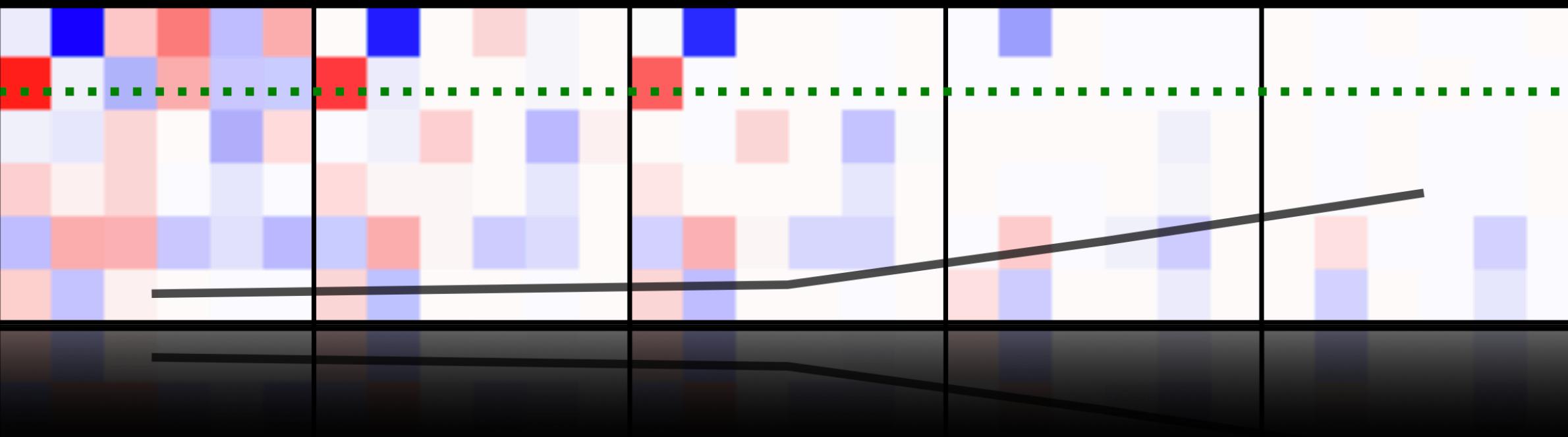
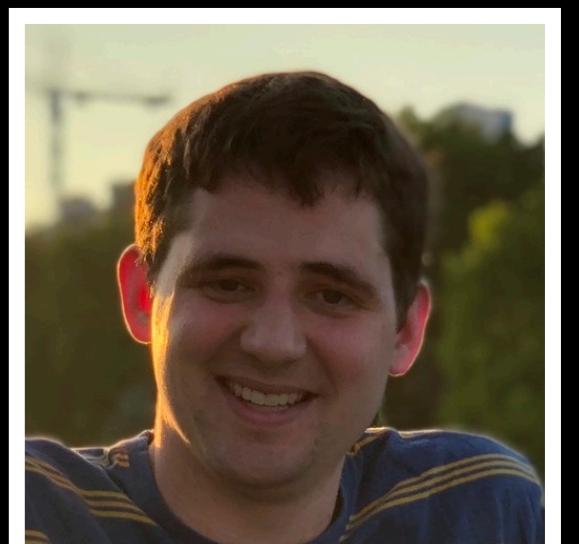


Code: [bit.ly/lcl-code](https://bit.ly/lcl-code)  
Data: [bit.ly/lcl-data](https://bit.ly/lcl-data)  
Slides: [bit.ly/lcl-kdd-slides](https://bit.ly/lcl-kdd-slides)



# Learning Interpretable Feature Context Effects in Discrete Choice

Kiran Tomlinson  
PhD Student, Cornell CS



research with Austin R. Benson

# Choices and context effects

# Discrete choices are everywhere



**amazon.com**

**Amazon's Choice**

KDD Chocolate Flavored Milk 180ML (18 PACK)  
6 FL Oz (Pack of 18)  
★★★★★ ~ 57  
\$27<sup>99</sup> (\$0.26/Fl Oz)  
Save \$2.00 with coupon  
✓prime FREE Delivery Thu, Jun 24

KDD Banana Flavored Milk 180ML (18 PACK)  
6.33 FL Oz (Pack of 18)  
★★★★★ ~ 31  
\$27<sup>99</sup> (\$0.26/Fl Oz)  
Save \$2.00 with coupon  
✓prime FREE Delivery Thu, Jun 24

KDD Original Milk 180ML (18 PACK)  
★★★★★ ~ 2  
\$27<sup>99</sup> (\$4.60/Ounce)  
✓prime FREE Delivery Thu, Jun 24

**Expedia®**

**Best Western University Inn**  
Ithaca

**Black Friday / Cyber Monday Deals Now**  
Free Shuttle Transportation, Grab & Go Breakfast, WiFi & Parking. Pet friendly, Outdoor Pool, Fitness Center. Sanitizing Daily

**Breakfast included**

**3.9/5 Good (999 reviews)**

**\$63**  
per night  
\$71 total  
Includes taxes & fees

**Quality Inn Ithaca - University Area**  
Ithaca

**Black Friday / Cyber Monday Deals Now**  
Complimentary Breakfast. Free Airport Shuttle, WiFi & parking. Close to Ithaca College & Cornell University. Pets welcome.

**Breakfast included**

**3.6/5 Good (694 reviews)**

**\$59**  
per night  
\$66 total  
Includes taxes & fees

**Hotel Ithaca**  
Ithaca

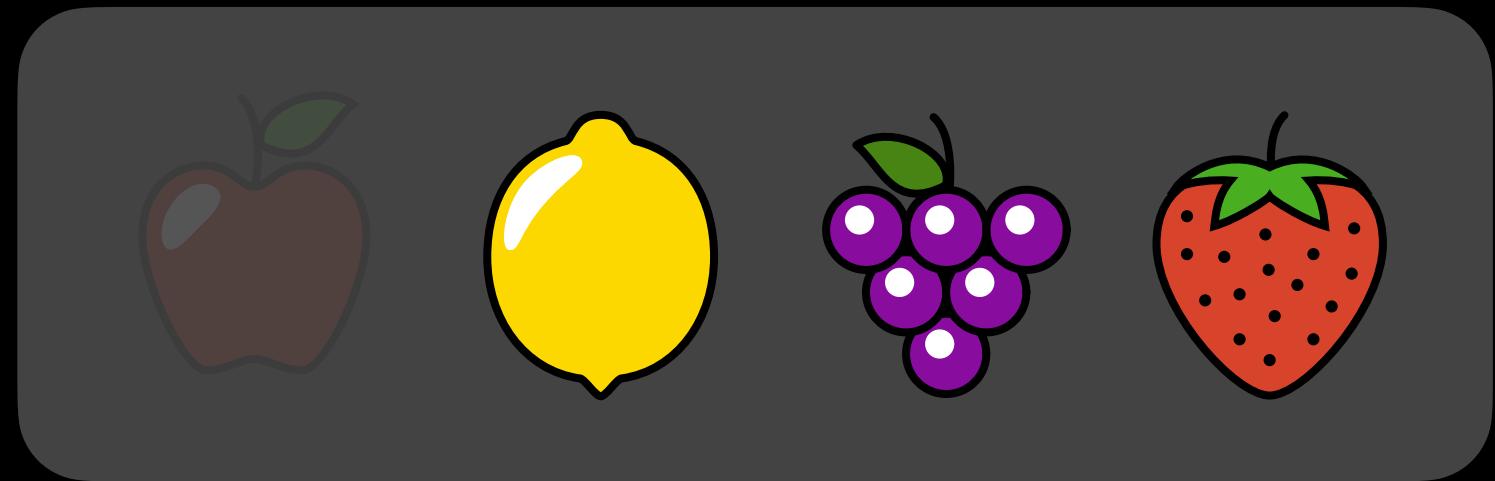
**Member Price available**

**\$94**  
per night  
\$106 total  
Includes taxes & fees

# “The fundamental problem of discrete choice”

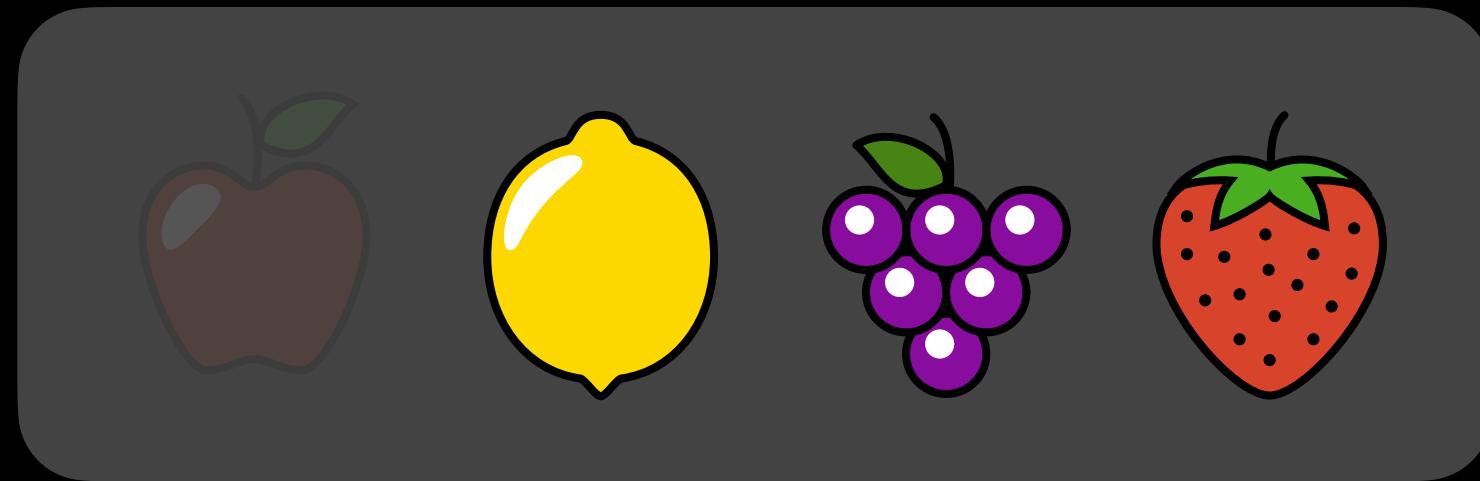
# “The fundamental problem of discrete choice”

