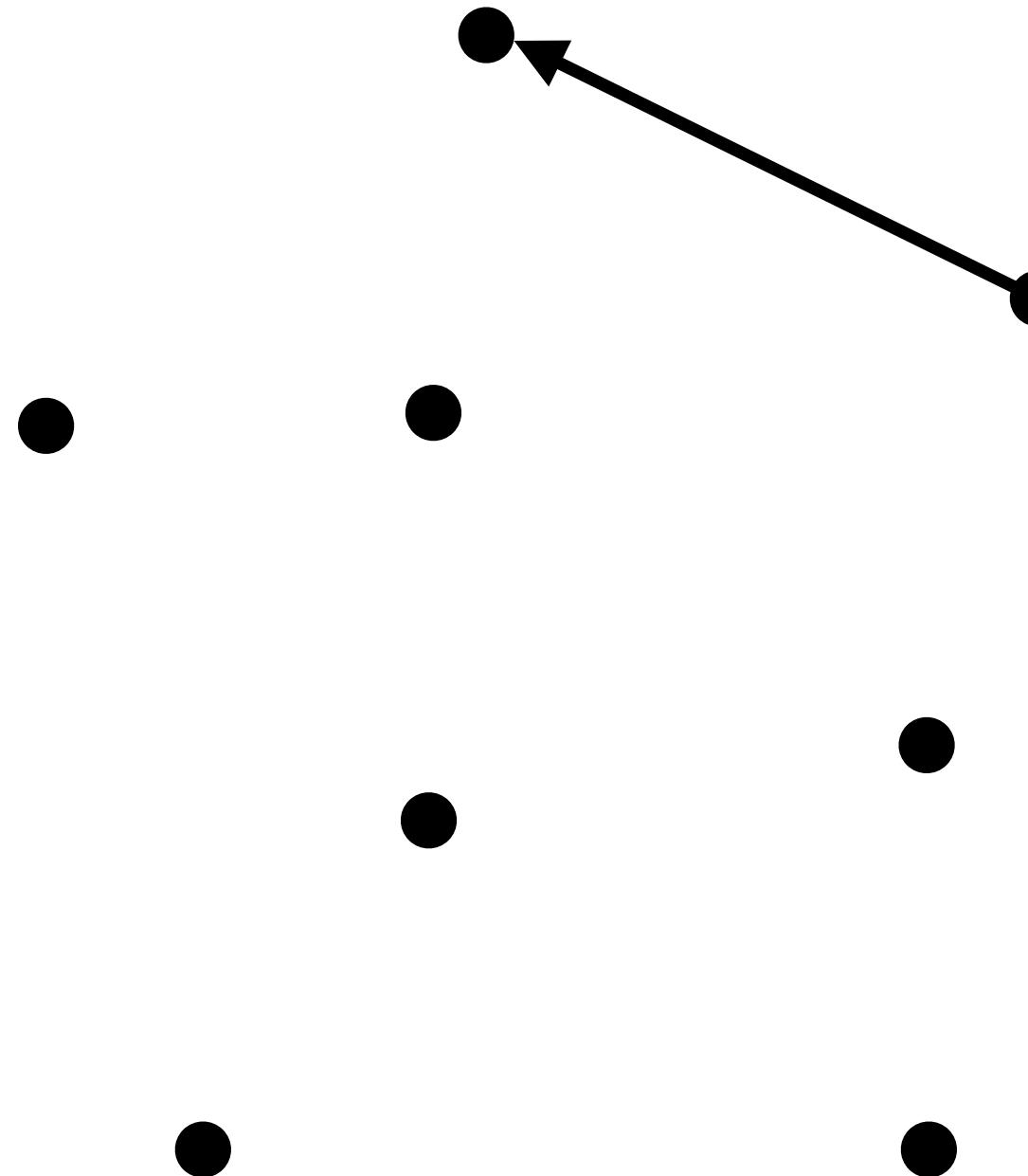
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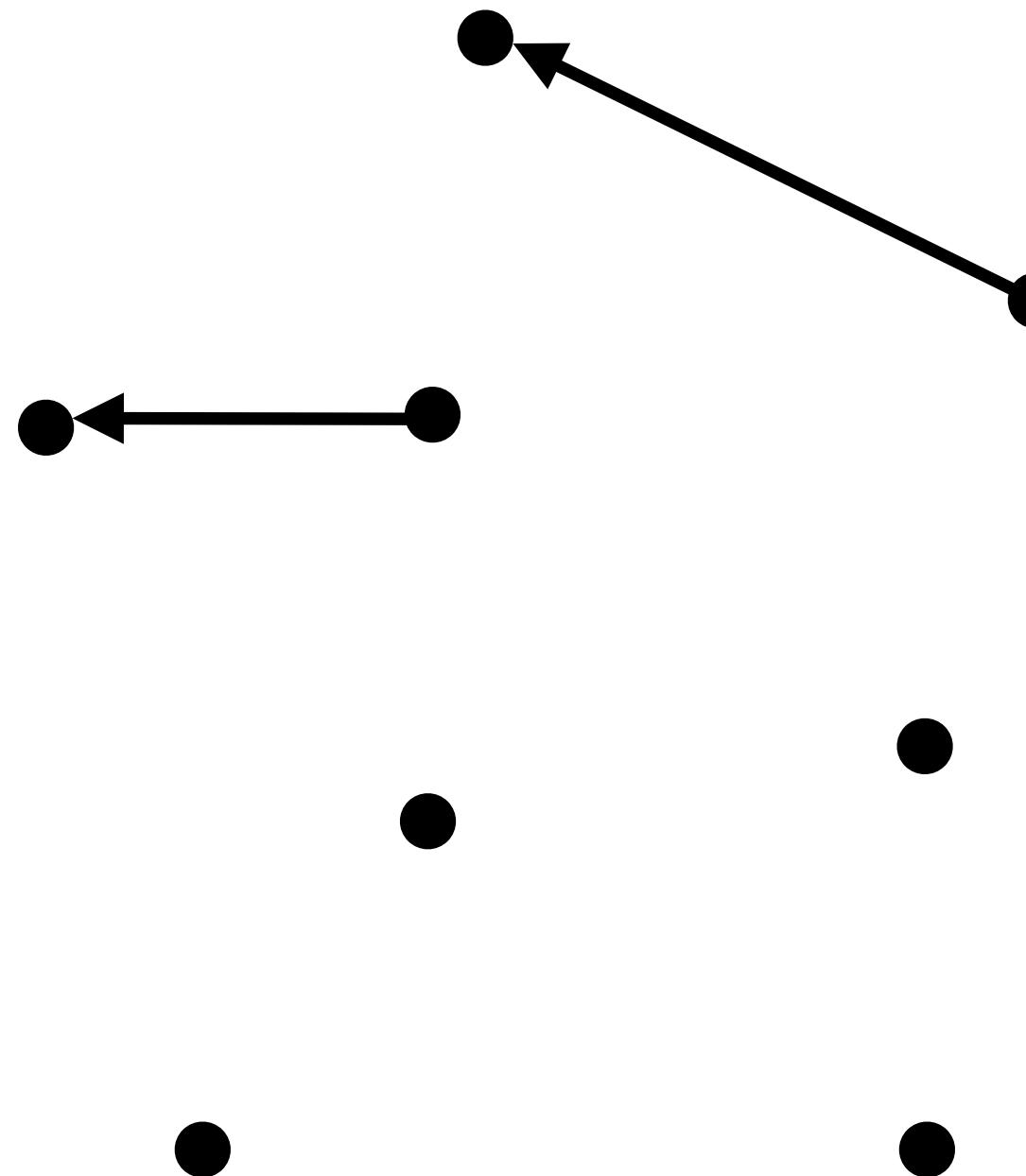
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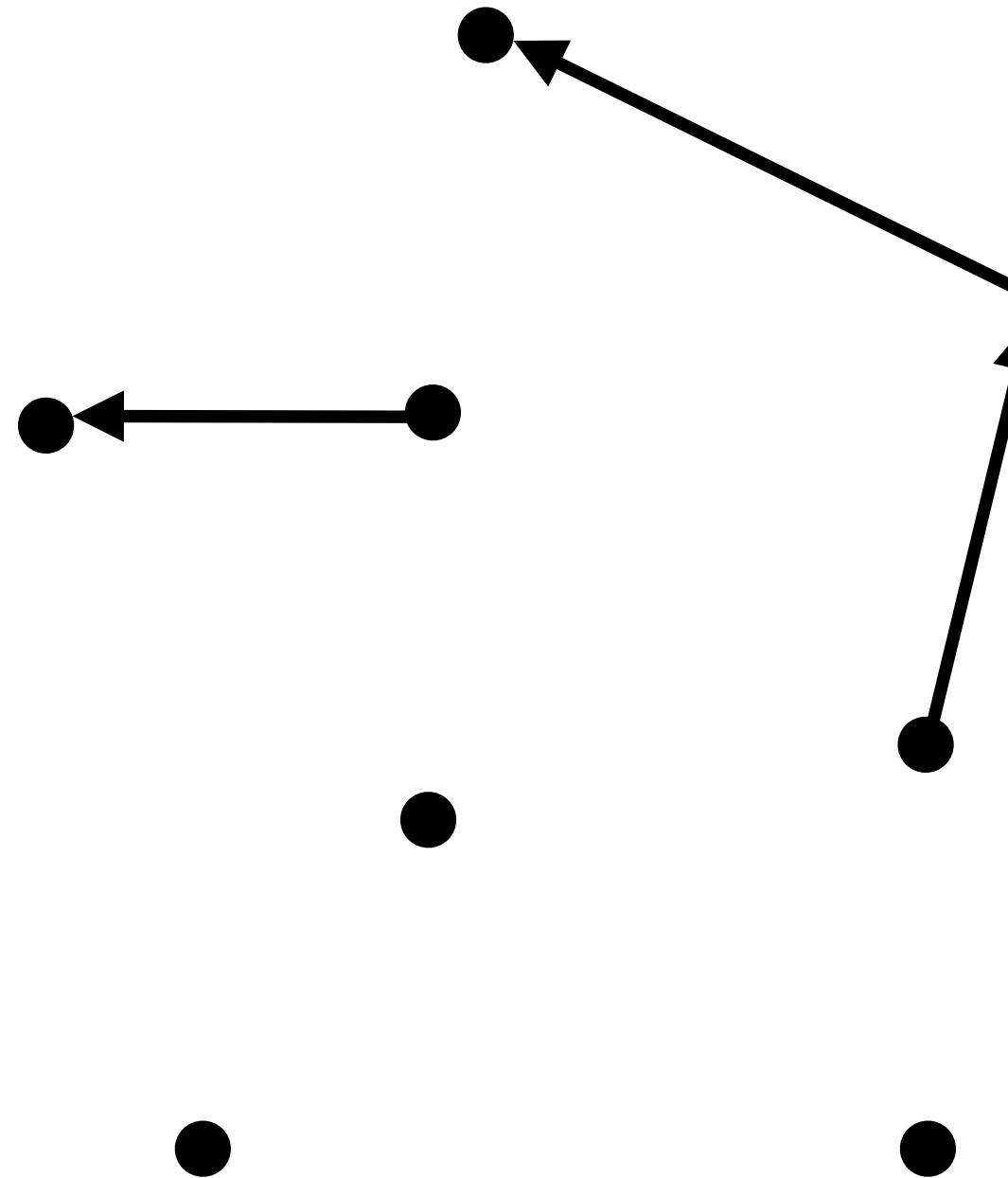
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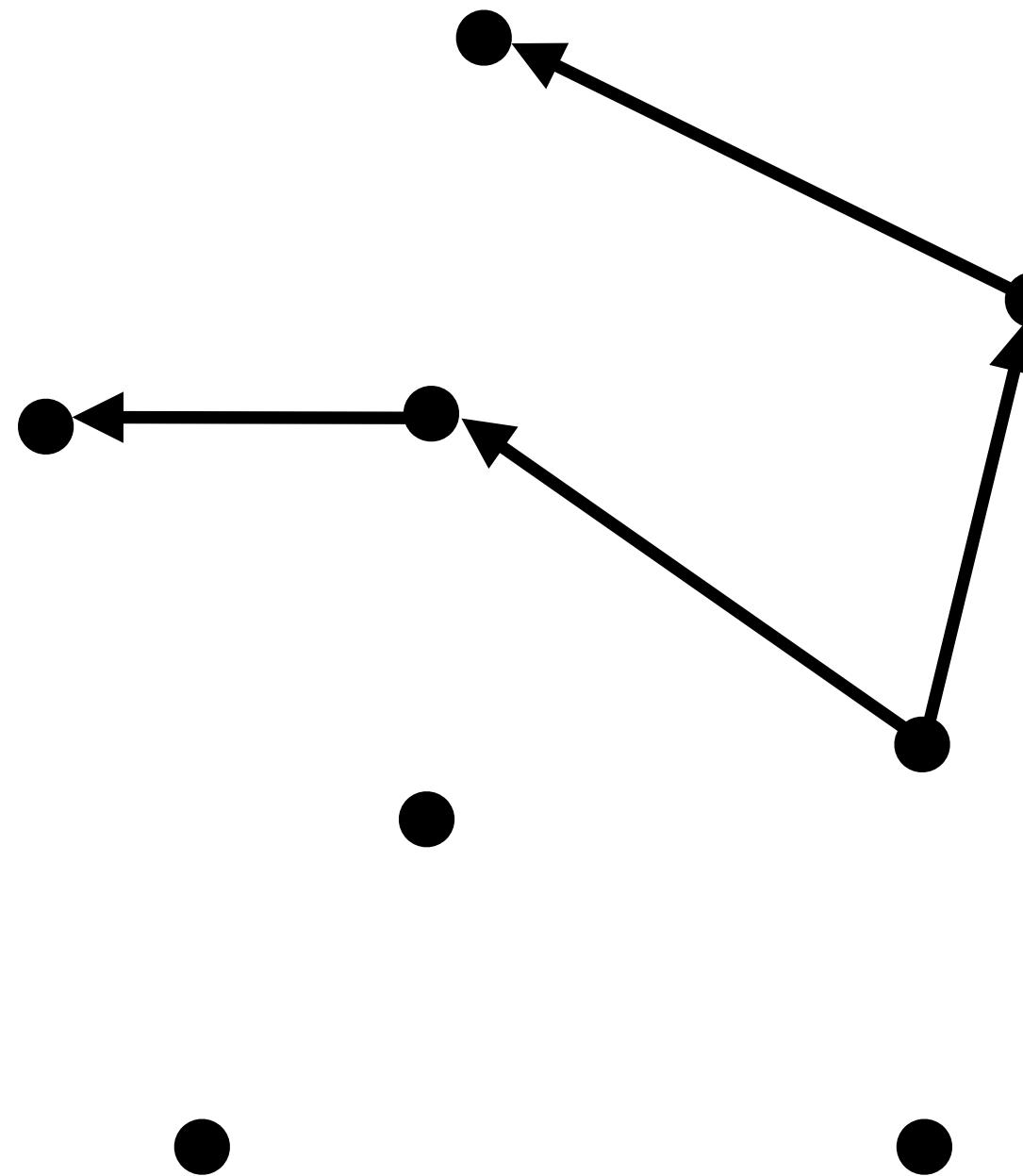
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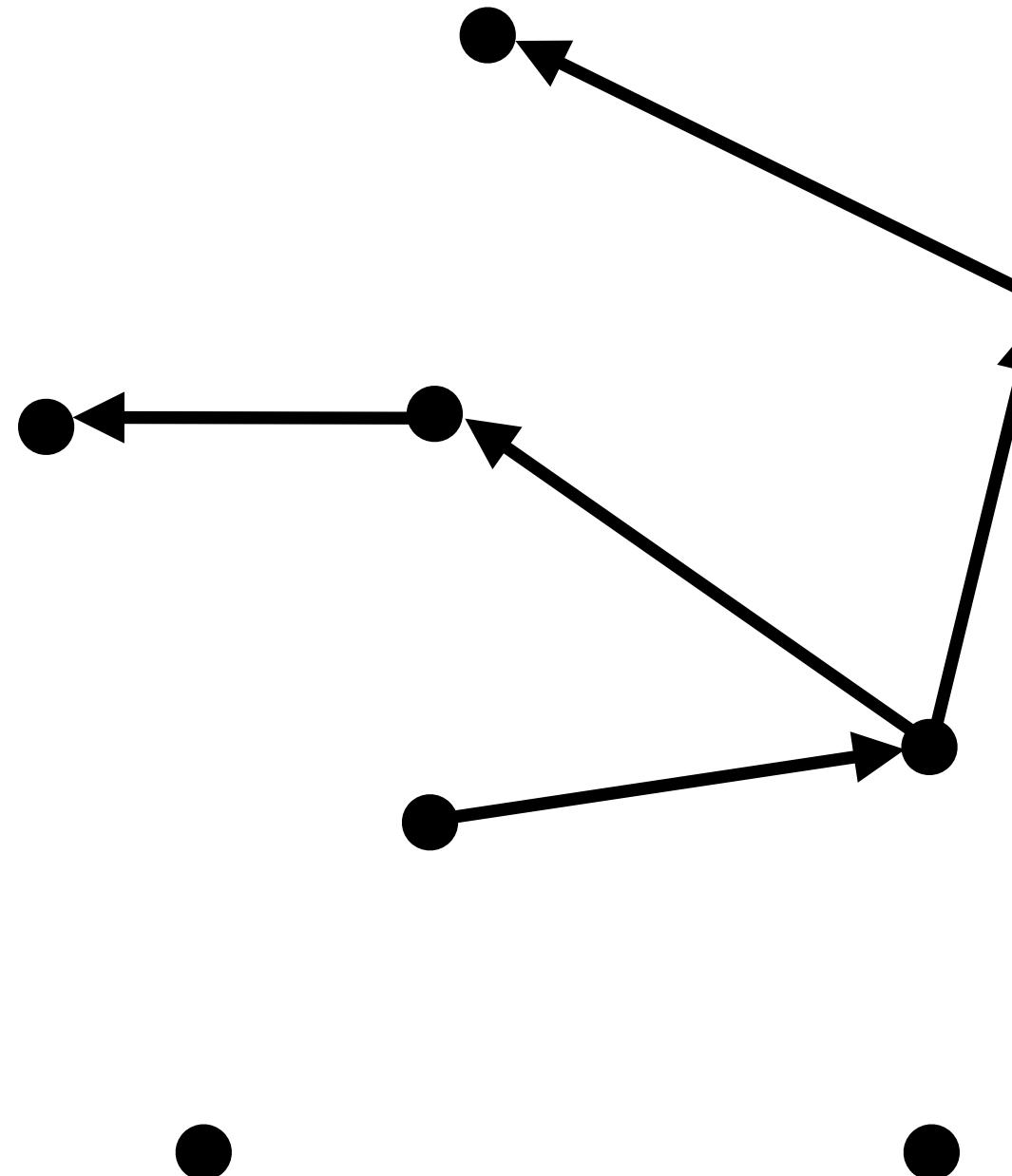
