

Ballot Length in Instant Runoff Voting

AAAI '23

Kiran Tomlinson



Johan Ugander

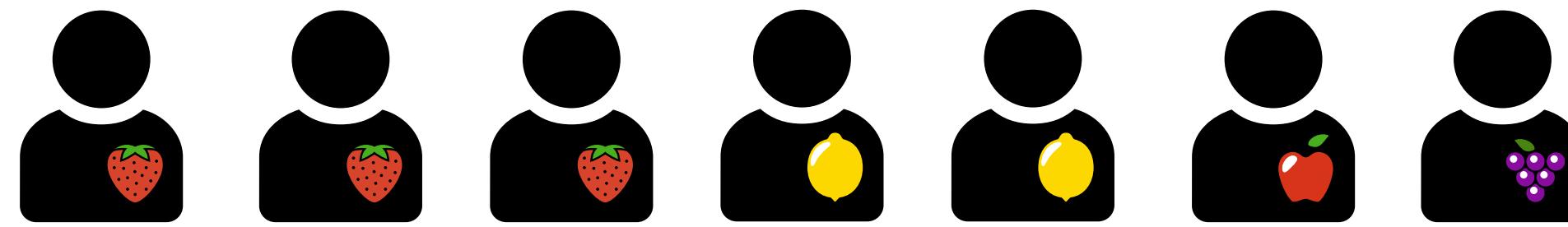


Jon Kleinberg

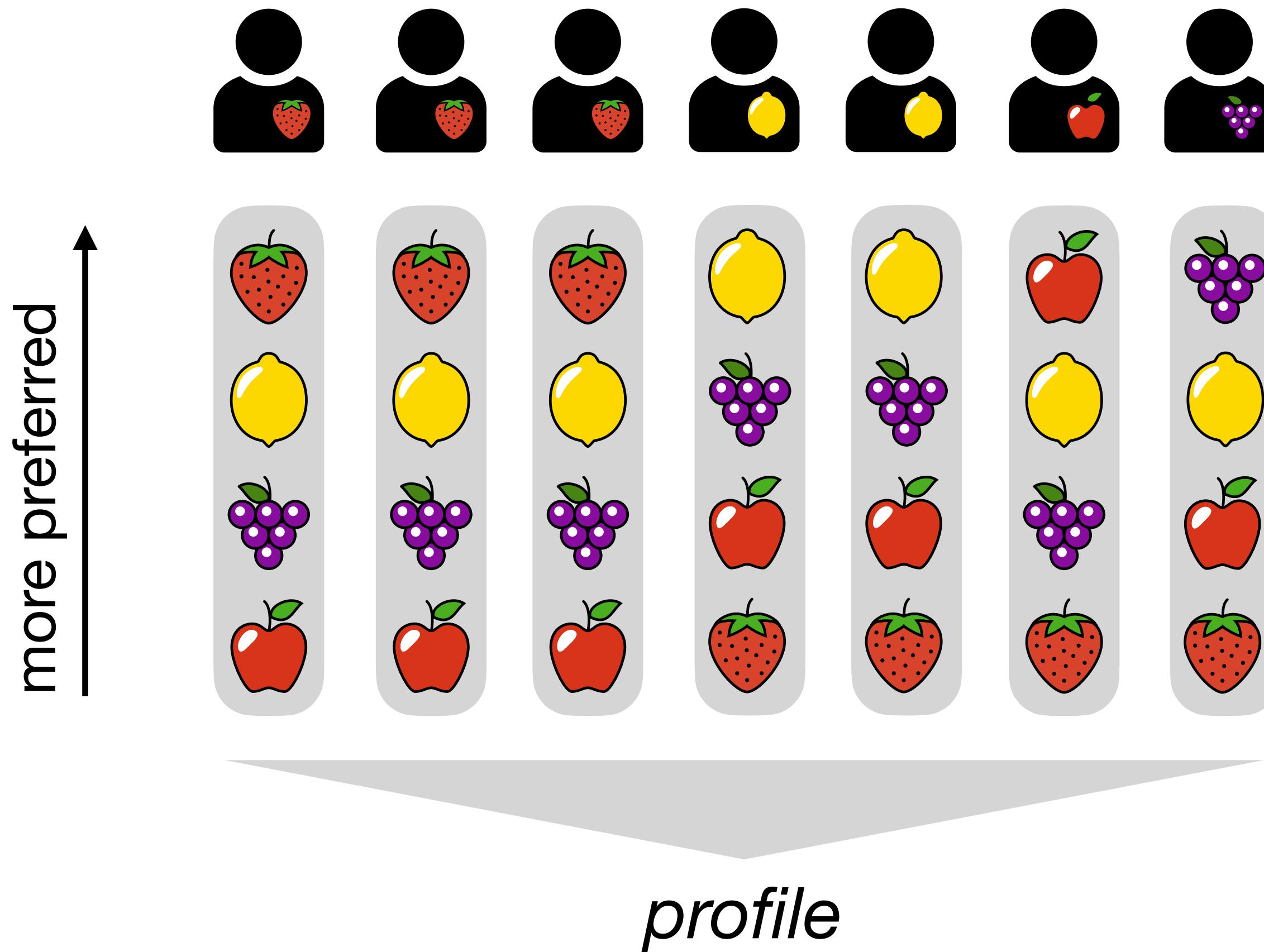


Given the preferences of voters, how do we pick a winner?

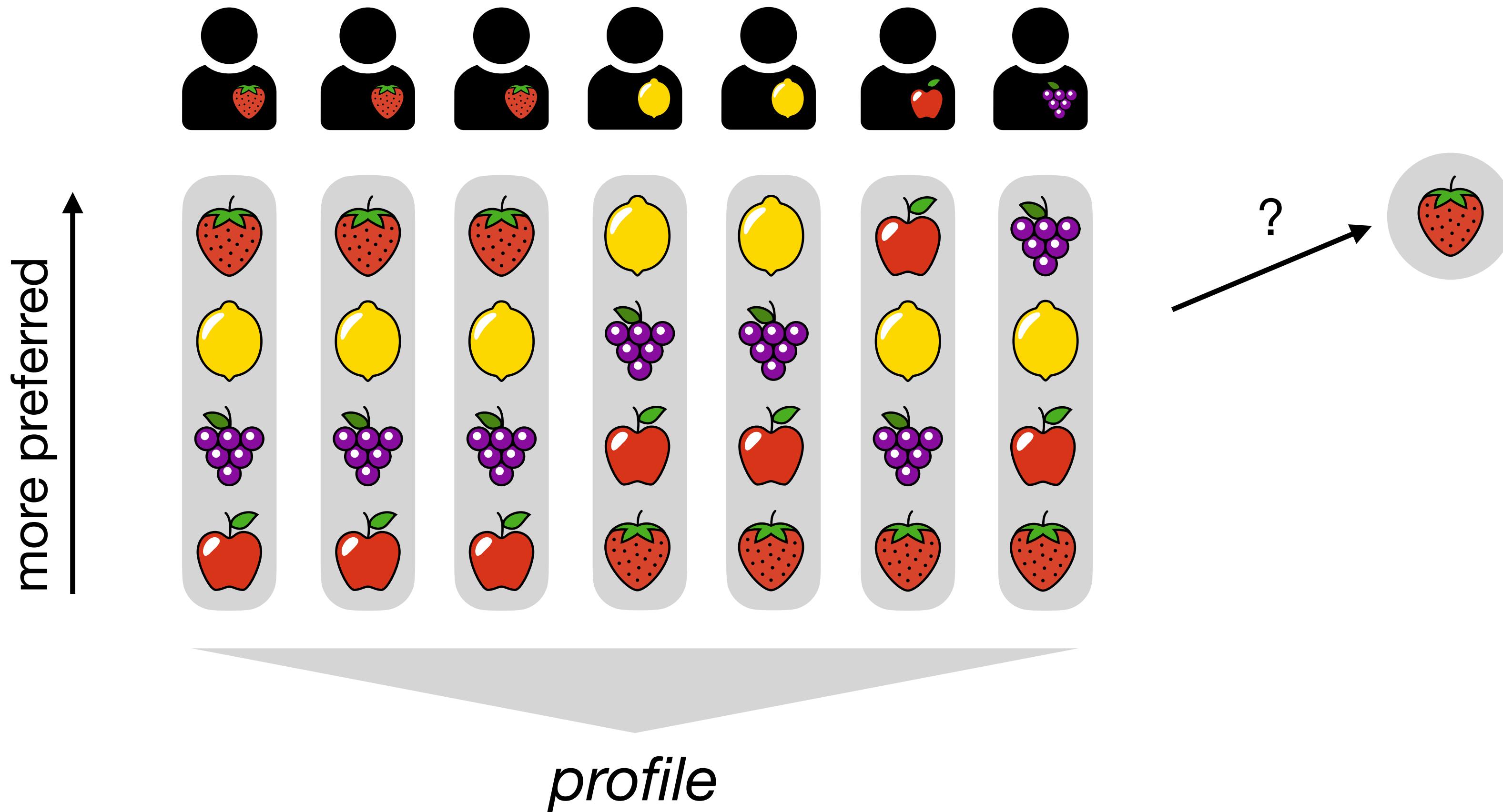
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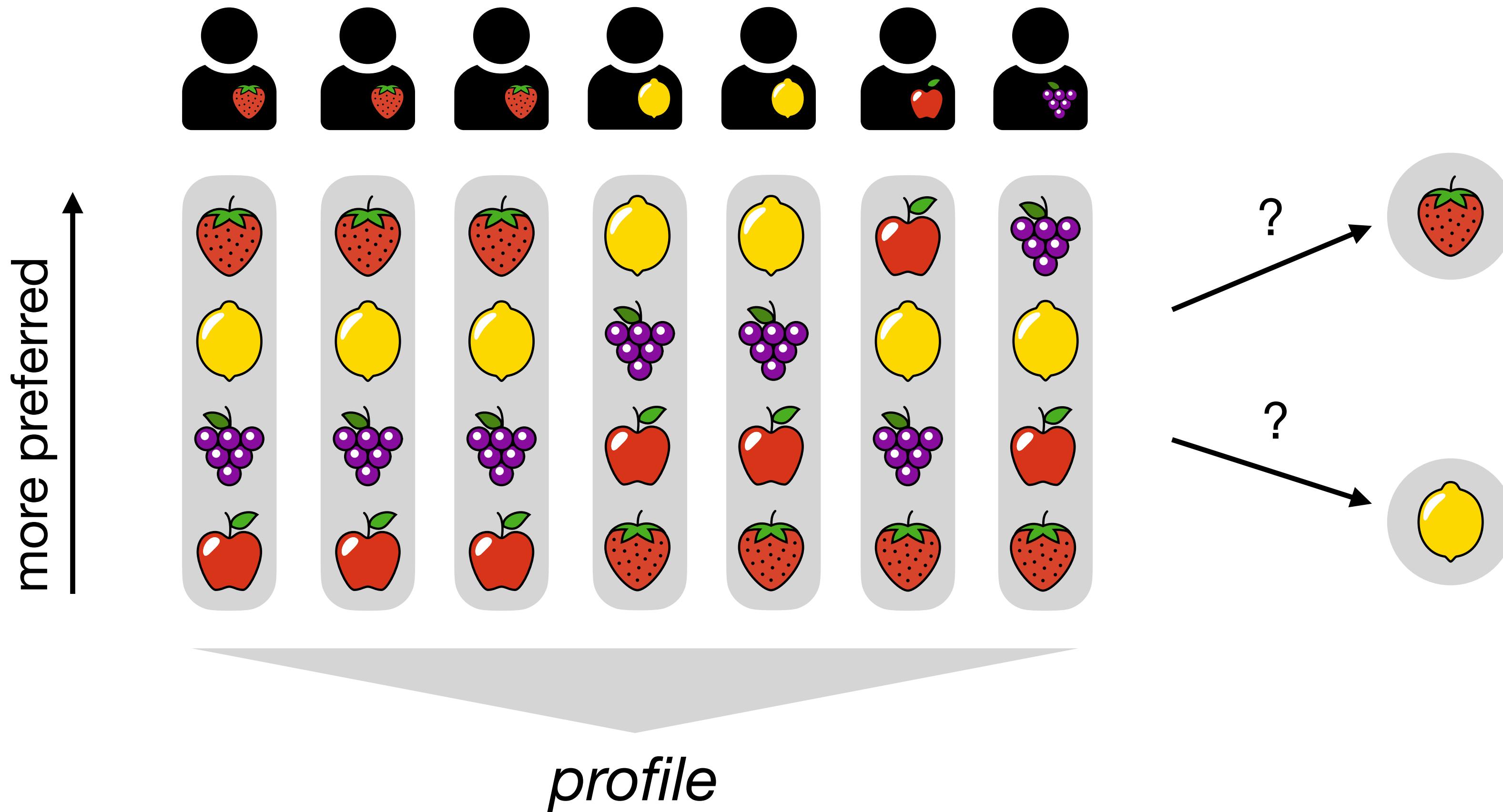
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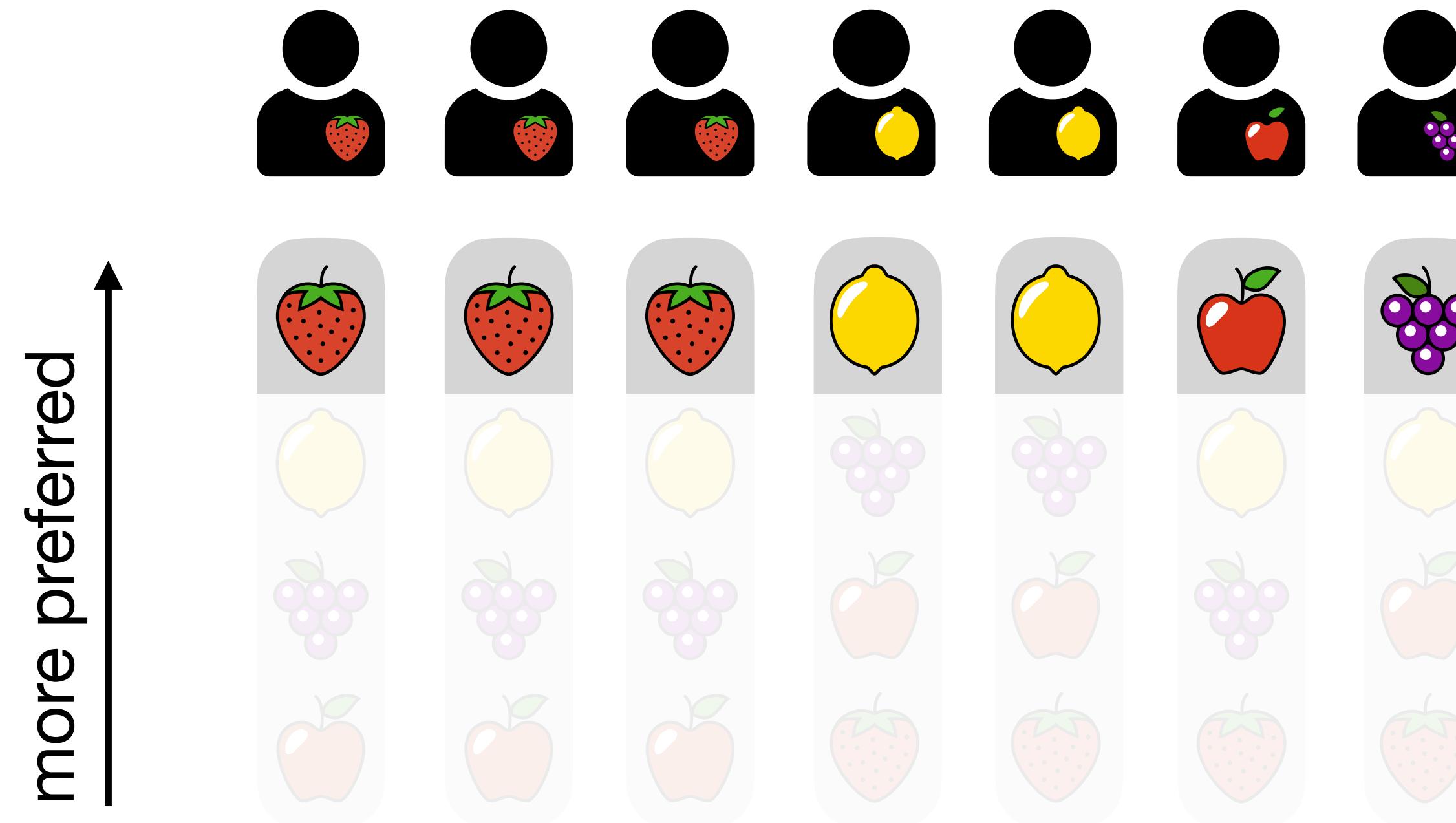


Given the preferences of voters, how do we pick a winner?



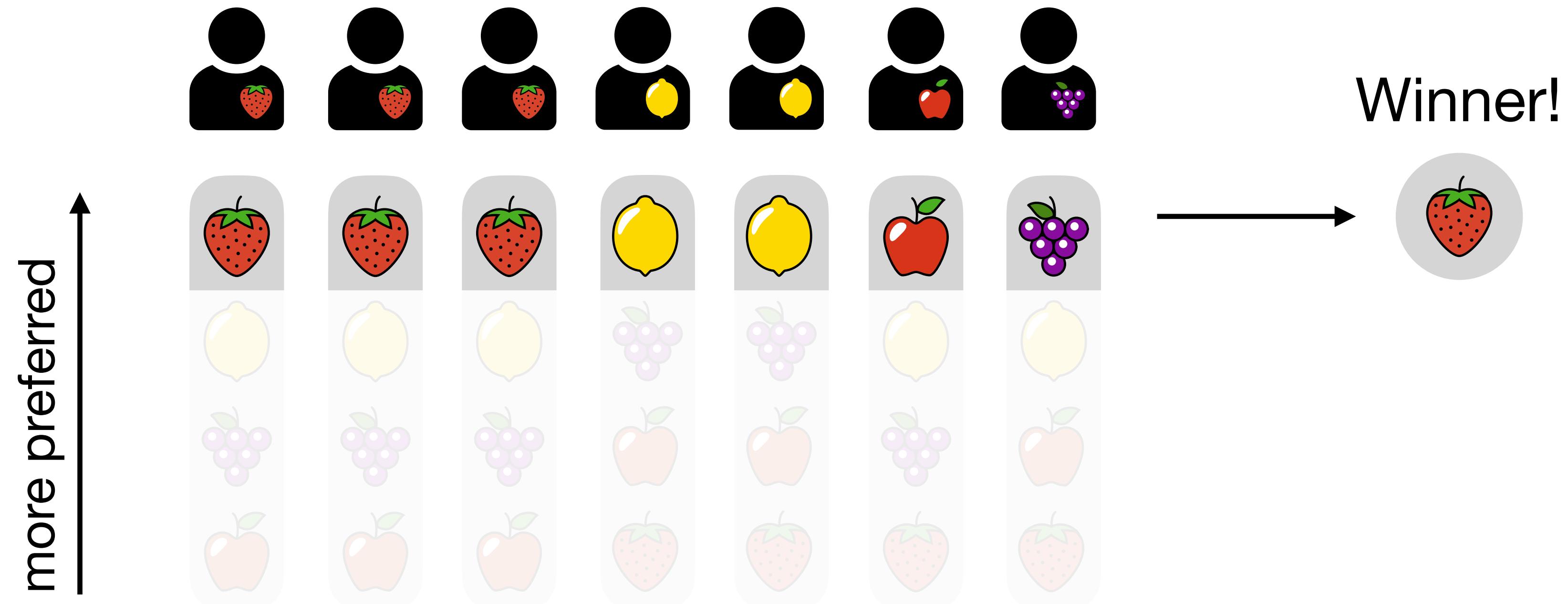
Plurality voting

choose the candidate with the most first-place votes



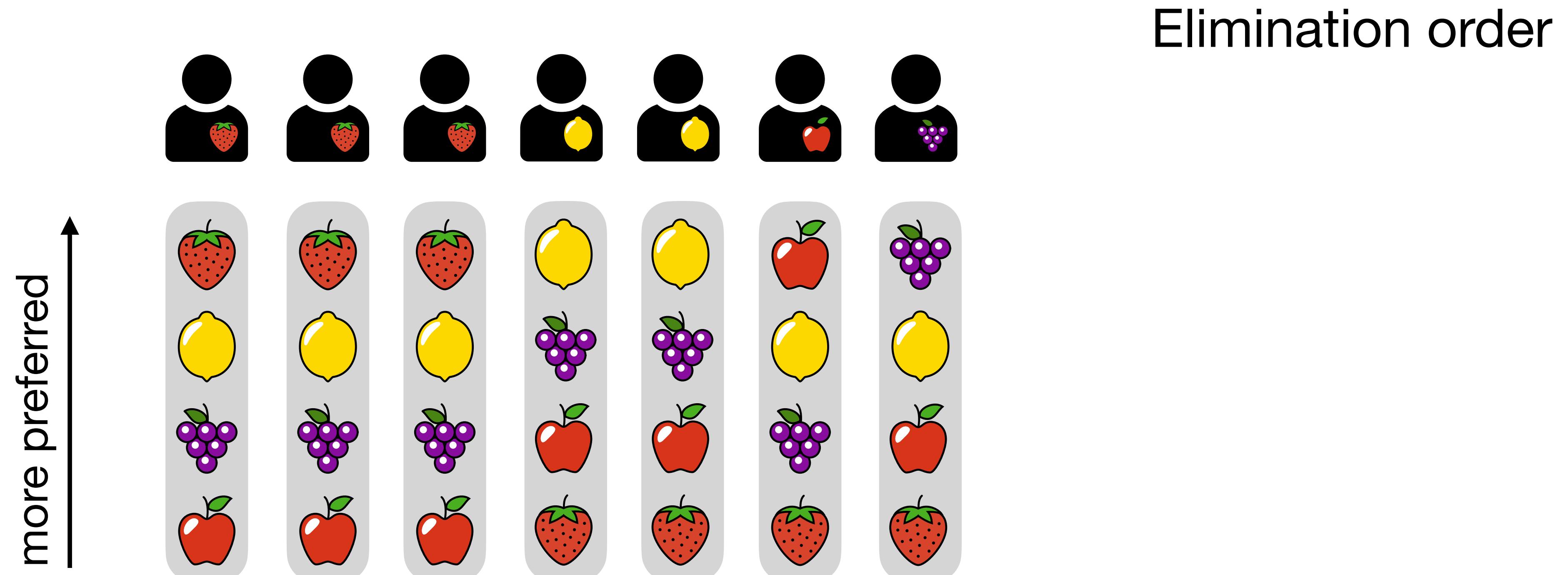
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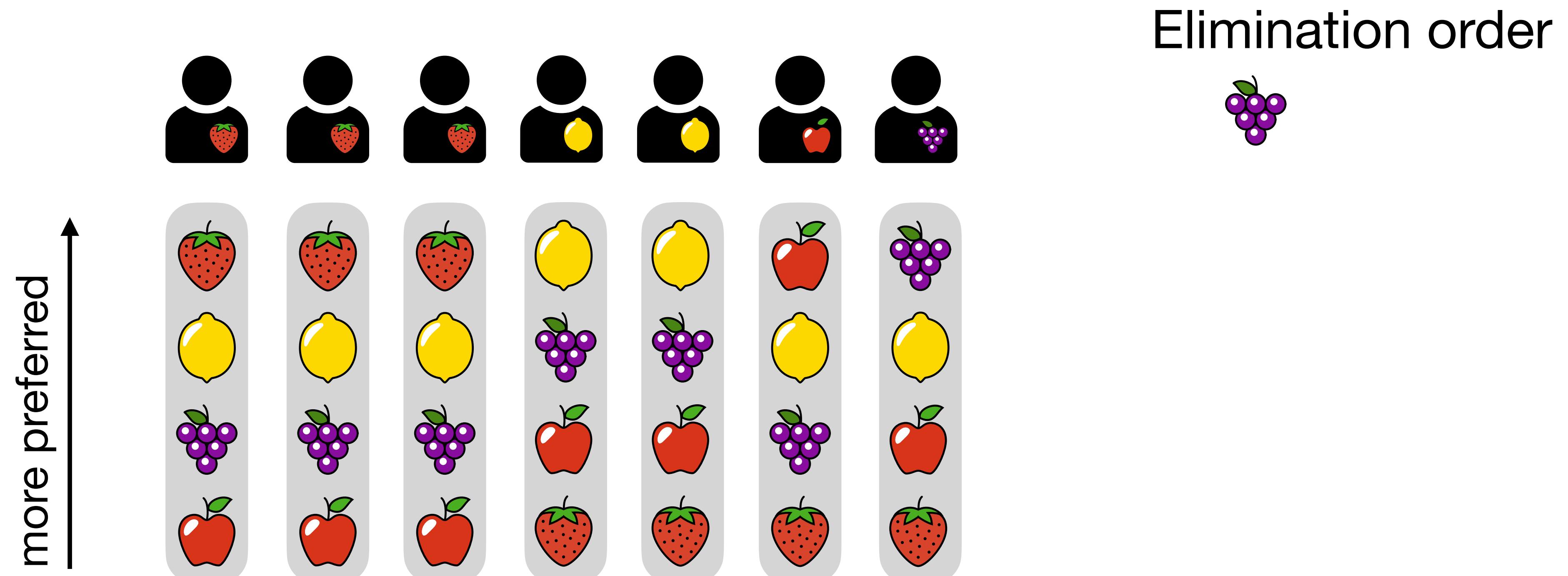
Instant runoff voting (IRV)