*choice set*

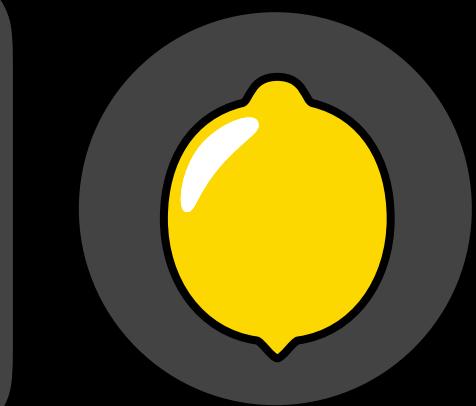


# “The fundamental problem of discrete choice”

*choice set*

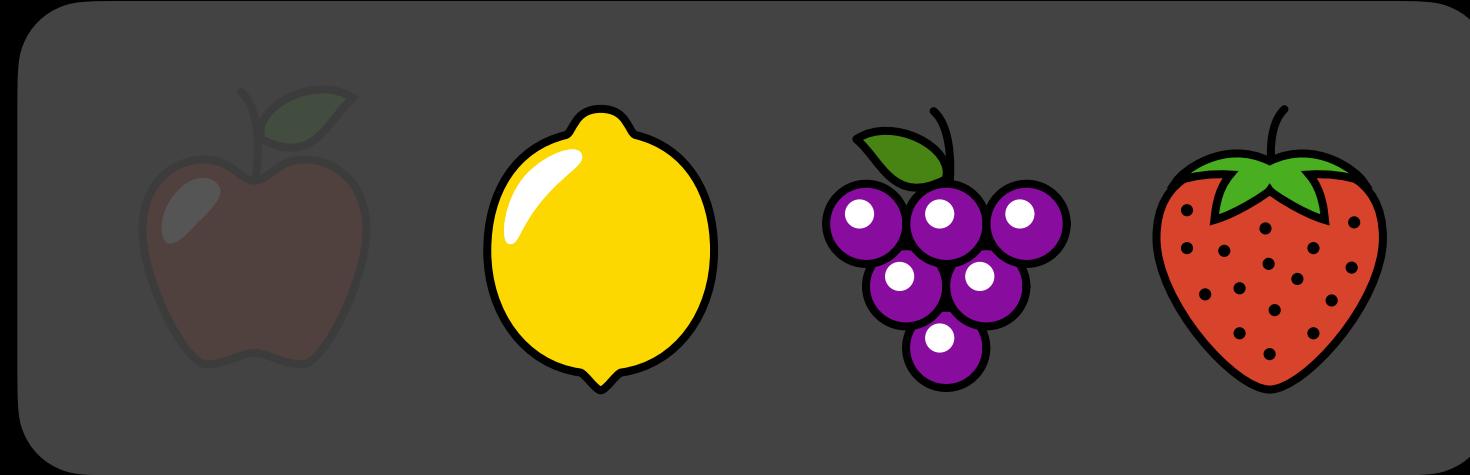


*choice*

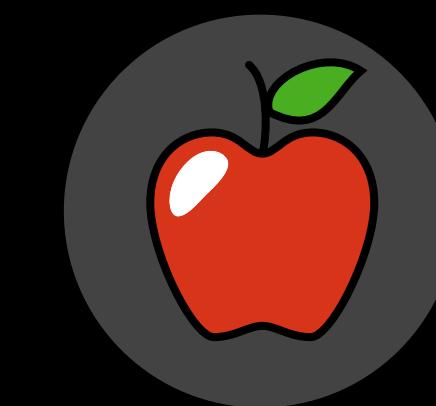
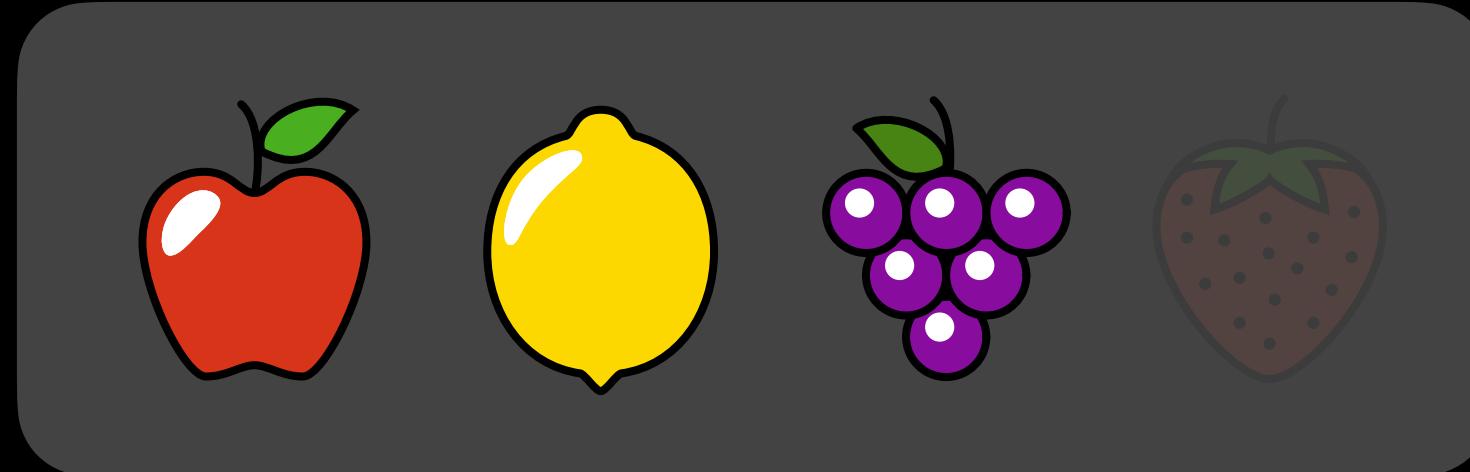
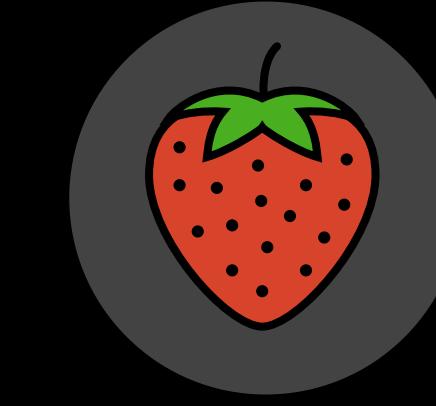
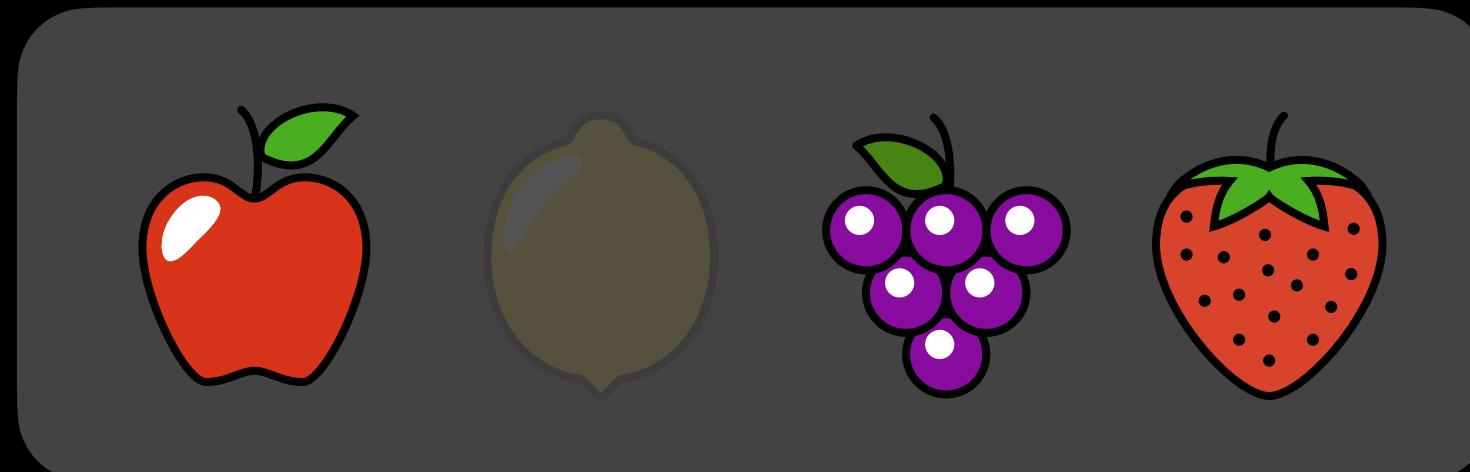
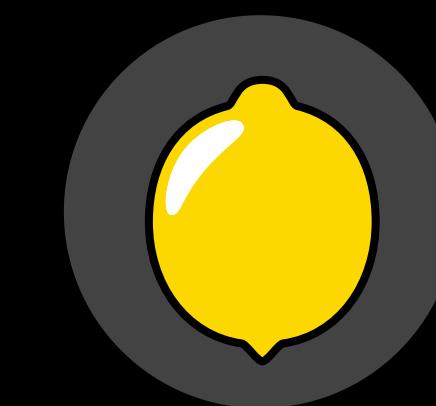


# “The fundamental problem of discrete choice”

*choice set*

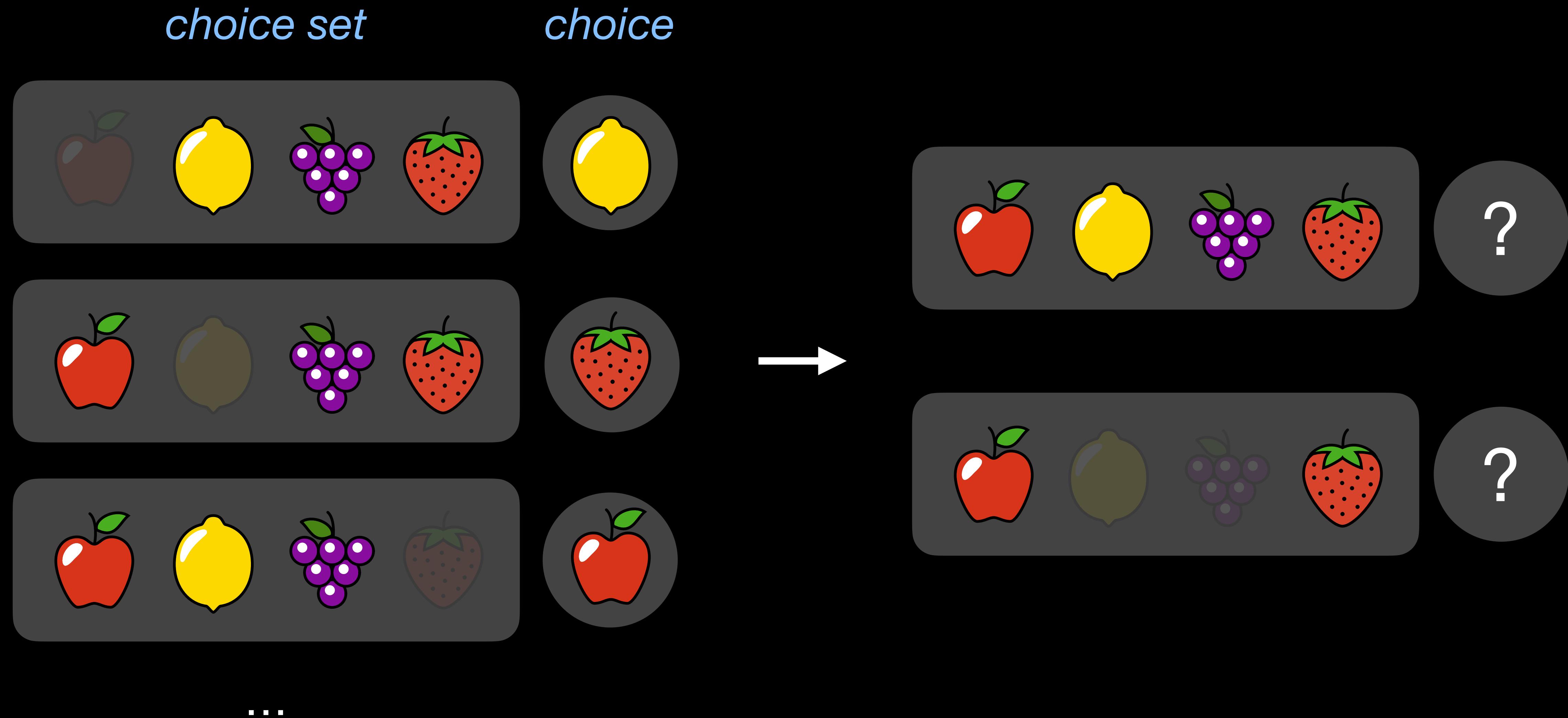


*choice*



...

# “The fundamental problem of discrete choice”



# The classic model: *multinomial logit (MNL)*

(McFadden, *Frontiers in Econometrics* 1973)

# The classic model: *multinomial logit (MNL)*

(McFadden, *Frontiers in Econometrics* 1973)

Assume *item  $i$*  has *utility  $u_i$*

$$\Pr(i \mid C) = \frac{\exp(u_i)}{\sum_{j \in C} \exp(u_j)}$$

# The classic model: *multinomial logit (MNL)*

(McFadden, *Frontiers in Econometrics* 1973)

Assume *item i* has *utility*  $u_i$

$$\Pr(i \mid C) = \frac{\exp(u_i)}{\sum_{j \in C} \exp(u_j)}$$

$C$				
$u_i$	1	-1	0	2
$\Pr(i \mid C)$	.24	.03	.09	.64

# The classic model: *multinomial logit (MNL)*

(McFadden, *Frontiers in Econometrics* 1973)

Assume *item i* has *utility*  $u_i$

$$\Pr(i \mid C) = \frac{\exp(u_i)}{\sum_{j \in C} \exp(u_j)}$$

$C$				
$u_i$	1	-1	0	2
$\Pr(i \mid C)$	.24	.03	.09	.64

Unique choice model satisfying  
*independence of irrelevant alternatives (IIA)*:

(Luce, *Individual Choice Behavior* 1959)

$$\frac{\Pr(i \mid C)}{\Pr(j \mid C)} = \frac{\Pr(i \mid C')}{\Pr(j \mid C')}$$

# Problem for MNL: *context effects*

# Problem for MNL: *context effects*

The choice set influences preferences.