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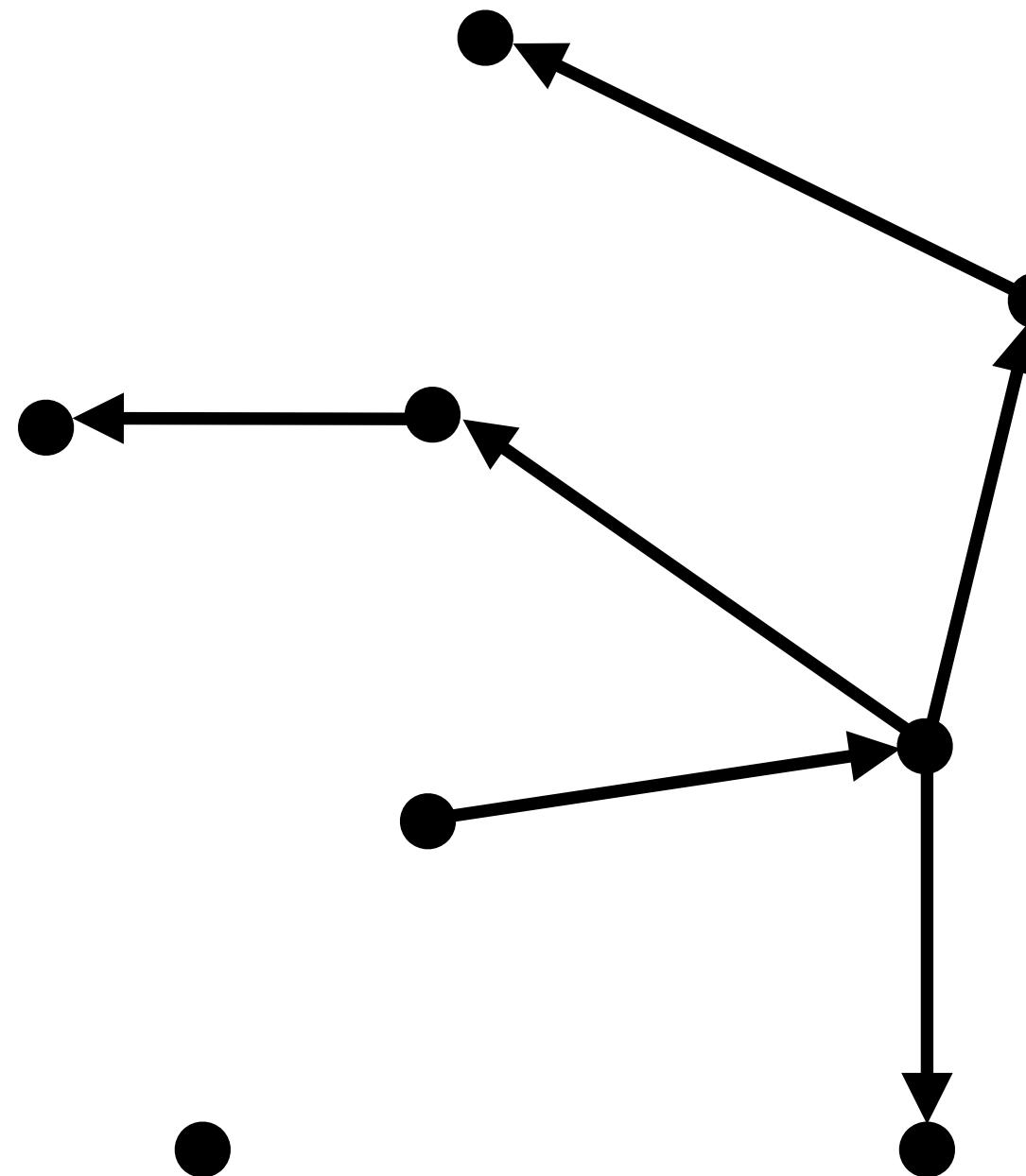
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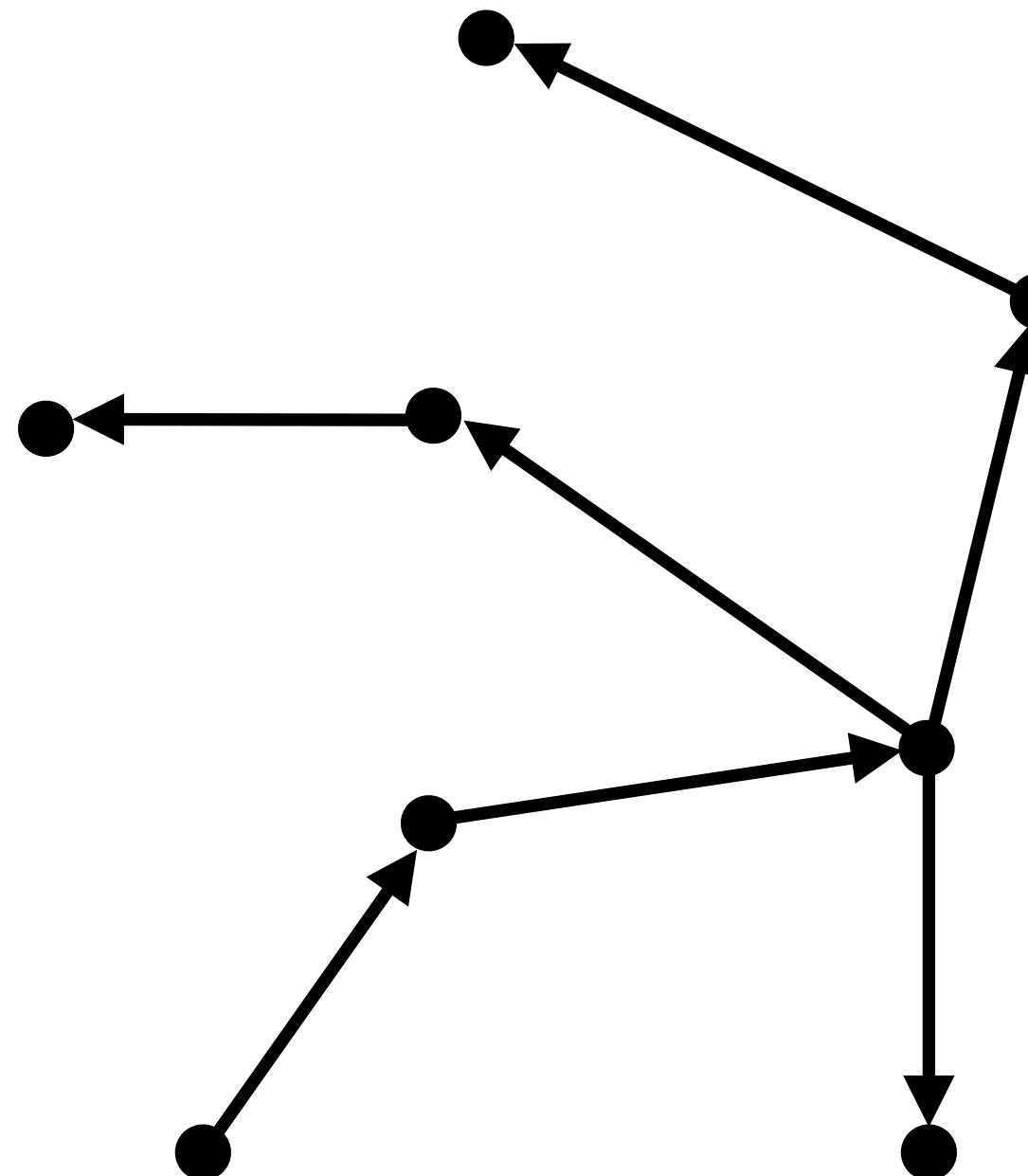
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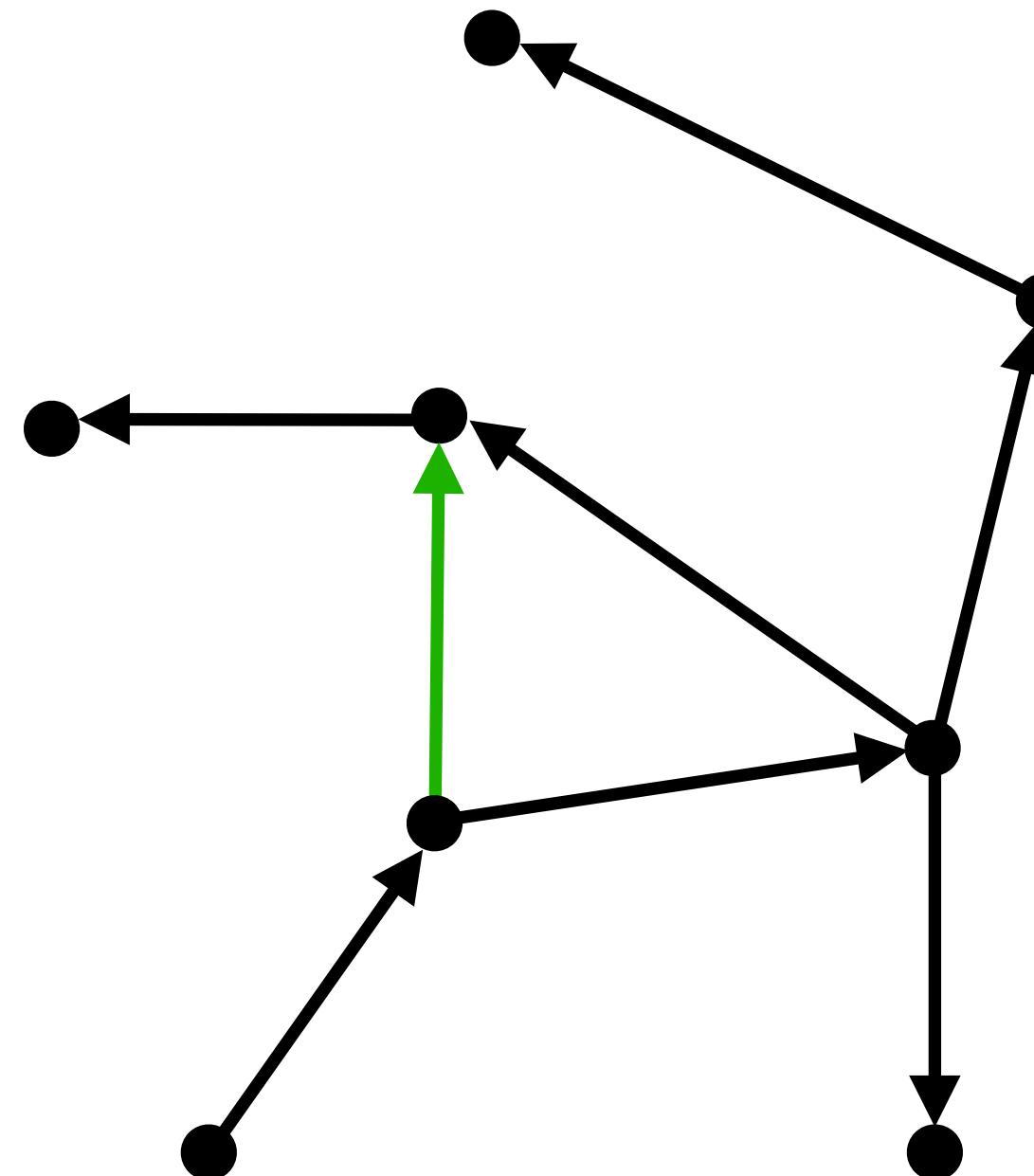
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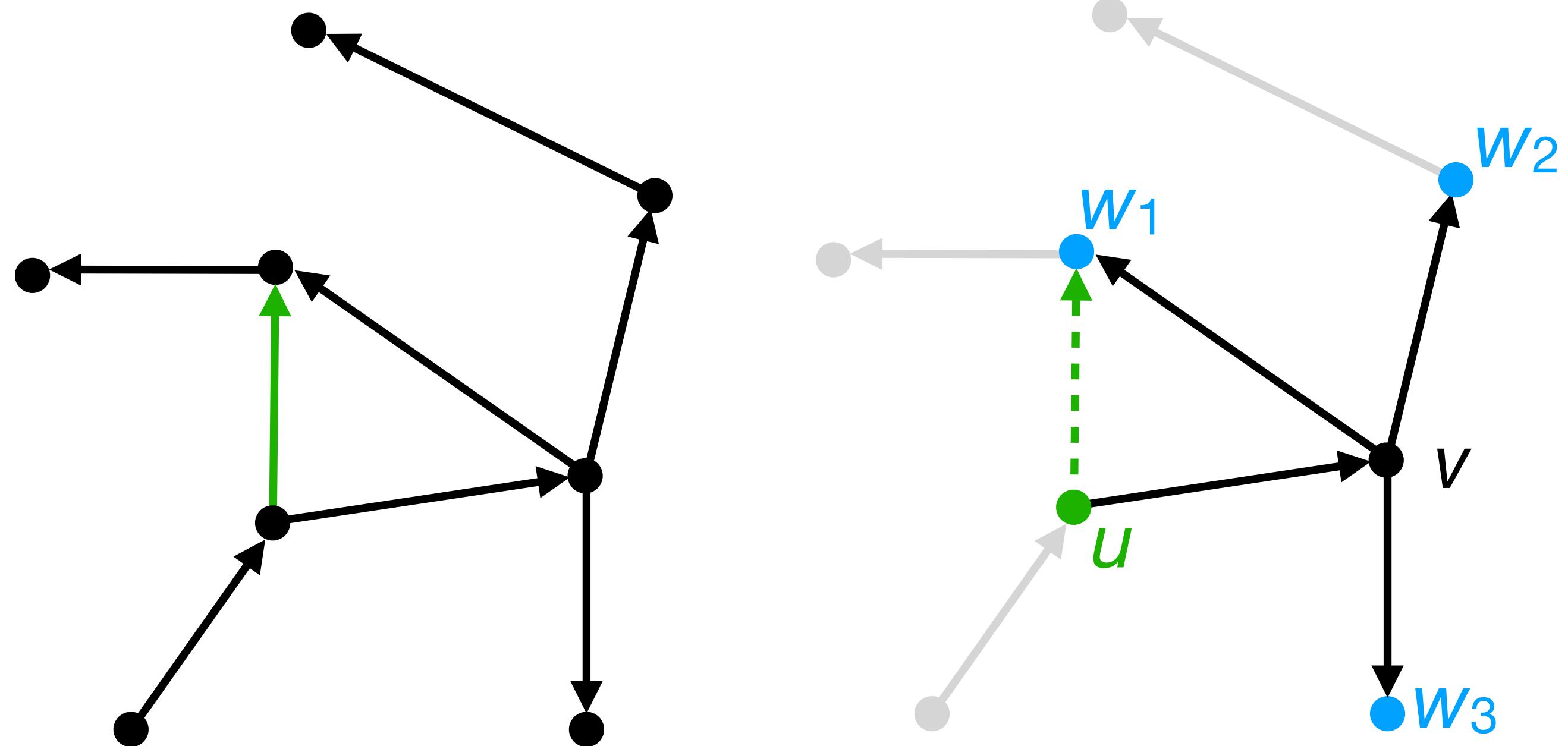