repeatedly eliminate the candidate with fewest first-place votes



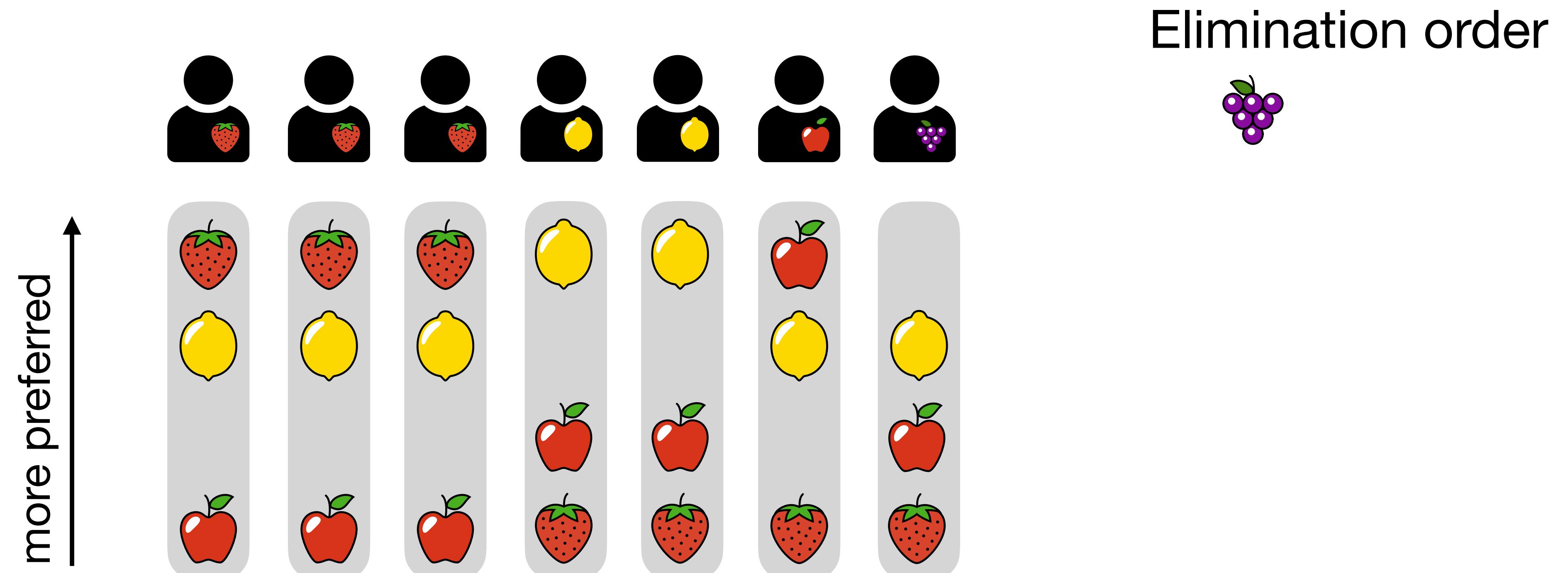
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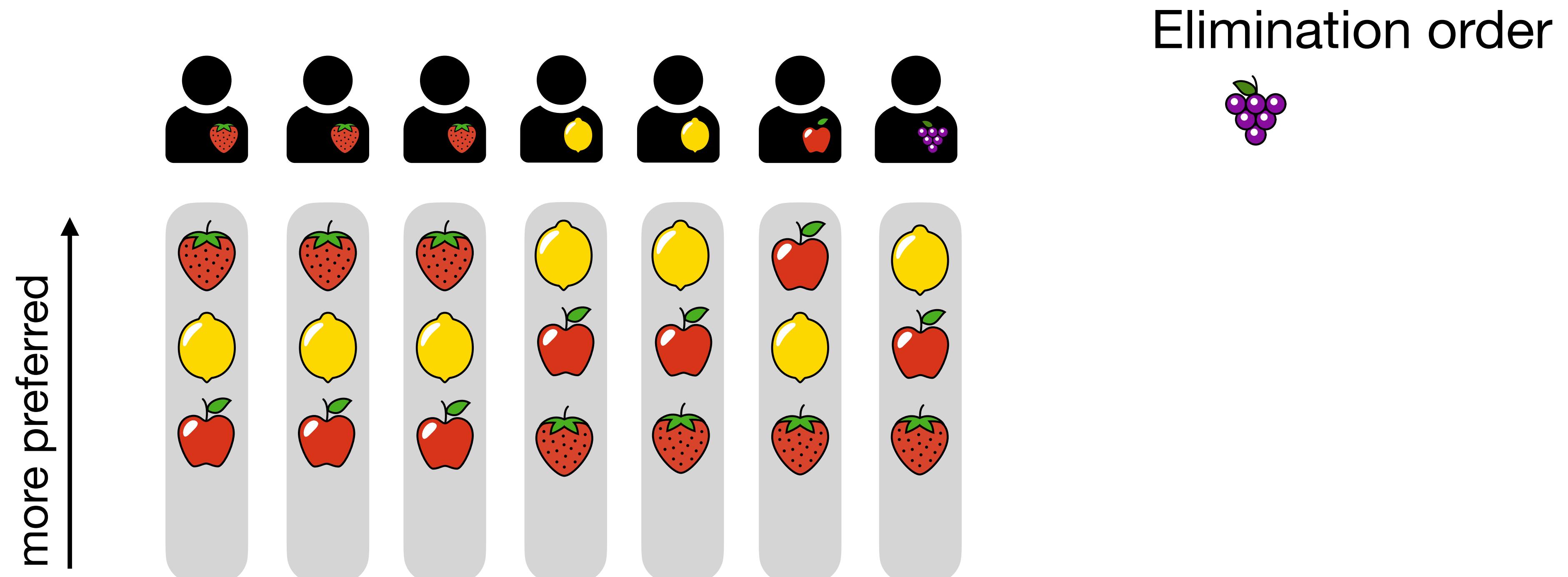
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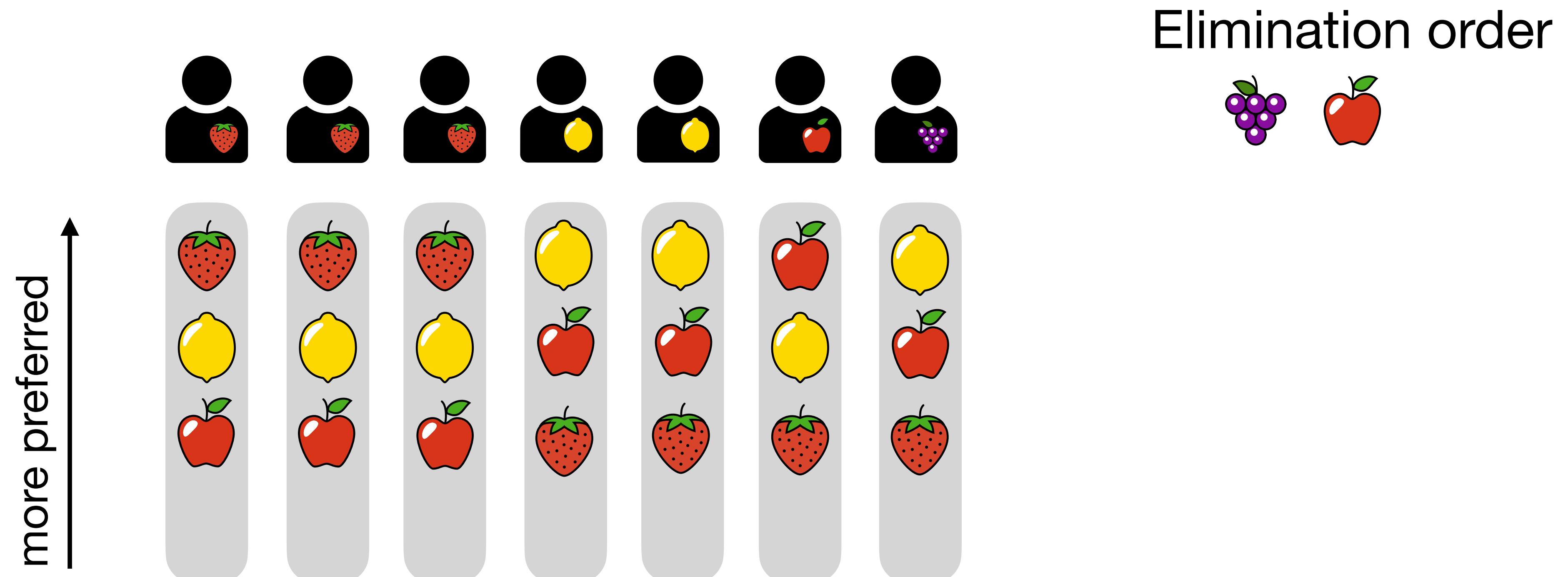
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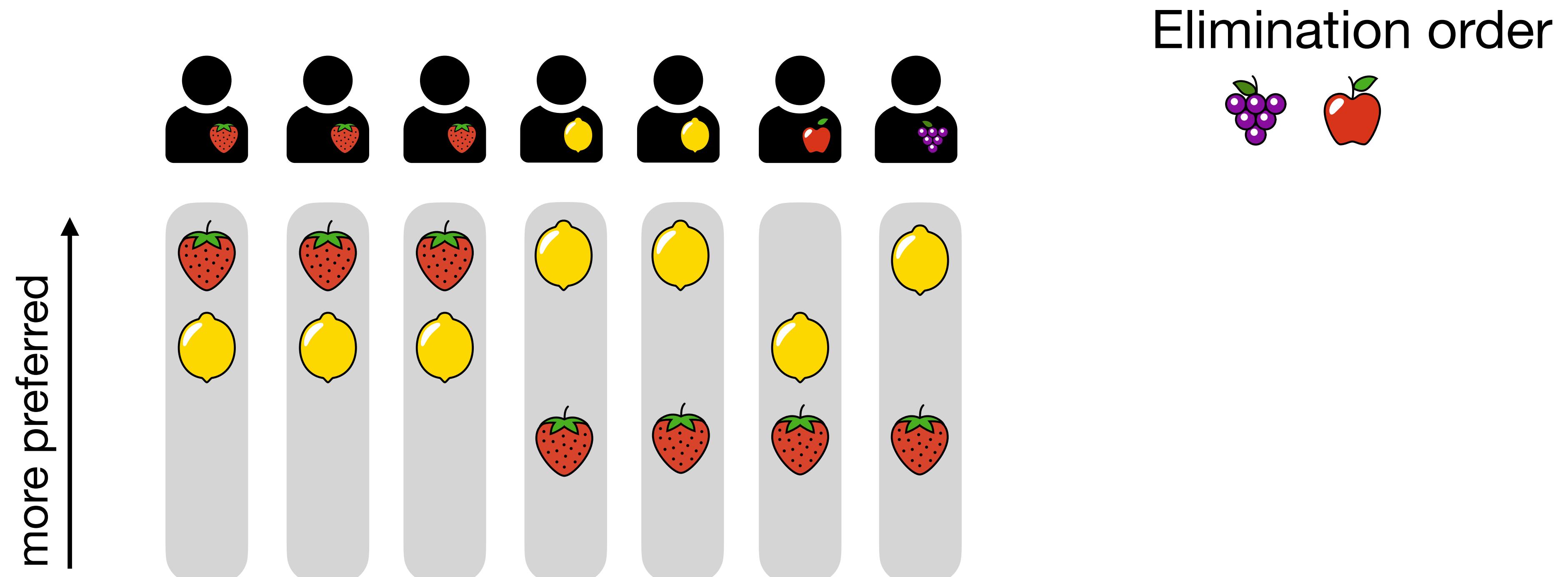
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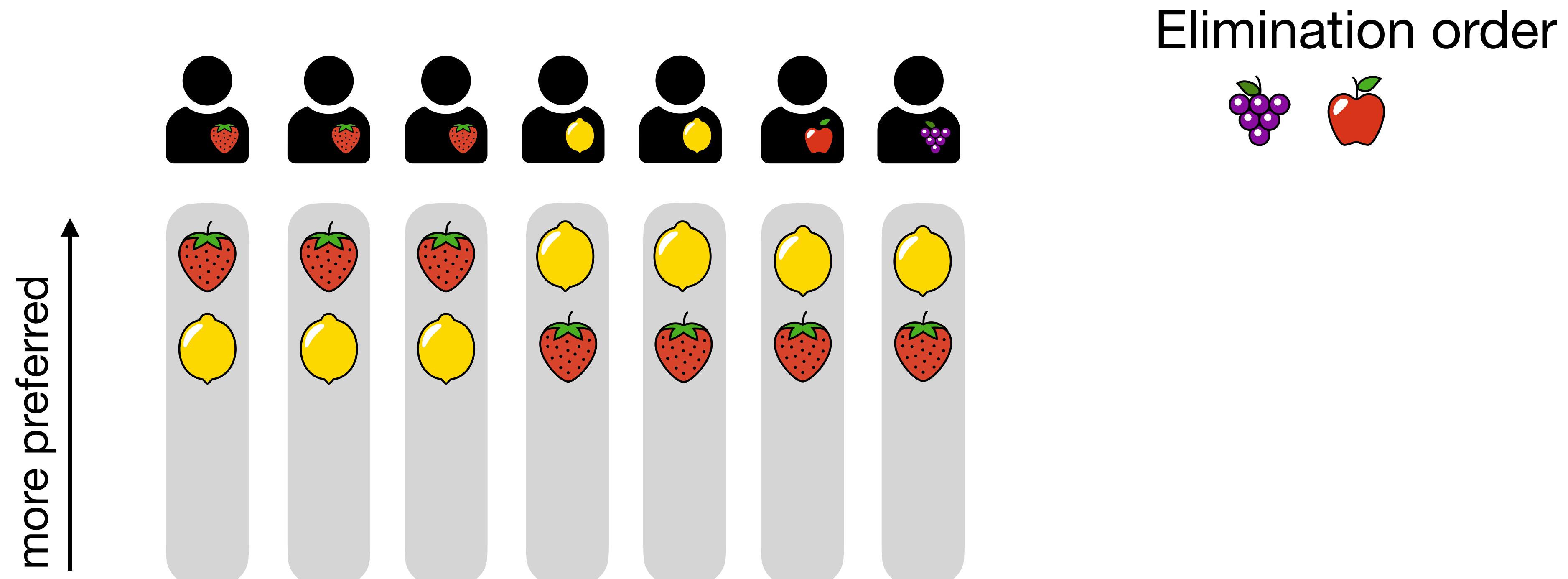
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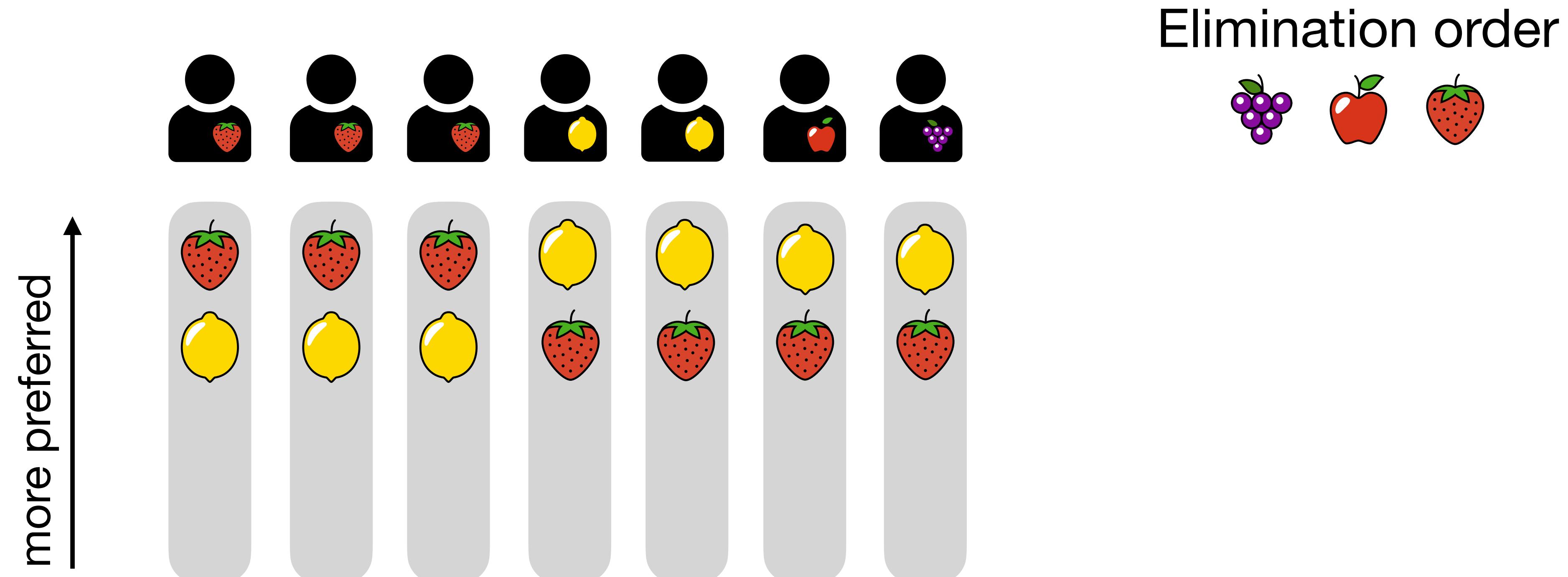
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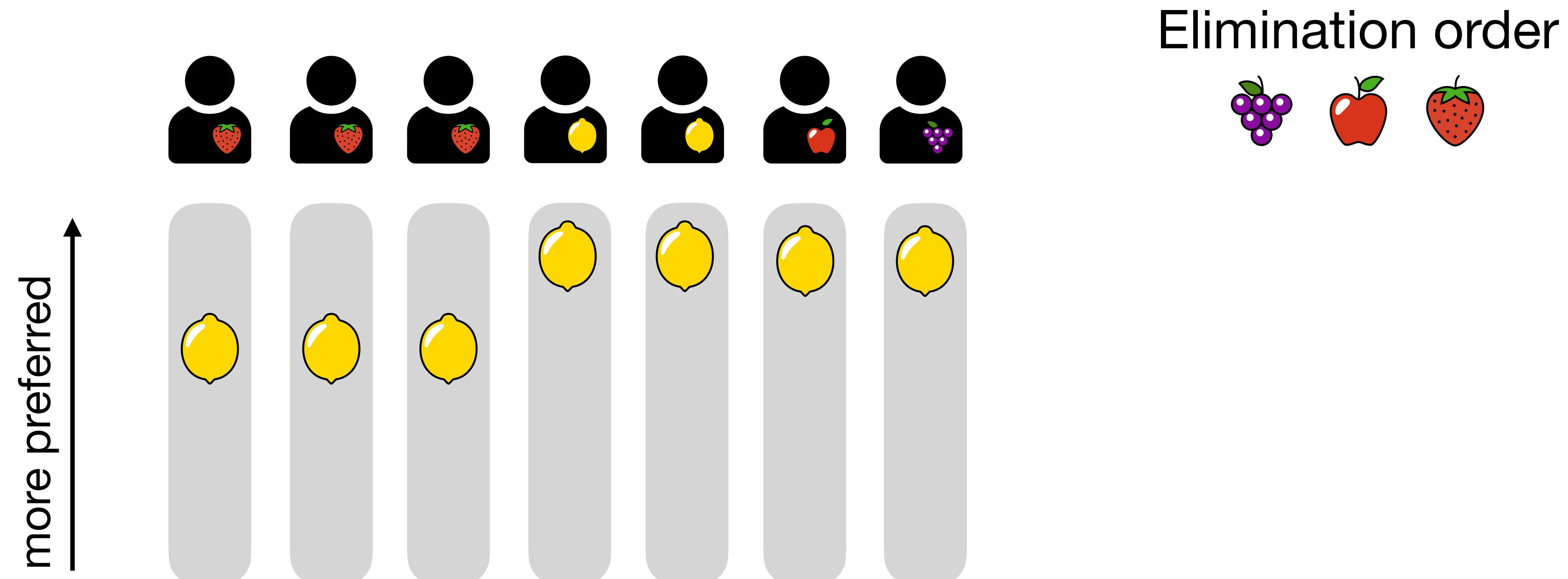
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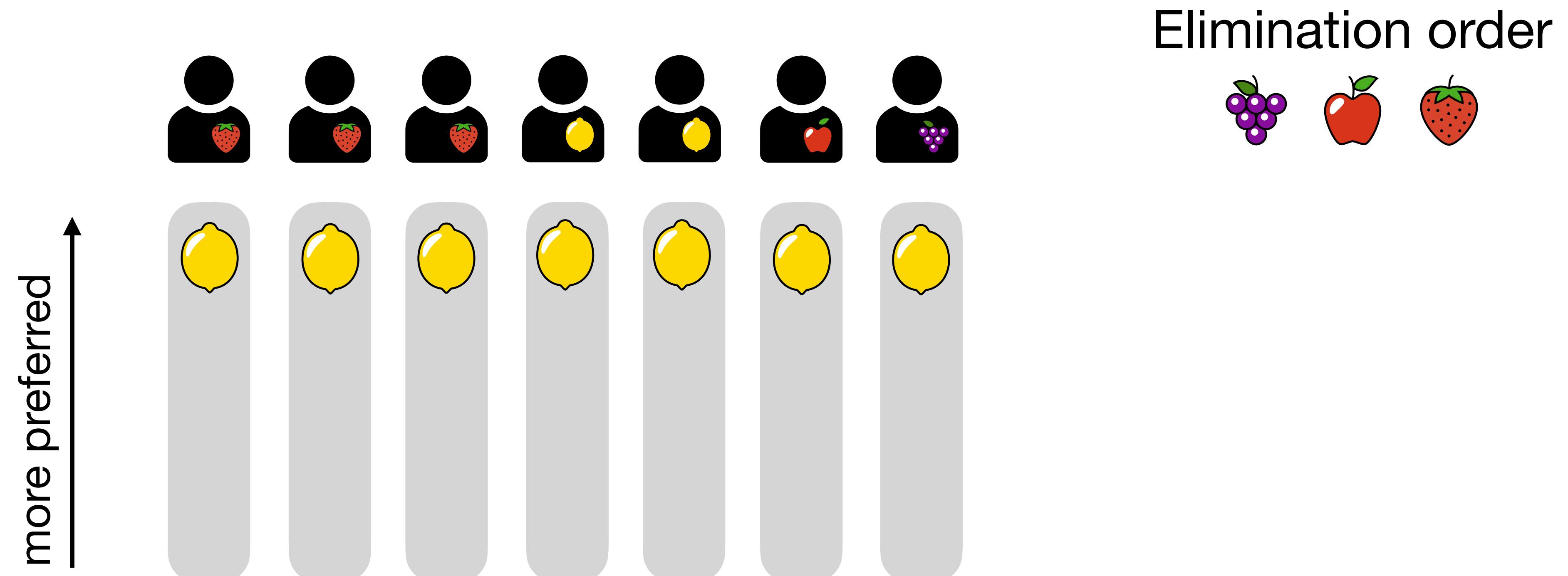
Instant runoff voting (IRV)

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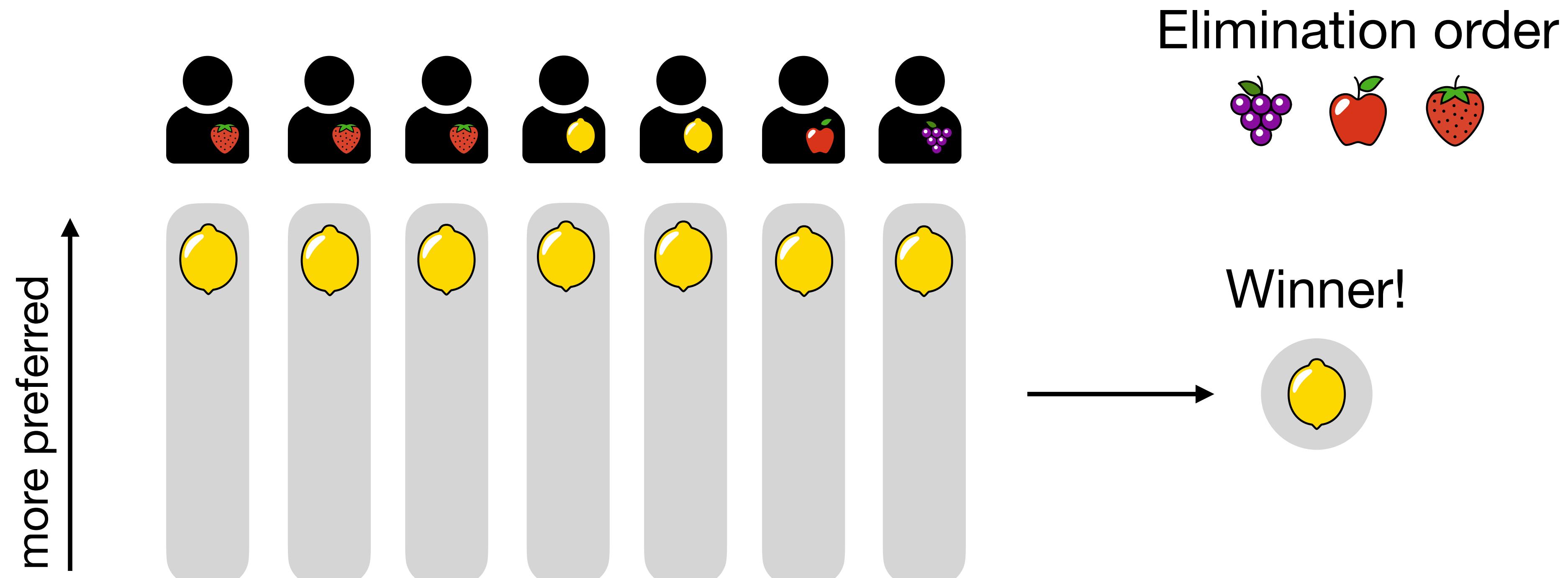
Instant runoff voting (IRV)

repeatedly eliminate the candidate with fewest first-place votes



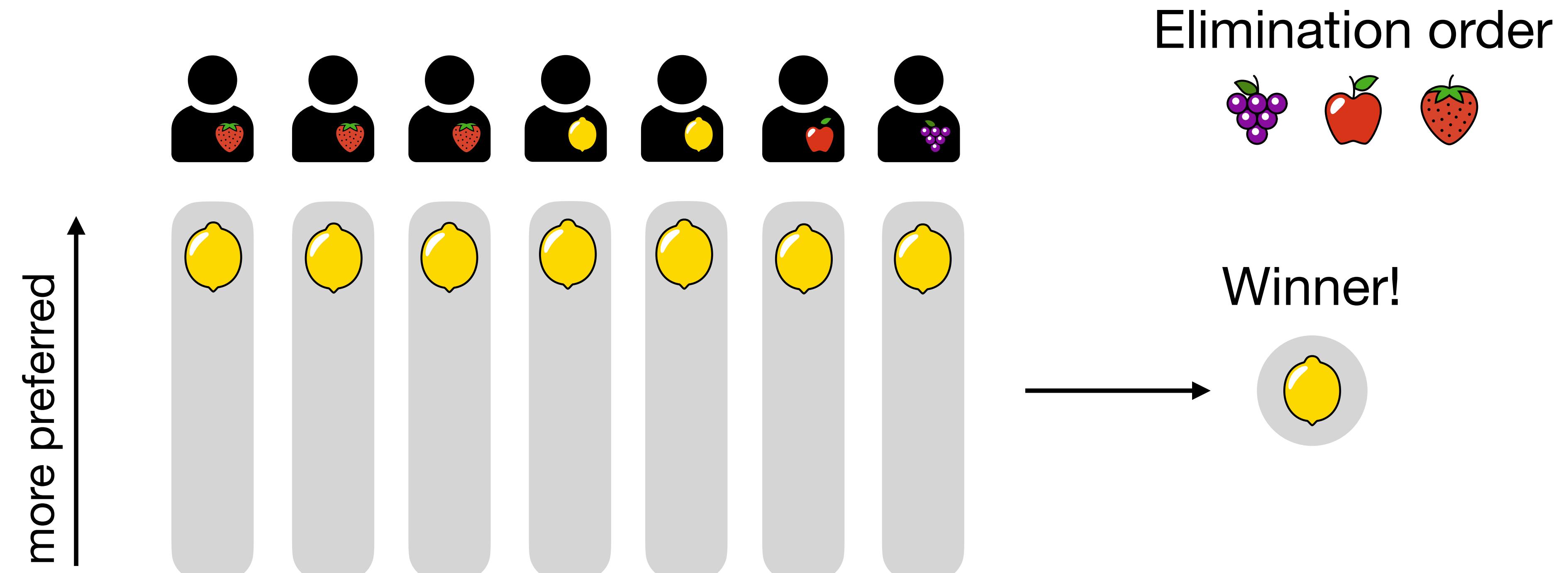
Instant runoff voting (IRV)