# Problem for MNL: *context effects*

The choice set influences preferences.

## ***Compromise***

(Simonson, 1989)

# Problem for MNL: *context effects*

The choice set influences preferences.

## Compromise

(Simonson, 1989)



# Problem for MNL: *context effects*

The choice set influences preferences.

## Compromise

(Simonson, 1989)

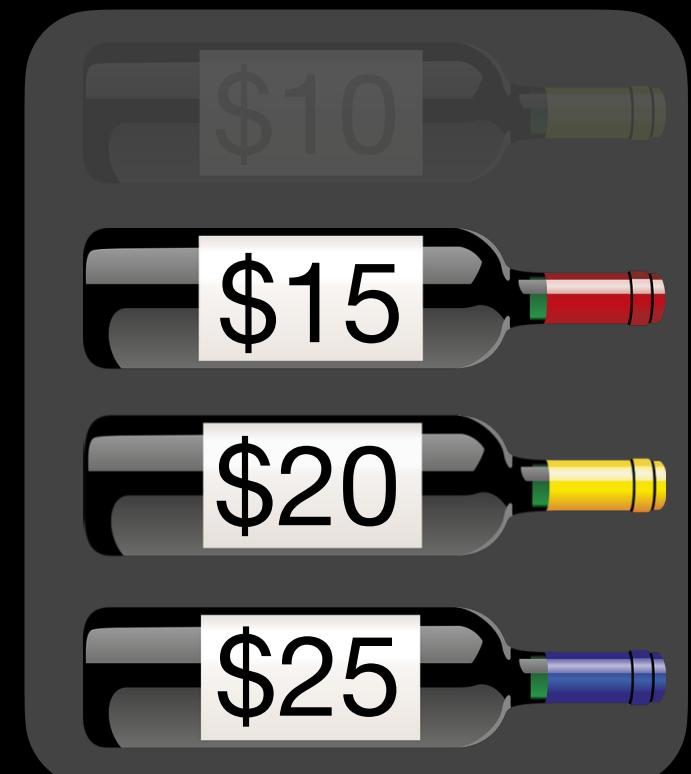


# Problem for MNL: *context effects*

The choice set influences preferences.

## Compromise

(Simonson, 1989)



# Problem for MNL: *context effects*

The choice set influences preferences.

## Compromise

(Simonson, 1989)



# Problem for MNL: *context effects*

The choice set influences preferences.

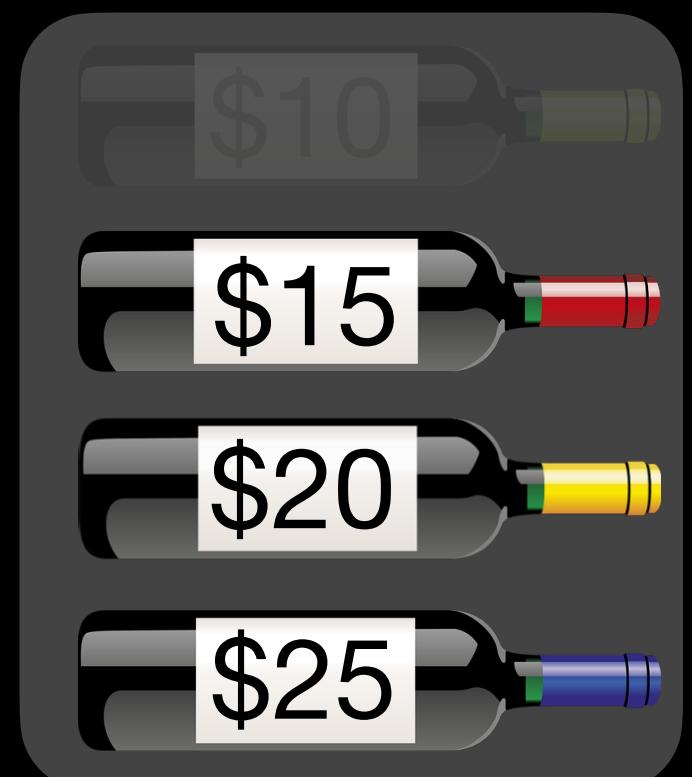
## Compromise

(Simonson, 1989)



## Similarity

(Tversky, 1972)



# Problem for MNL: *context effects*

The choice set influences preferences.

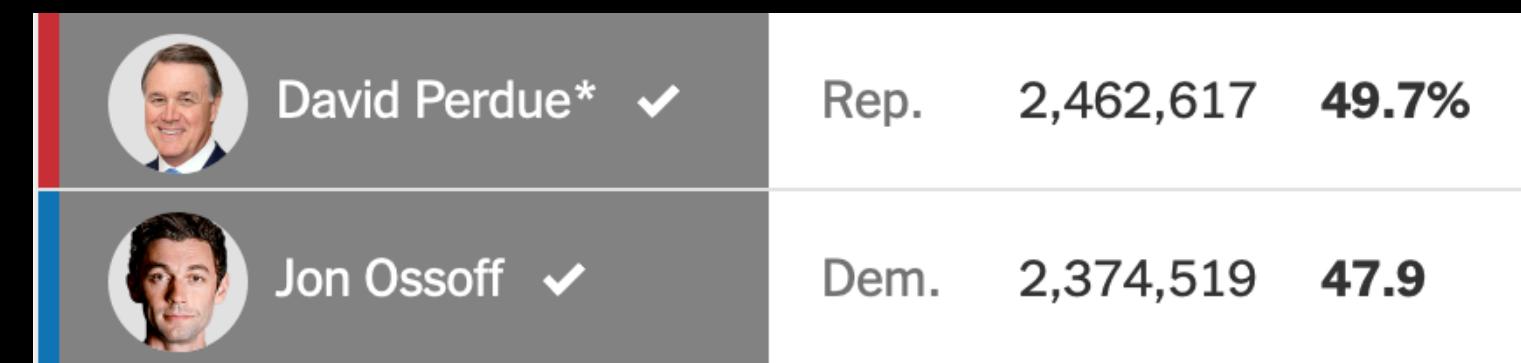
## Compromise

(Simonson, 1989)



## Similarity

(Tversky, 1972)



# Problem for MNL: *context effects*

The choice set influences preferences.

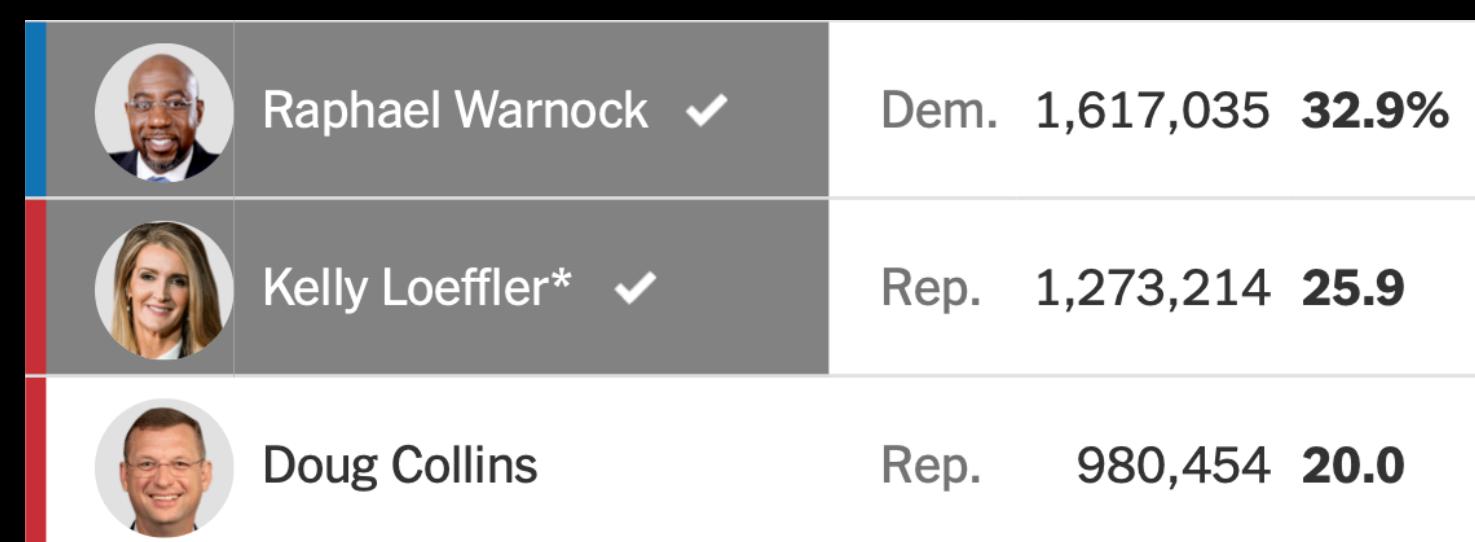
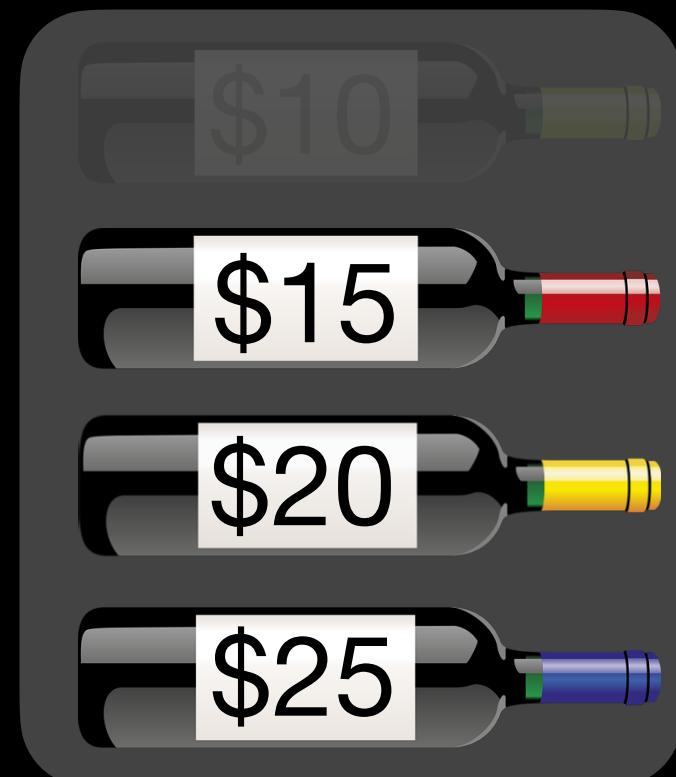
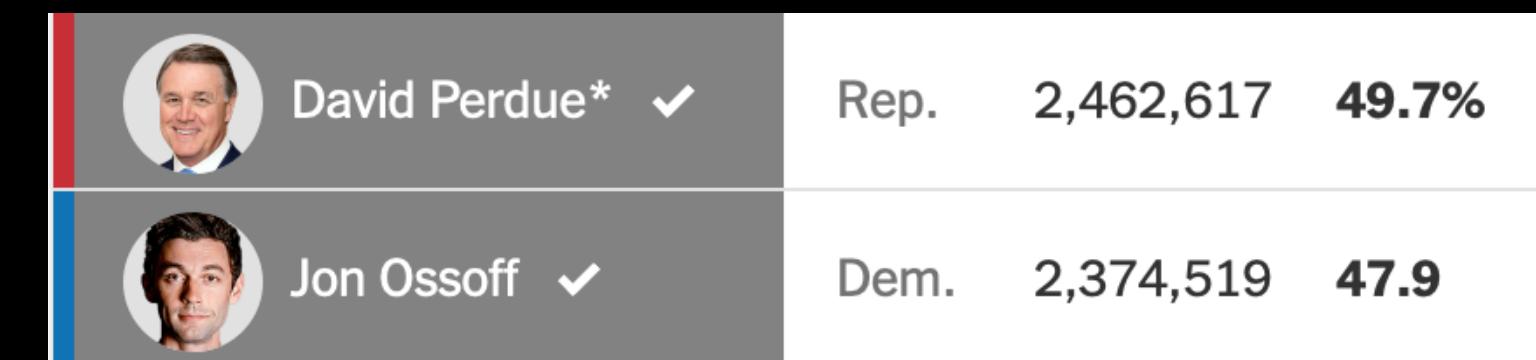
## Compromise

(Simonson, 1989)



## Similarity

(Tversky, 1972)



# Problem for MNL: *context effects*

The choice set influences preferences.