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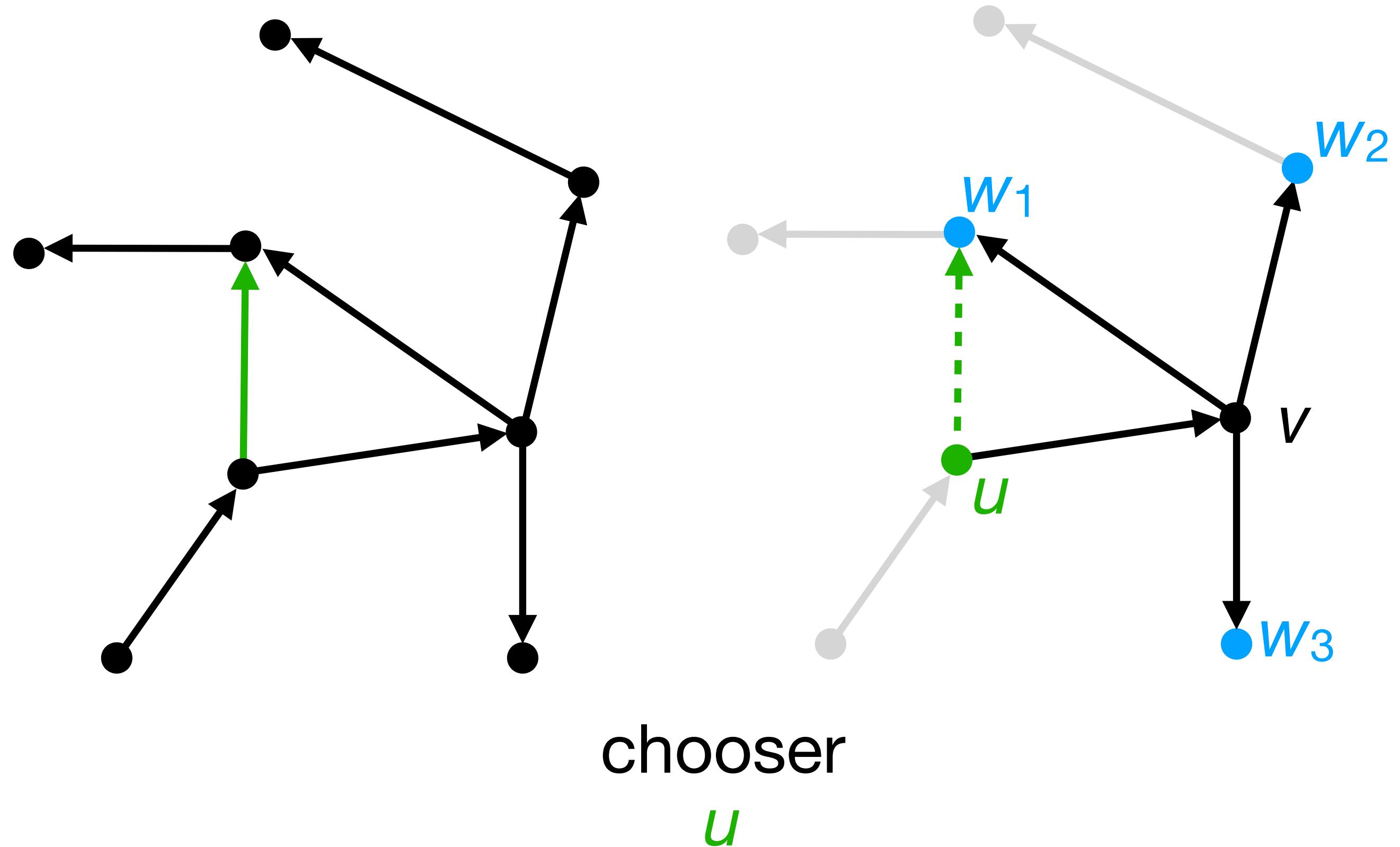
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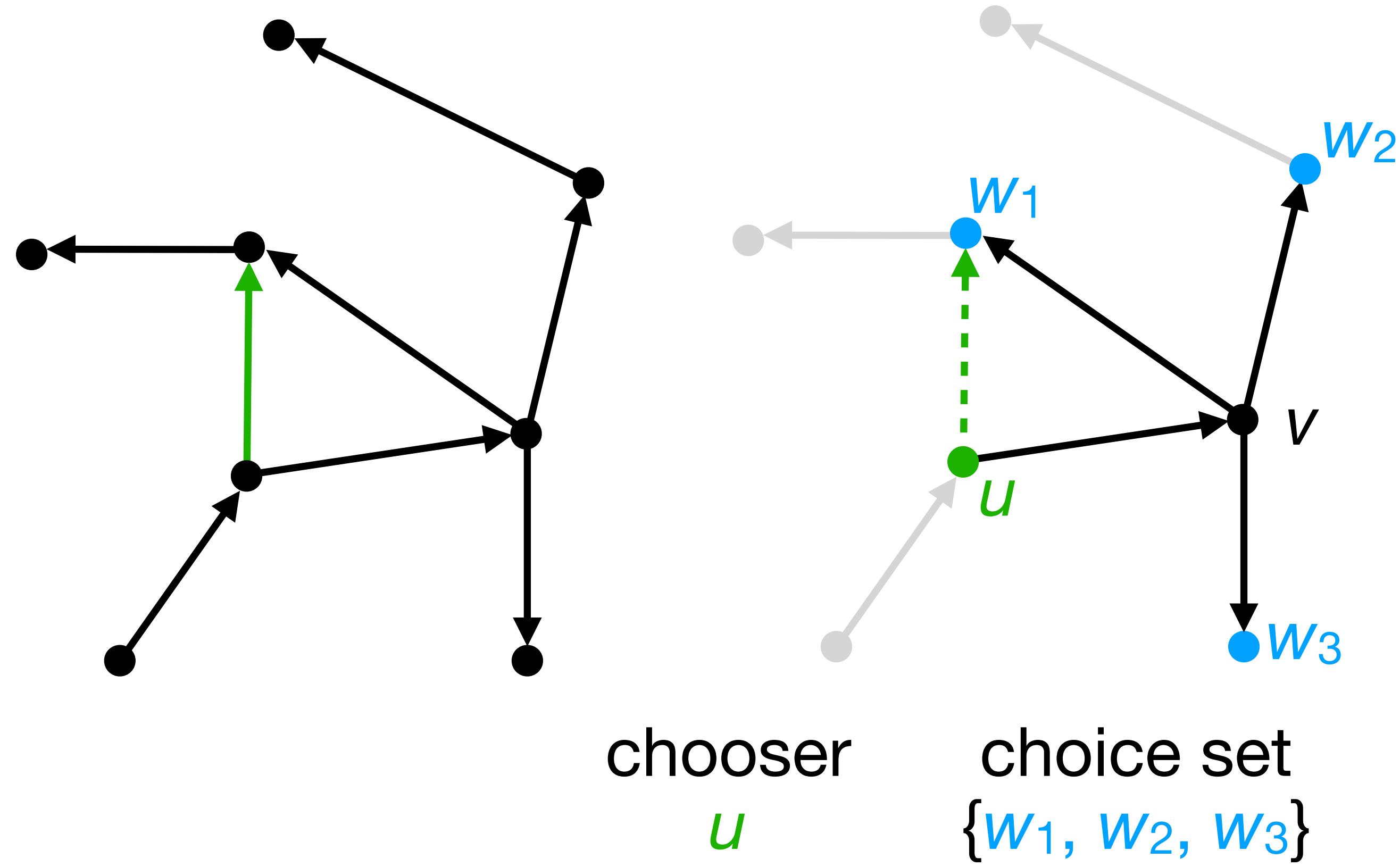
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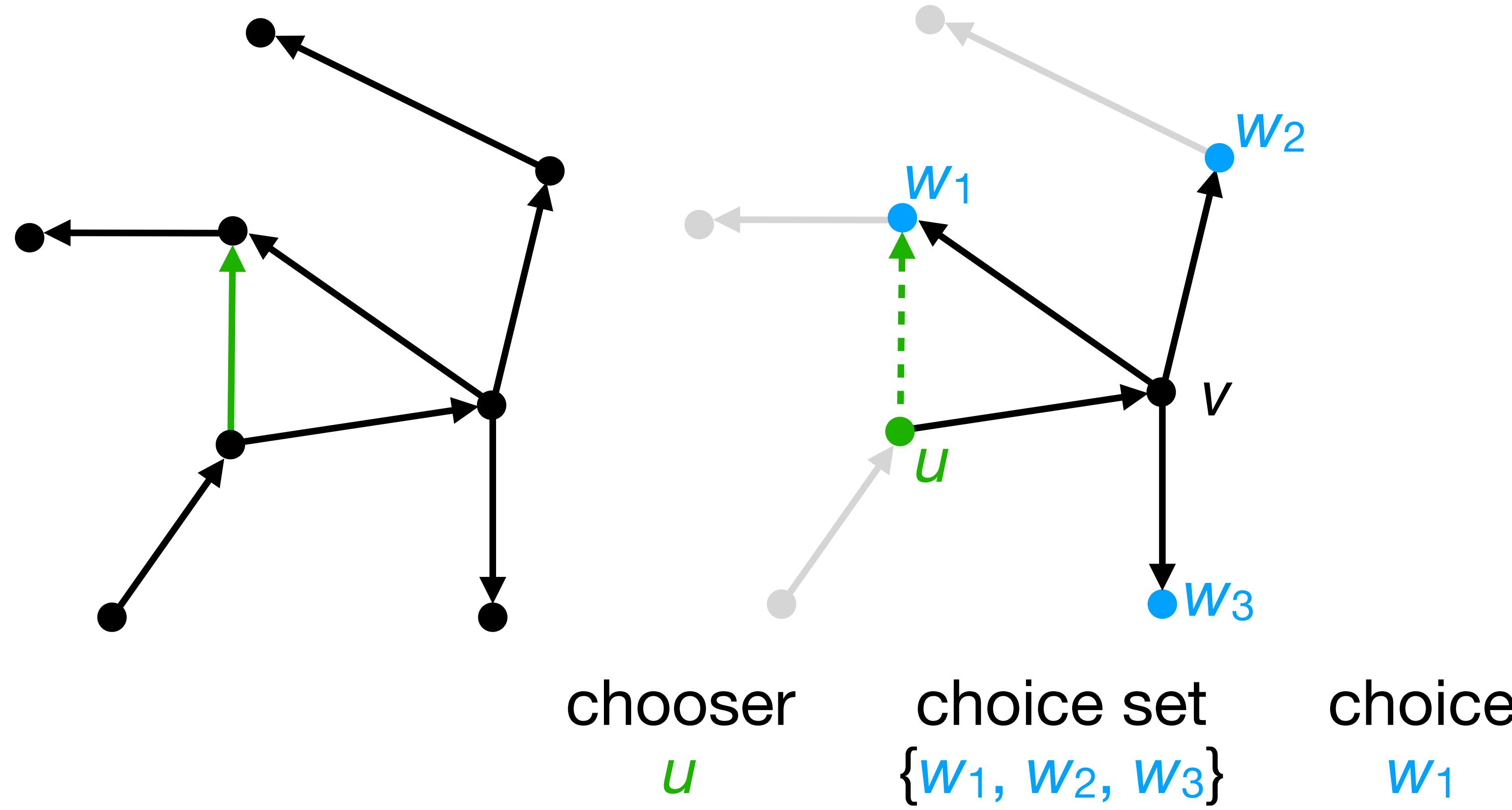
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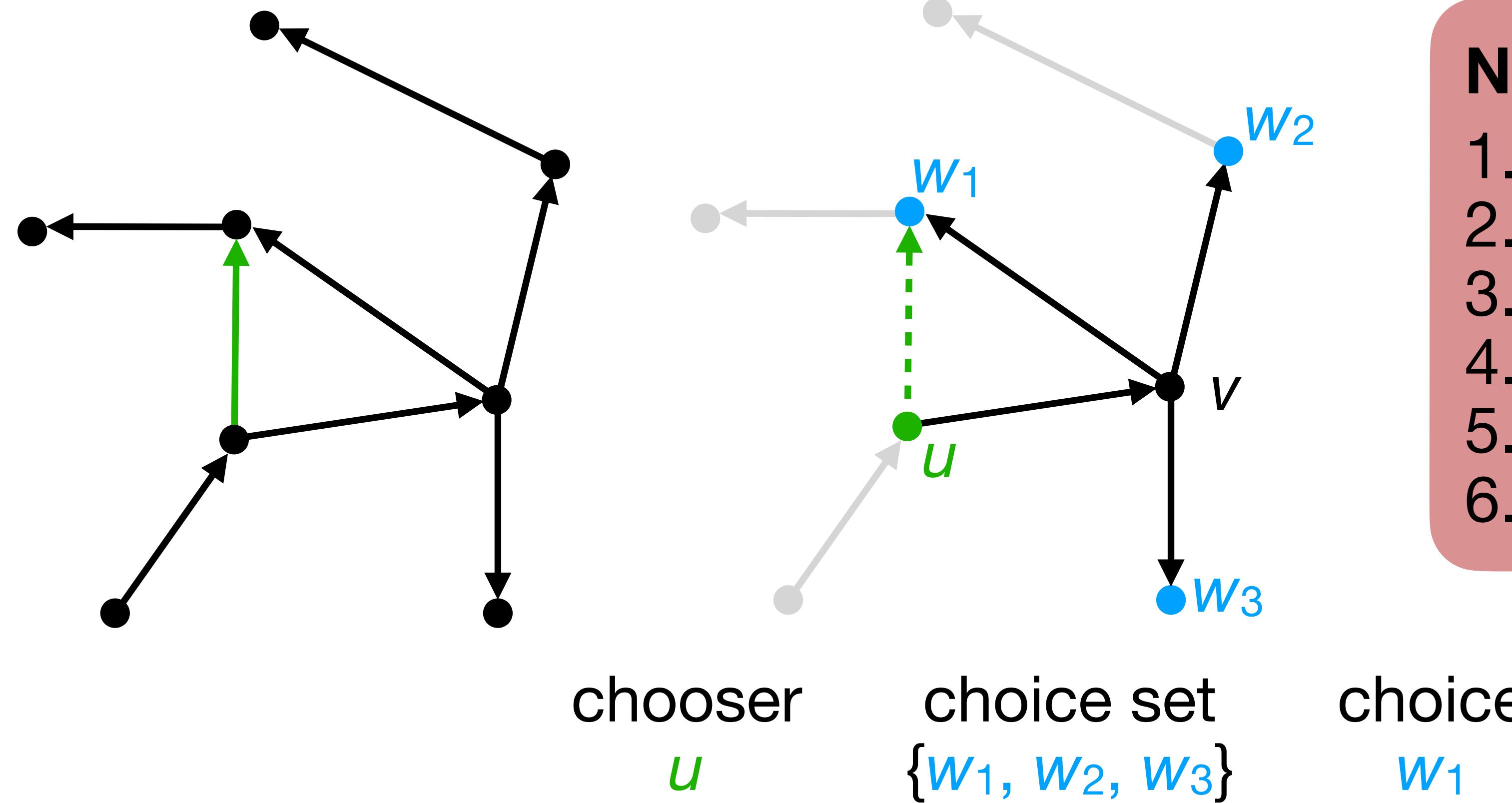