repeatedly eliminate the candidate with fewest first-place votes



Instant runoff voting (IRV)

repeatedly eliminate the candidate with fewest first-place votes



a.k.a. STV, AV, RCV, Hare method, preferential voting

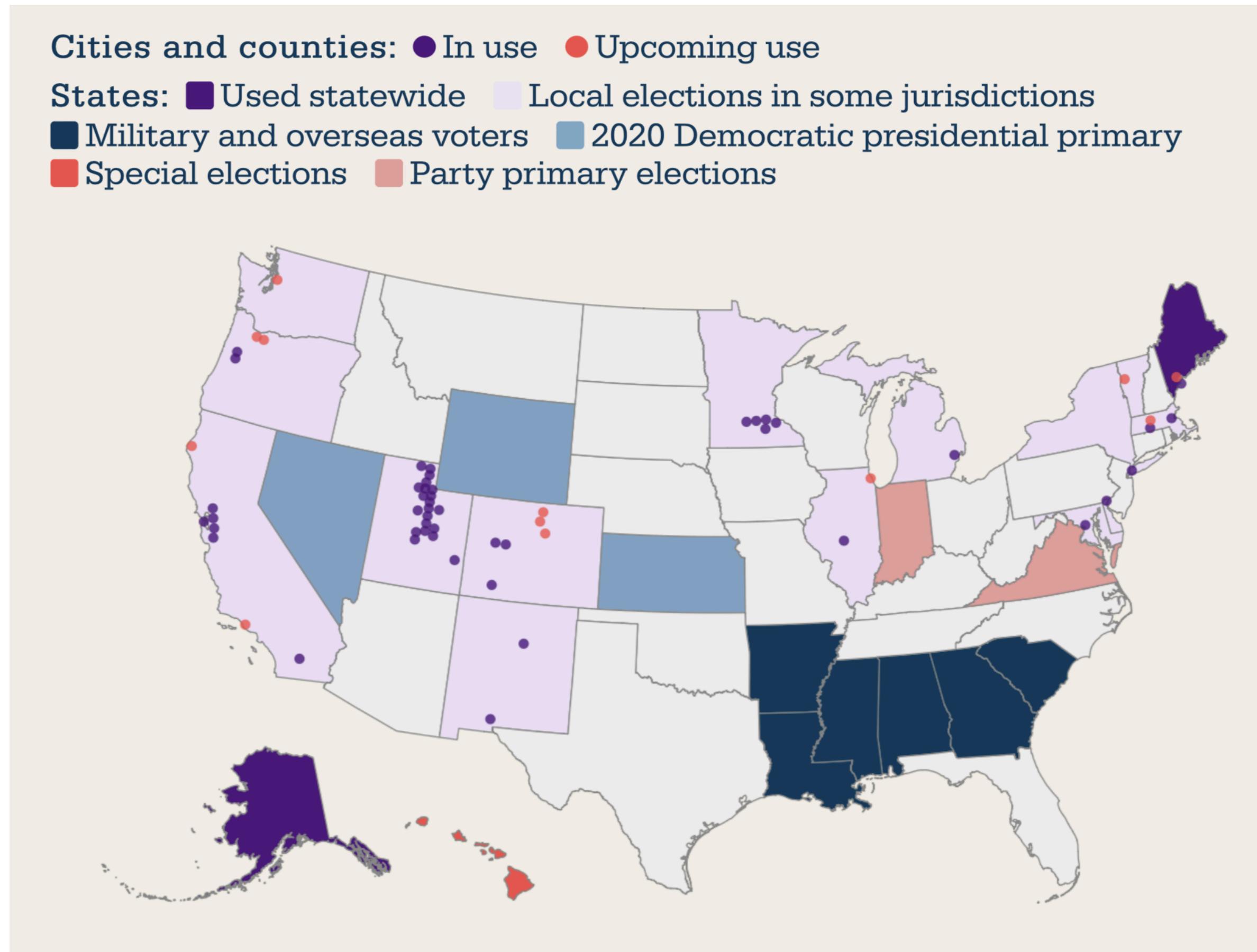
Who uses IRV?

Cities and counties: ● In use ● Upcoming use

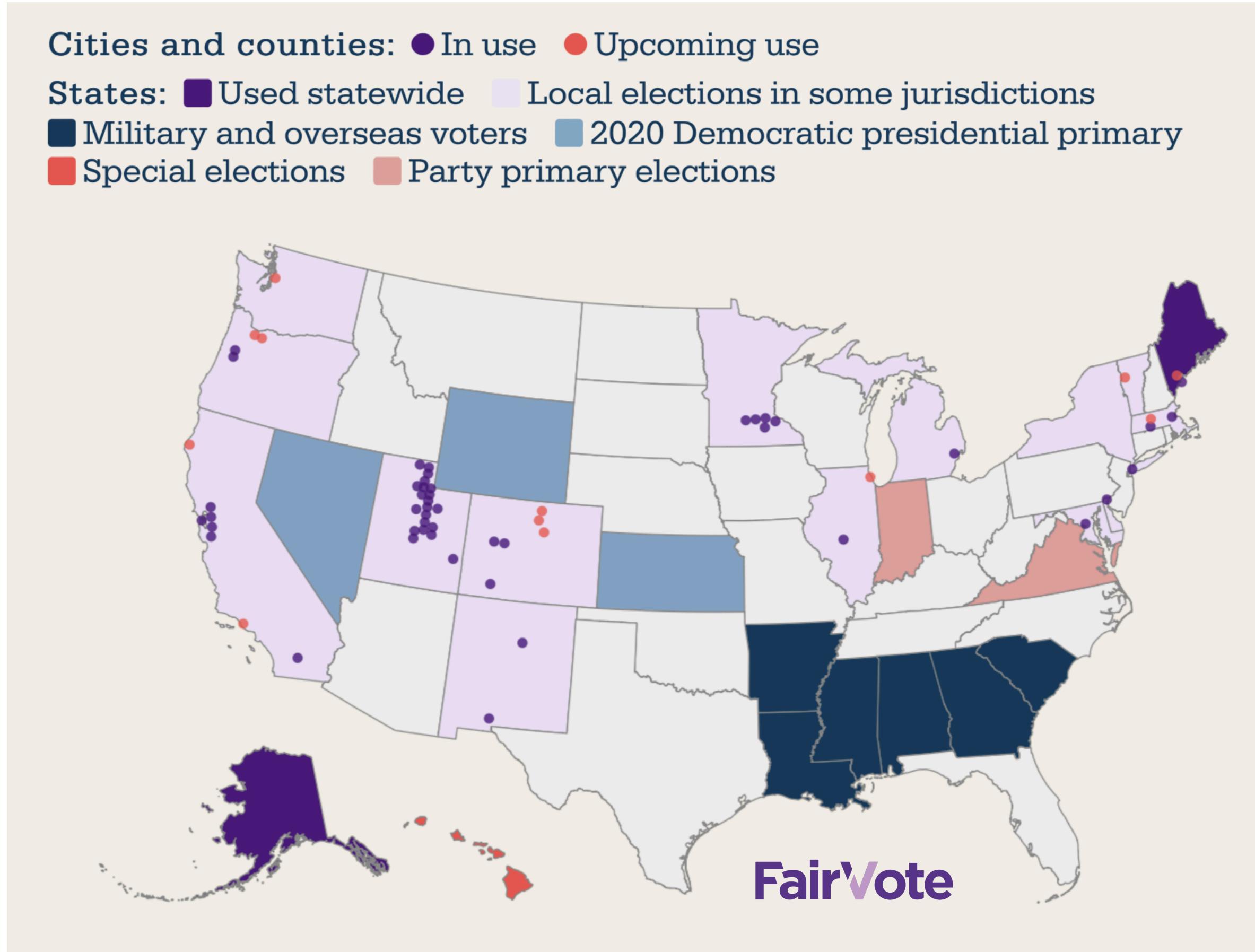
States: ■ Used statewide □ Local elections in some jurisdictions

■ Military and overseas voters ■ 2020 Democratic presidential primary

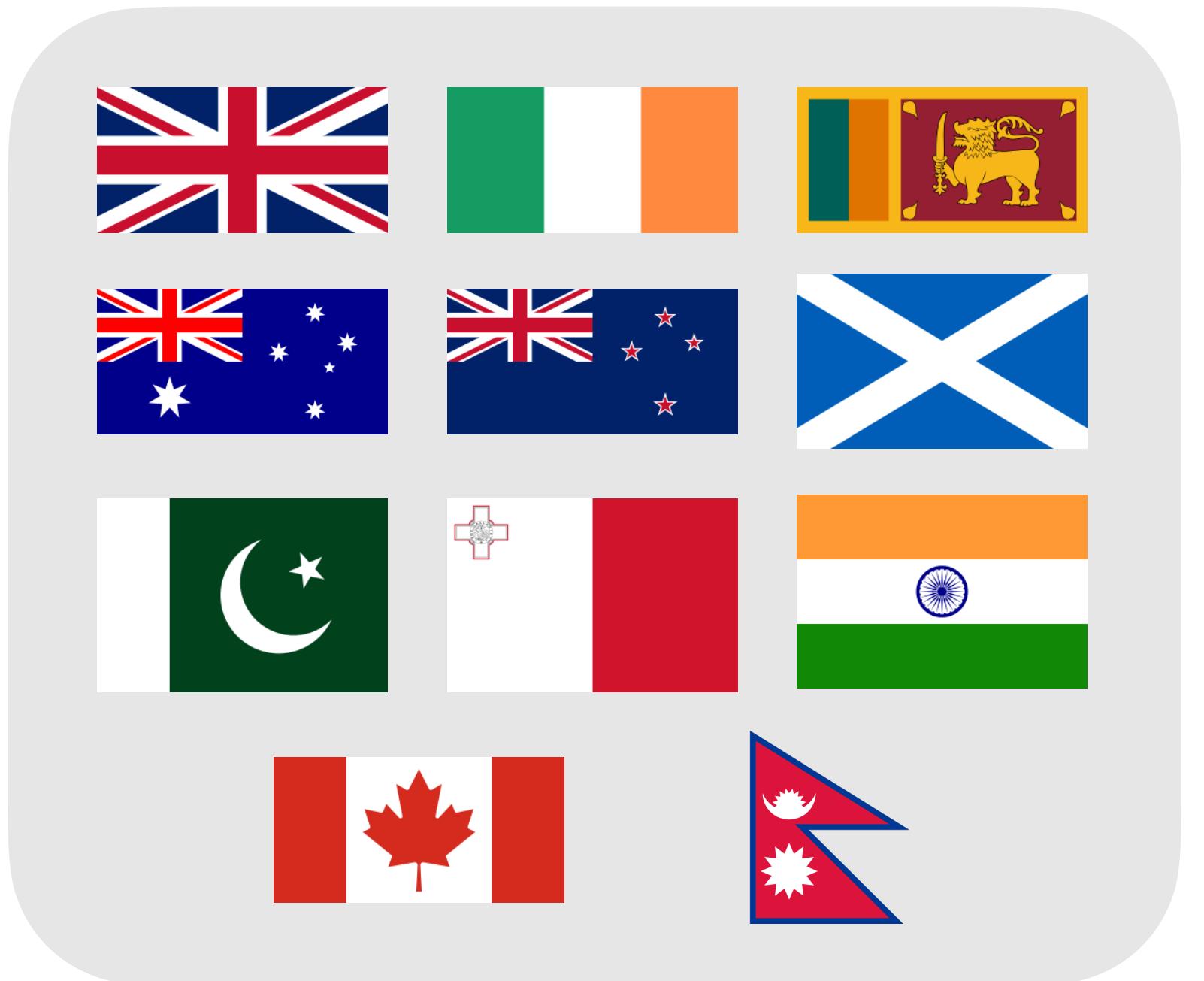
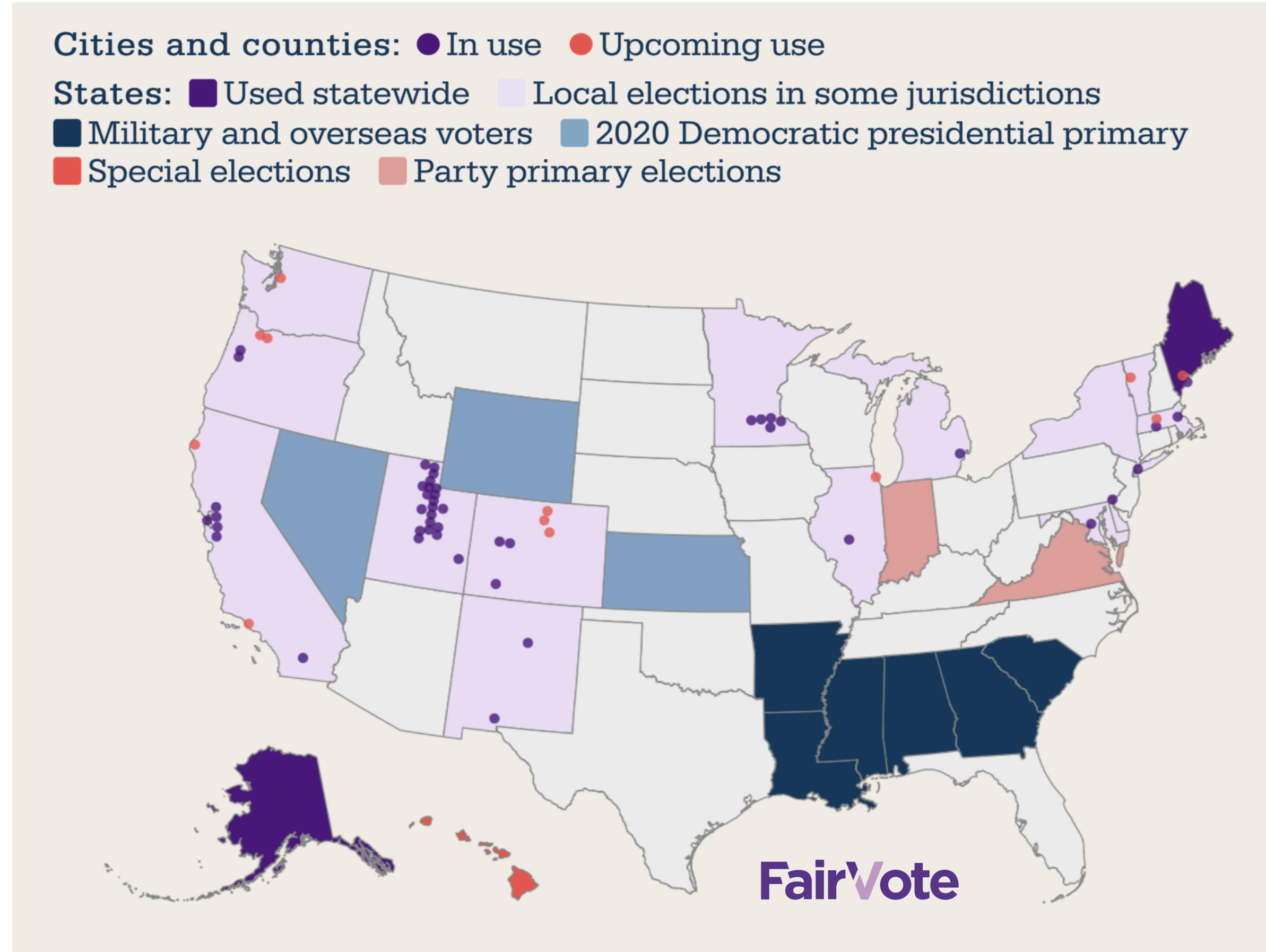
Special elections Party primary elections



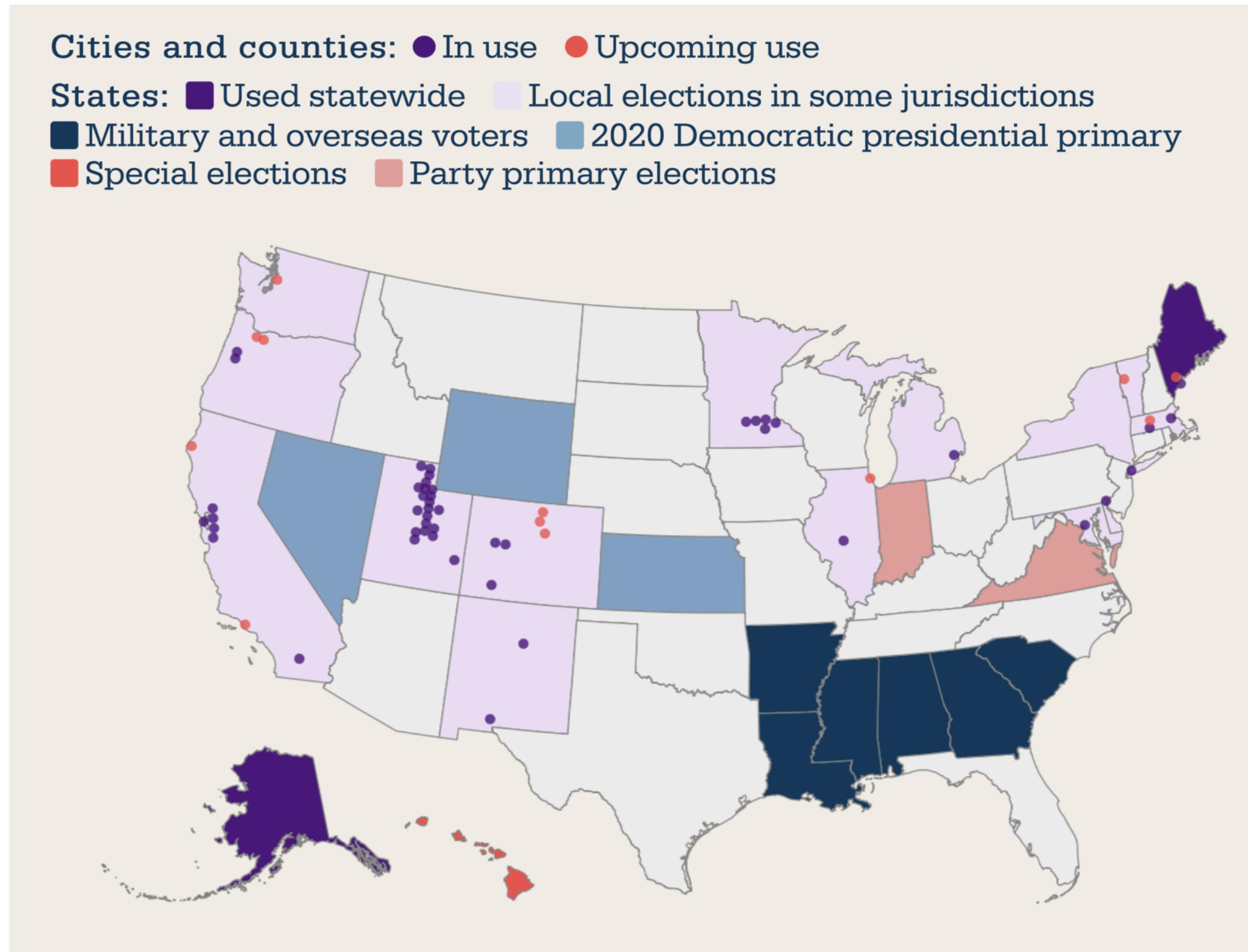
Who uses IRV?



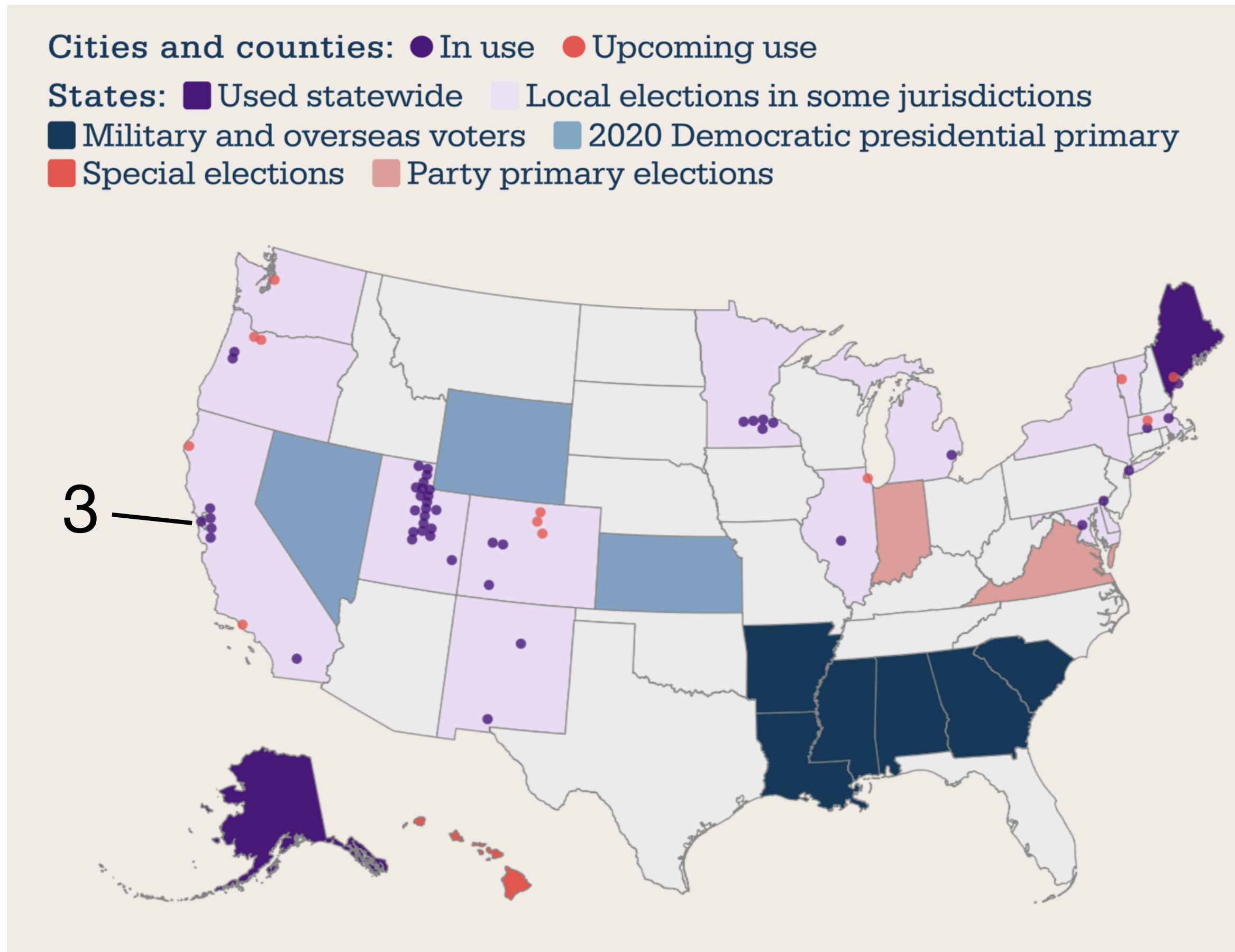
Who uses IRV?



Ballot length: how many candidates can you rank?



Ballot length: how many candidates can you rank?



San Fransisco

DEMONSTRATION BALLOT / BALOTA DE MUESTRA / 模擬選票
CONSOLIDATED GENERAL ELECTION / ELECCIONES GENERALES CONSOLIDADAS / 聯合普選
CITY AND COUNTY OF SAN FRANCISCO / CIUDAD Y CONDADO DE SAN FRANCISCO / 三藩市市縣
NOVEMBER 2, 2004 / 2 DE NOVIEMBRE DE 2004 / 2004年11月2日

CONGRESSIONAL DISTRICT 8, SENATE DISTRICT 8, ASSEMBLY DISTRICT 12, SUPERVISORIAL DISTRICT 00, BART DISTRICT 9

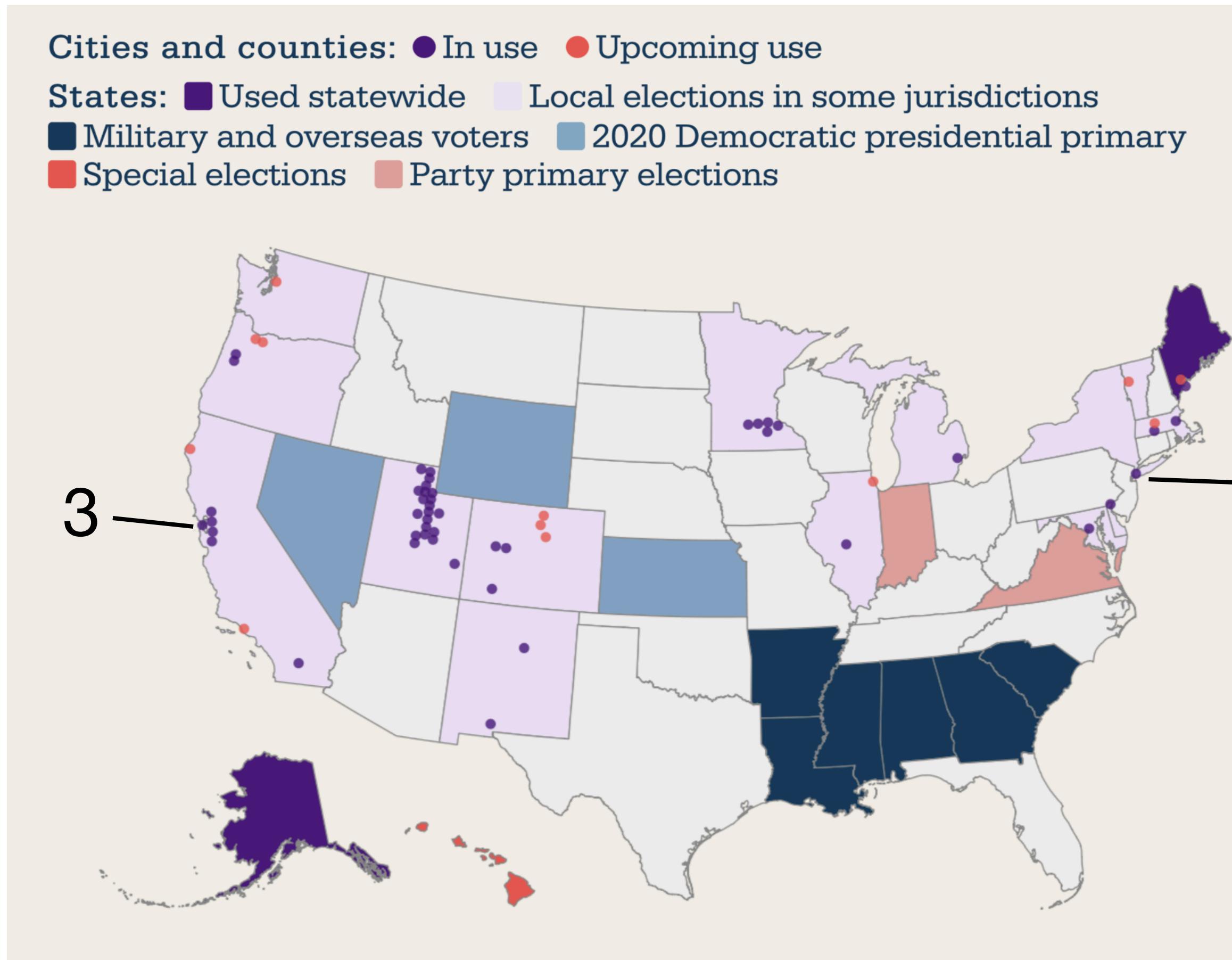
INSTRUCTIONS TO VOTERS: Mark your first choice in the first column by completing the arrow pointing to your choice, as shown in the picture. To indicate a second choice, select a different candidate in the second column. To indicate a third choice, select a different candidate in the third column. To vote for a qualified write-in candidate, write the person's name on the blank line provided and complete the arrow.