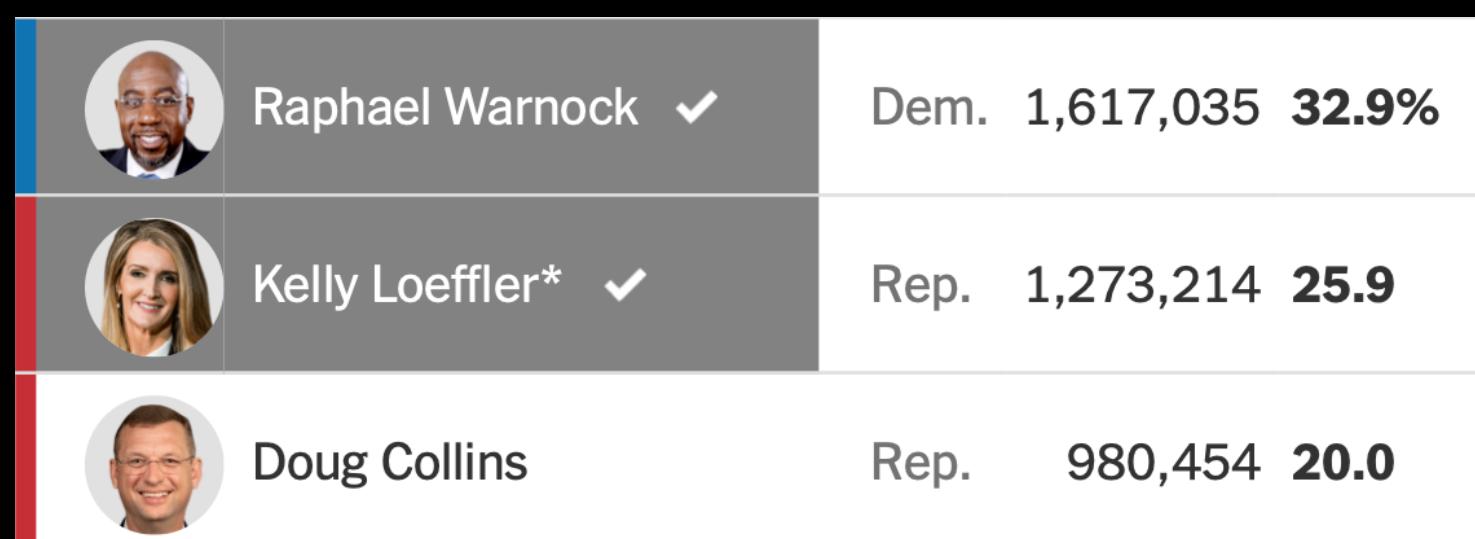
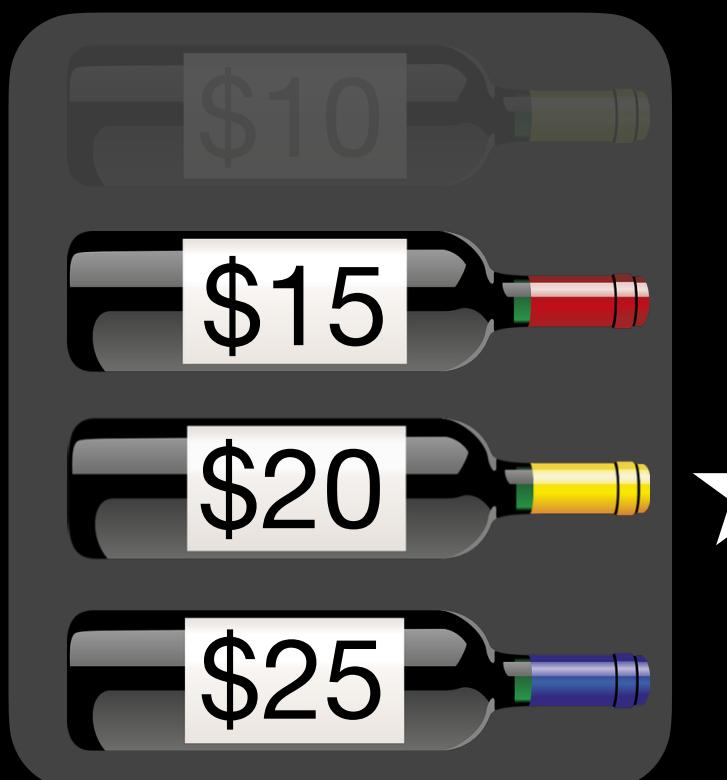
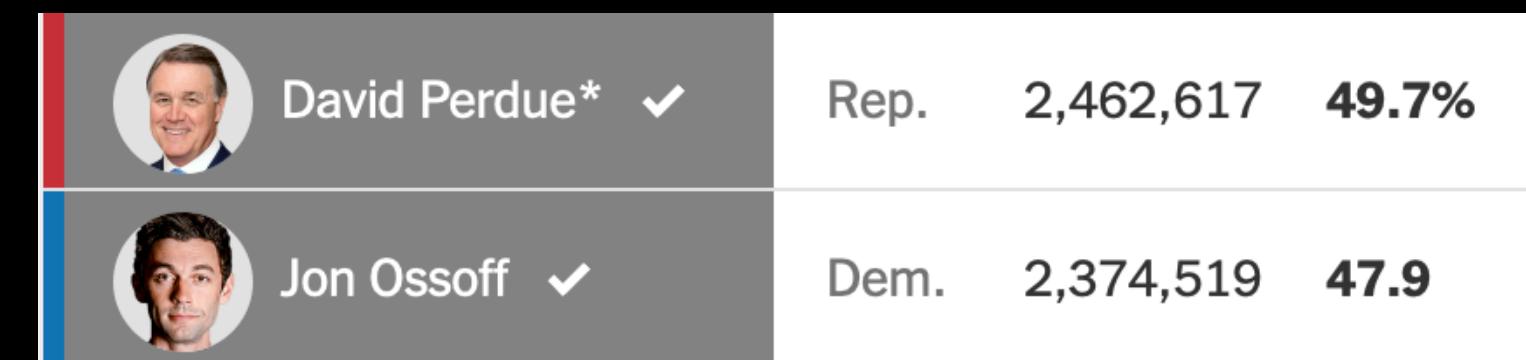
## Compromise

(Simonson, 1989)



## Similarity

(Tversky, 1972)



IIA violations:

$$\frac{\Pr(i \mid C)}{\Pr(j \mid C)} \neq \frac{\Pr(i \mid C')}{\Pr(j \mid C')}$$

# Natural context effect model: *CDM*

(Seshadri, Peysakhovich, & Ugander, ICML 2019)

# Natural context effect model: *CDM*

(Seshadri, Peysakhovich, & Ugander, ICML 2019)

Item  $j$  exerts *pull*  $u_{ij}$  on item  $i$ , item utility is sum of pulls:

$$\Pr(i \mid C) = \frac{\exp\left(\sum_{k \in C \setminus i} u_{ik}\right)}{\sum_{j \in C} \exp\left(\sum_{k \in C \setminus i} u_{jk}\right)}$$

# Natural context effect model: *CDM*

(Seshadri, Peysakhovich, & Ugander, ICML 2019)

Item  $j$  exerts *pull*  $u_{ij}$  on item  $i$ , item utility is sum of pulls:

$$\Pr(i \mid C) = \frac{\exp\left(\sum_{k \in C \setminus i} u_{ik}\right)}{\sum_{j \in C} \exp\left(\sum_{k \in C \setminus i} u_{jk}\right)}$$

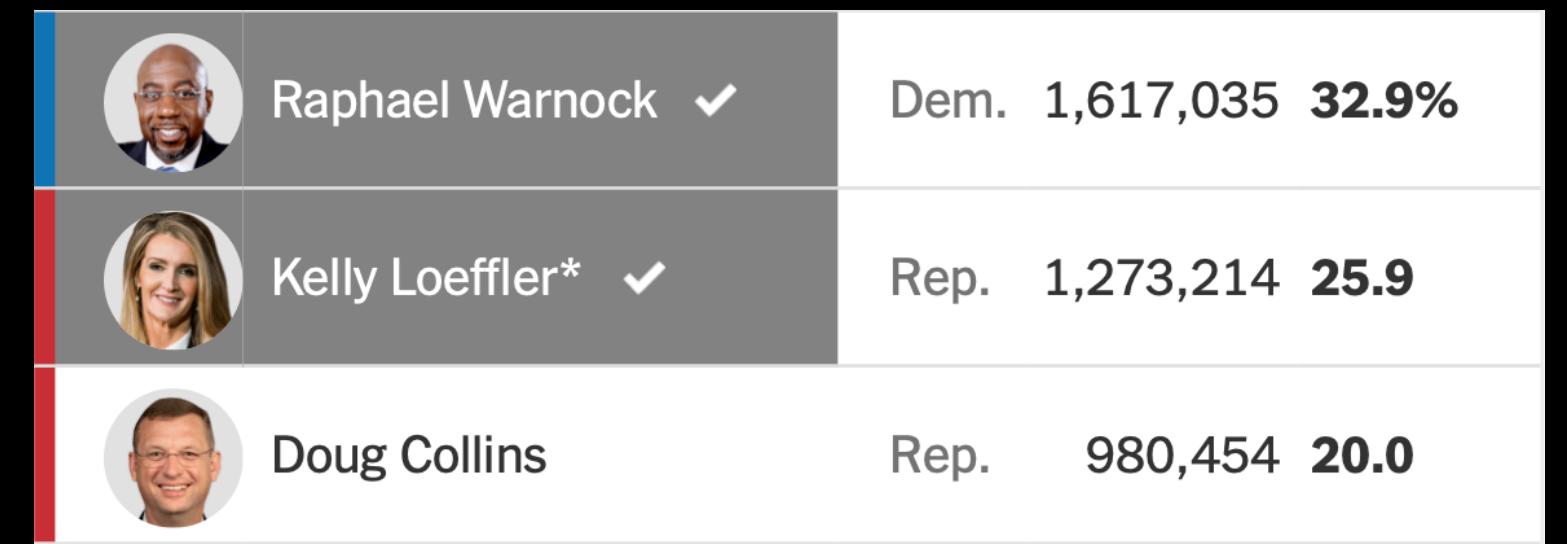
Assumes no higher-order effects  
(i.e., context effects decompose additively into effects of items)

# Natural context effect model: *CDM*

(Seshadri, Peysakhovich, & Ugander, ICML 2019)

Item  $j$  exerts *pull*  $u_{ij}$  on item  $i$ , item utility is sum of pulls:

$$\Pr(i \mid C) = \frac{\exp\left(\sum_{k \in C \setminus i} u_{ik}\right)}{\sum_{j \in C} \exp\left(\sum_{k \in C \setminus i} u_{jk}\right)}$$



$$u_{\text{Loeffler, Collins}} < 0$$
$$u_{\text{Collins, Loeffler}} < 0$$

Assumes no higher-order effects  
(i.e., context effects decompose additively into effects of items)

# Item features and the LCL

# Choice models with *item features*

# Choice models with *item features*

So far, models have per-item parameters

# Choice models with *item features*

So far, models have per-item parameters

→ can't generalize to new items not in training set

# Choice models with *item features*

So far, models have per-item parameters

- can't generalize to new items not in training set
- hard to learn utilities for rare items