# Choosing to close triangles

Triadic closure offers small choice sets  
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## Our data

Timestamped edges  
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## Node features

1. in-degree of  $w$
2. # shared neighbors of  $u, w$
3. weight of edge  $w \rightarrow u$
4. time since last edge into  $w$
5. time since last edge out of  $w$
6. time since last  $w \rightarrow u$  edge

# **Context matters in triadic closure**

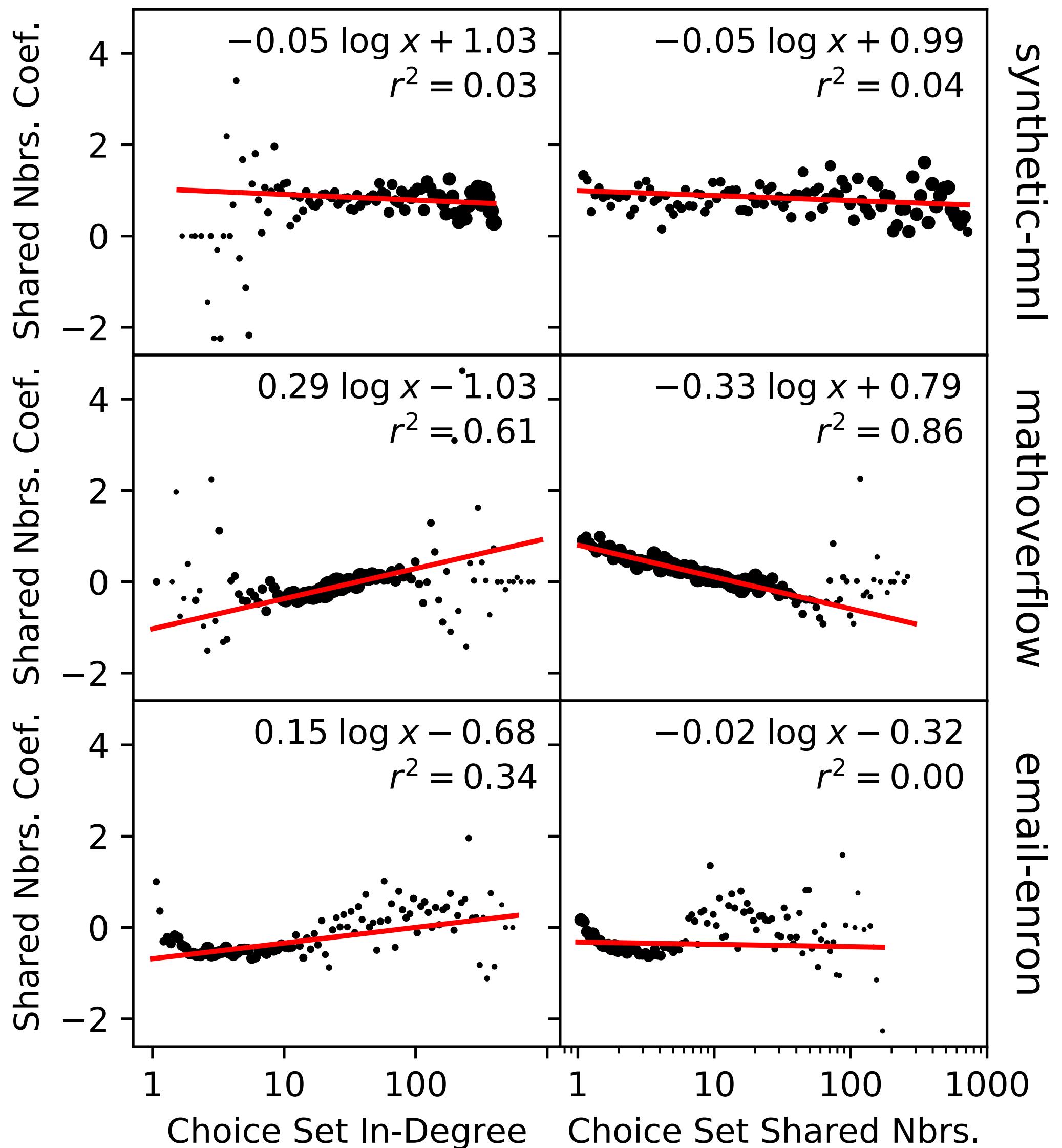
# Context matters in triadic closure

## Datasets

email-enron  
email-eu  
email-w3c  
wiki-talk  
reddit-hyperlink  
bitcoin-alpha  
bitcoin-otc  
mathoverflow  
college-msg  
facebook-wall  
sms-a  
sms-b  
sms-c

[bit.ly/lcl-data](http://bit.ly/lcl-data)

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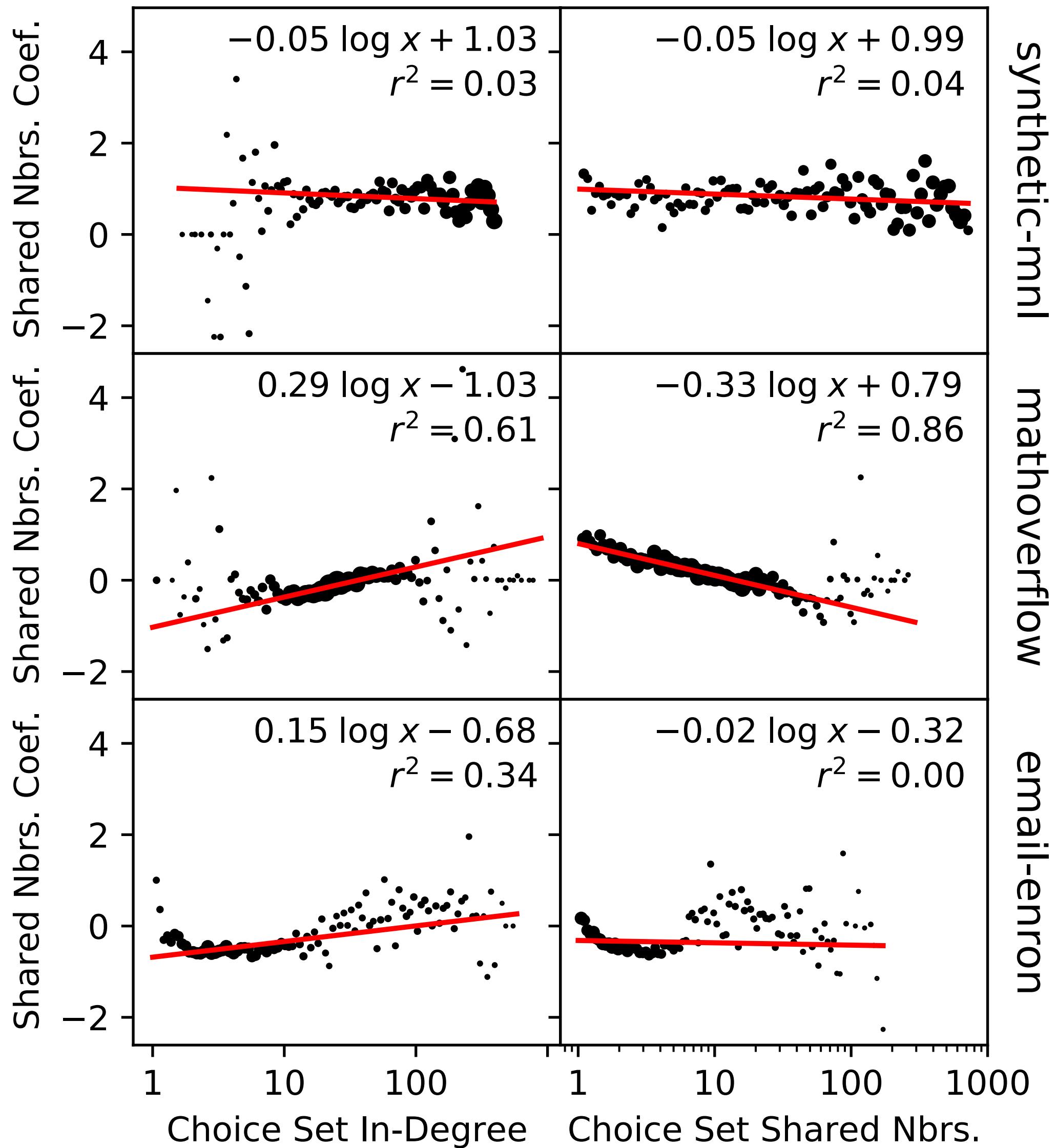