INSTRUCCIONES PARA LOS ELECTORES: Para marcar su primera opción en la primera columna, complete la flecha que apunta hacia su selección, tal como se indica en la imagen. Para indicar una segunda opción, seleccione un candidato distinto en la segunda columna. Para indicar una tercera opción, seleccione un candidato distinto en la tercera columna. Para votar por un candidato calificado no listado, escriba el nombre de la persona en el espacio en blanco provisto, y complete la flecha.

選民指南：在第一列中標記你第一個選擇，將箭頭向你的選擇的箭頭盡頭連接起來。如圖所示。
標記第二個選擇，在第二列中選擇一位不同的候選人。標記第三個選擇時，在第三列中選擇一位不同的候選人。投票給合格寫入候選人時，在提供的空線上填寫此人的姓名，並將箭頭盡頭連接起來。

MEMBER, BOARD OF SUPERVISORS / MIEMBRO, CONSEJO DE SUPERVISORES / 由參議員 DISTRICT 00 - DISTRITO 00 - 第參議員選區		
VOTE YOUR FIRST, SECOND AND THIRD CHOICES / VOTE POR SU PRIMERA, SEGUNDA Y TERCERA SELECCIÓN / 行使你的第一、第二和第三選擇		
1 FIRST CHOICE PRIMERA SELECCIÓN 第一選擇	2 SECOND CHOICE SEGUNDA SELECCIÓN 第二選擇	3 THIRD CHOICE TERCERA SELECCIÓN 第三選擇
Vote for One Vote por Uno 第一選擇	Vote for One - Must be different than your first choice Vote por Uno - Debe ser diferente de su primera selección 第一選擇 / 必須與第一個選擇 和第二個選擇不同	Vote for One - Must be different than your first and second choices Vote por Uno - Debe ser diferente de su primera y segunda selección 第一選擇 / 必須與第一個選擇 和第二個選擇不同
ELEANOR ROOSEVELT Incumbent Treasurer RETIRE	ELEANOR ROOSEVELT Incumbent Treasurer RETIRE	ELEANOR ROOSEVELT Incumbent Treasurer RETIRE
CESAR CHAVEZ 2004 • 劳工组织者 Labor Organizer Organizador Laboral 2004 • 劳工组织者	CESAR CHAVEZ 2004 • 劳工组织者 Labor Organizer Organizador Laboral 2004 • 劳工组织者	CESAR CHAVEZ 2004 • 劳工组织者 Labor Organizer Organizador Laboral 2004 • 劳工组织者
WALTER LUM 1998 • 1996 Publisher Editor 编辑者	WALTER LUM 1998 • 1996 Publisher Editor 编辑者	WALTER LUM 1998 • 1996 Publisher Editor 编辑者
JOHN HANCOCK 1998 • 1996 Physician Majestic 医生	JOHN HANCOCK 1998 • 1996 Physician Majestic 医生	JOHN HANCOCK 1998 • 1996 Physician Majestic 医生
MARTIN LUTHER KING, JR. 1998 • 1996 Minister Pastor 牧师	MARTIN LUTHER KING, JR. 1998 • 1996 Minister Pastor 牧师	MARTIN LUTHER KING, JR. 1998 • 1996 Minister Pastor 牧师
ANNA MAE PICTOU AQUASH 1998 • 1996 Indigenous Rights Activist Organizadora para Derechos Indígenas 土著權益倡議者	ANNA MAE PICTOU AQUASH 1998 • 1996 Indigenous Rights Activist Organizadora para Derechos Indígenas 土著權益倡議者	ANNA MAE PICTOU AQUASH 1998 • 1996 Indigenous Rights Activist Organizadora para Derechos Indígenas 土著權益倡議者

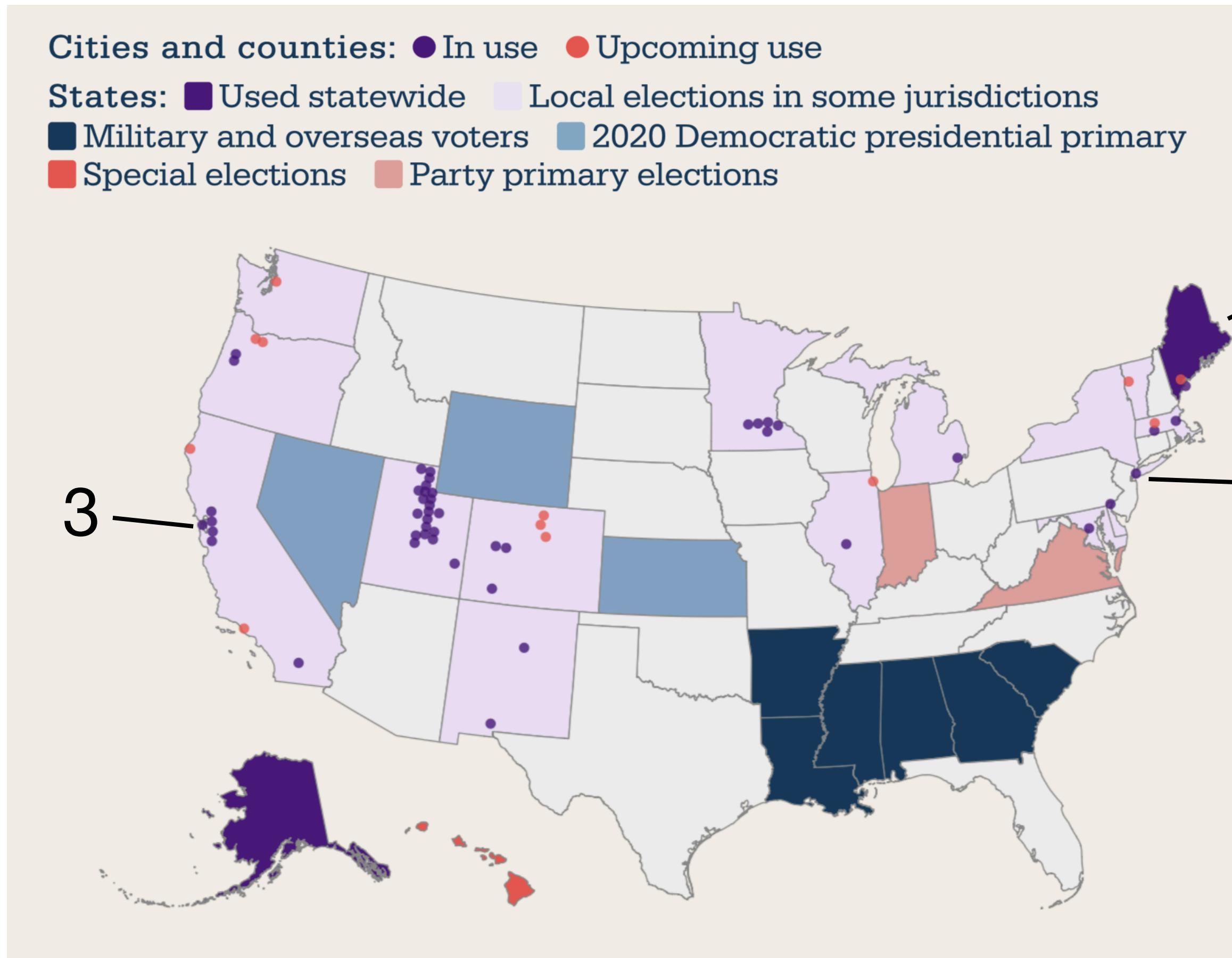
Ballot length: how many candidates can you rank?



New York City

Council Member Rank up to 5 choices Mark no more than 1 oval in each column Miembro del Consejo Clasifique hasta 5 opciones Marque no más de un óvalo en cada columna	Choice Option				
	Choice Option				
	1st	2nd	3rd	4th	5th
John E. Sanchez Community First	A	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Oswald Feliz People United	B	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Elisa Crespo Jobs & Justice	C	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Kenny G. Agosto Empower People	D	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Ischia J. Bravo We Matter	E	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Ariel Rivera-Diaz Second Choice	F	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Bernadette Ferrara Fifteen Forward	G	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Latchmi Devi Gopal Go For The Bronx	H	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Jose A. Padilla Jr. Safe & Stable	I	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Altagracia Soldevilla People First	J	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
White-in candidato por escrito		<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Ballot length: how many candidates can you rank?



Maine

**State of Maine Sample Ballot
Democratic Primary Election, June 12, 2018
for**

Instructions to Voters

To vote, fill in the oval like this ●

To rank your candidate choices, fill in the oval:

- In the 1st column for your 1st choice candidate.
- In the 2nd column for your 2nd choice candidate, and so on.

Continue until you have ranked as many or as few candidates as you like.

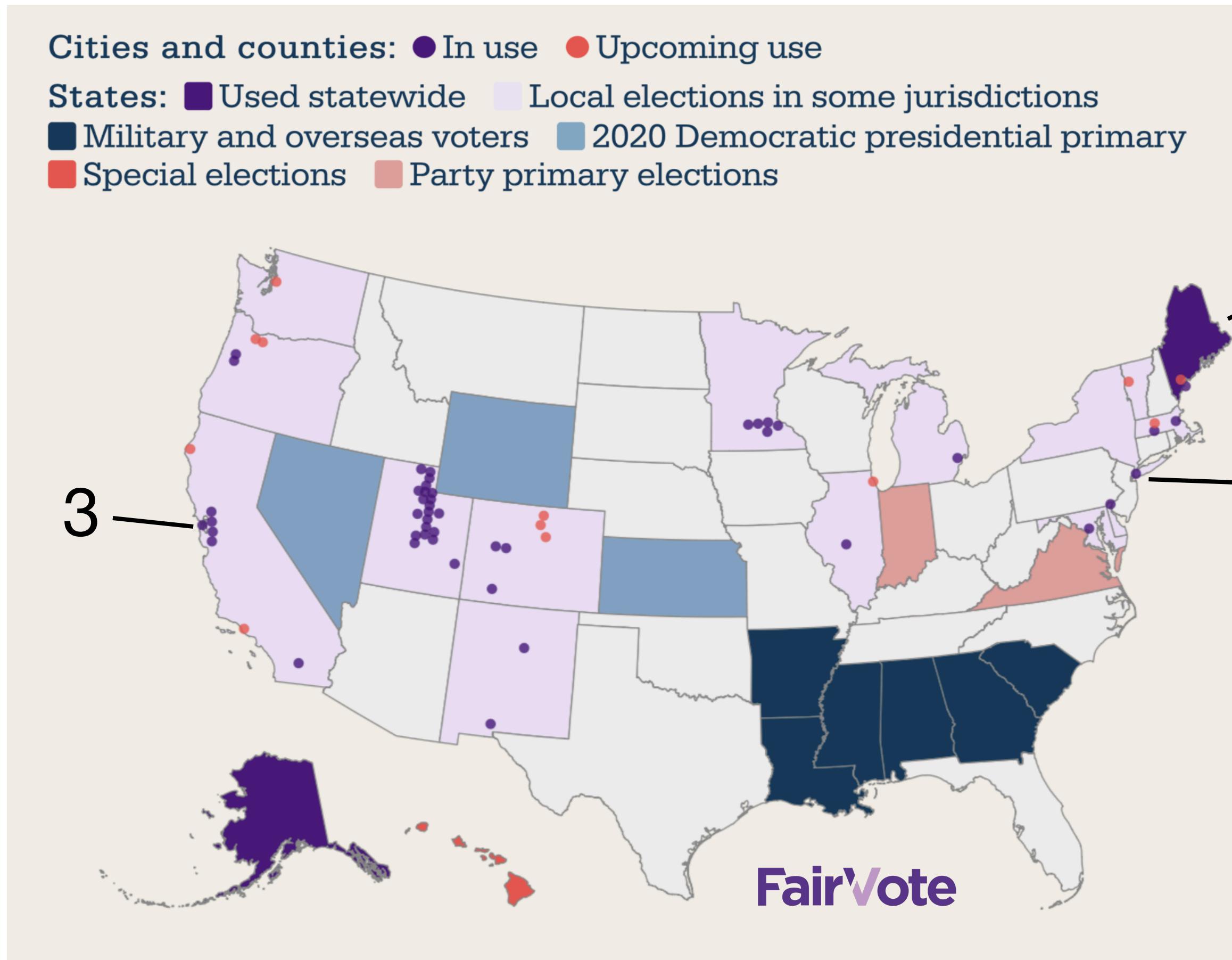
Fill in no more than one oval for each candidate or column.

To rank a write-in candidate, write the person's name in the write-in space and fill in the oval for the ranking of your choice.

Governor	1st Choice	2nd Choice	3rd Choice	4th Choice	5th Choice	6th Choice	7th Choice	8th Choice
Cote, Adam Roland Sanford	0	0	0	0	0	0	0	0
Dion, Donna J. Biddeford	0	0	0	0	0	0	0	0
Dion, Mark N. Portland	0	0	0	0	0	0	0	0
Eves, Mark W. North Berwick	0	0	0	0	0	0	0	0
Mills, Janet T. Farmington	0	0	0	0	0	0	0	0
Russell, Diane Marie Portland	0	0	0	0	0	0	0	0
Sweet, Elizabeth A. Hallowell	0	0	0	0	0	0	0	0
Write-in	0	0	0	0	0	0	0	0

SOURCE: Maine Secretary of State Office

Ballot length: how many candidates can you rank?



DEMONSTRATION BALLOT / BALOTA DE MUESTRA / 模擬選票
CONSOLIDATED GENERAL ELECTION / ELECCIONES GENERALES CONSOLIDADAS / 聯合大選
CITY AND COUNTY OF SAN FRANCISCO / CIUDAD Y CONDADO DE SAN FRANCISCO / 三藩市
NOVEMBER 2, 2004 / 2 DE NOVIEMBRE DE 2004 / 11月2日

CONGRESSIONAL DISTRICT 4, SENATE DISTRICT 8, ASSEMBLY DISTRICT 12, SUPERVISORIAL DISTRICT 00, BART DISTRICT 9

INSTRUCTIONS TO VOTERS: Mark your first choice in the first column by completing the arrow pointing to your choice. If you would like to rank more than one candidate, continue to the second column. If you would like to rank more than third choice, select a different candidate in the third column. To vote for a qualified write-in candidate, write the person's name in the blank space provided.

INSTRUCCIONES PARA LOS ELECTORES: Para marcar su primera opción en la primera columna, complete la flecha que apunta hacia su selección, tal como se indica en la imagen. Para indicar una segunda opción, seleccione un candidato diferente en la segunda columna. Para votar por un candidato calificado no listado, escriba el nombre de la persona en el espacio en blanco proporcionado.

選民指南：在第一列中標記您的第一選擇。若要繼續標記其他候選人，請繼續到第二列。若要標記第三個候選人，請選擇另一個候選人。若要為合格的寫入候選人投票，請在提供的空格中輸入候選人姓名。

FIRST CHOICE PRIMERA SELECCIÓN	SECOND CHOICE SEGUNDA SELECCIÓN	THIRD CHOICE TERCERA SELECCIÓN
Vote for One Voto por Uno 1º - 1º	Vote for One - Must be different than your Voto por Uno - Debe ser diferente de 1º - 1º	ELEANOR ROOSEVELT ELEANOR ROOSEVELT 1º - 1º
CEASAR CHAVEZ CEASAR CHAVEZ Labor Organiza Organización del Trabajo	CEASAR CHAVEZ CEASAR CHAVEZ Labor Organiza Organización del Trabajo	CEASAR CHAVEZ CEASAR CHAVEZ Labor Organiza Organización del Trabajo
WALTER LUM WALTER LUM Pacifist Pacifista	WALTER LUM WALTER LUM Pacifist Pacifista	WALTER LUM WALTER LUM Pacifist Pacifista
JOHN HANCOCK JOHN HANCOCK Physician Médico	JOHN HANCOCK JOHN HANCOCK Physician Médico	JOHN HANCOCK JOHN HANCOCK Physician Médico
MARTIN LUTHER KING, JR. MARTIN LUTHER KING, JR. Minister Misionero	MARTIN LUTHER KING, JR. MARTIN LUTHER KING, JR. Minister Misionero	MARTIN LUTHER KING, JR. MARTIN LUTHER KING, JR. Minister Misionero
ANNA MAR PICTOV ADAMS ANNA MAR PICTOV ADAMS Organizadora para Derechos Indigenas Organizadora para Derechos Indigenas	ANNA MAR PICTOV ADAMS ANNA MAR PICTOV ADAMS Organizadora para Derechos Indigenas Organizadora para Derechos Indigenas	ANNA MAR PICTOV ADAMS ANNA MAR PICTOV ADAMS Organizadora para Derechos Indigenas Organizadora para Derechos Indigenas

**State of Maine Sample Ballot
Democratic Primary Election, June 12, 2018**

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- In the 1st column for your 1st choice candidate.
- In the 2nd column for your 2nd choice candidate, and so on.

Continue until you have ranked as many or as few candidates as you like.

Fill in no more than one oval for each candidate or column.

To rank a write-in candidate, write the person's name in the write-in space and fill in the oval for the ranking of your choice.

Governor	1st Choice	2nd Choice	3rd Choice	4th Choice	5th Choice	6th Choice	7th Choice	8th Choice
Cole, Adam Roland Senate	○	○	○	○	○	○	○	○
Dion, Donna J. Senate	○	○	○	○	○	○	○	○
Dion, Mark N. Portland	○	○	○	○	○	○	○	○
Eves, Mark W. Senate	○	○	○	○	○	○	○	○
Mills, Janet T. Farmington	○	○	○	○	○	○	○	○
Peterson, Diane Marie Portland	○	○	○	○	○	○	○	○
Sweet, Elizabeth A. Hancock	○	○	○	○	○	○	○	○
Write-in	○	○	○	○	○	○	○	○

SOURCE: Maine Secretary of State Office

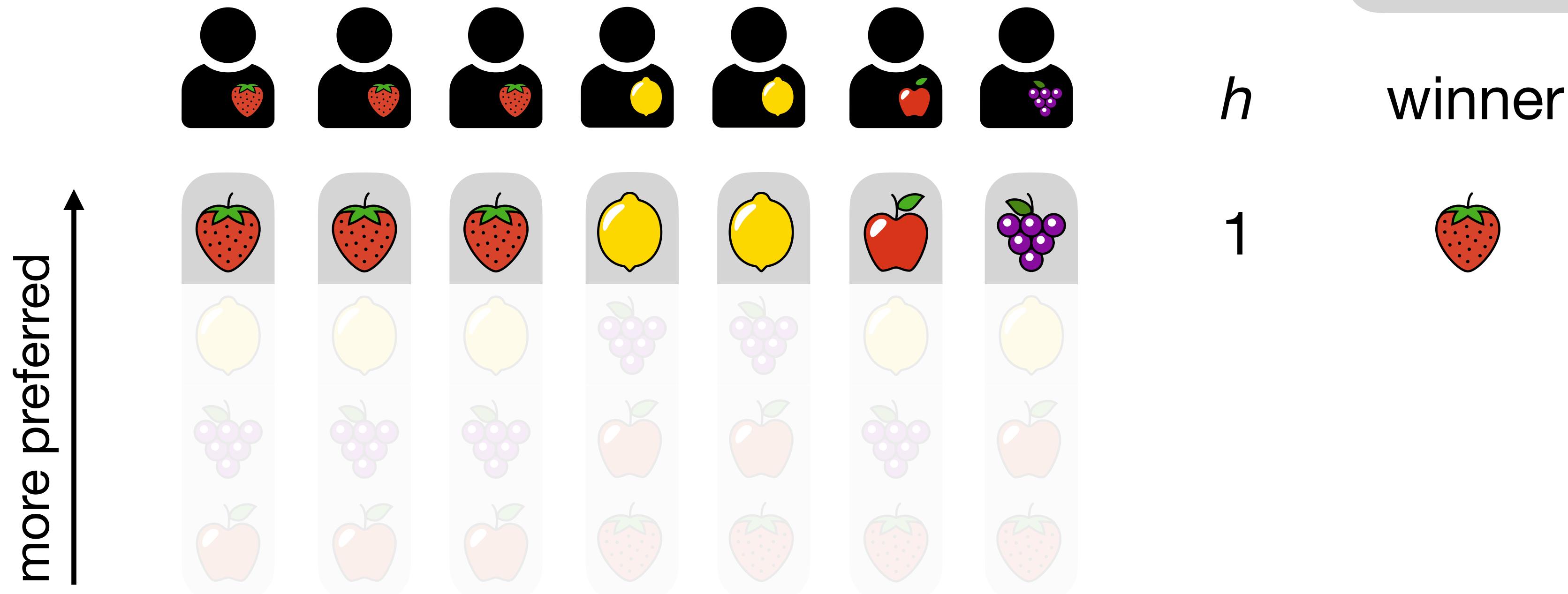
Council Member Miembro del Consejo	Rank up to 5 choices Clasifique hasta 5 opciones Marque no más de un óvalo en cada columna				
	1st Choice 1er Choice	2nd Choice 2da Choice	3rd Choice 3ra Choice	4th Choice 4ta Choice	5th Choice 5ta Choice
John E. Sanchez Community First	A ○	○	○	○	○
Oswald Feliz People United	B ○	○	○	○	○
Elisa Crespo Jobs & Justice	C ○	○	○	○	○
Kenny G. Agosto Empower People	D ○	○	○	○	○
Ischia J. Bravo We Matter	E ○	○	○	○	○
Ariel Rivera-Diaz Second Choice	F ○	○	○	○	○
Bernadette Ferrara Fifteen Forward	G ○	○	○	○	○
Latchmi Devi Gopal Go For The Bronx	H ○	○	○	○	○
Jose A. Padilla Jr. Safe & Stable	I ○	○	○	○	○
Altagracia Soldevilla People First	J ○	○	○	○	○
Write-in candidate por escrito	○	○	○	○	○

How much does ballot length matter?

Ballot length can change the winner

fix the profile, truncate all rankings

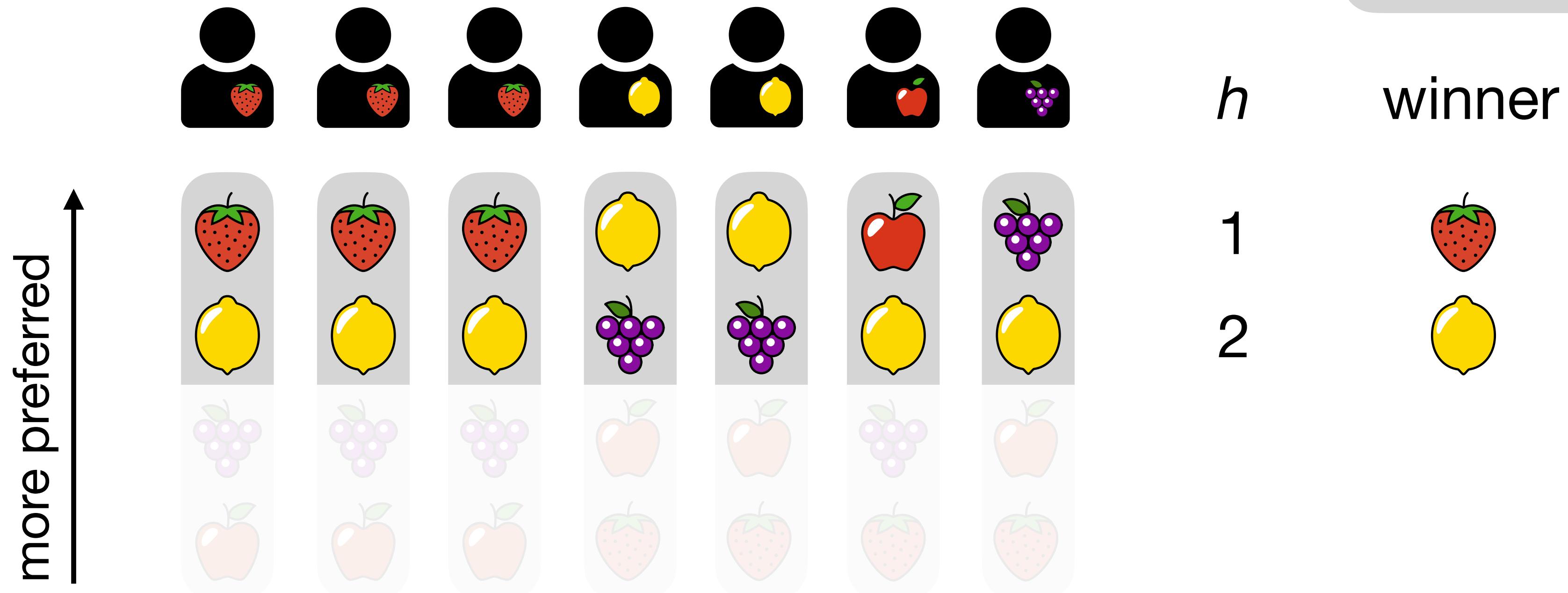
h = ballot length
 k = # candidates



Ballot length can change the winner

fix the profile, truncate all rankings

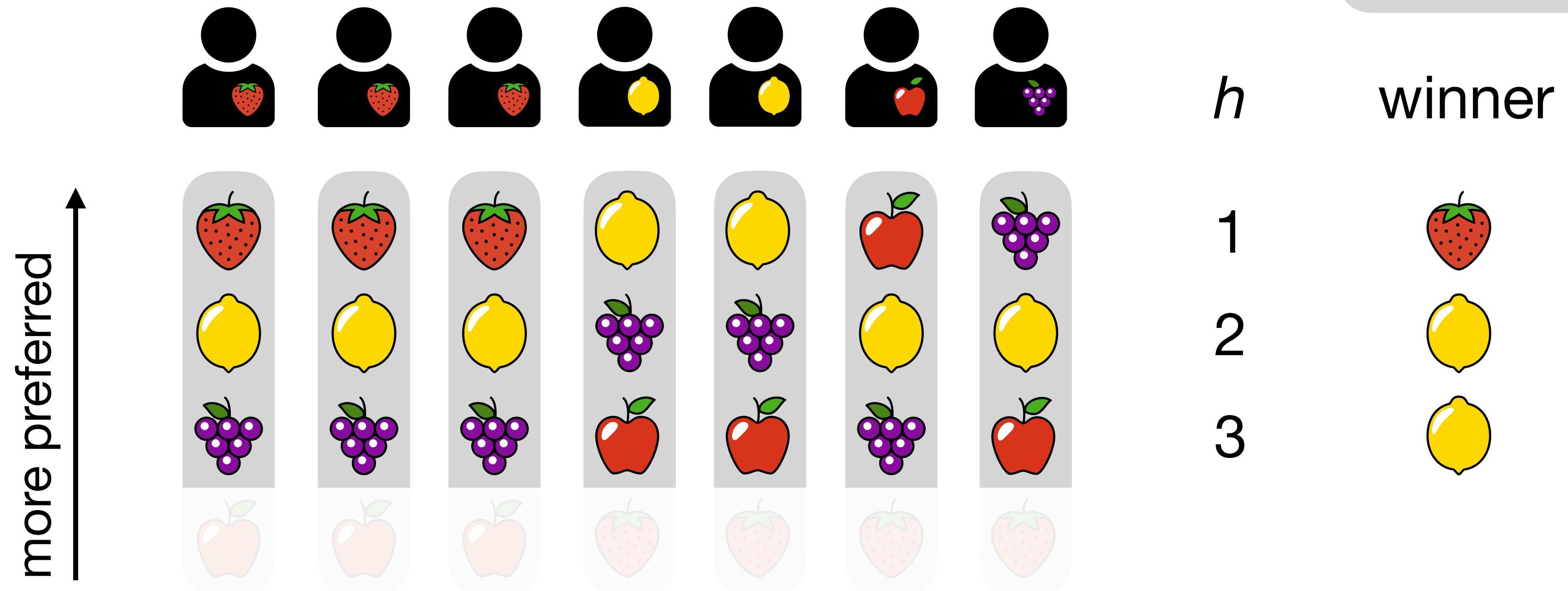
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Ballot length can change the winner