# Choice models with *item features*

So far, models have per-item parameters

- can't generalize to new items not in training set
- hard to learn utilities for rare items
- too many parameters with many items

# Choice models with *item features*

So far, models have per-item parameters

- can't generalize to new items not in training set
- hard to learn utilities for rare items
- too many parameters with many items

Use item features:



*genre*: drama,  
*in\_top\_10*: True,  
*has\_new\_episodes*: True,  
*producer*: Netflix



*genre*: comedy,  
*in\_top\_10*: False,  
*has\_new\_episodes*: False,  
*producer*: NBC



*genre*: drama,  
*in\_top\_10*: True,  
*has\_new\_episodes*: False,  
*producer*: Netflix



*genre*: reality,  
*in\_top\_10*: True,  
*has\_new\_episodes*: False,  
*producer*: Banijay

# MNL with item features: *conditional logit*

# MNL with item features: *conditional logit*

Feature vector  $x_i \in \mathbb{R}^d$  for each item  $i$

Preference vector  $\theta \in \mathbb{R}^d$

# MNL with item features: *conditional logit*

Feature vector  $x_i \in \mathbb{R}^d$  for each item  $i$

Preference vector  $\theta \in \mathbb{R}^d$

MNL:

$$\Pr(i \mid C) = \frac{\exp(u_i)}{\sum_{j \in C} \exp(u_j)}$$

# MNL with item features: *conditional logit*

Feature vector  $x_i \in \mathbb{R}^d$  for each item  $i$

Preference vector  $\theta \in \mathbb{R}^d$

MNL:

$$\Pr(i | C) = \frac{\exp(u_i)}{\sum_{j \in C} \exp(u_j)} \rightarrow$$

Conditional logit:

$$\Pr(i | C) = \frac{\exp(\theta^T x_i)}{\sum_{j \in C} \exp(\theta^T x_j)}$$

# MNL with item features: *conditional logit*

Feature vector  $x_i \in \mathbb{R}^d$  for each item  $i$

Preference vector  $\theta \in \mathbb{R}^d$

MNL:

$$\Pr(i | C) = \frac{\exp(u_i)}{\sum_{j \in C} \exp(u_j)} \rightarrow$$

Conditional logit:

$$\Pr(i | C) = \frac{\exp(\theta^T x_i)}{\sum_{j \in C} \exp(\theta^T x_j)}$$

Preference coefficient  $\theta_k$  is easy to interpret: importance of the  $k^{\text{th}}$  feature

# Incorporating *feature context effects* into conditional logit

# Incorporating *feature context effects* into conditional logit

Conditional logit utility:  $u_i = \theta^T x_i$

# Incorporating *feature context effects* into conditional logit

Conditional logit utility:  $u_i = \theta^T x_i$  → *Contextual utility:*  $u_{i,C} = [\theta + F(C)]^T x_i$

# Incorporating *feature context effects* into conditional logit

Conditional logit utility:  $u_i = \theta^T x_i \rightarrow$  *Contextual utility:*  $u_{i,C} = [\theta + F(C)]^T x_i$

Simplifying assumptions on  $F(C)$ :

# Incorporating *feature context effects* into conditional logit

Conditional logit utility:  $u_i = \theta^T x_i$   $\rightarrow$  Contextual utility:  $u_{i,C} = [\theta + F(C)]^T x_i$

Simplifying assumptions on  $F(C)$ :

1. *Additivity*:  $F(C) \propto \sum_{j \in C} f(x_j)$  for some function  $f$

# Incorporating *feature context effects* into conditional logit

Conditional logit utility:  $u_i = \theta^T x_i \rightarrow$  Contextual utility:  $u_{i,C} = [\theta + F(C)]^T x_i$

Simplifying assumptions on  $F(C)$ :

1. *Additivity*:  $F(C) \propto \sum_{j \in C} f(x_j)$  for some function  $f$
2. *Dilution*:  $F(C) = \frac{1}{|C|} \sum_{j \in C} f(x_j)$

# Incorporating *feature context effects* into conditional logit

Conditional logit utility:  $u_i = \theta^T x_i \rightarrow$  Contextual utility:  $u_{i,C} = [\theta + F(C)]^T x_i$

Simplifying assumptions on  $F(C)$ :

1. *Additivity*:  $F(C) \propto \sum_{j \in C} f(x_j)$  for some function  $f$
2. *Dilution*:  $F(C) = \frac{1}{|C|} \sum_{j \in C} f(x_j)$
3. *Linearity*:  $f(x_j) = Ax_j$  for some matrix  $A \in \mathbb{R}^{d \times d}$

# Incorporating *feature context effects* into conditional logit

Conditional logit utility:  $u_i = \theta^T x_i \rightarrow$  Contextual utility:  $u_{i,C} = [\theta + F(C)]^T x_i$

Simplifying assumptions on  $F(C)$ :

1. *Additivity*:  $F(C) \propto \sum_{j \in C} f(x_j)$  for some function  $f$

2. *Dilution*:  $F(C) = \frac{1}{|C|} \sum_{j \in C} f(x_j)$

3. *Linearity*:  $f(x_j) = Ax_j$  for some matrix  $A \in \mathbb{R}^{d \times d}$

$\rightarrow u_{i,C} = (\theta + Ax_C)^T x_i$  ( $x_C = \frac{1}{|C|} \sum_{j \in C} x_j$  is the *mean feature vector*)

# The *Linear Context Logit (LCL)*

# The *Linear Context Logit (LCL)*

$$\Pr(i \mid C) = \frac{\exp([\theta + Ax_C]^T x_i)}{\sum_{j \in C} \exp([\theta + Ax_C]^T x_j)}$$

# The *Linear Context Logit (LCL)*

$$\Pr(i \mid C) = \frac{\exp([\theta + Ax_C]^T x_i)}{\sum_{j \in C} \exp([\theta + Ax_C]^T x_j)}$$

→ convex negative log-likelihood

# The *Linear Context Logit (LCL)*

$$\Pr(i \mid C) = \frac{\exp([\theta + Ax_C]^T x_i)}{\sum_{j \in C} \exp([\theta + Ax_C]^T x_j)}$$

- convex negative log-likelihood
- $\theta$ : *base preference coefficients*

# The *Linear Context Logit (LCL)*

$$\Pr(i \mid C) = \frac{\exp([\theta + Ax_C]^T x_i)}{\sum_{j \in C} \exp([\theta + Ax_C]^T x_j)}$$

- convex negative log-likelihood
- $\theta$ : *base preference coefficients*
- $A_{pq} > 0$ : when  $q$  is *high* in the choice set,  $p$  is *more preferred*

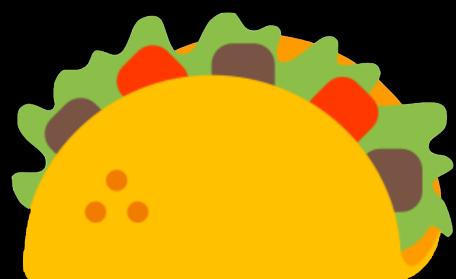
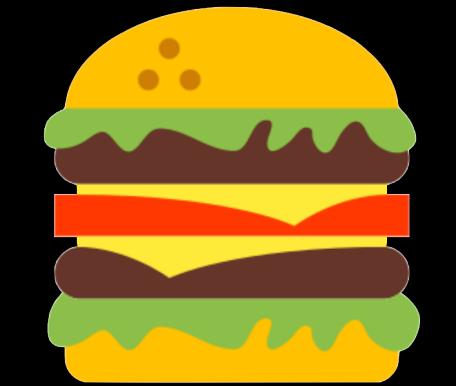
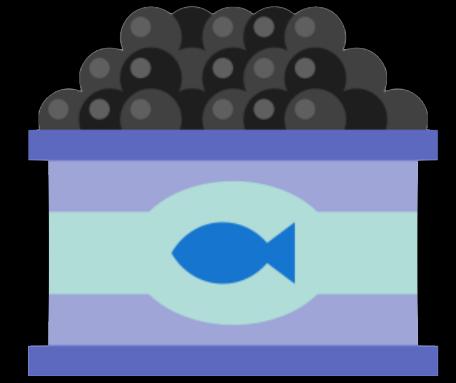
# The *Linear Context Logit (LCL)*

$$\Pr(i \mid C) = \frac{\exp([\theta + Ax_C]^T x_i)}{\sum_{j \in C} \exp([\theta + Ax_C]^T x_j)}$$

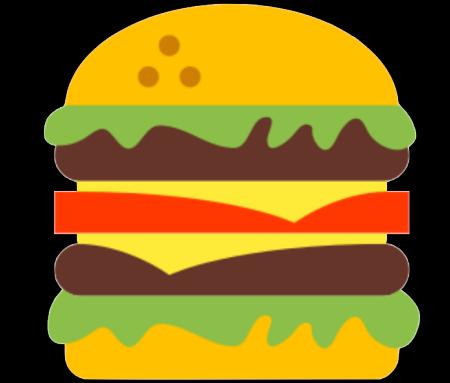
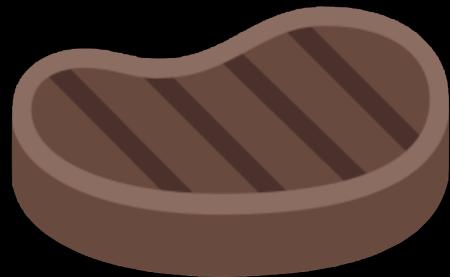
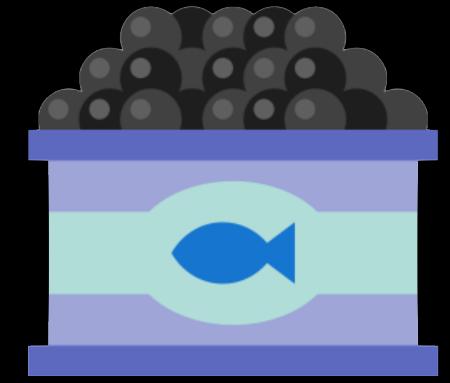
- convex negative log-likelihood
- $\theta$ : *base preference coefficients*
- $A_{pq} > 0$ : when  $q$  is *high* in the choice set,  $p$  is *more* preferred
- $A_{pq} < 0$ : when  $q$  is *high* in the choice set,  $p$  is *less* preferred

# LCL example: restaurant selection

# LCL example: restaurant selection



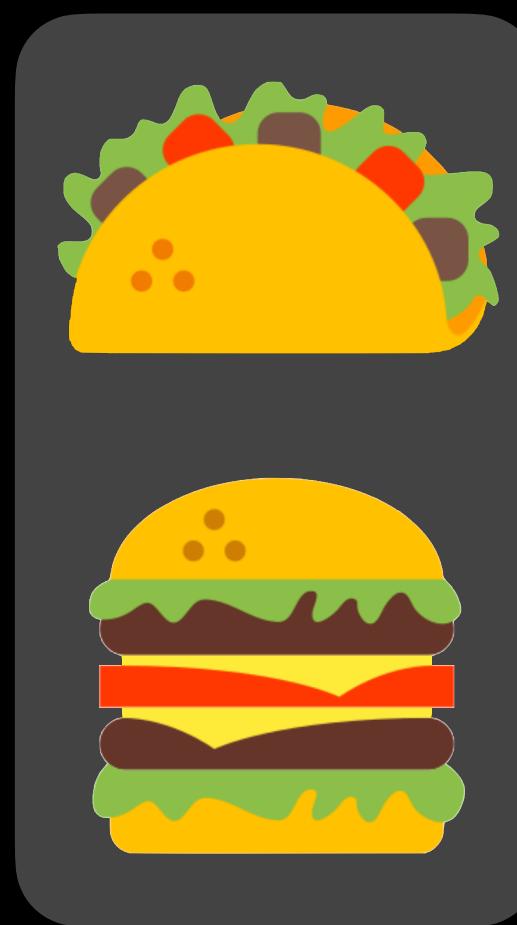
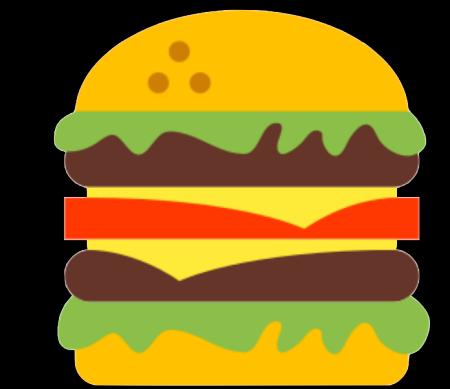
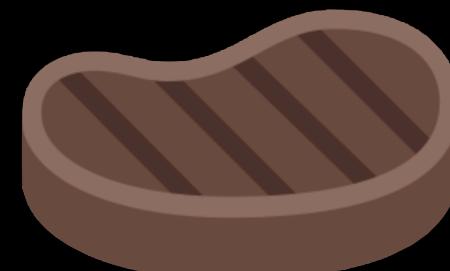
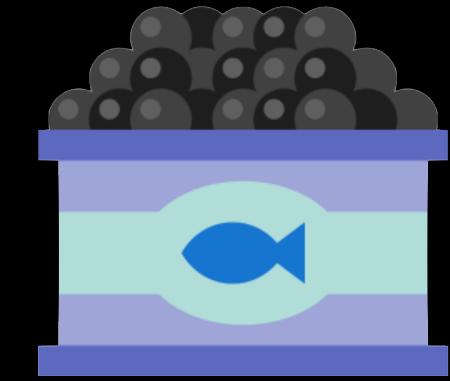
# LCL example: restaurant selection