synthetic-mnl      mathoverflow      email-enron

- Datasets**
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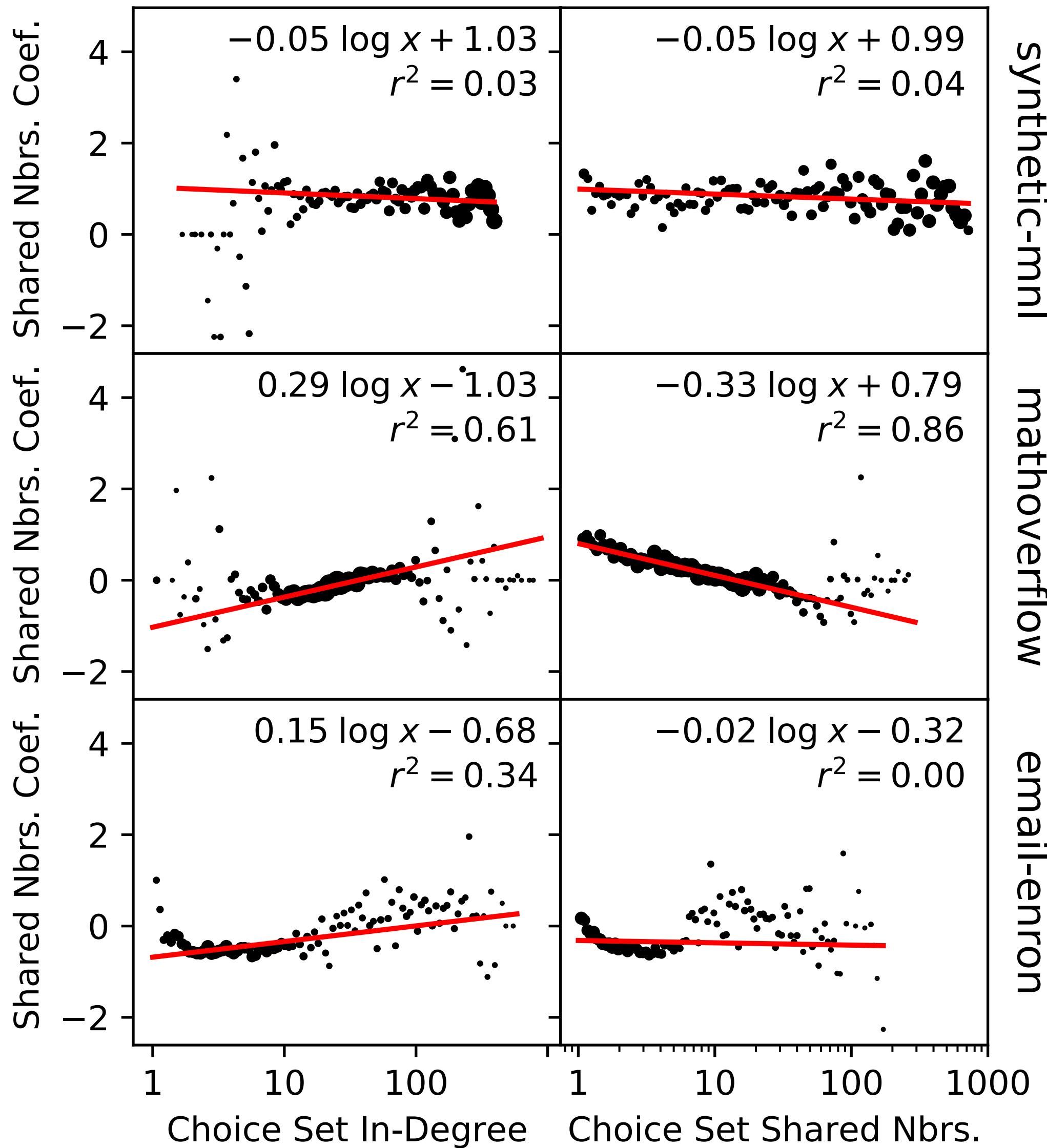
synthetic-mnl  
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Synthetic data,  
no context effects

- Datasets**
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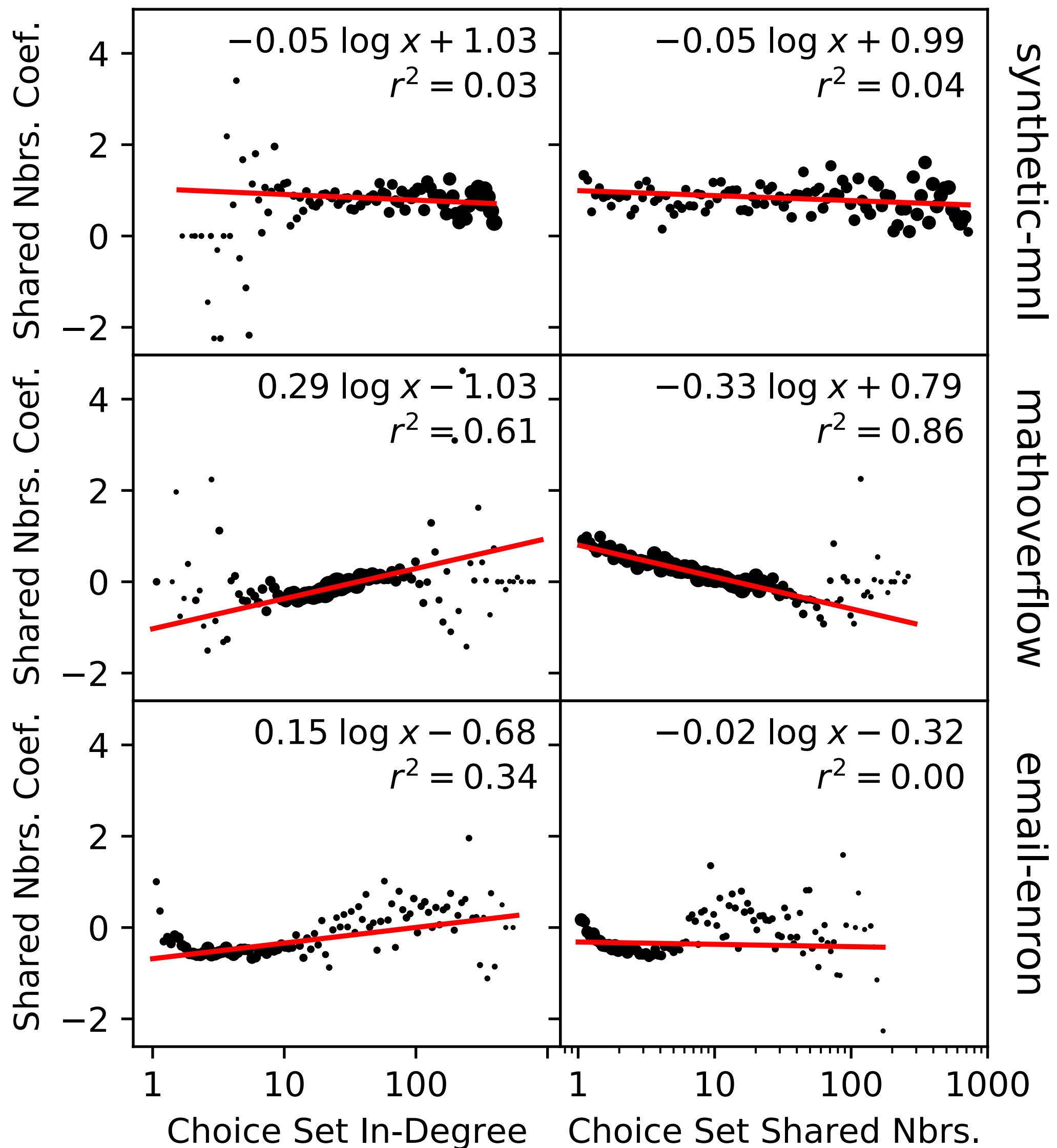
Commenting network,  
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synthetic-mnl  
mathoverflow  
email-enron

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# Context matters in triadic closure



Synthetic data,  
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Commenting network,  
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Email network,  
nonlinear context effects?

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# **LCL reveals interpretable context effects**

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## Estimation

MLE to infer LCL

$$\ell(\theta, A; \mathcal{D}) = \sum_{(i,C) \in \mathcal{D}} (\theta + Ax_C)^T x_i - \log \sum_{j \in C} \exp([\theta + Ax_C]^T x_j)$$

(concave)

# LCL reveals interpretable context effects



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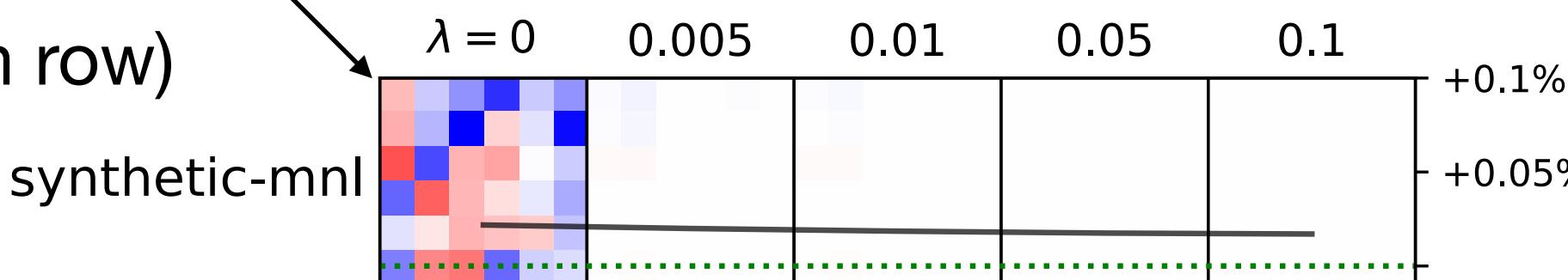
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# LCL reveals interpretable context effects

**Node features**  
(left-right, top-bottom)

1. in-degree
2. shared neighbors
3. reciprocal weight
4. send recency
5. receive recency
6. reciprocal recency

context effect matrix  $A$   
red: +, blue: -, white: 0  
(column acts on row)



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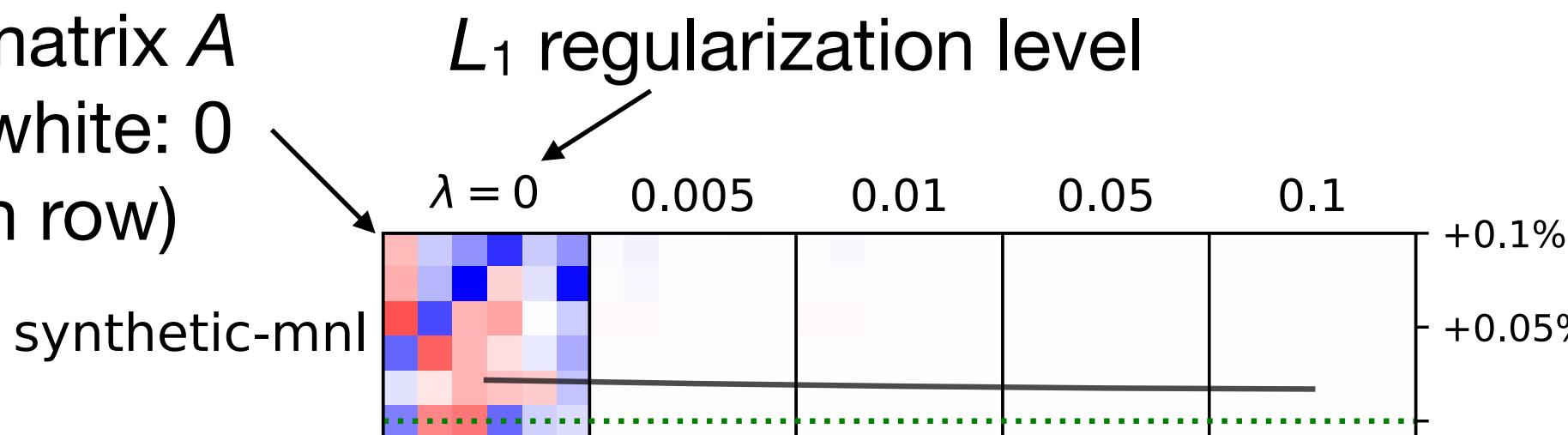
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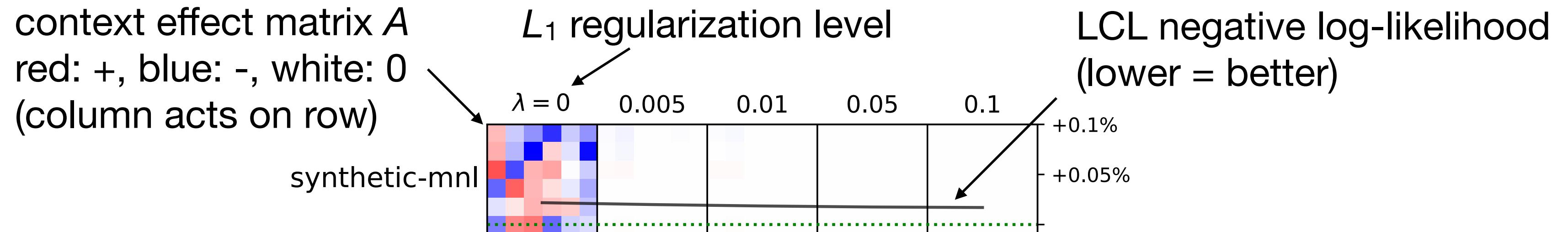
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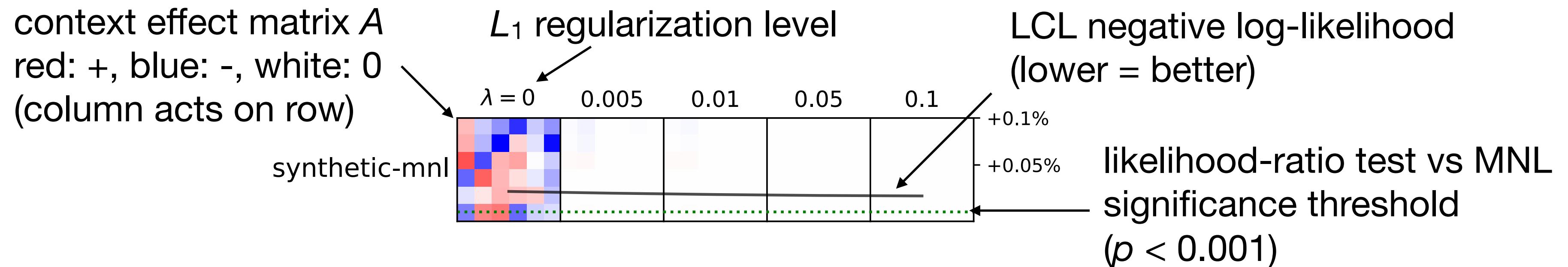
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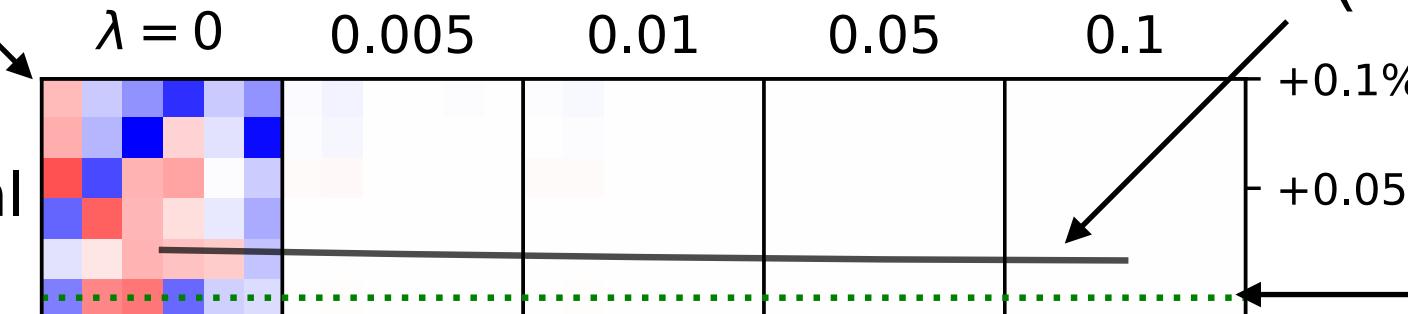
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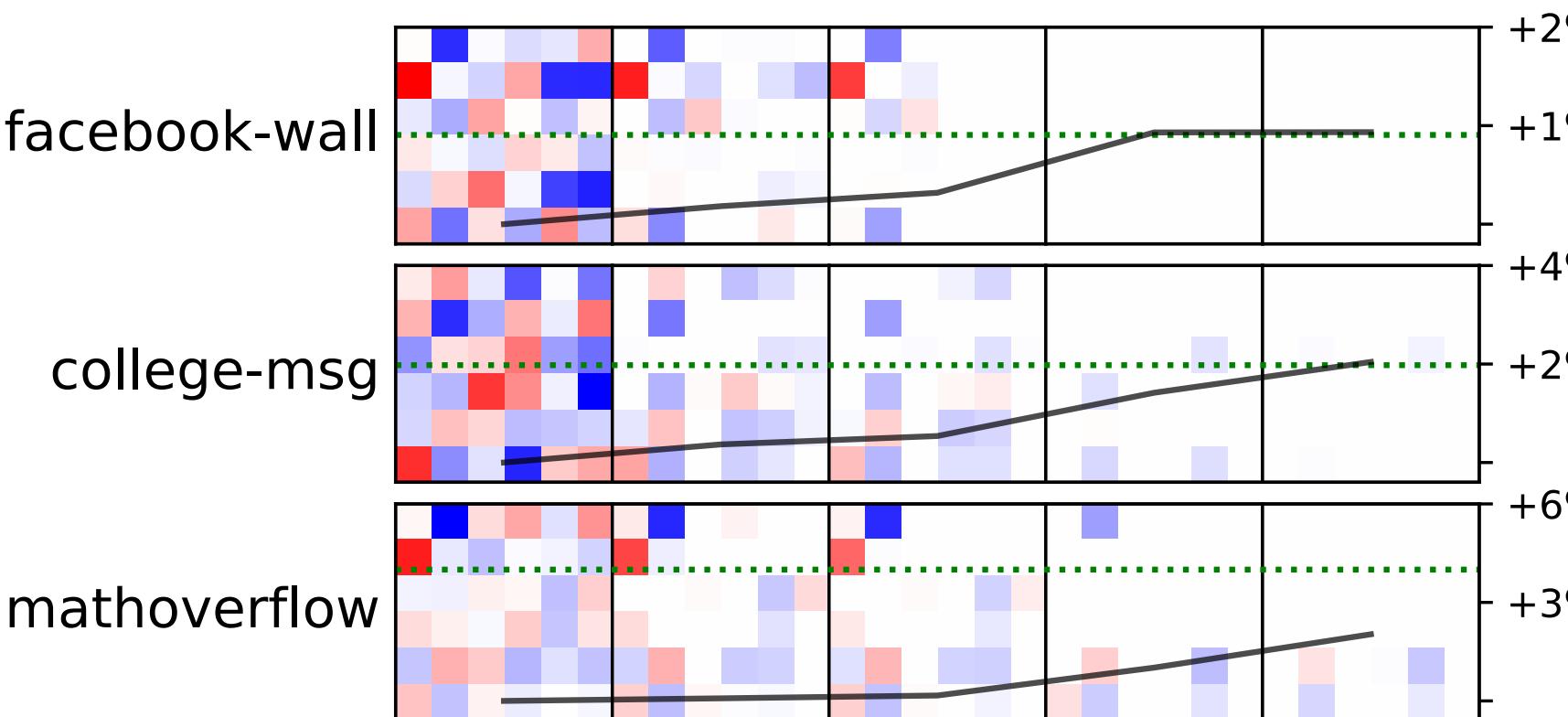
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$L_1$  regularization level