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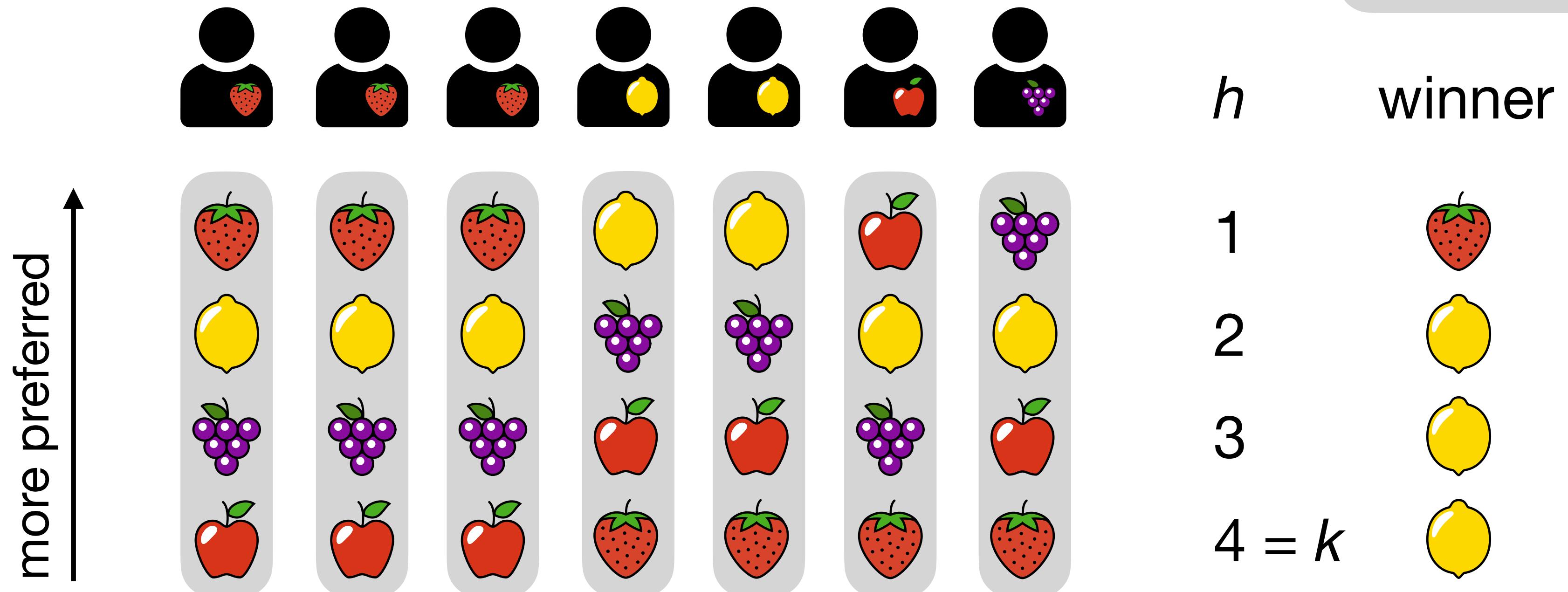
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Ballot length can change the winner

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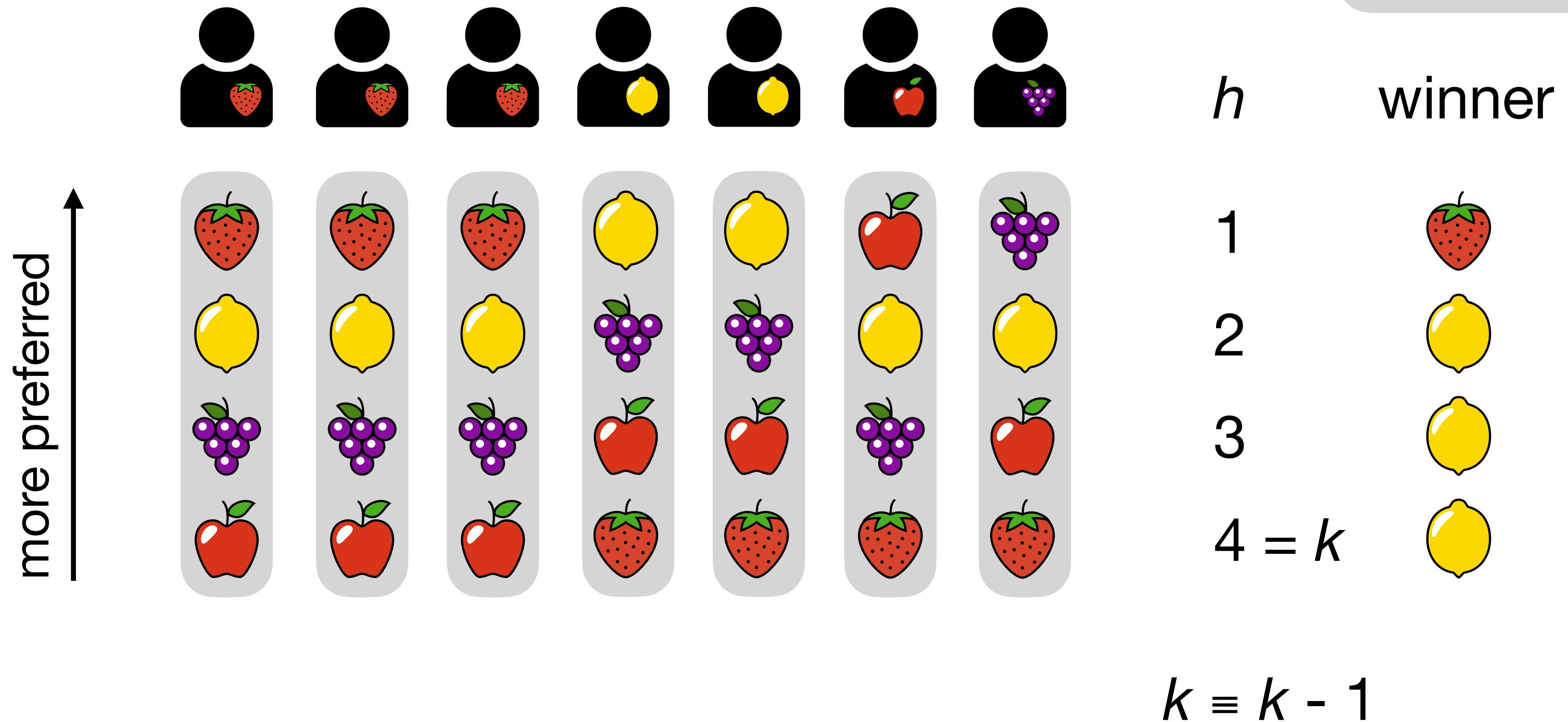
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Ballot length can change the winner

fix the profile, truncate all rankings

h = ballot length
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Prior work

Prior work

voluntary truncation

[Saari & Newenhizen, *Public Choice* 1988]

[Baumeister et al, *AAMAS* '12]

[Narodytska & Walsh, *ECAI* '14]

[Menon & Larson, *JAAMAS* 2017]

Prior work

voluntary truncation

[Saari & Newenhizen, *Public Choice* 1988]

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forced truncation (i.e., ballot length)

[Ayadi et al., *AAMAS* '19]

**The prevalence and consequences of ballot truncation
in ranked-choice elections**

D. Marc Kilgour¹ · Jean-Charles Grégoire² · Angèle M. Foley¹ 

Prior work

voluntary truncation

[Saari & Newenhizen, *Public Choice* 1988]

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**The prevalence and consequences of ballot truncation
in ranked-choice elections**

D. Marc Kilgour¹ · Jean-Charles Grégoire² · Angèle M. Foley¹ 

“A natural question [...] is whether the outcome of the election stays the same as the extent of truncation increases from 0 (complete ballots) to $k - 1$. If not, how many different winners are possible?”

Prior work

voluntary truncation

[Saari & Newenhizen, *Public Choice* 1988]

[Baumeister et al, *AAMAS* '12]

[Narodytska & Walsh, *ECAI* '14]

[Menon & Larson, *JAAMAS* 2017]

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[Ayadi et al., *AAMAS* '19]

Public Choice (2020) 184:197–218

**The prevalence and consequences of ballot truncation
in ranked-choice elections**

D. Marc Kilgour¹ · Jean-Charles Grégoire² · Angèle M. Foley¹ 

“A natural question [...] is whether the outcome of the election stays the same as the extent of truncation increases from 0 (complete ballots) to $k - 1$. If not, how many different winners are possible?”

“In thousands of simulations involving $k = 4, 5$, and 6 candidates, we found instances of up to $k - 2$ different winners.”

A $k-1$ winner construction for $k = 4$

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<i>voter count</i>	2	5	6	6	3	2
	<div style="border: 1px solid black; padding: 2px; display: inline-block;">A D C</div>	<div style="border: 1px solid black; padding: 2px; display: inline-block;">A</div>	<div style="border: 1px solid black; padding: 2px; display: inline-block;">B D A</div>	<div style="border: 1px solid black; padding: 2px; display: inline-block;">C</div>	<div style="border: 1px solid black; padding: 2px; display: inline-block;">D B</div>	<div style="border: 1px solid black; padding: 2px; display: inline-block;">D C</div>

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we generalize this construction to any k

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Theorem 2

For every $k > 3$, there are consequential-tie-free profiles with $2k^2 - 2k$ voters and $k - 1$ truncation winners.

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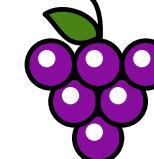
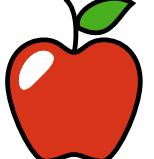
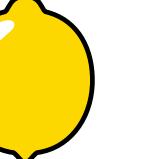
Theorem 1

For every $k > 3$, a consequential-tie-free profile needs at least $2k^2 - 2k$ voters to have $k - 1$ truncation winners.

Actually, it's even worse....

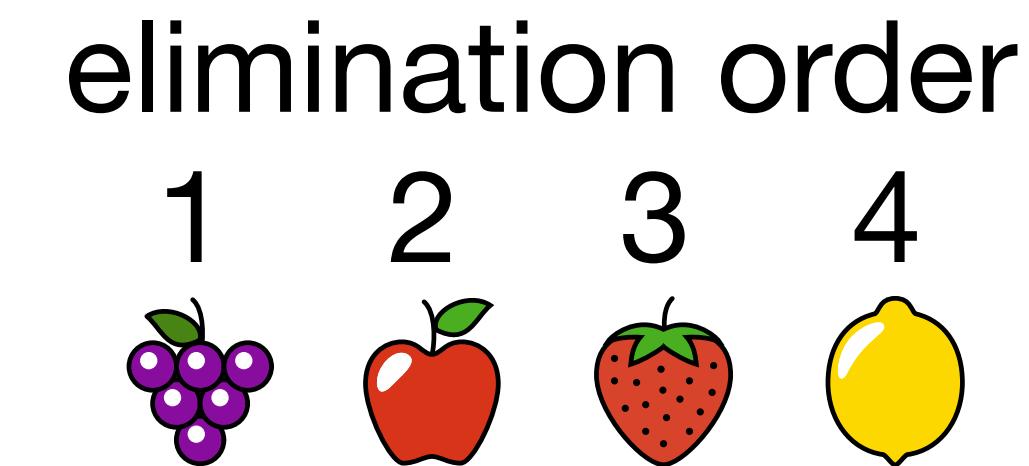
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elimination order			
1	2	3	4
			

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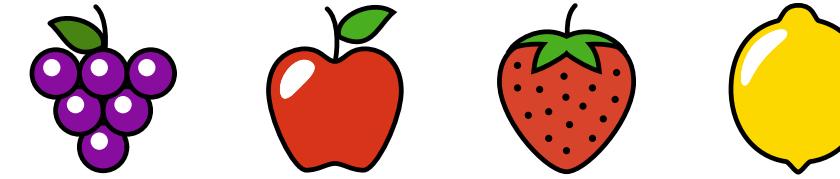
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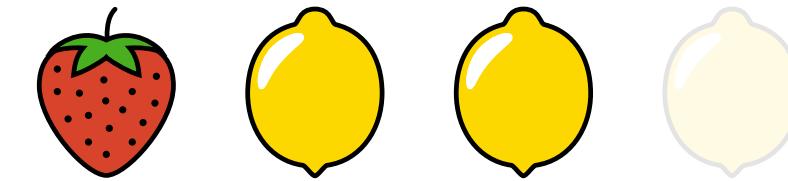
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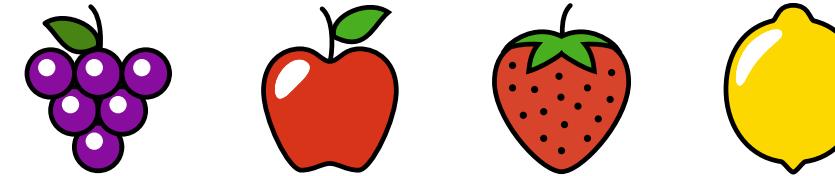
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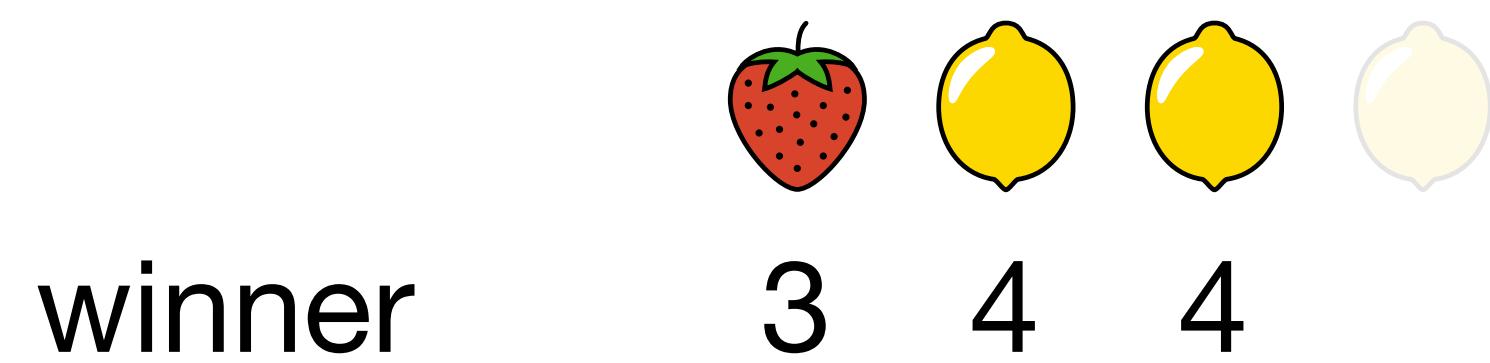
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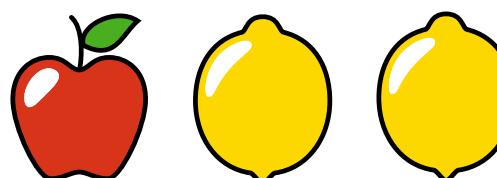
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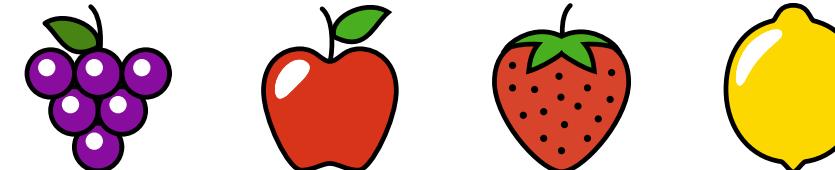
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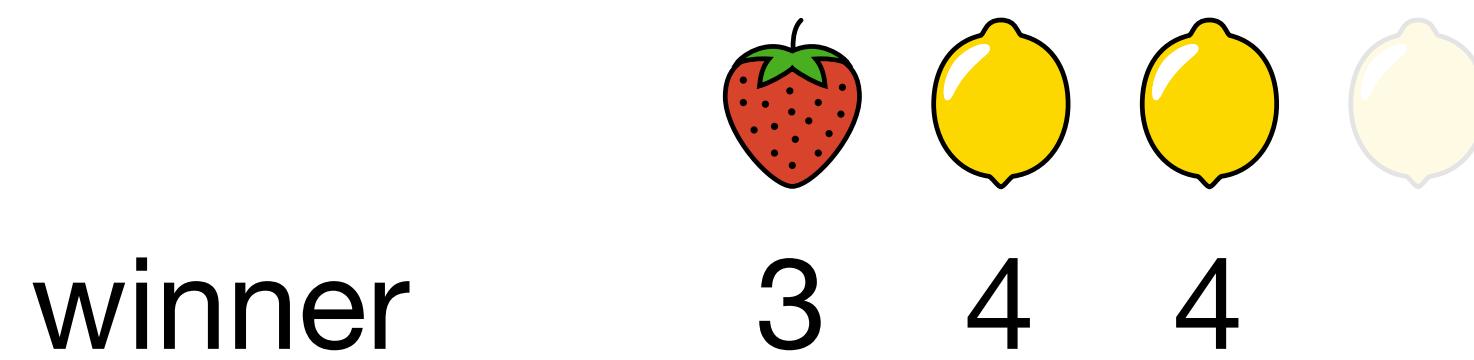
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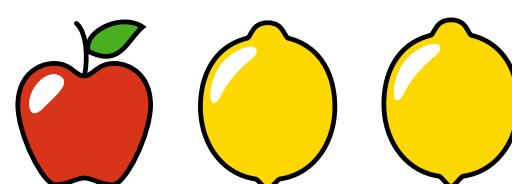
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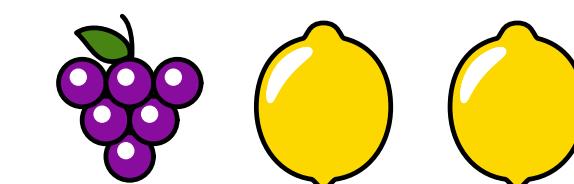
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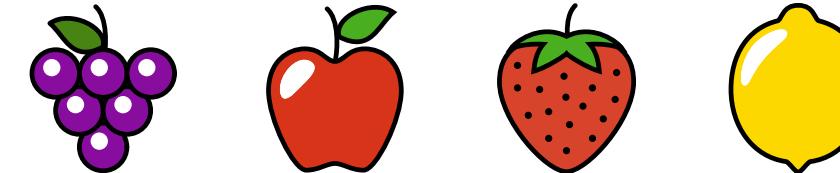
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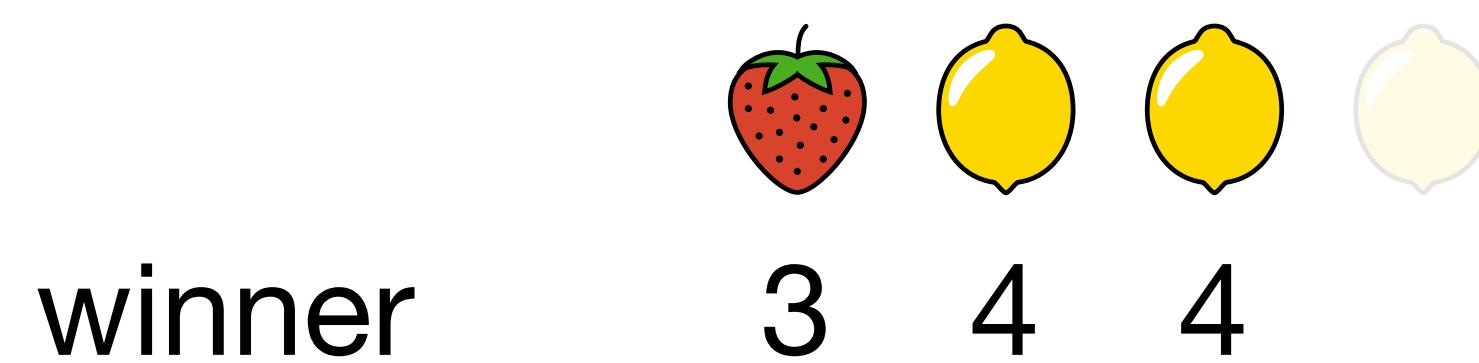
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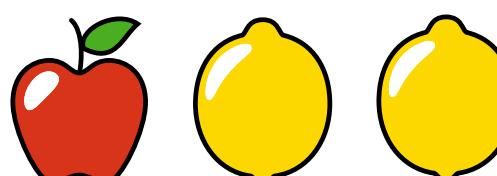
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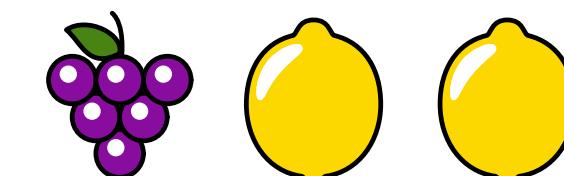
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Theorem 2

For every $k > 3$ and **every feasible truncation winner sequence**, there is a consequential-tie-free profile with $2k^2 - 2k$ voters achieving that sequence.