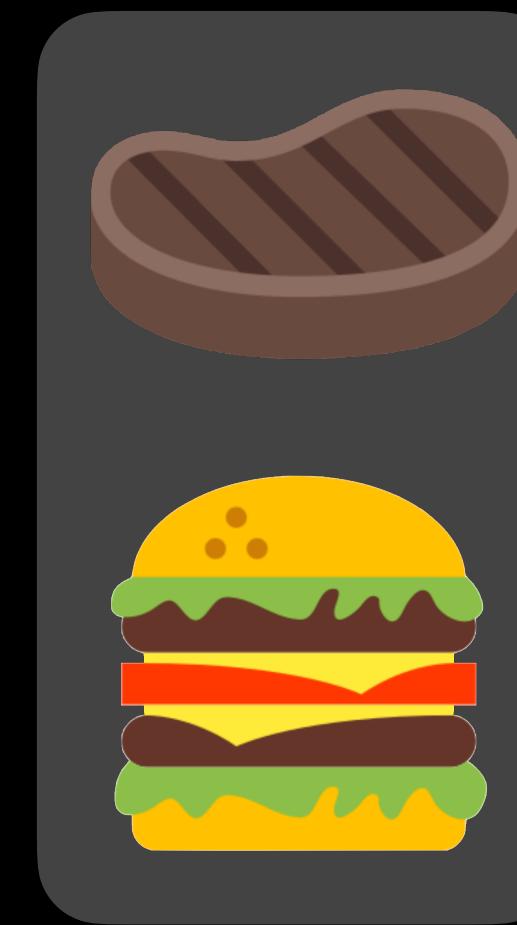
item features:

- price
- service speed
- wine selection

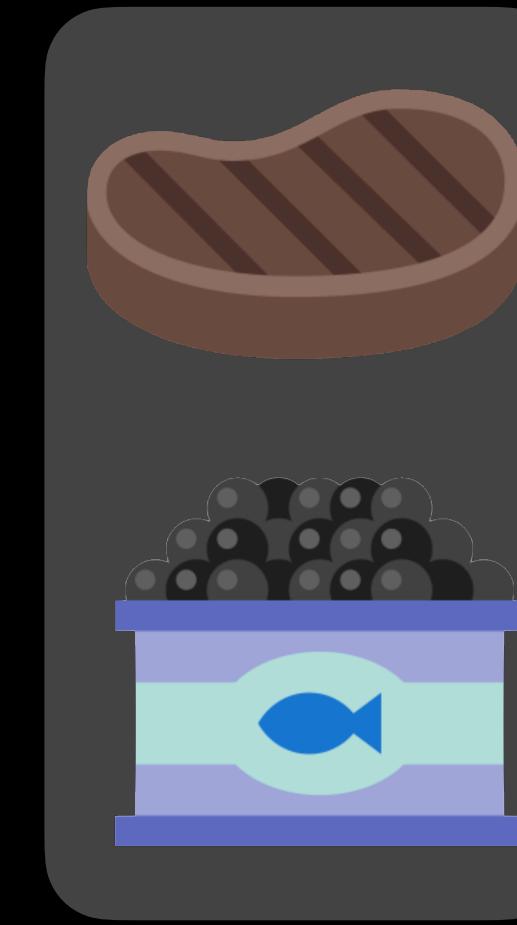
# LCL example: restaurant selection



$C_1$



$C_2$

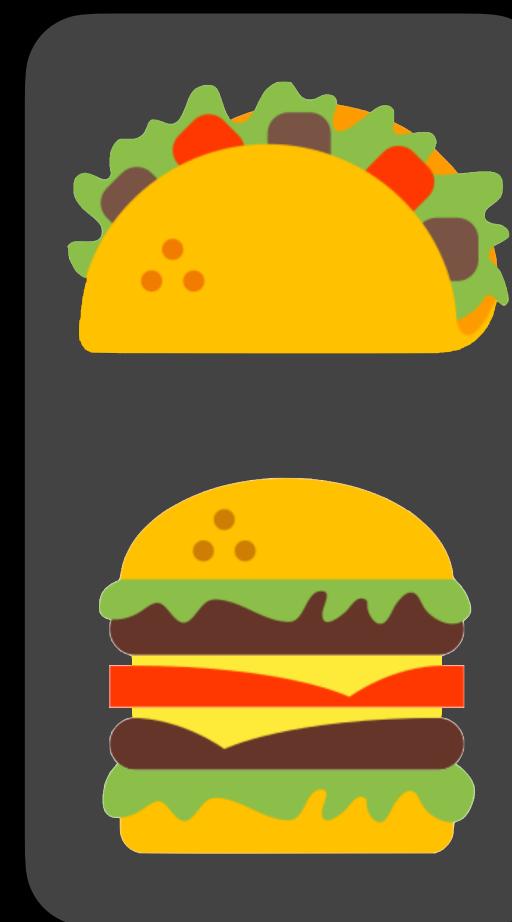
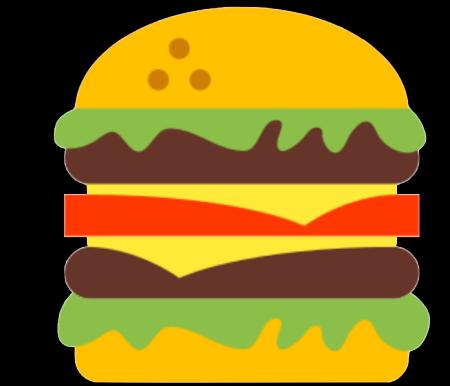
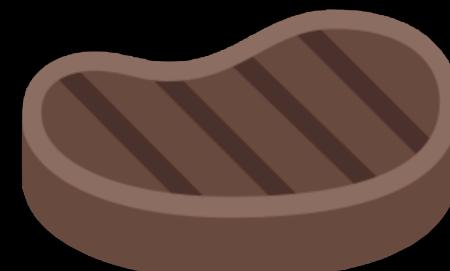
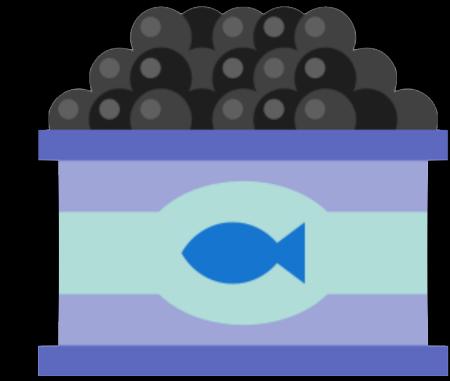
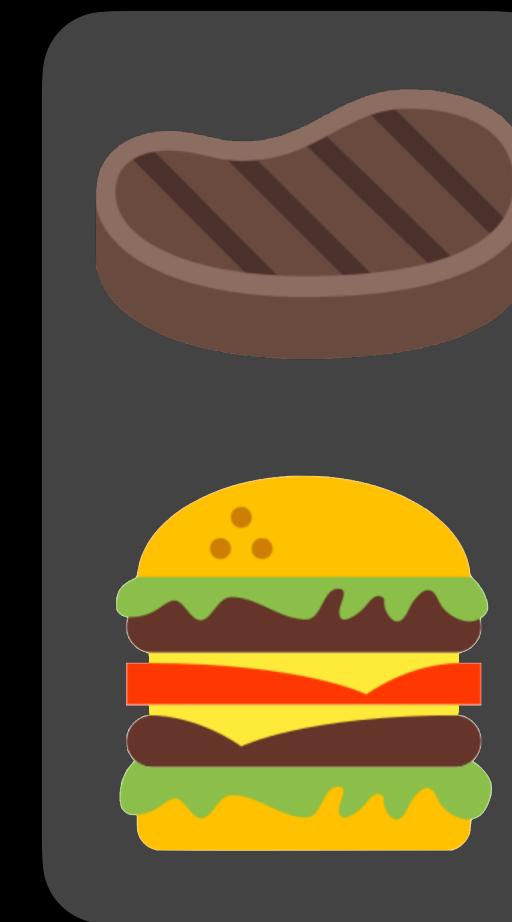
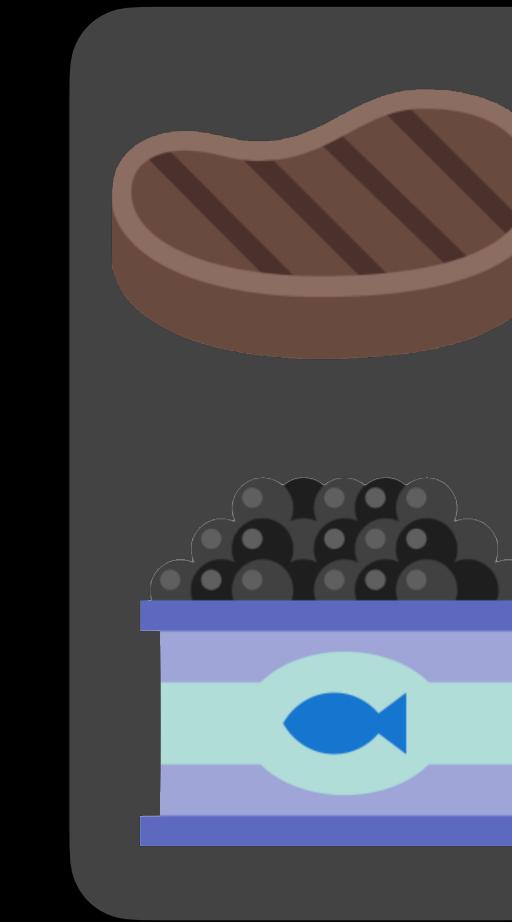


$C_3$

item features:

- price
- service speed
- wine selection

# LCL example: restaurant selection

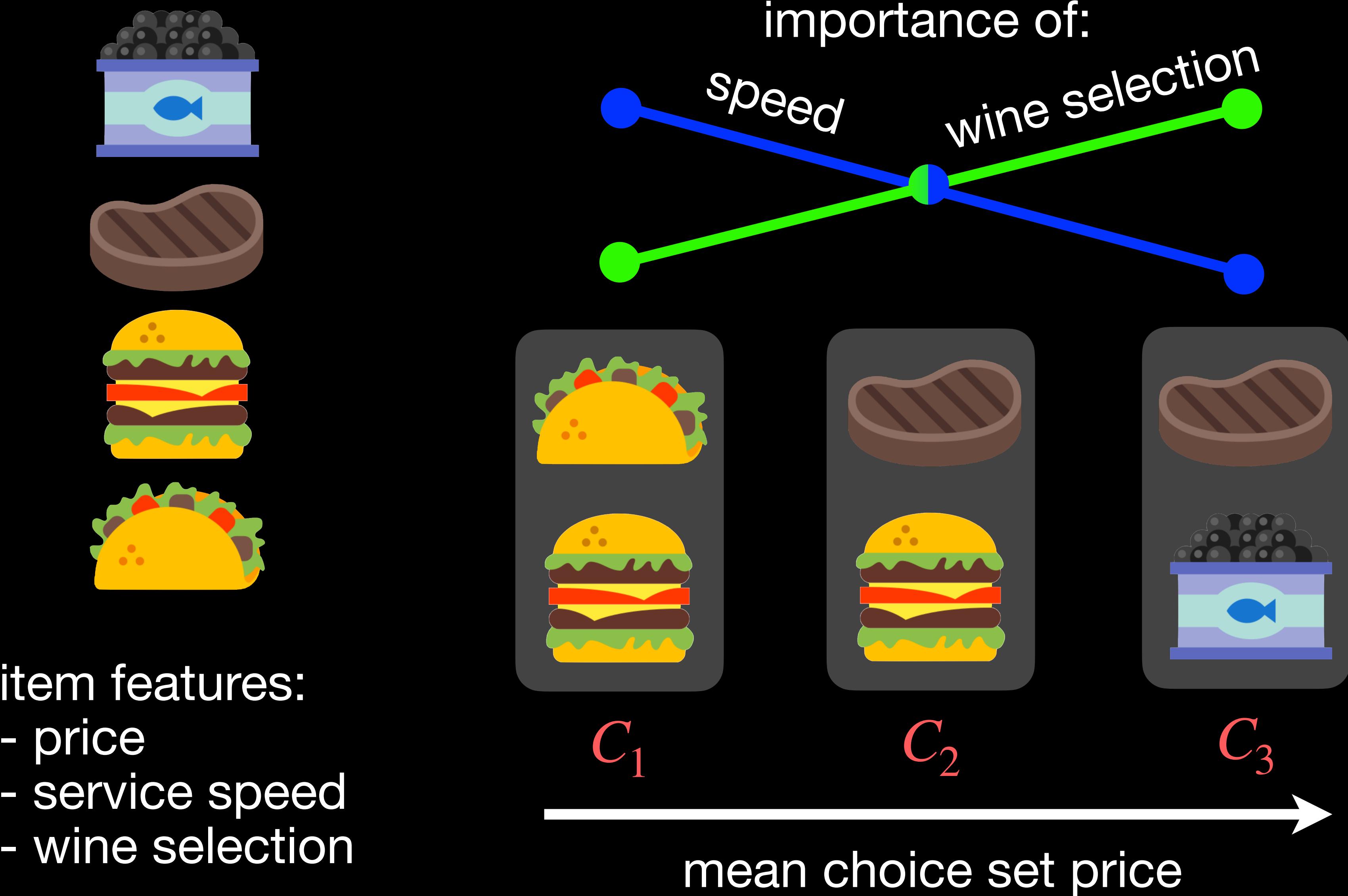
 $C_1$  $C_2$  $C_3$ 

item features:

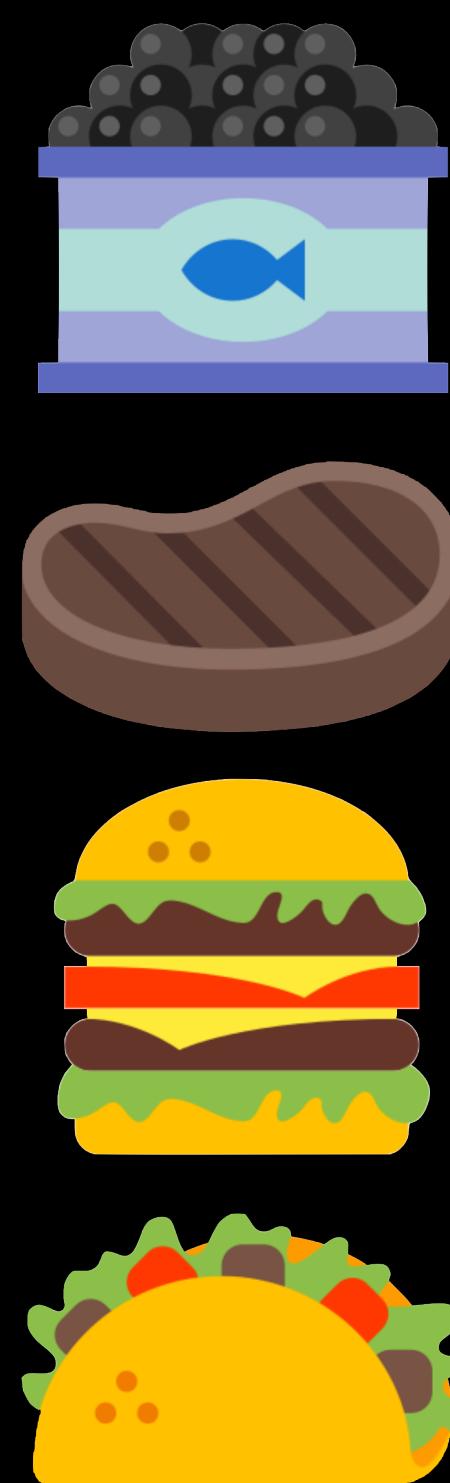
- price
- service speed
- wine selection

mean choice set price

# LCL example: restaurant selection

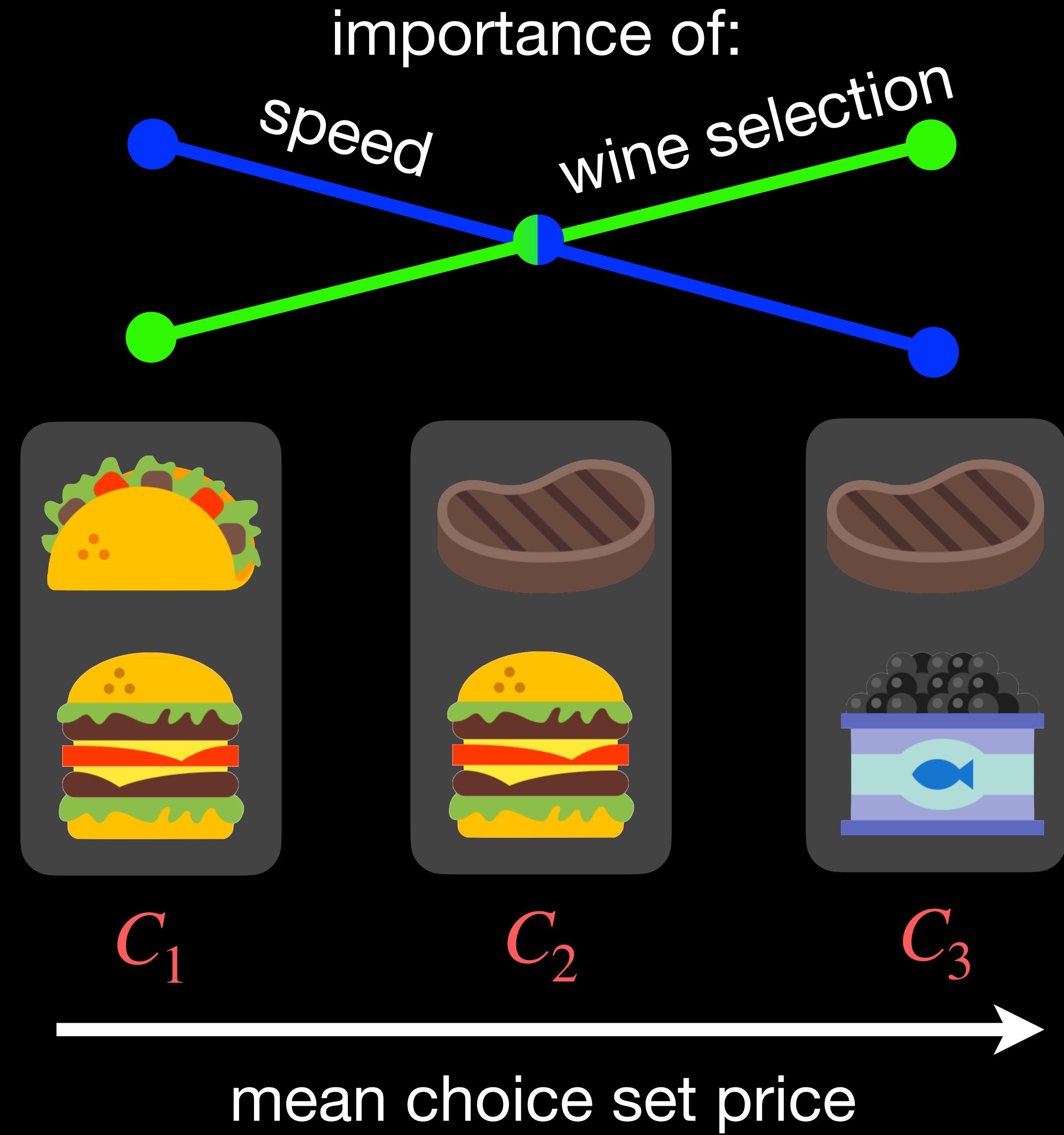


# LCL example: restaurant selection



item features:

- price
- service speed
- wine selection



$$A = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

# LCL identifiability, fully characterized

# LCL identifiability, fully characterized

model is *identifiable* from dataset  $\mathcal{D}$  if no two parameter values result in the same probability distribution

# LCL identifiability, fully characterized

model is *identifiable* from dataset  $\mathcal{D}$  if no two parameter values result in the same probability distribution

→ important for inference and interpretation

# LCL identifiability, fully characterized

model is *identifiable* from dataset  $\mathcal{D}$  if no two parameter values result in the same probability distribution

→ important for inference and interpretation

*Theorem 1.* A  $d$ -feature linear context logit is identifiable from a dataset  $\mathcal{D}$  if and only if

$$\text{span} \left\{ \begin{bmatrix} x_C \\ 1 \end{bmatrix} \otimes (x_i - x_C) \mid C \in \mathcal{C}_{\mathcal{D}}, i \in C \right\} = \mathbb{R}^{d^2+d}. \quad (6)$$

( $\mathcal{C}_{\mathcal{D}}$ : unique choice sets in  $\mathcal{D}$ ,  $\otimes$ : Kronecker product)

# LCL identifiability, fully characterized

model is *identifiable* from dataset  $\mathcal{D}$  if no two parameter values result in the same probability distribution

→ important for inference and interpretation

*Theorem 1.* A  $d$ -feature linear context logit is identifiable from a dataset  $\mathcal{D}$  if and only if

$$\text{span} \left\{ \begin{bmatrix} x_C \\ 1 \end{bmatrix} \otimes (x_i - x_C) \mid C \in \mathcal{C}_{\mathcal{D}}, i \in C \right\} = \mathbb{R}^{d^2+d}. \quad (6)$$

( $\mathcal{C}_{\mathcal{D}}$ : unique choice sets in  $\mathcal{D}$ ,  $\otimes$ : Kronecker product)

*intuition:* need varied choice sets containing varied items

# LCL extension: *Decomposed LCL (DLCL)*

# LCL extension: *Decomposed LCL (DLCL)*

- combines *mixed logit* with LCL
- more flexible but harder to train (*expectation-maximization*)

# LCL extension: *Decomposed LCL (DLCL)*

- combines *mixed logit* with LCL
- more flexible but harder to train (*expectation-maximization*)

$$\Pr(i \mid C) = \sum_{k=1}^d \pi_k \frac{\exp([B_k + A_k(x_C)_k]^T x_i)}{\sum_{j \in C} \exp([B_k + A_k(x_C)_k]^T x_j)}$$

## LCL extension: *Decomposed LCL (DLCL)*

- combines *mixed logit* with LCL
- more flexible but harder to train (*expectation-maximization*)

$$\Pr(i \mid C) = \sum_{k=1}^d \pi_k \frac{\exp([B_k + A_k(x_C)_k]^T x_i)}{\sum_{j \in C} \exp([B_k + A_k(x_C)_k]^T x_j)}$$