LCL negative log-likelihood  
(lower = better)

likelihood-ratio test vs MNL  
significance threshold  
( $p < 0.001$ )



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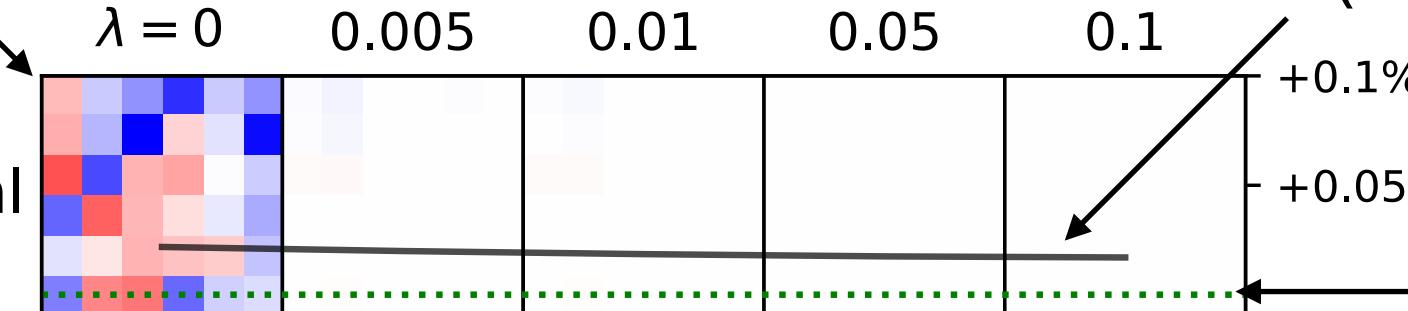
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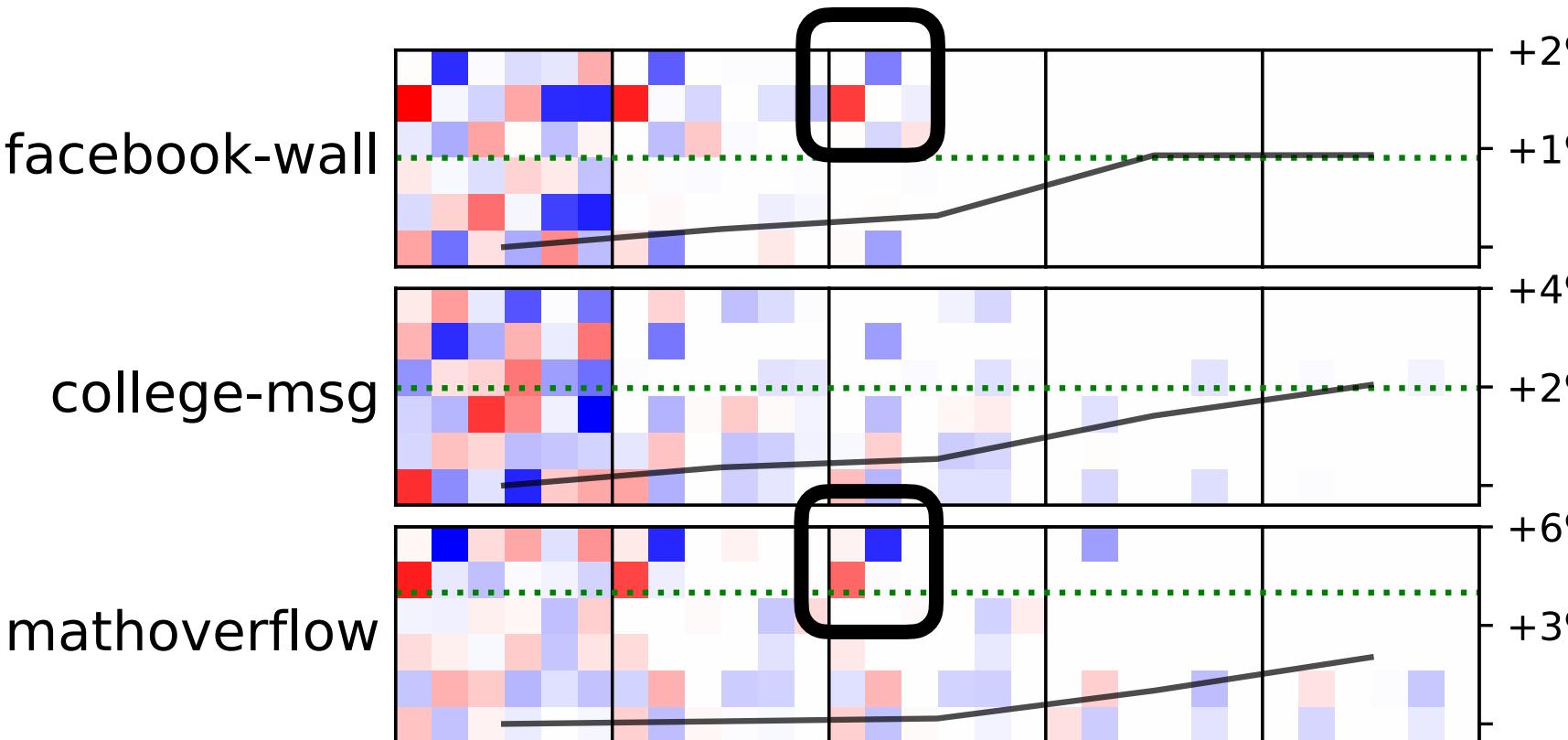
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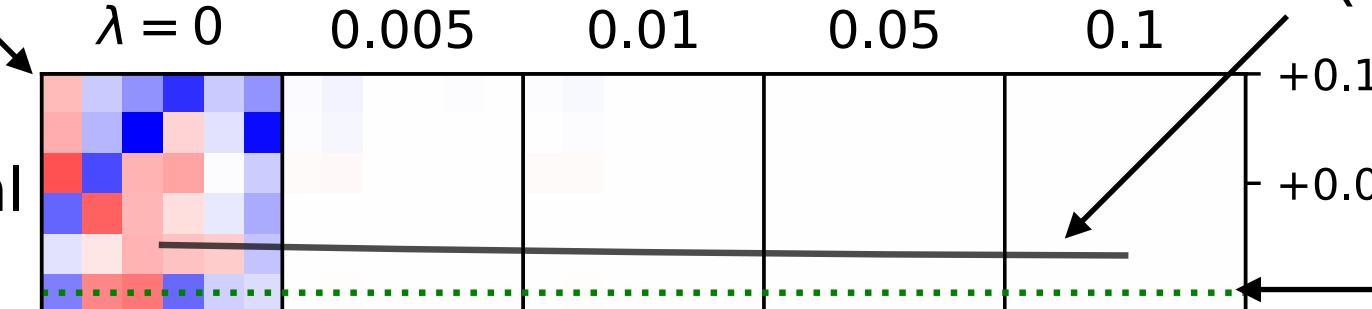
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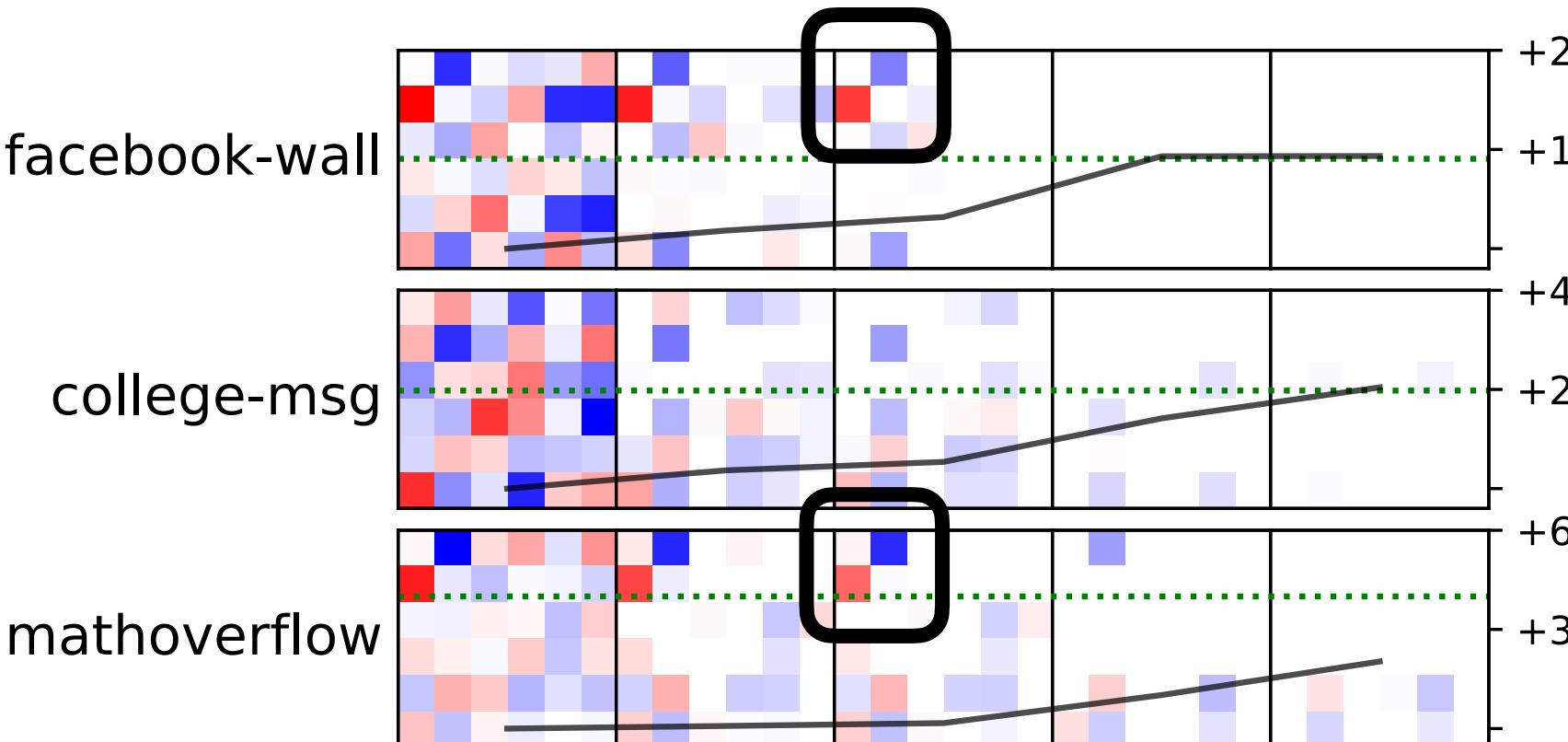
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“popularity matters less when  
 choosing from close connections”