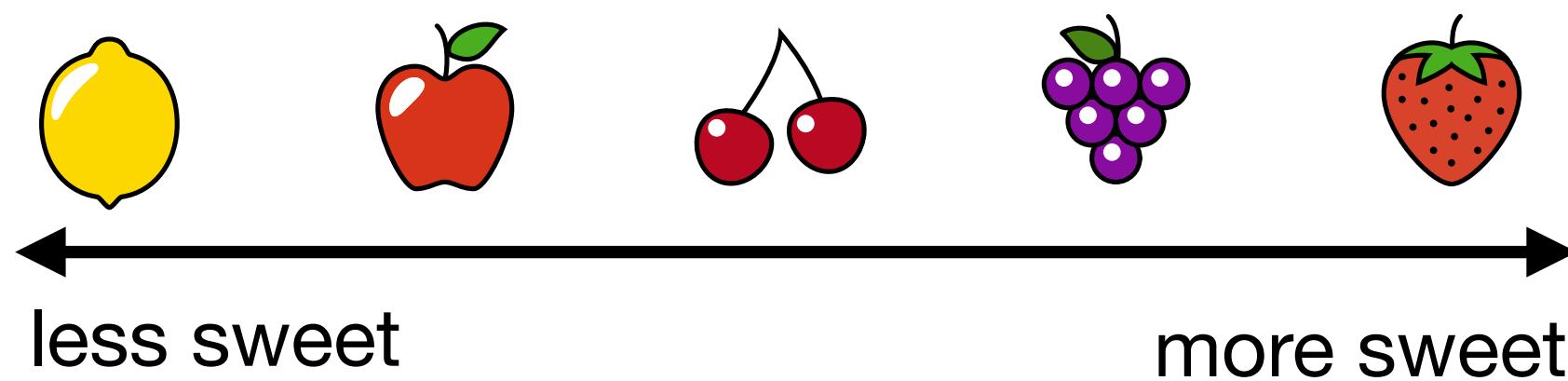
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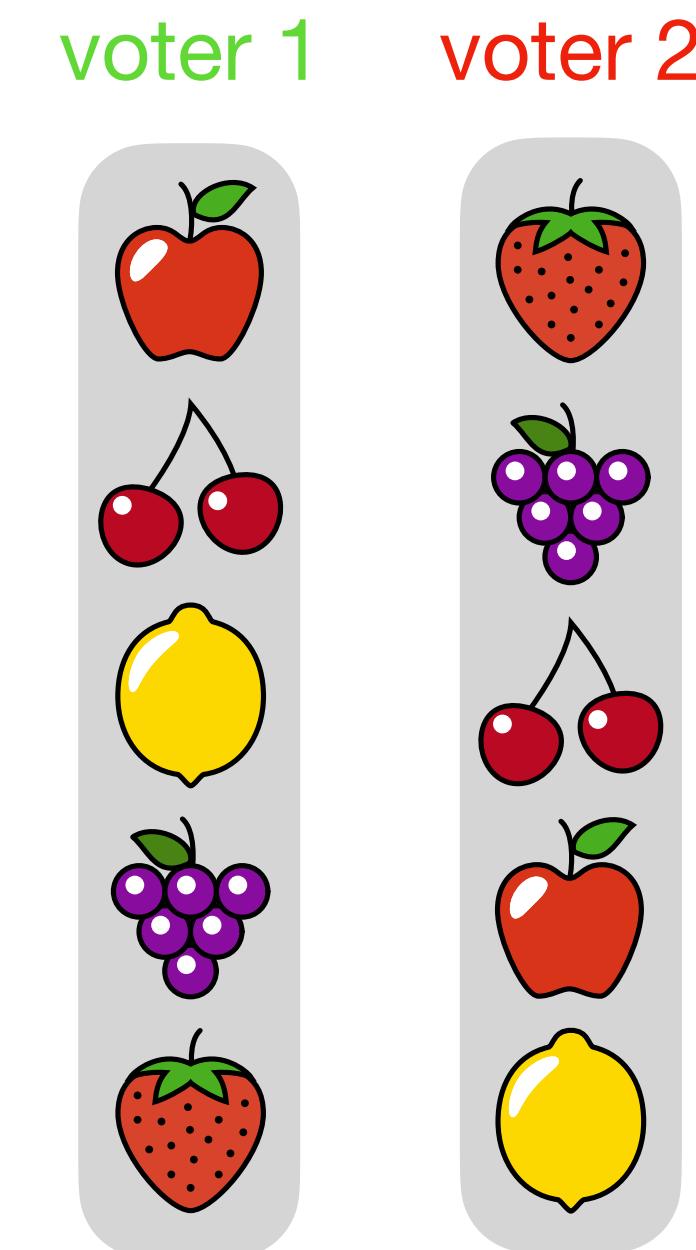
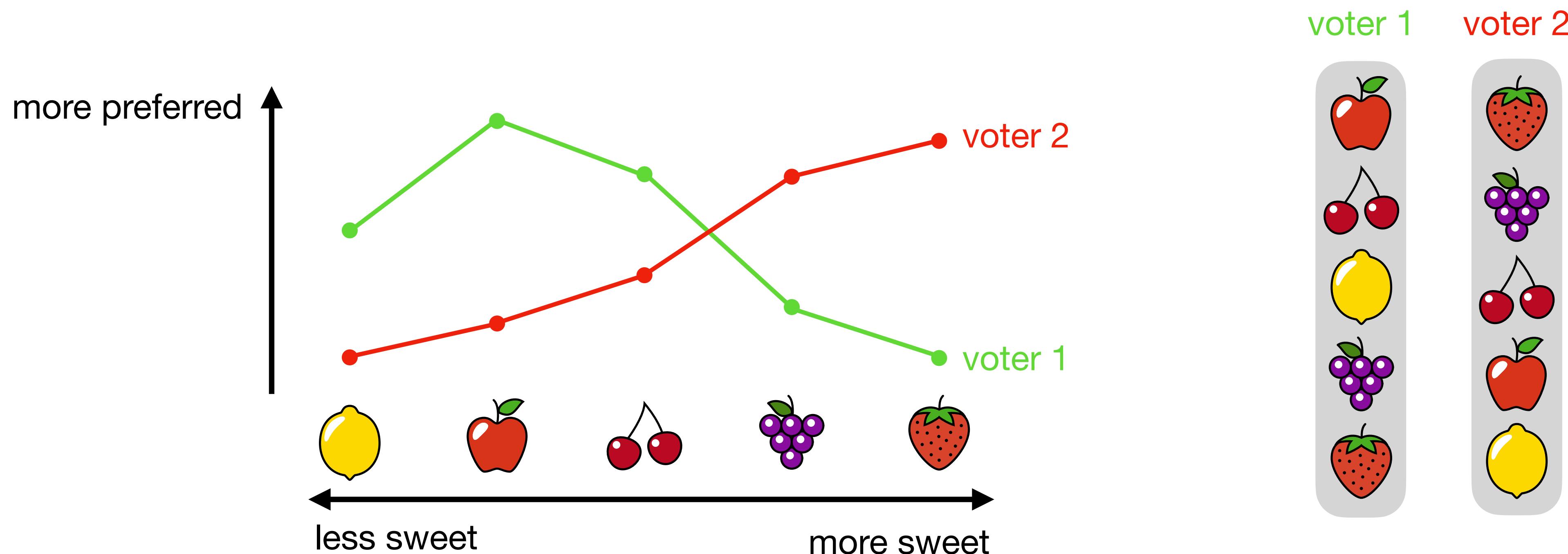
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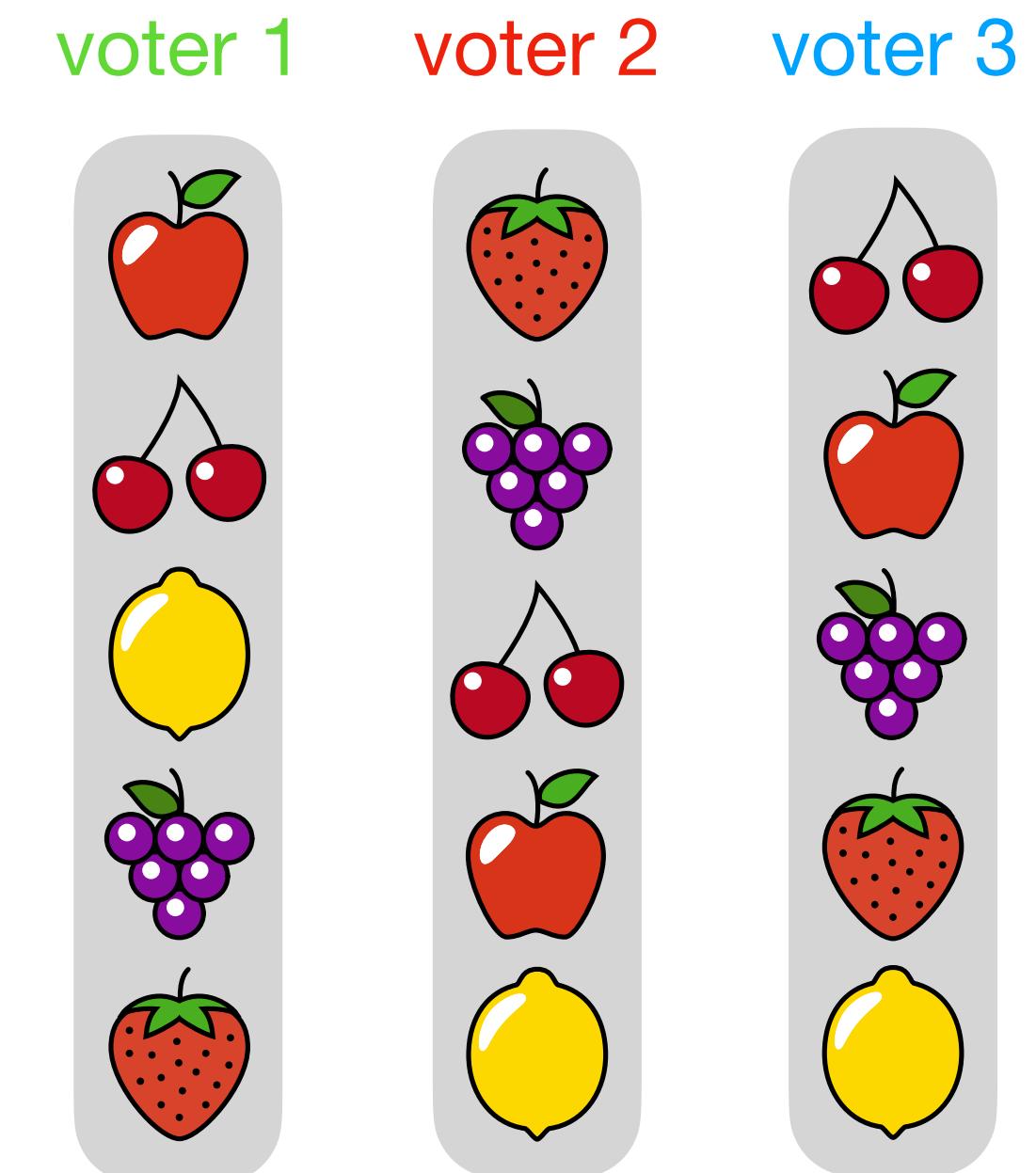
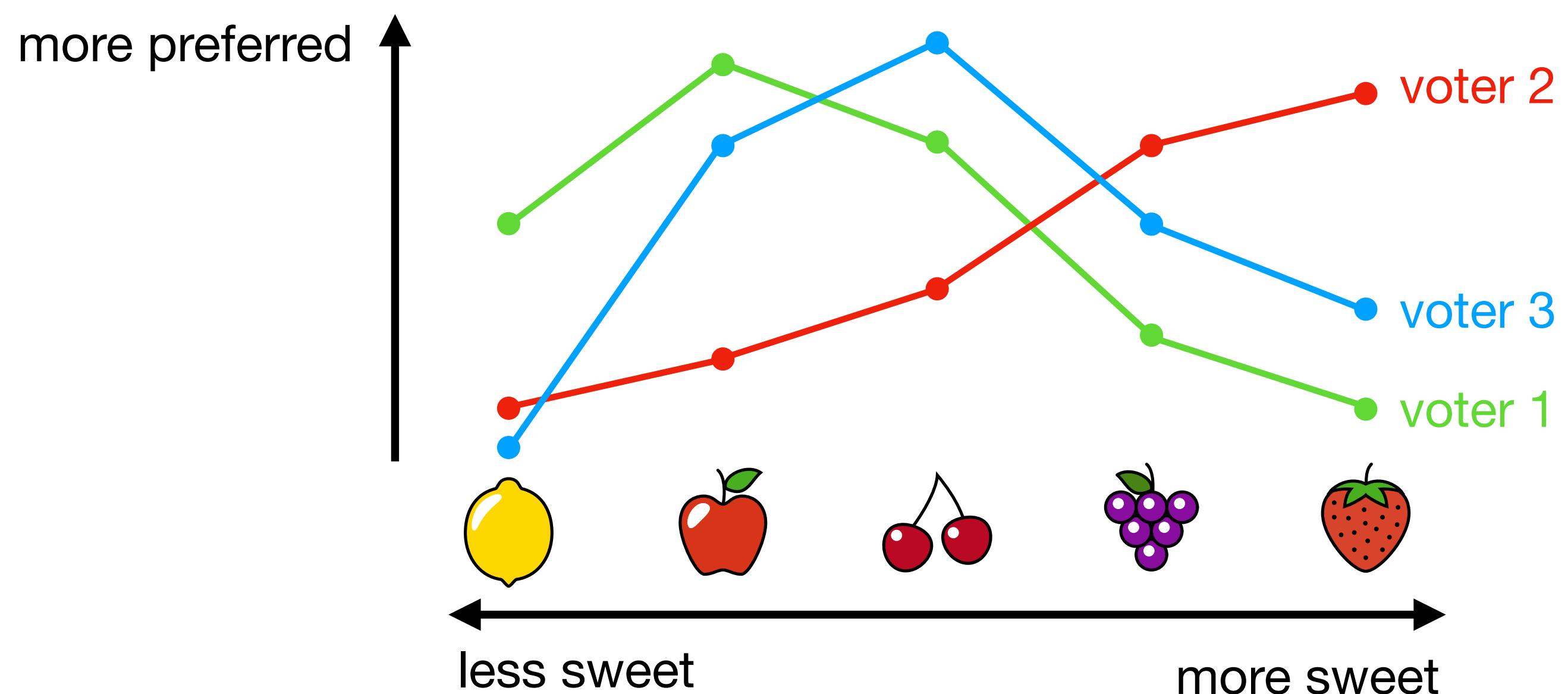
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For every $k = c(c + 1)/2$, where $c \geq 3$, there is a single-peaked profile with $3k$ voters and c truncation winners.

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Open question: more than $\Theta(\sqrt{k})$ truncation winners with single-peaked profiles?

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168 elections (PrefLib.org)

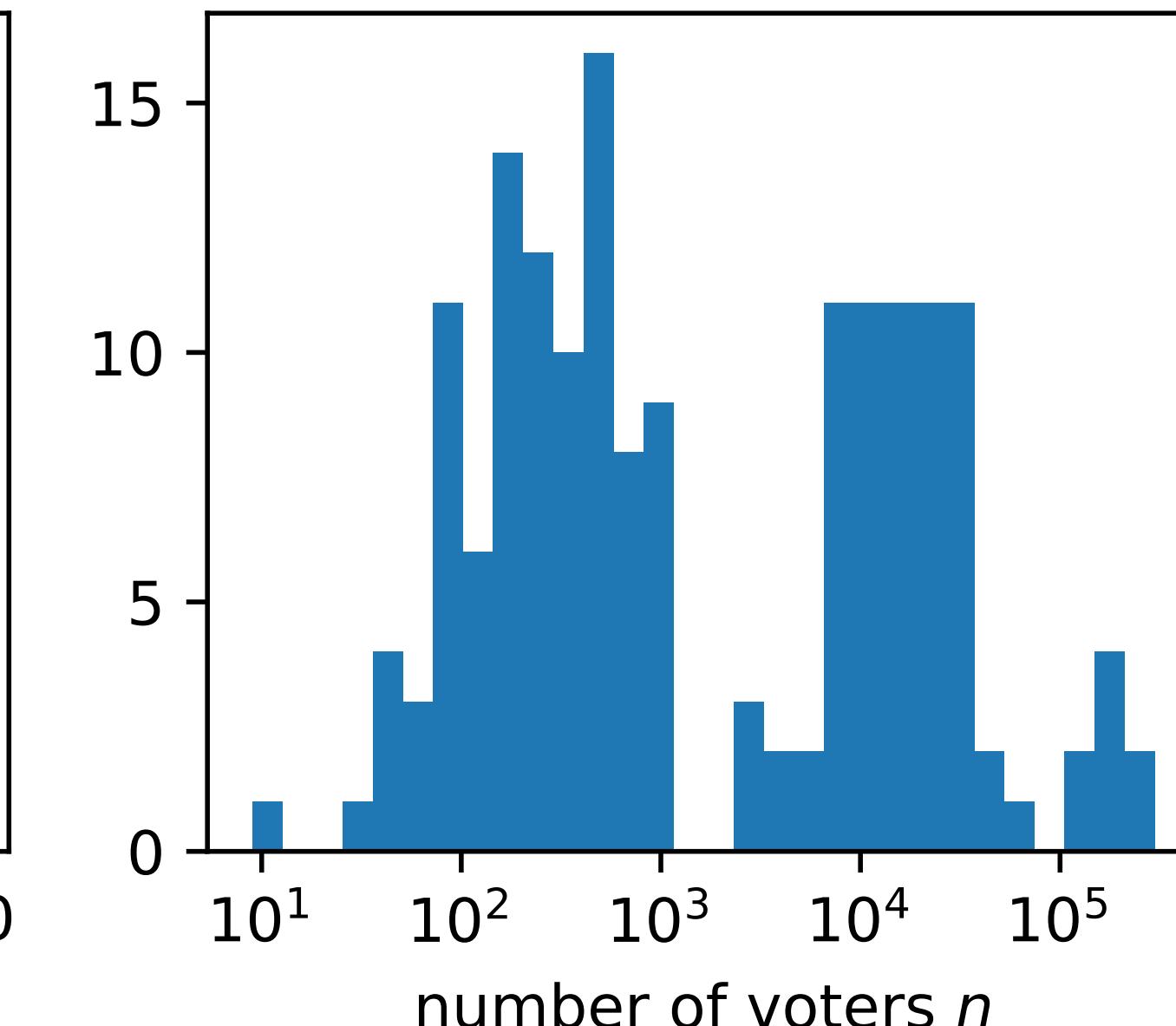
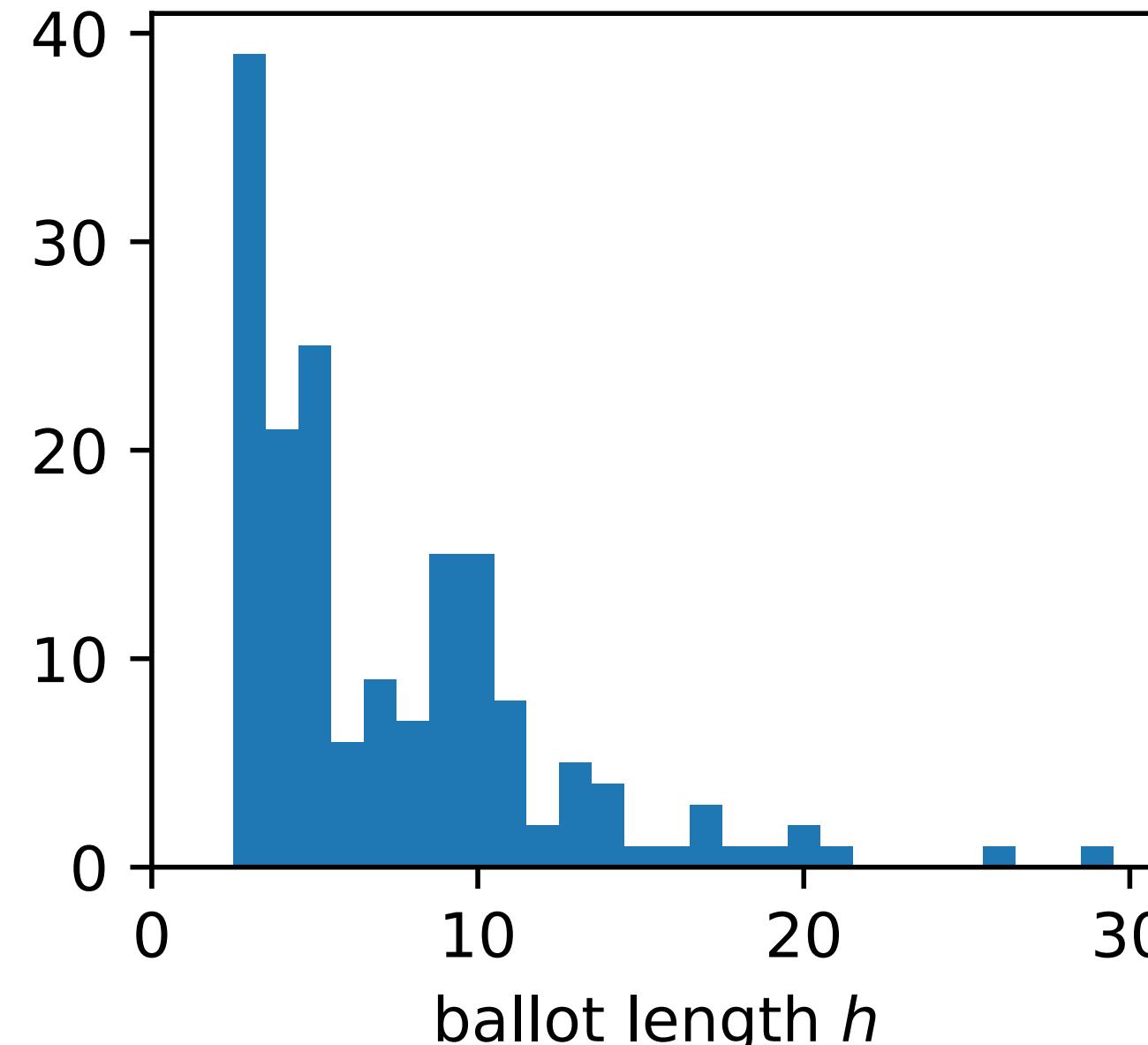
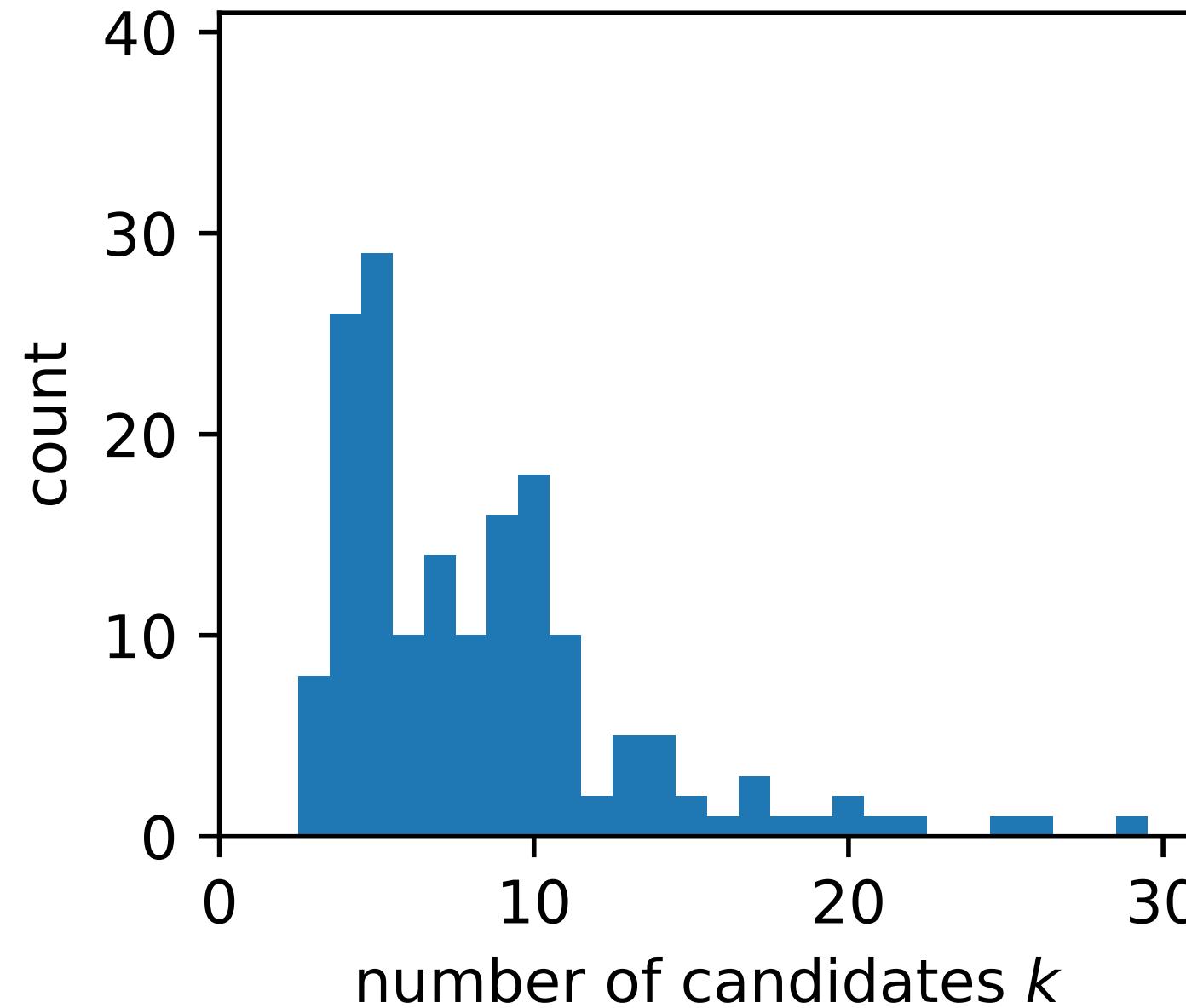
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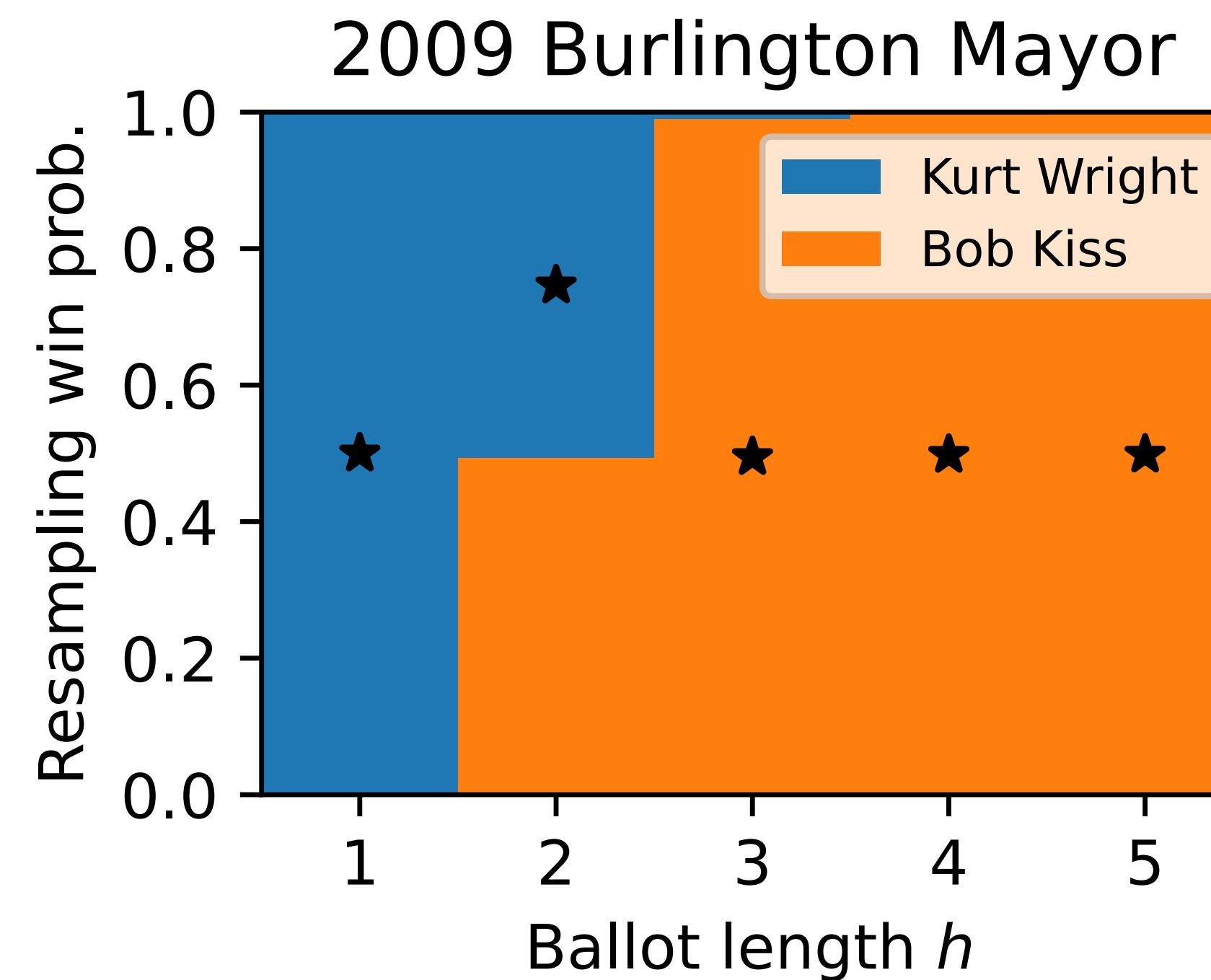
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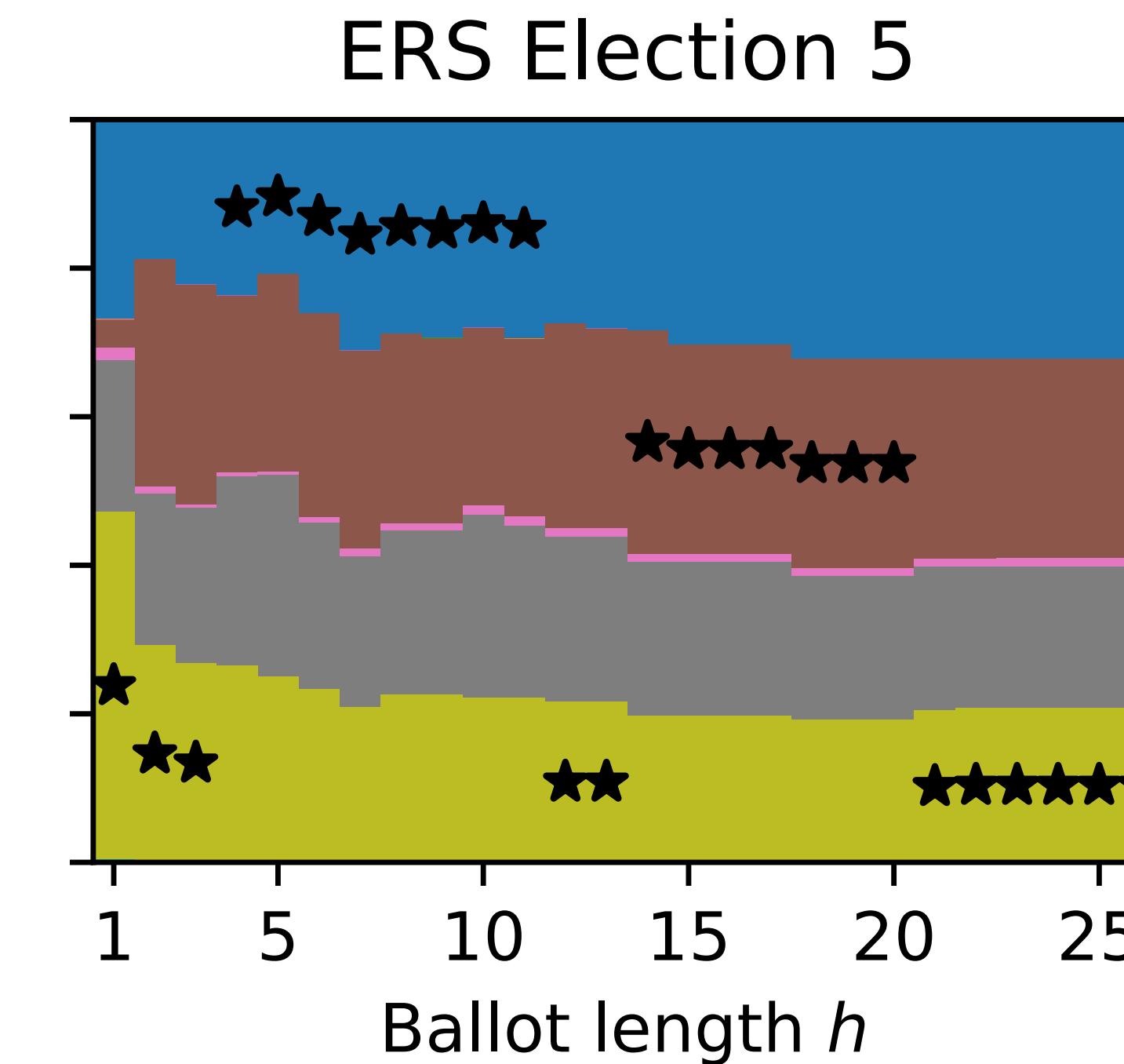
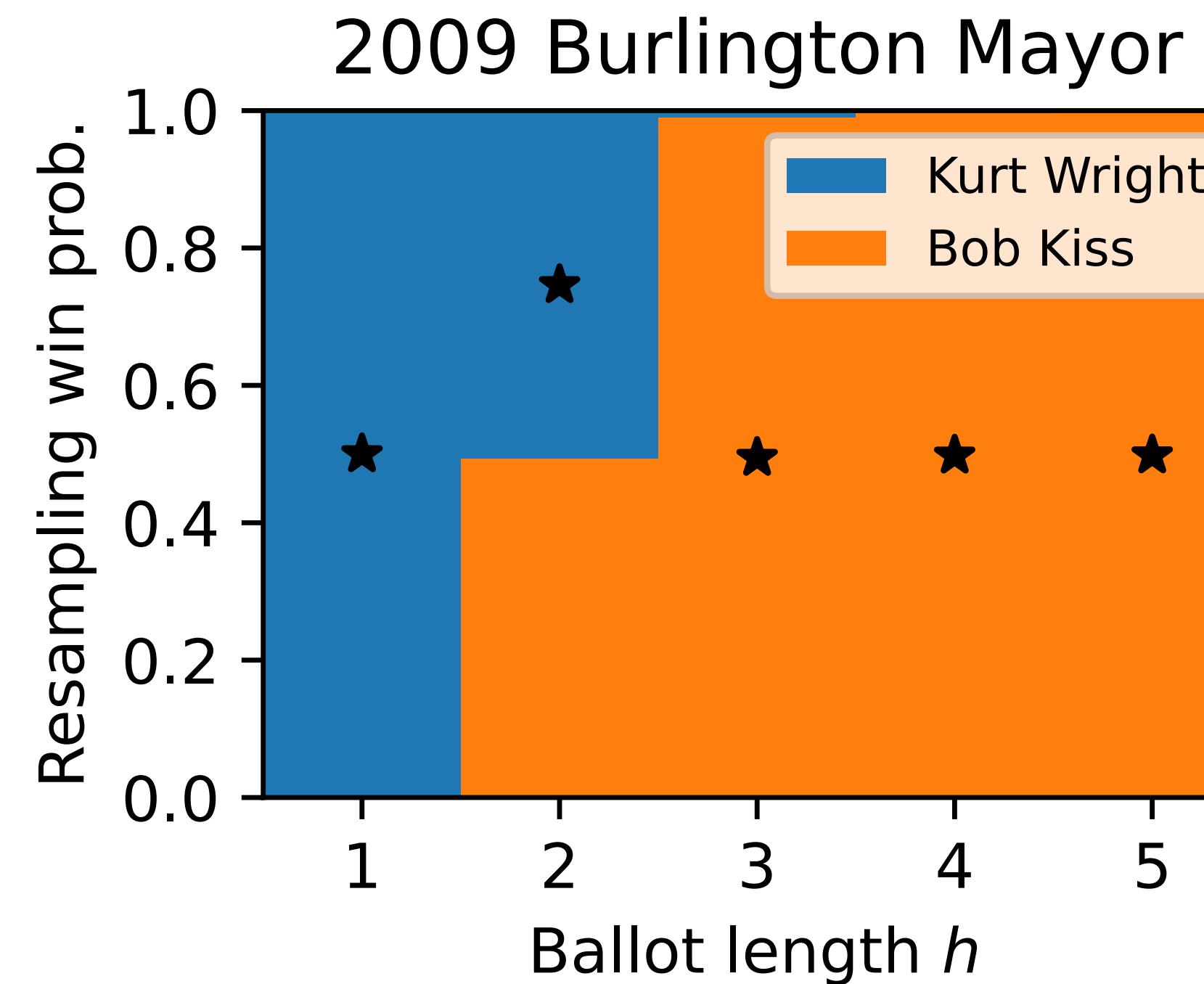
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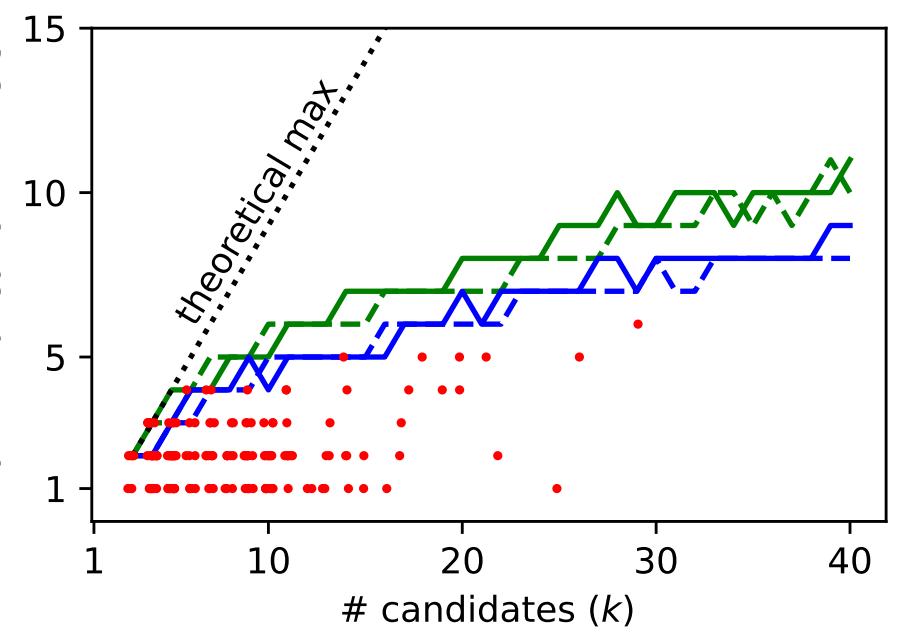
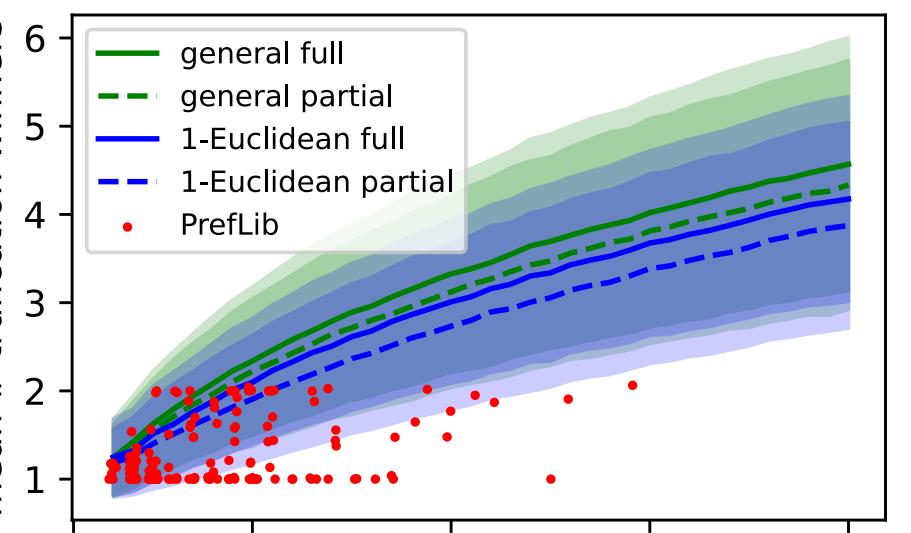
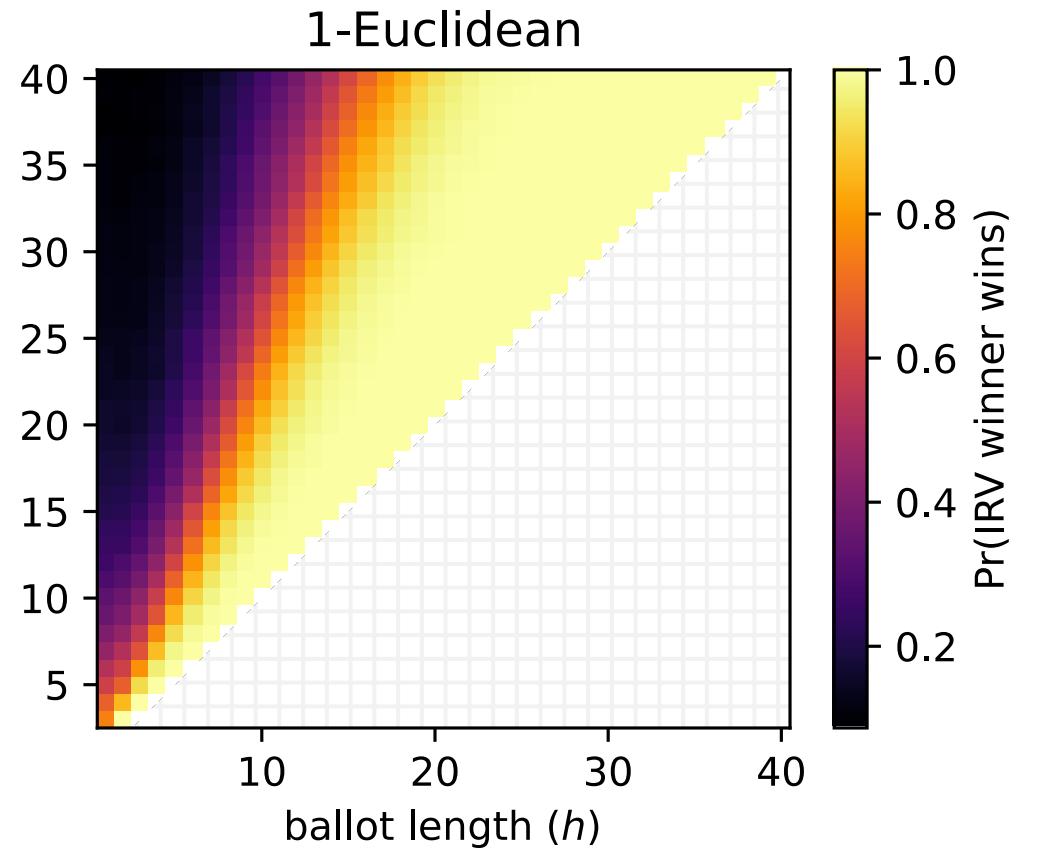
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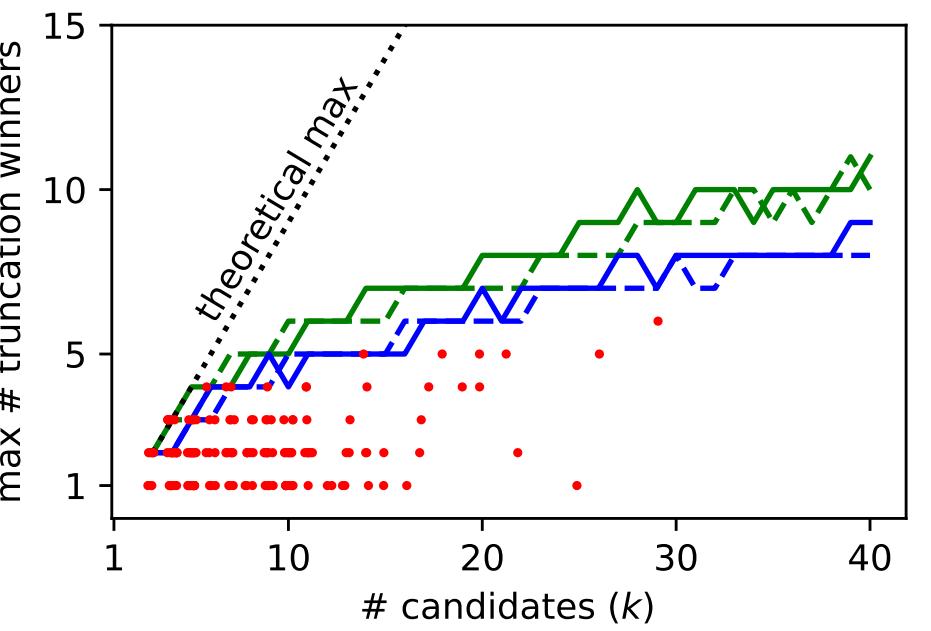
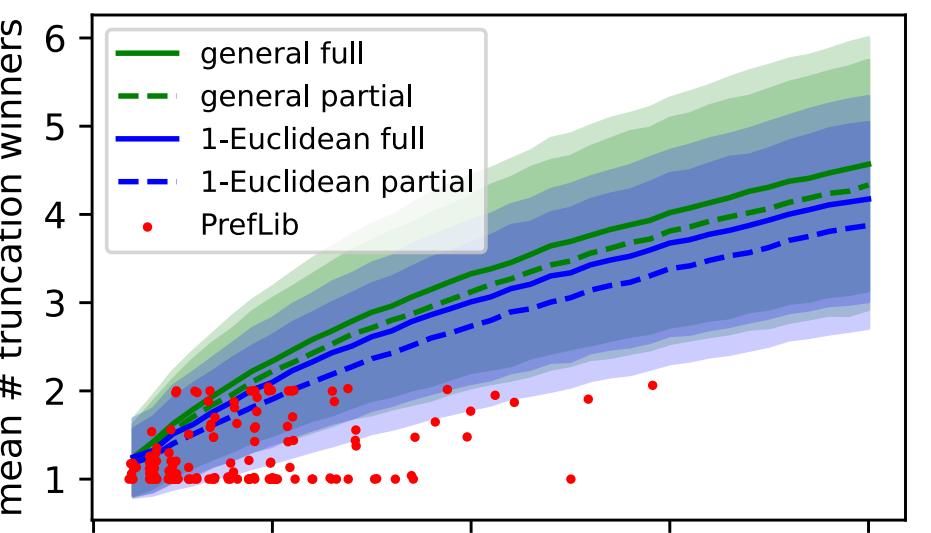
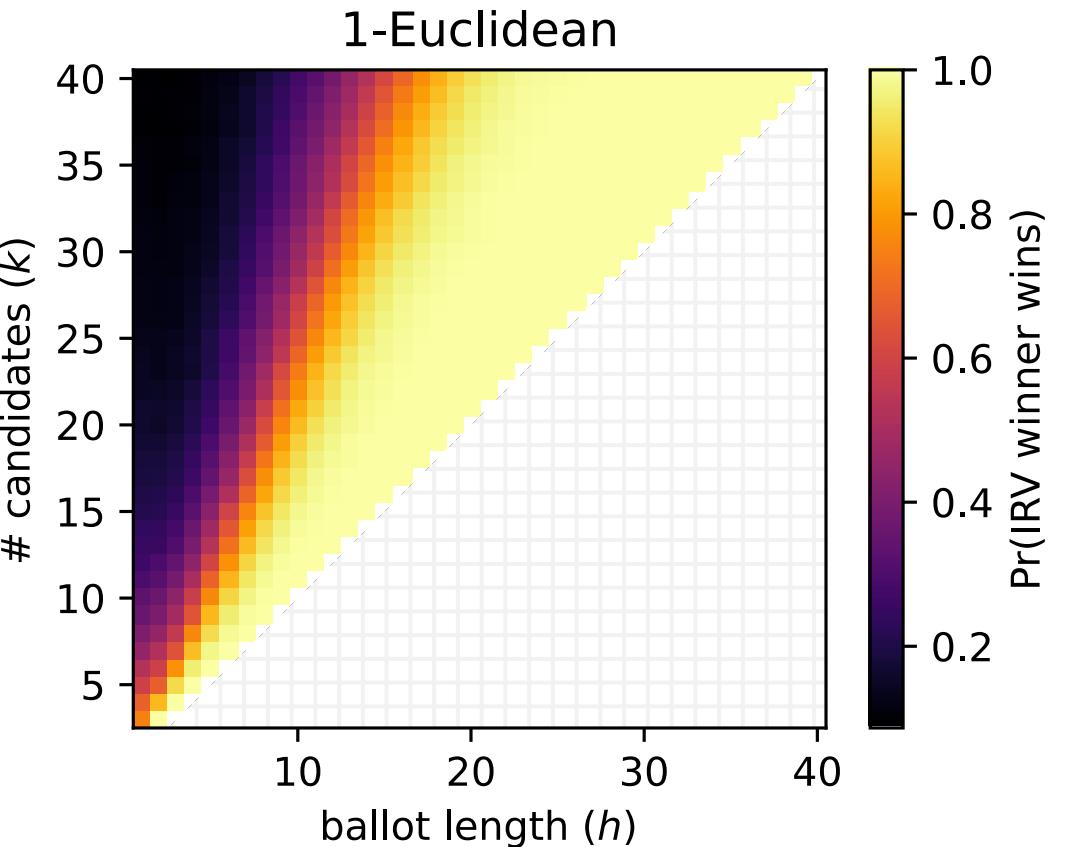