“close connections matter more  
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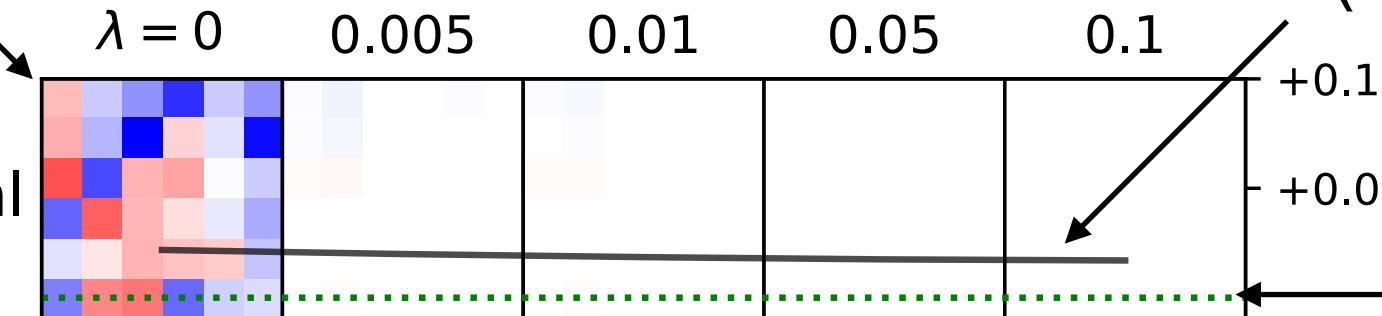
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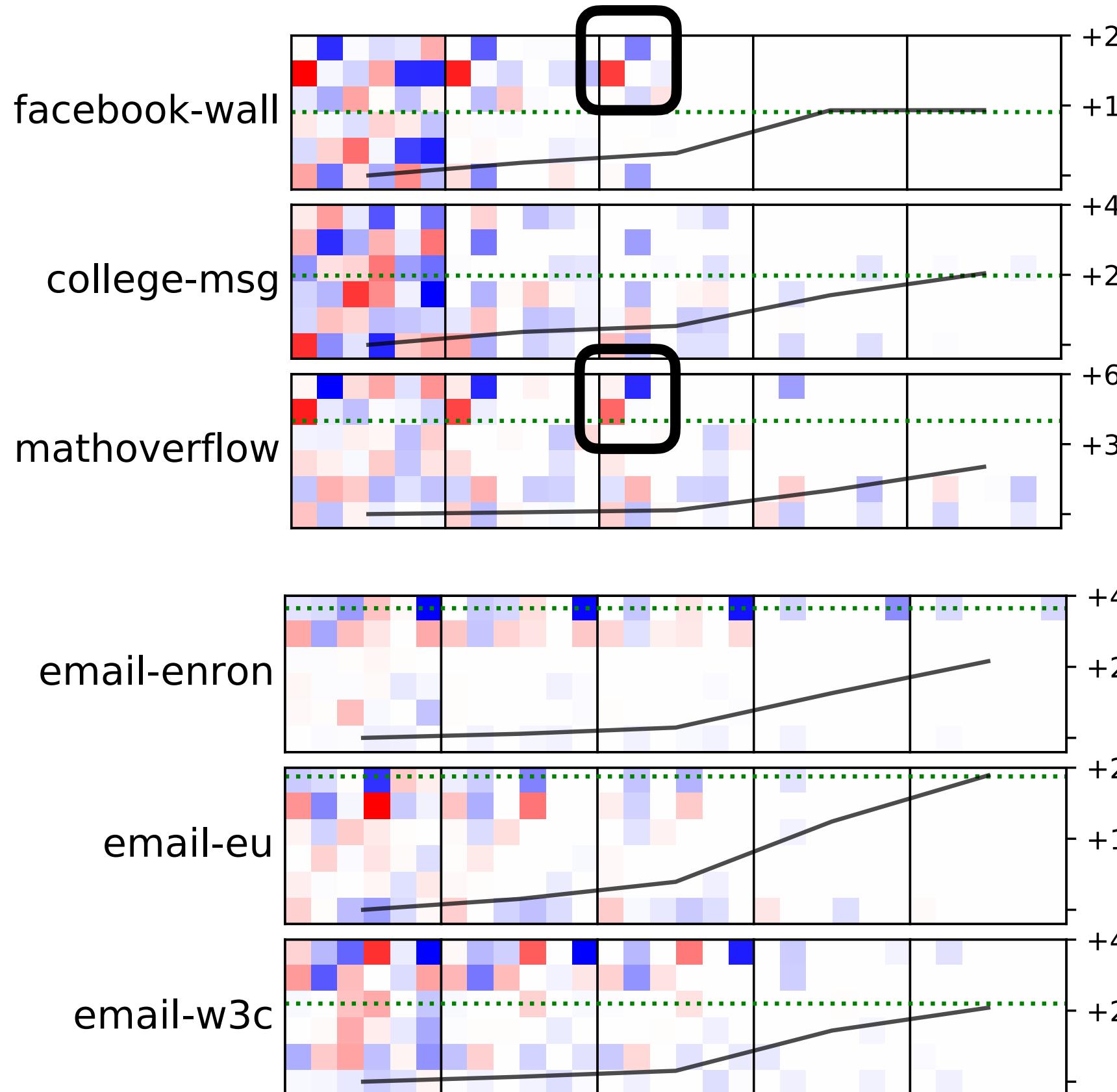
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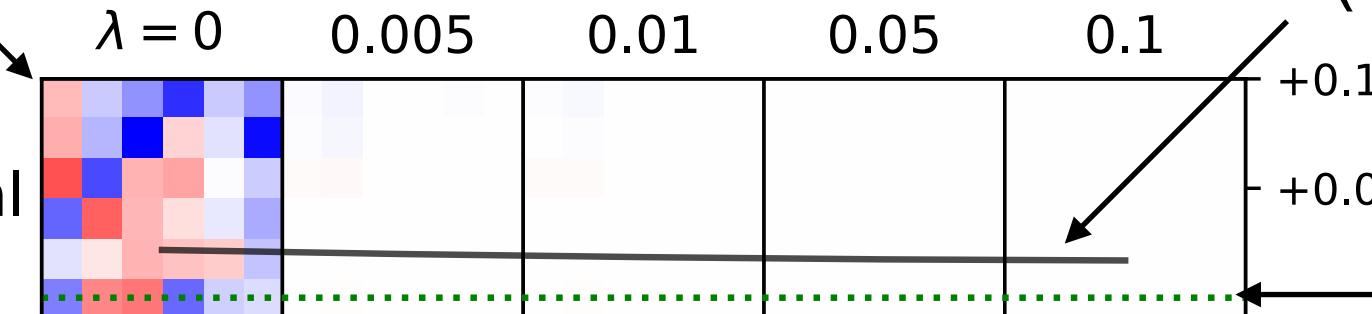
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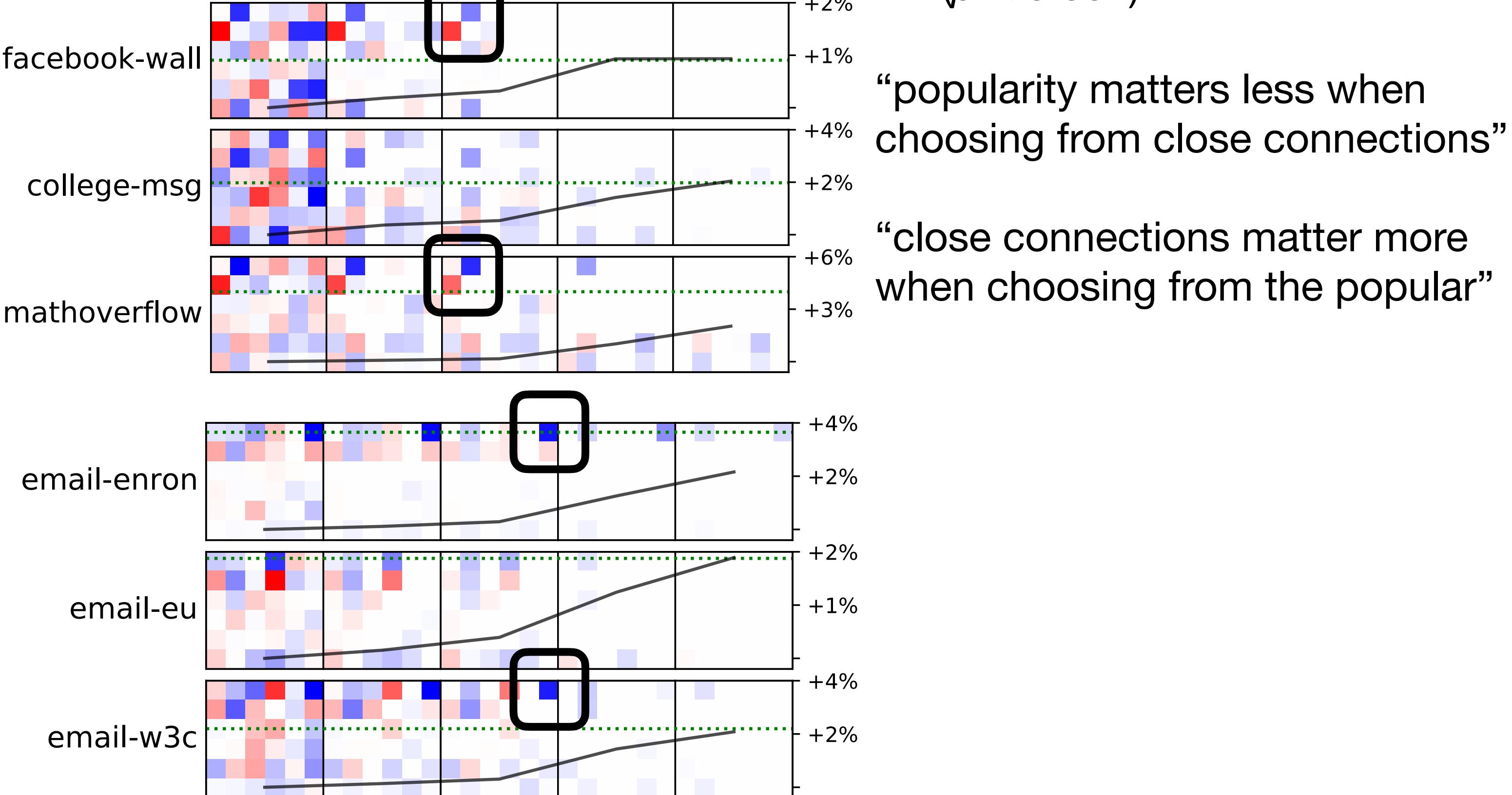
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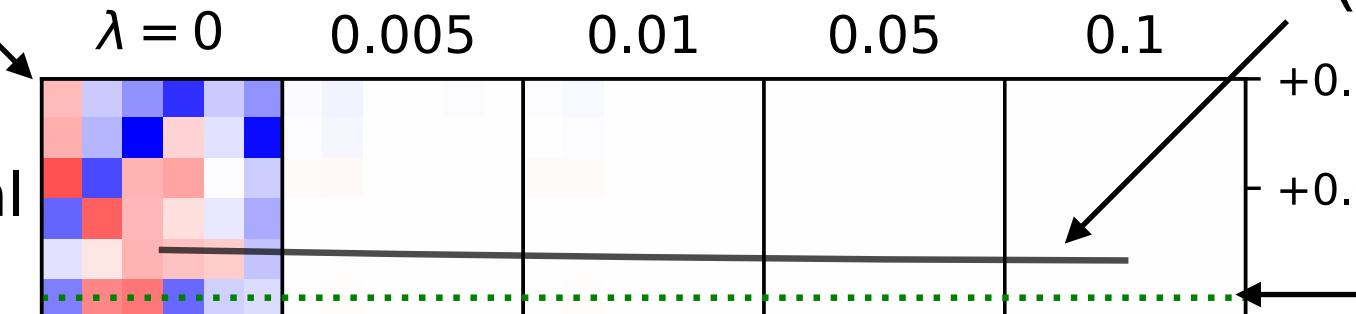
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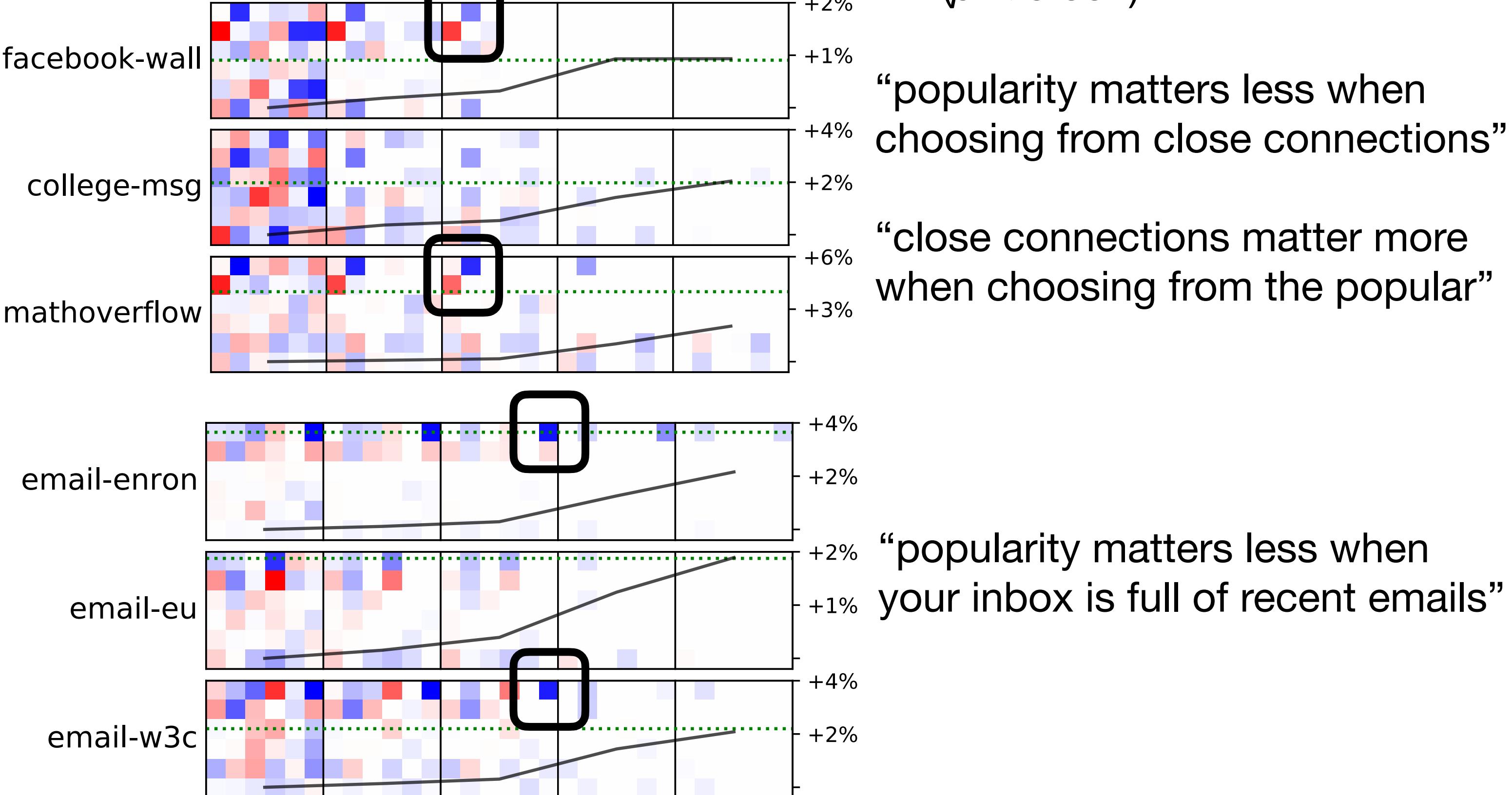
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# Other things in our paper

Kiran Tomlinson and Austin R. Benson

Learning Interpretable Feature Context Effects in Discrete Choice

*arXiv: 2009.03417*, September 2020

[bit.ly/lcl-paper](https://bit.ly/lcl-paper)

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- LCL derivation from simple assumptions
- More flexible model: decomposed LCL

$$\Pr(i, C) = \sum_{k=1}^d \pi_k \frac{\exp([B_k + A_k(x_C)_k]^T x_i)}{\sum_{j \in C} \exp([B_k + A_k(x_C)_k]^T x_j)}$$

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$$\text{span} \left\{ \begin{bmatrix} x_C \\ 1 \end{bmatrix} \otimes (x_i - x_C) \mid C \in \mathcal{C}_{\mathcal{D}}, i \in C \right\} = \mathbb{R}^{d^2+d}. \quad (6)$$

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- LCL derivation from simple assumptions
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- Application to general choice data

$$\Pr(i, C) = \sum_{k=1}^d \pi_k \frac{\exp([B_k + A_k(x_C)_k]^T x_i)}{\sum_{j \in C} \exp([B_k + A_k(x_C)_k]^T x_j)}$$

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---

## Dataset

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DISTRICT  
DISTRICT-SMART  
SUSHI  
EXPEDIA  
CAR-A  
CAR-B  
CAR-ALT

---

# Other things in our paper

Kiran Tomlinson and Austin R. Benson

Learning Interpretable Feature Context Effects in Discrete Choice

arXiv: 2009.03417, September 2020

[bit.ly/lcl-paper](https://bit.ly/lcl-paper)

- LCL derivation from simple assumptions
- More flexible model: decomposed LCL
- LCL identifiability condition
- Application to general choice data
- Accounting for context improves prediction

$$\Pr(i, C) = \sum_{k=1}^d \pi_k \frac{\exp([B_k + A_k(x_C)_k]^T x_i)}{\sum_{j \in C} \exp([B_k + A_k(x_C)_k]^T x_j)}$$

*Theorem 1.* A  $d$ -feature linear context logit is identifiable from a dataset  $\mathcal{D}$  if and only if

$$\text{span} \left\{ \begin{bmatrix} x_C \\ 1 \end{bmatrix} \otimes (x_i - x_C) \mid C \in \mathcal{C}_{\mathcal{D}}, i \in C \right\} = \mathbb{R}^{d^2+d}. \quad (6)$$

Dataset	MNL	LCL
DISTRICT	.3680 (.4823)	.3327 (.4712)
DISTRICT-SMART	.4006 (.4900)	.3894 (.4876)
EXPEDIA	.3859 (.2954)	.3696* (.2926)
SUSHI	.2727 (.2751)	.2741 (.2771)
CAR-A	.3570 (.4791)	.3514 (.4774)
CAR-B	.3326 (.4711)	.3326 (.4711)
CAR-ALT	.2944 (.2875)	.2650* (.2804)
SYNTHETIC-MNL	.1513 (.1865)	.1512 (.1864)
SYNTHETIC-LCL	.1360 (.1684)	.1357* (.1683)
WIKI-TALK	.2946 (.2916)	.2666* (.2773)
REDDIT-HYPERLINK	.2859 (.2611)	.2761* (.2606)
BITCOIN-ALPHA	.2724 (.3246)	.2591* (.3178)
BITCOIN-OTC	.1891 (.2756)	.1529* (.2468)
SMS-A	.2825 (.3250)	.2661* (.3193)
SMS-B	.3045 (.3419)	.2848* (.3273)
SMS-C	.3115 (.3455)	.3070 (.3477)
EMAIL-ENRON	.1265 (.2068)	.1244* (.2115)
EMAIL-EU	.2683 (.3021)	.2665 (.3037)
EMAIL-W3C	.1332 (.2070)	.1210* (.1845)
FACEBOOK-WALL	.2176 (.2895)	.2109* (.2871)
COLLEGE-MSG	.1850 (.2726)	.1723* (.2655)
MATHOVERFLOW	.1385 (.2503)	.1153* (.2200)

# Concluding thoughts

## Key takeaway

Context effects matter in triadic closure

## Challenges

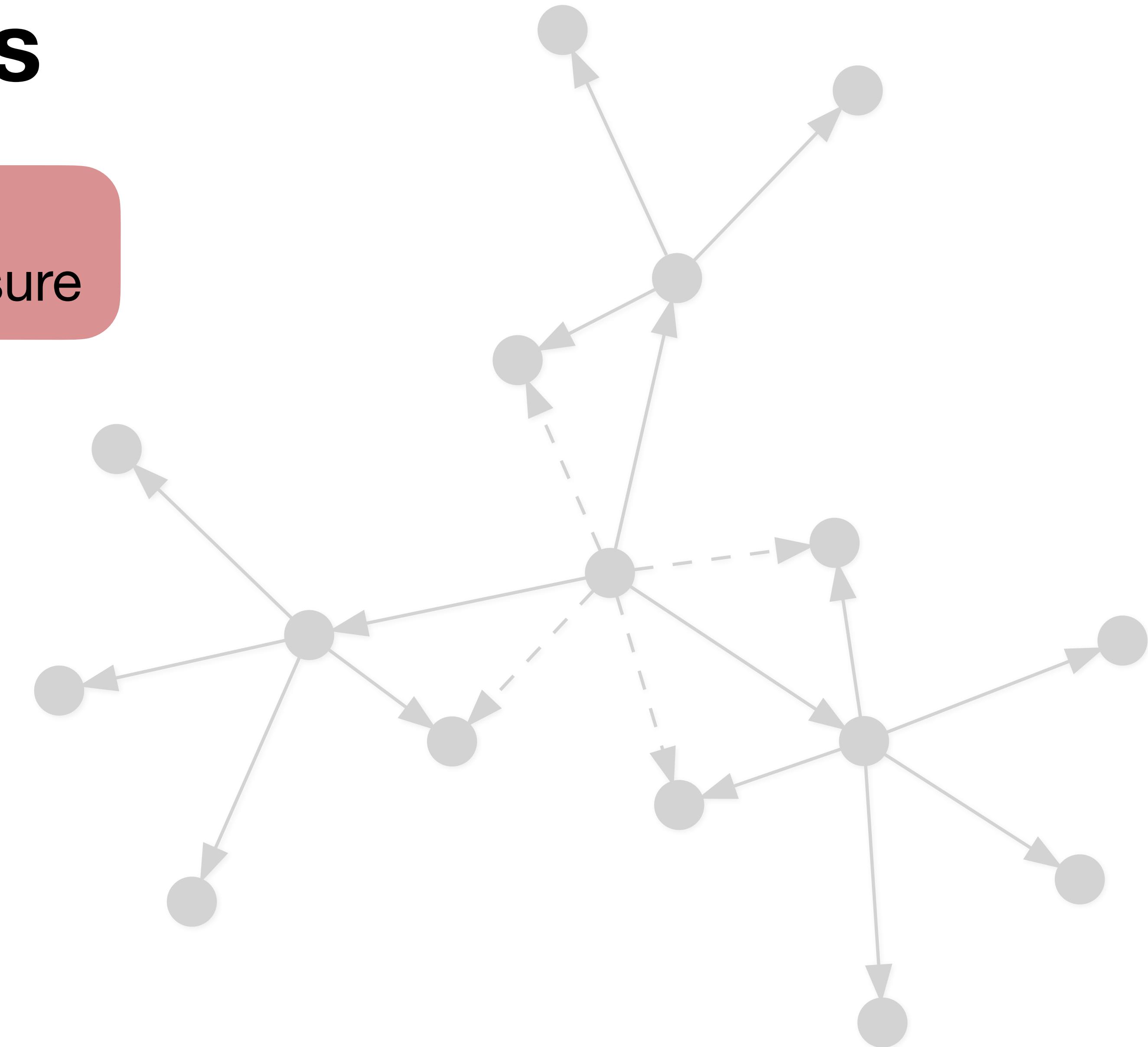
Features correlate

Causal context effects?

Handling nonlinearity?

Global edge formation modes?

Missing timestamps?



# Concluding thoughts

Slides: [bit.ly/lcl-slides](https://bit.ly/lcl-slides)  
Preprint: [bit.ly/lcl-paper](https://bit.ly/lcl-paper)  
Code: [bit.ly/lcl-code](https://bit.ly/lcl-code)  
Data: [bit.ly/lcl-data](https://bit.ly/lcl-data)

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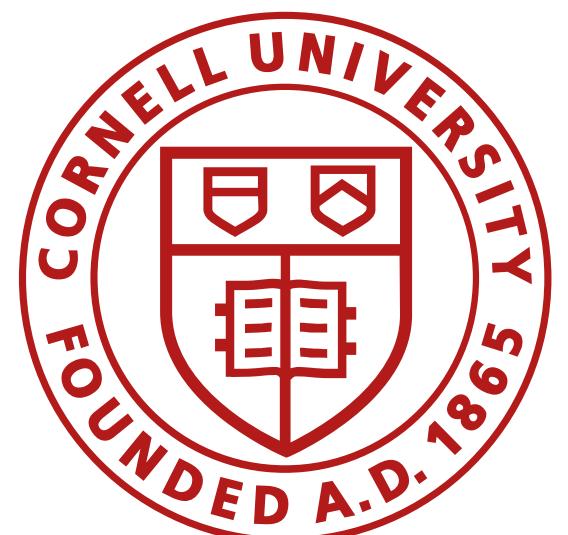
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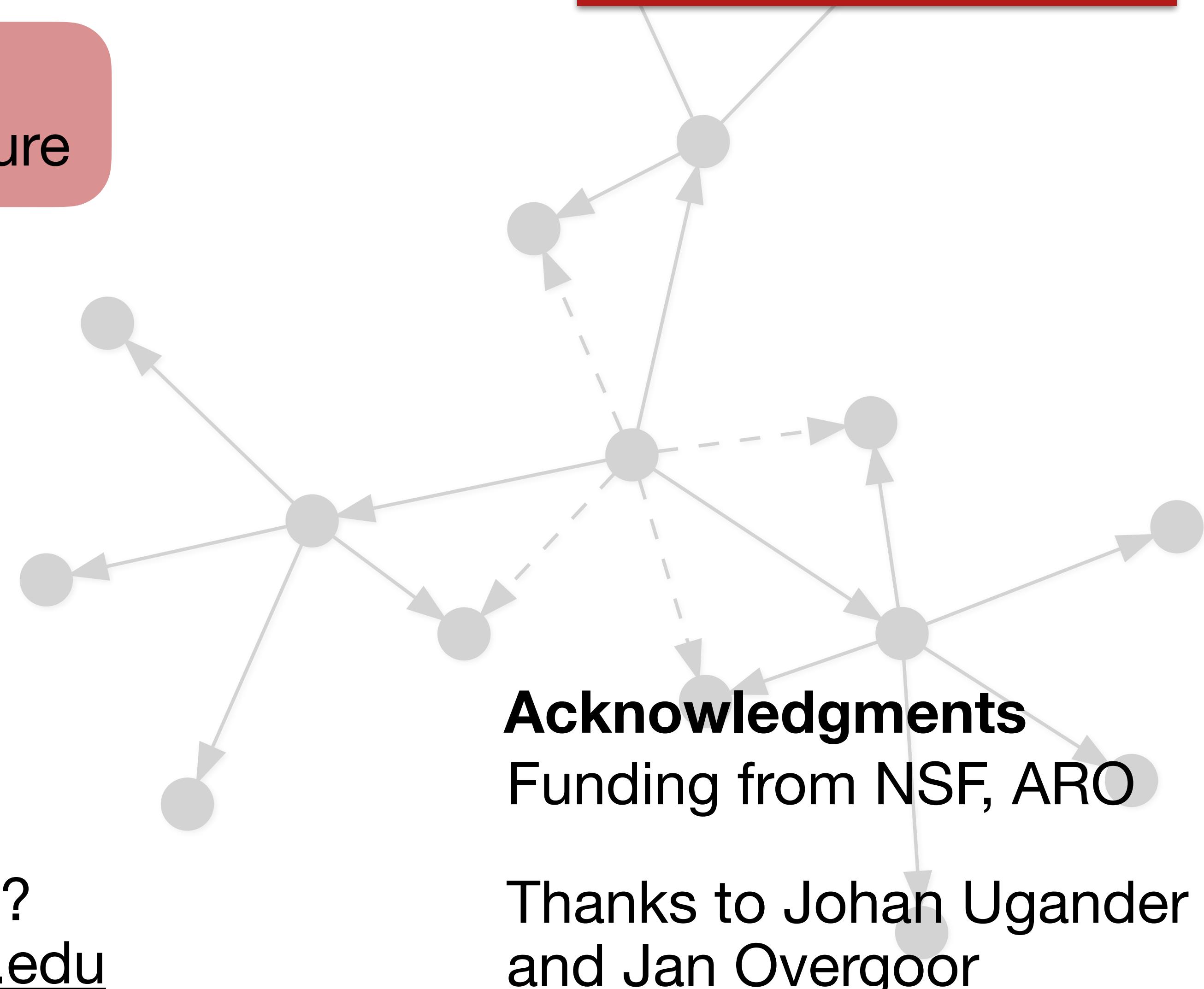
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# Thank you!

More questions or ideas?

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