More things in the paper



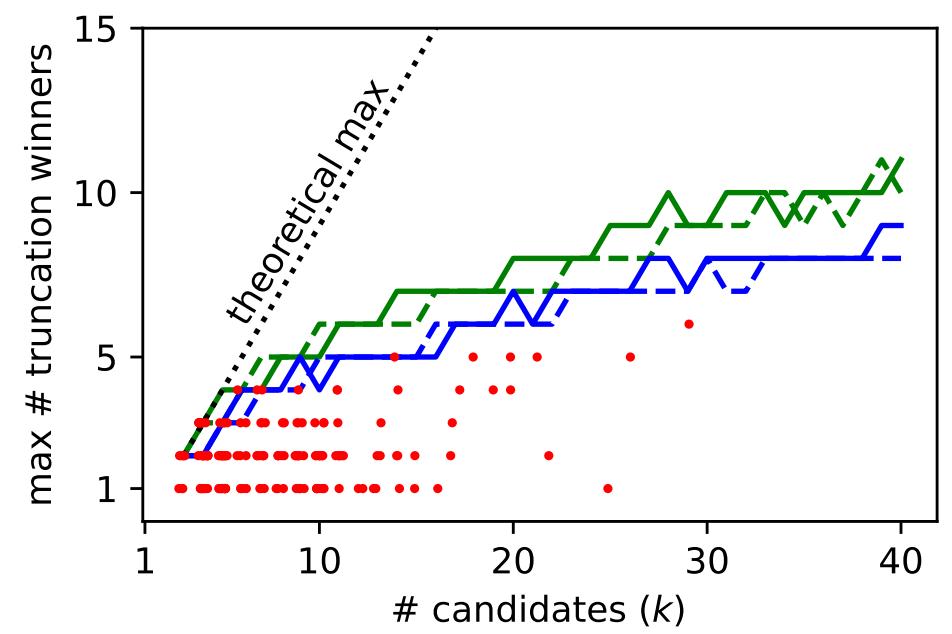
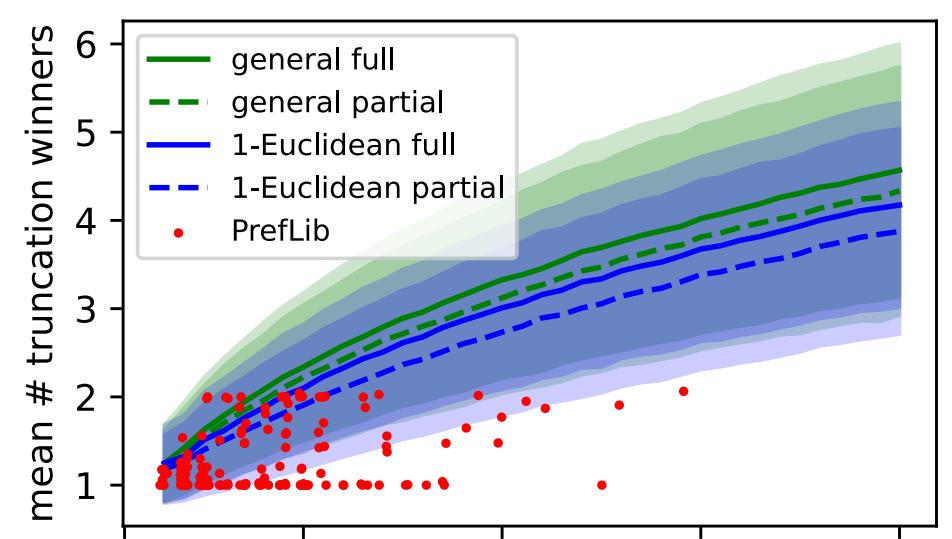
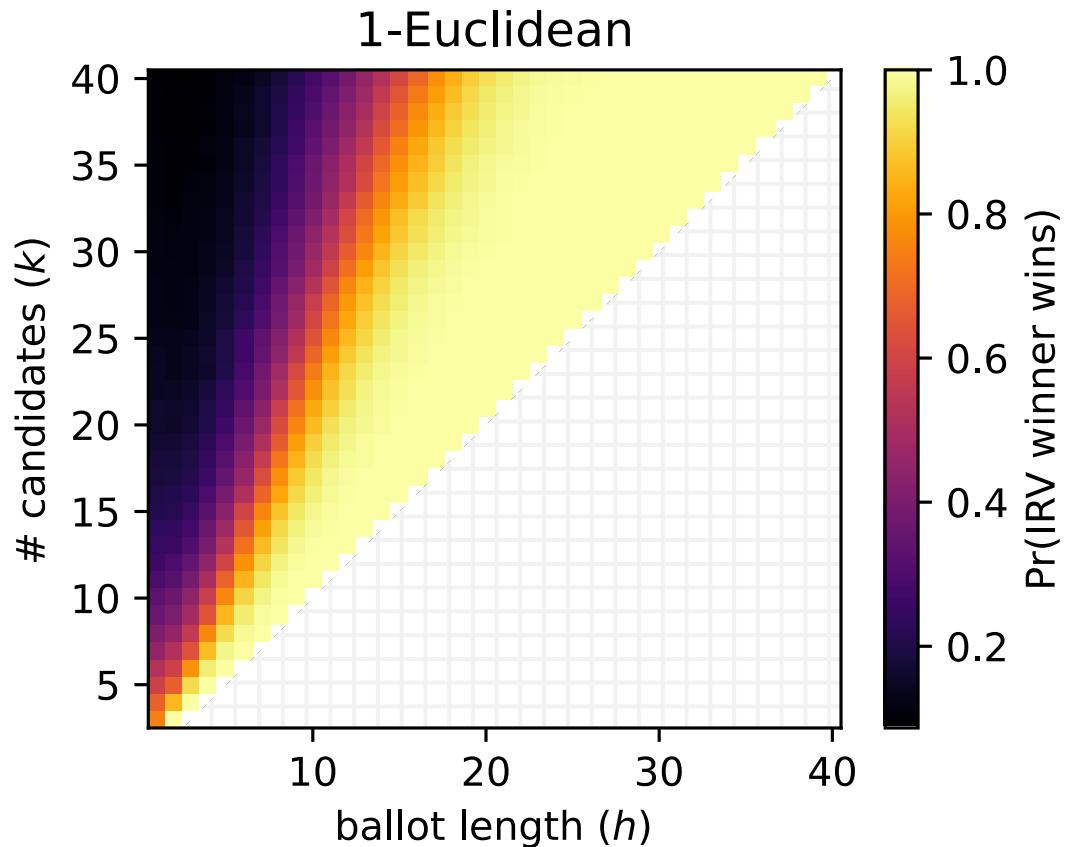
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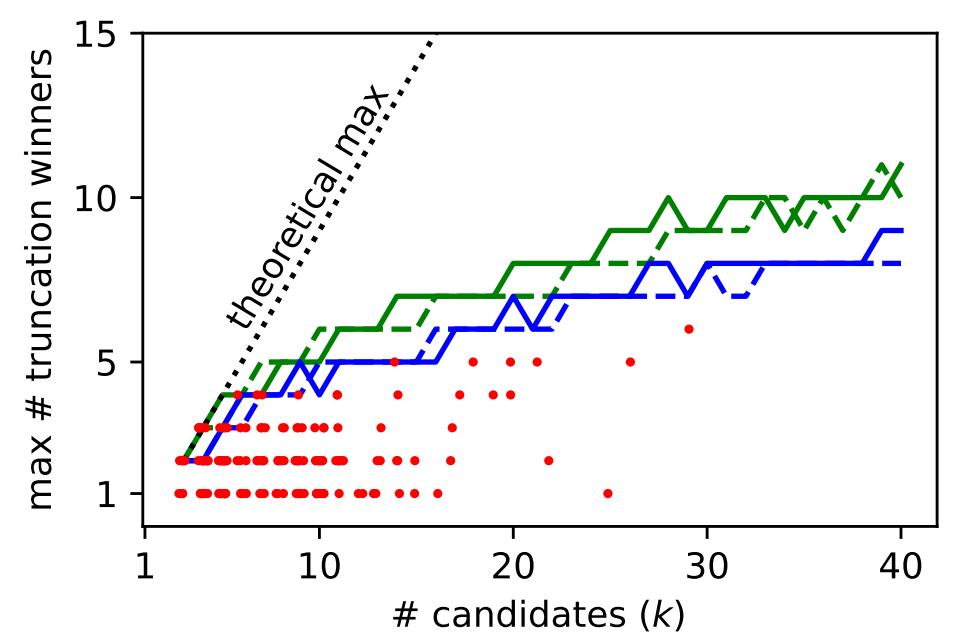
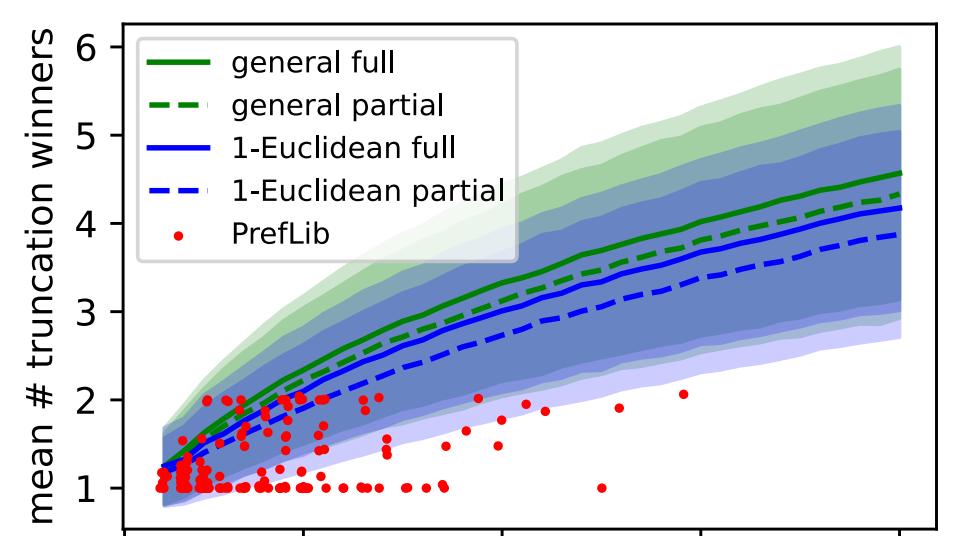
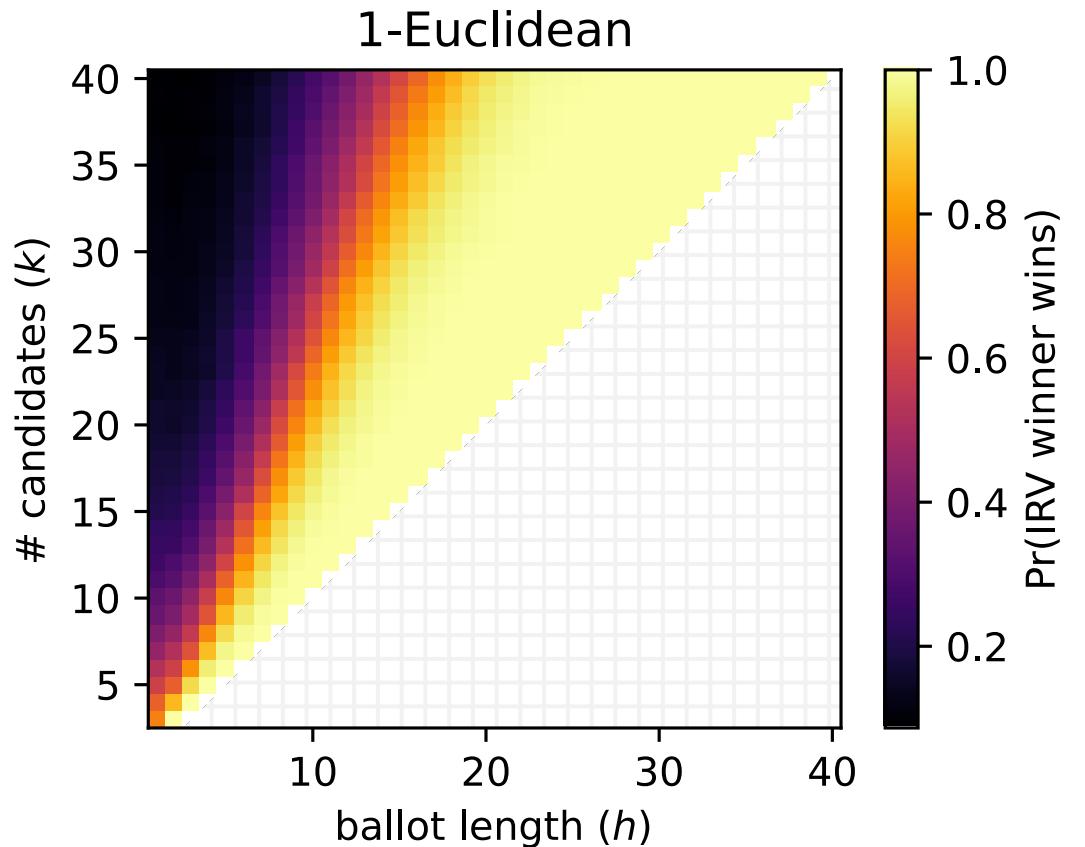
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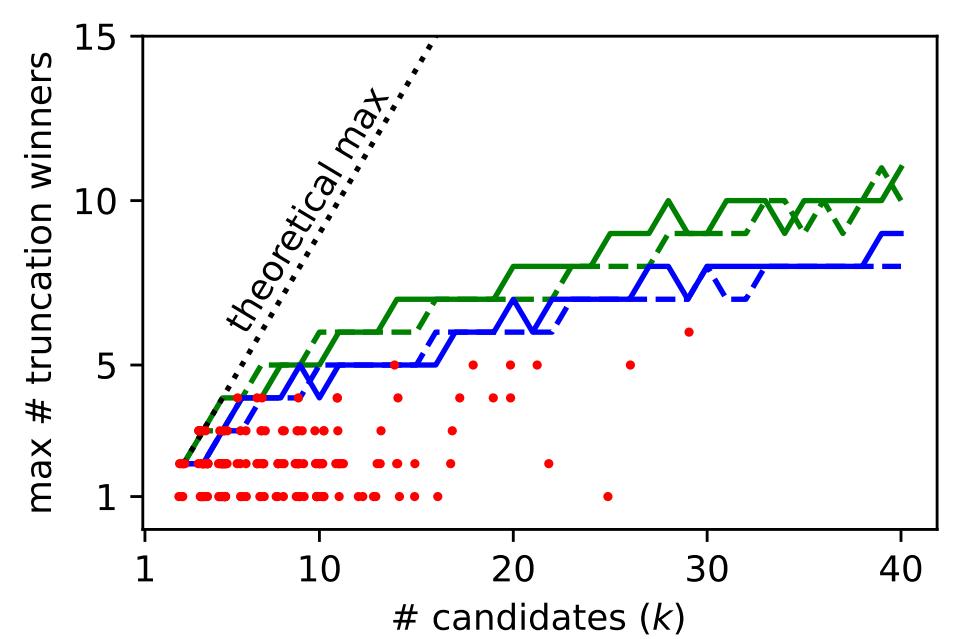
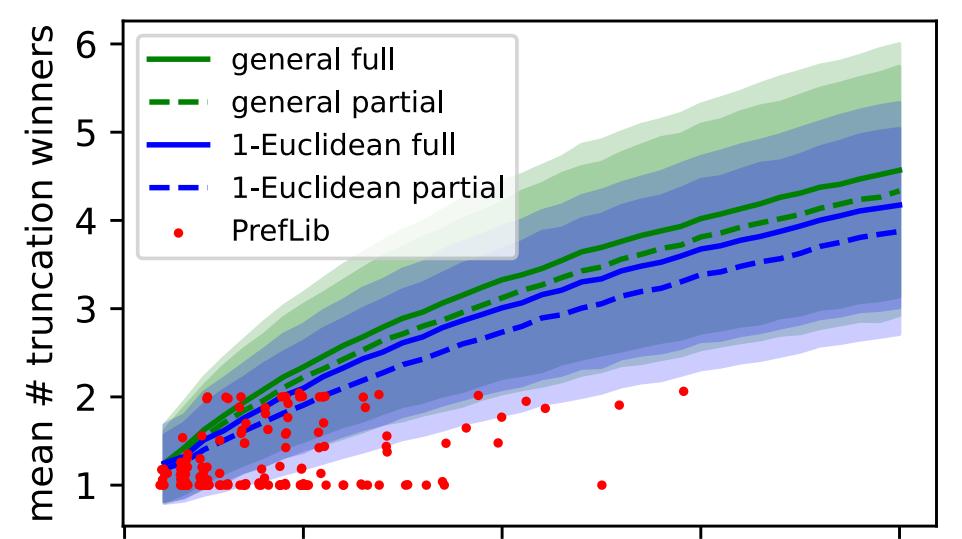
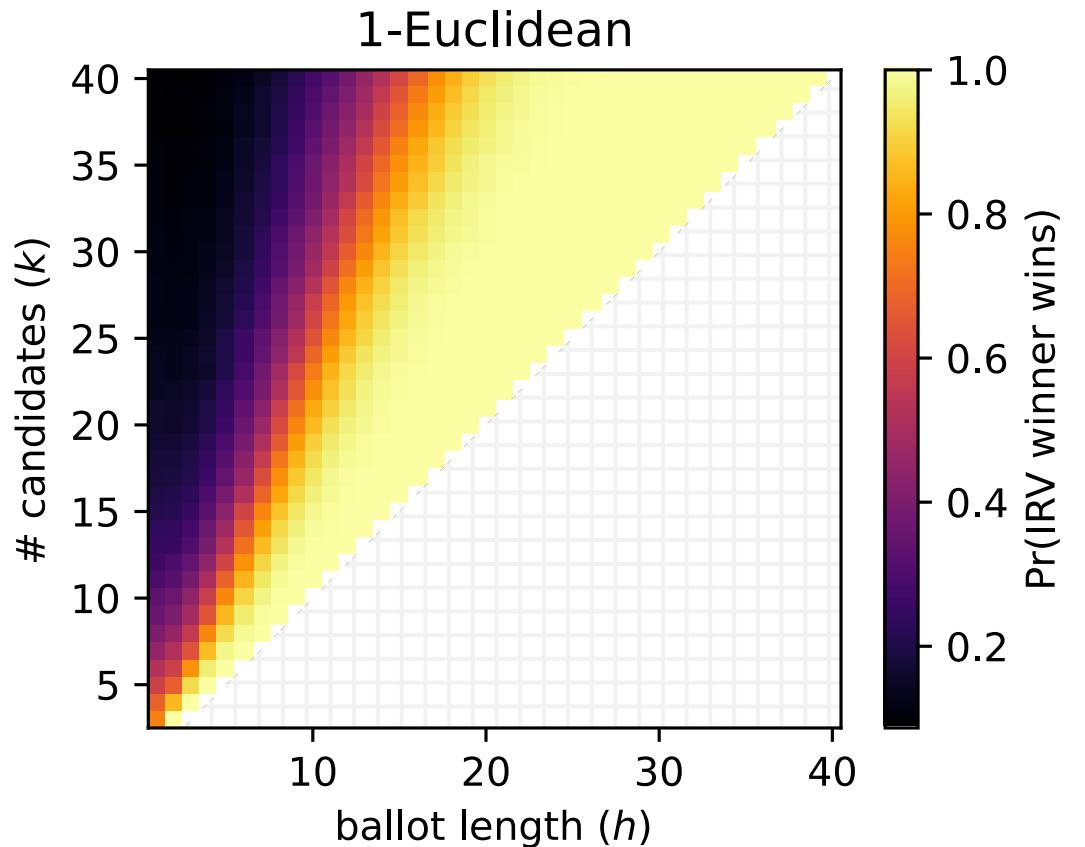
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- Construction with $k - 1$ truncation winners and only $\Theta(k)$ voter types



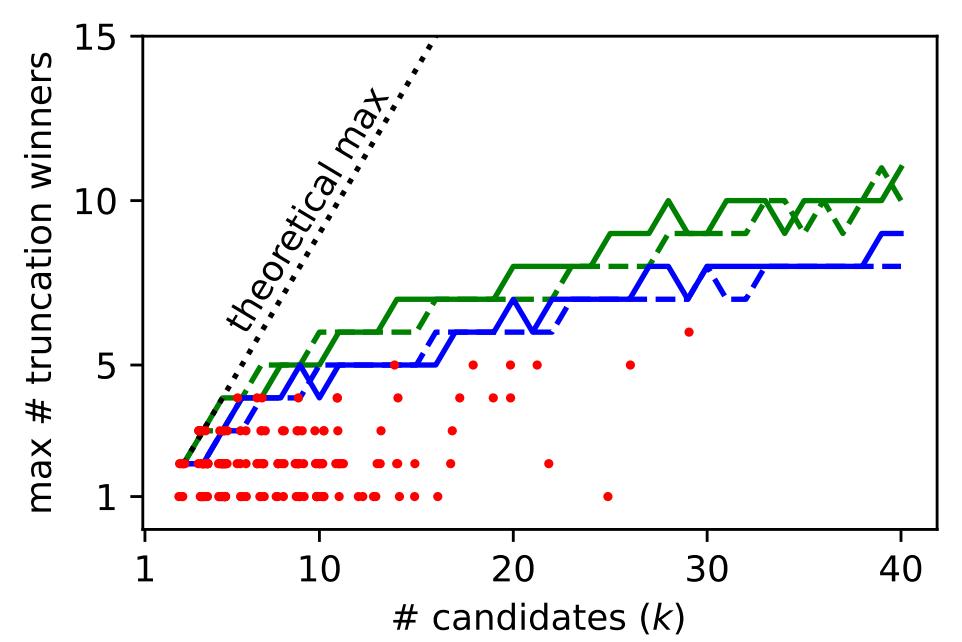
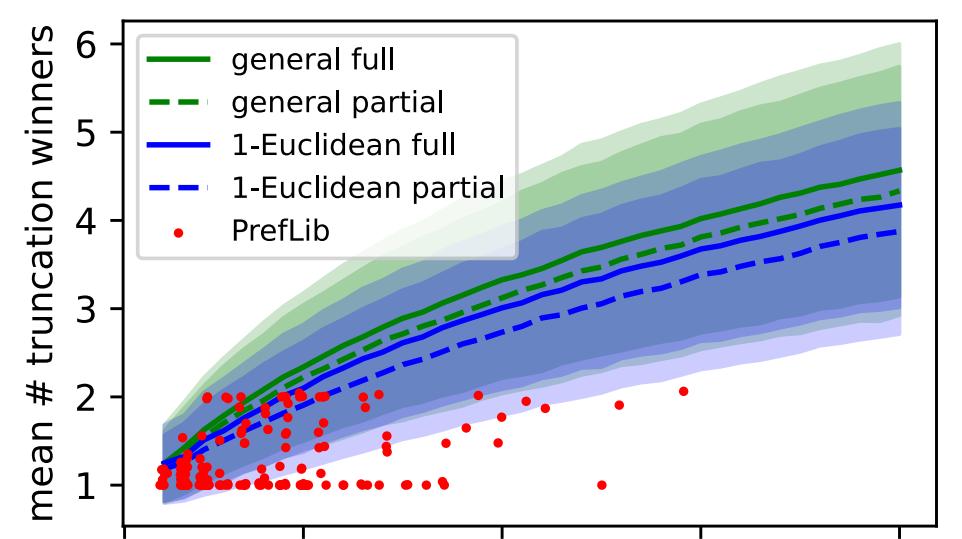
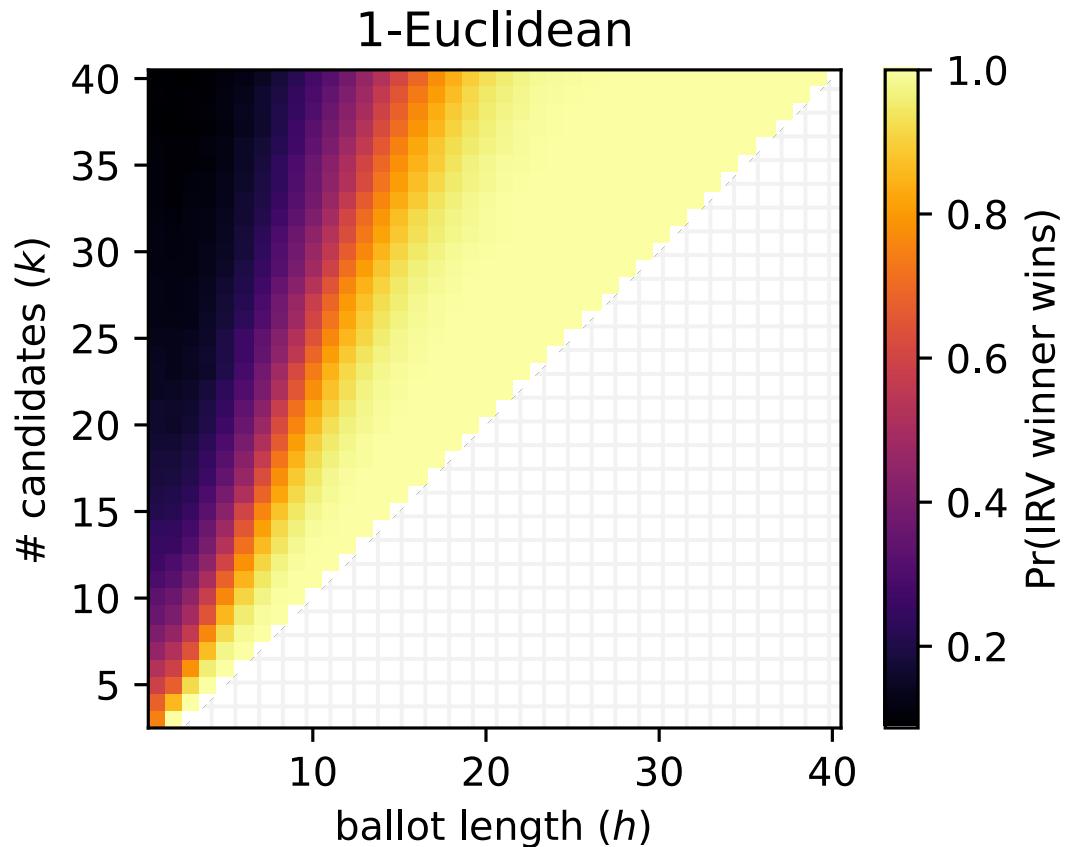
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- Full-ballot construction with $k/2$ truncation winners



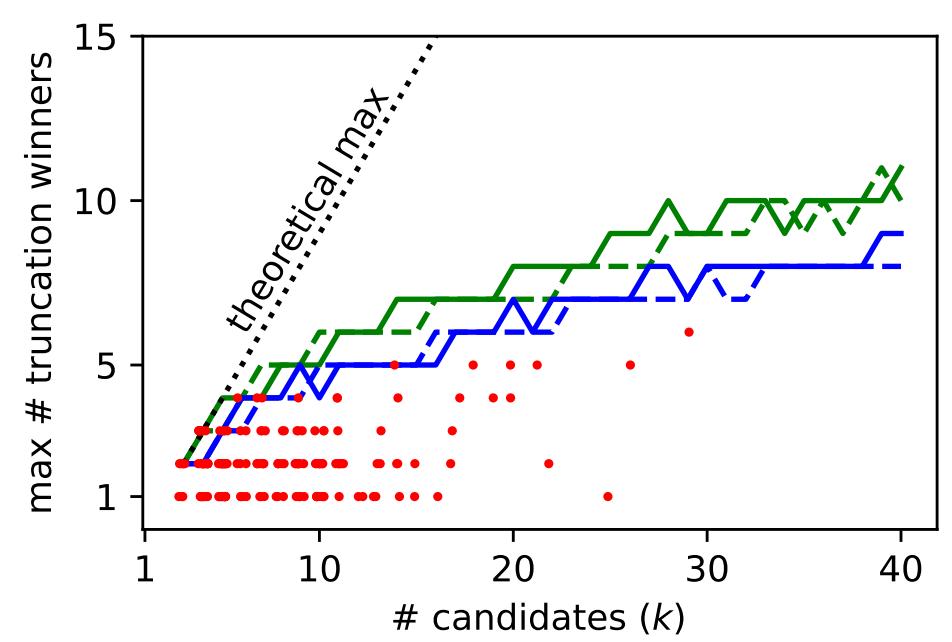
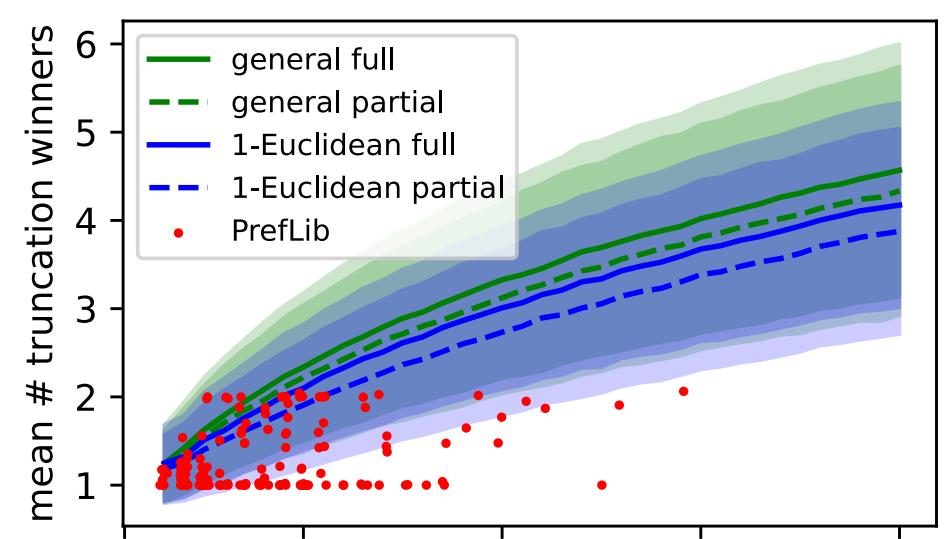
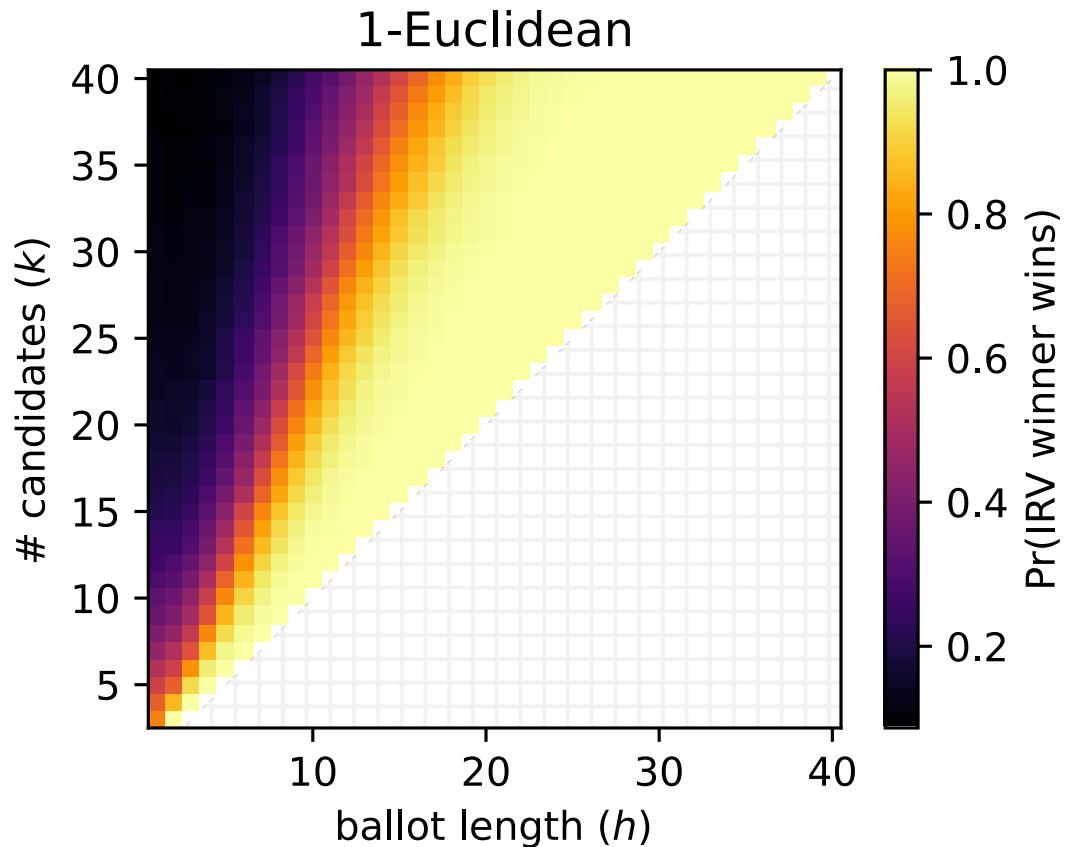
More things in the paper

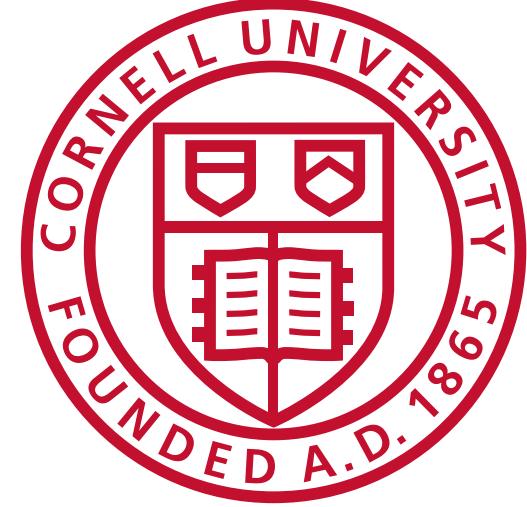
- Single-crossing preferences: $k - 1$ truncation winners impossible
- Other restrictions on ties with voter lower bounds and matching constructions
- Construction with $k - 1$ truncation winners and only $\Theta(k)$ voter types
- Full-ballot construction with $k/2$ truncation winners
- Linear program for finding full-ballot $k - 1$ truncation winner profiles



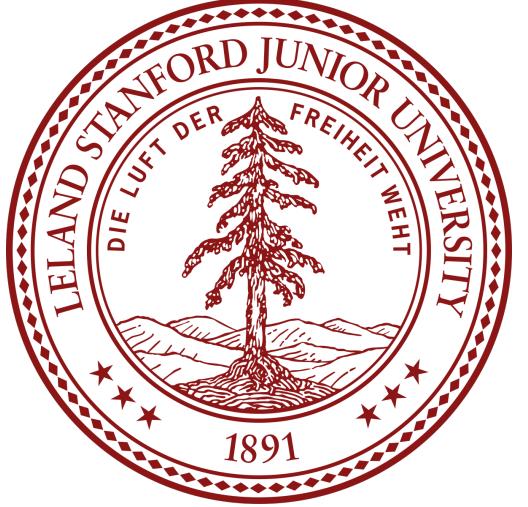
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- Simulations





Thank you!



Code and data:
github.com/tomlinsonk/irv-ballot-length

Extended version:
arxiv.org/abs/2207.08958

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