- see paper for details

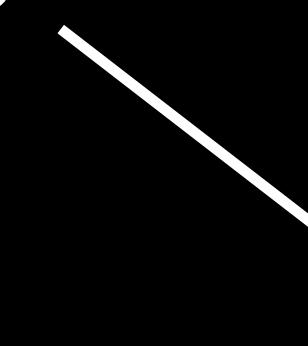
# Results on choice data

# Choice datasets

Dataset	Choices	Features	Largest Choice Set
DISTRICT	5376	27	2
DISTRICT-SMART	5376	6	2
SUSHI	5000	6	10
EXPEDIA	276593	5	38
CAR-A	2675	4	2
CAR-B	2206	5	2
CAR-ALT	4654	21	6

# Choice datasets

favorite sushi types



Dataset	Choices	Features	Largest Choice Set
DISTRICT	5376	27	2
DISTRICT-SMART	5376	6	2
SUSHI	5000	6	10
EXPEDIA	276593	5	38
CAR-A	2675	4	2
CAR-B	2206	5	2
CAR-ALT	4654	21	6

# Choice datasets

favorite sushi types

hotel bookings

Dataset	Choices	Features	Largest Choice Set
DISTRICT	5376	27	2
DISTRICT-SMART	5376	6	2
SUSHI	5000	6	10
EXPEDIA	276593	5	38
CAR-A	2675	4	2
CAR-B	2206	5	2
CAR-ALT	4654	21	6

# LCL improves model fit

whole-dataset negative log-likelihood (lower = better)

	CL	LCL	Mixed logit	DLCL
DISTRICT	3313	<b>3130</b>	3258	3206
DISTRICT-SMART	3426	<b>3278*</b>	3351	3303 <sup>†</sup>
EXPEDIA	839505	837649*	839055	<b>837569<sup>†</sup></b>
SUSHI	9821	9773*	9793	<b>9764</b>
CAR-A	1702	1694	1696	<b>1692</b>
CAR-B	1305	1295	1297	<b>1284</b>
CAR-ALT	7393	<b>6733*</b>	7301	7011 <sup>†</sup>

\*significant likelihood ratio test vs MNL ( $p < 0.001$ )

†significant likelihood ratio test vs mixed logit ( $p < 0.001$ )

# LCL can improve out-of-sample prediction performance

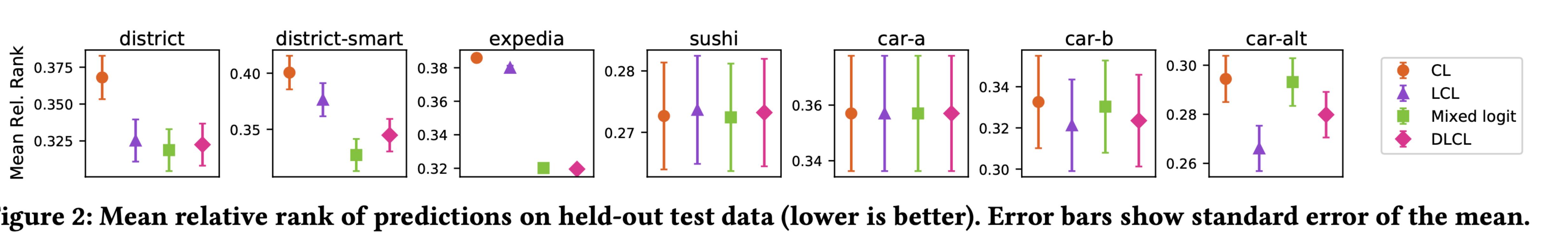


Figure 2: Mean relative rank of predictions on held-out test data (lower is better). Error bars show standard error of the mean.

# LCL can test individual effects for significance

# LCL can test individual effects for significance

Compute std. errs. (and z-scores) for each parameter estimate using MLE *asymptotic normality*

# LCL can test individual effects for significance

Compute std. errs. (and z-scores) for each parameter estimate using MLE *asymptotic normality*

**Table 4: Five largest context effects in SUSHI.**

Effect ( $q$ on $p$ )	$A_{pq}$ (std. err.)	$p$ -value
<i>popularity</i> on <i>popularity</i>	−0.28 (0.15)	0.066
<i>availability</i> on <i>is maki</i>	0.24 (0.14)	0.087
<i>oiliness</i> on <i>oiliness</i>	−0.20 (0.08)	0.0089
<i>popularity</i> on <i>availability</i>	0.19 (0.14)	0.16
<i>availability</i> on <i>oiliness</i>	−0.18 (0.10)	0.064

# LCL can test individual effects for significance

Compute std. errs. (and z-scores) for each parameter estimate using MLE *asymptotic normality*

**Table 4: Five largest context effects in SUSHI.**

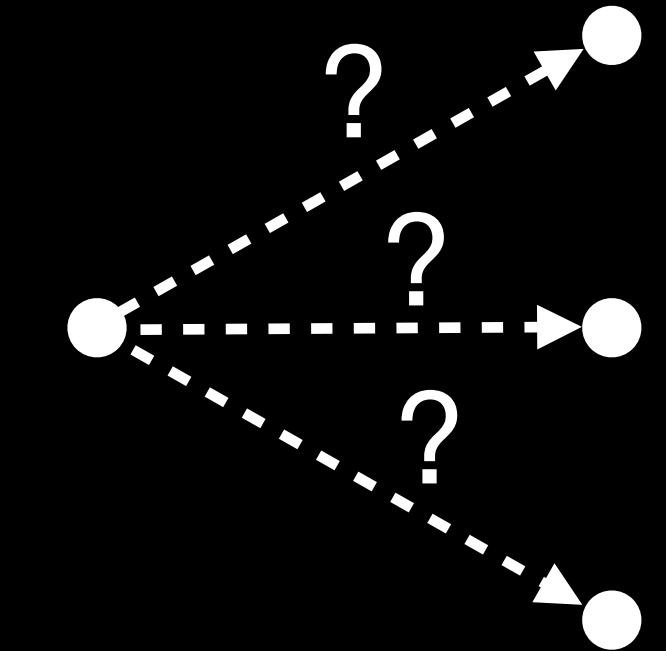
Effect ( $q$ on $p$ )	$A_{pq}$ (std. err.)	$p$ -value
<i>popularity</i> on <i>popularity</i>	-0.28 (0.15)	0.066
<i>availability</i> on <i>is maki</i>	0.24 (0.14)	0.087
<i>oiliness</i> on <i>oiliness</i>	-0.20 (0.08)	0.0089
<i>popularity</i> on <i>availability</i>	0.19 (0.14)	0.16
<i>availability</i> on <i>oiliness</i>	-0.18 (0.10)	0.064

**Table 5: Five largest context effects in EXPEDIA.**

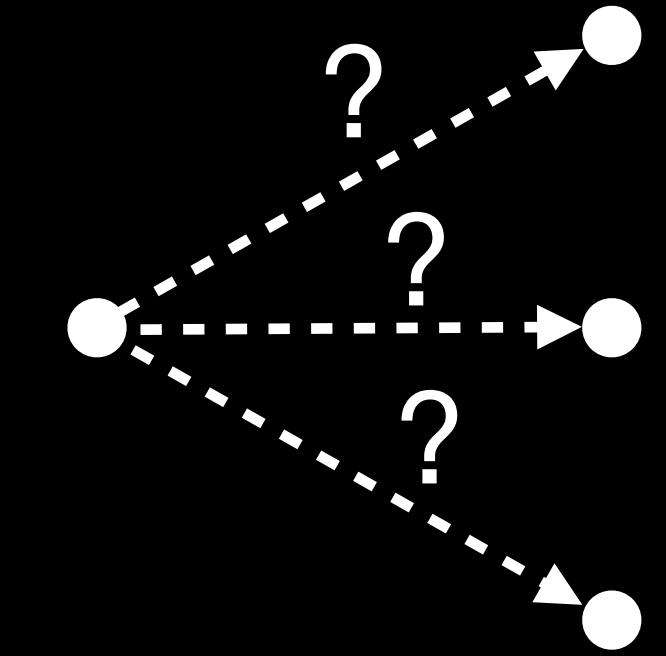
Effect ( $q$ on $p$ )	$A_{pq}$ (std. err.)	$p$ -value
<i>location score</i> on <i>price</i>	-0.47 (0.05)	$< 10^{-16}$
<i>on promotion</i> on <i>price</i>	0.27 (0.03)	$< 10^{-16}$
<i>review score</i> on <i>price</i>	-0.19 (0.03)	$1.4 \times 10^{-9}$
<i>star rating</i> on <i>price</i>	0.15 (0.04)	$6.7 \times 10^{-5}$
<i>price</i> on <i>star rating</i>	0.10 (0.00)	$< 10^{-16}$

# Social network application

# What factors drive edge formation?

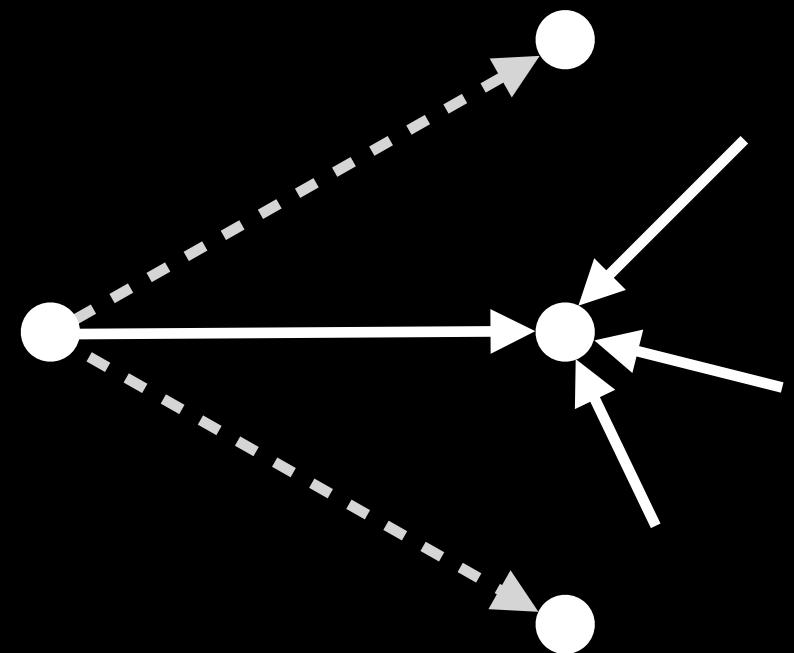


# What factors drive edge formation?



## *Preferential attachment*

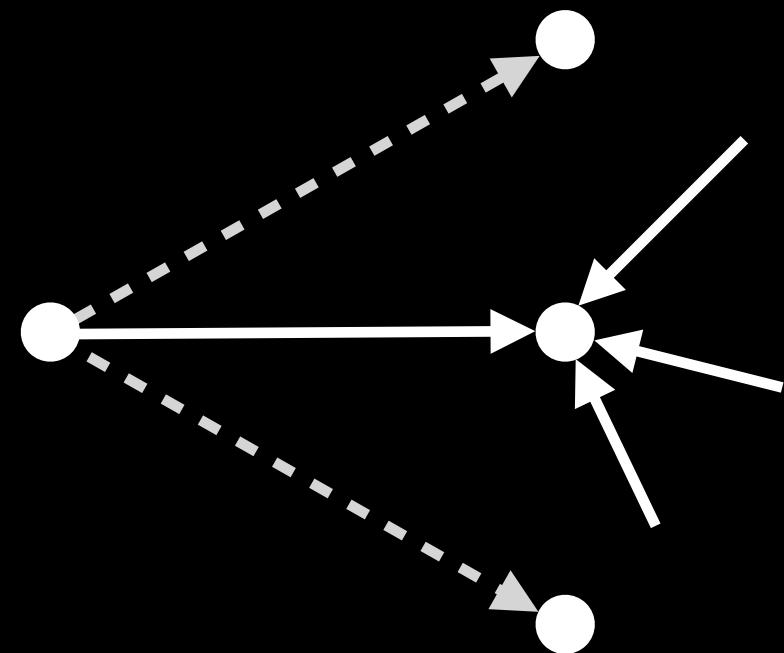
(Barabási & Albert, *Science* 1999)



# What factors drive edge formation?

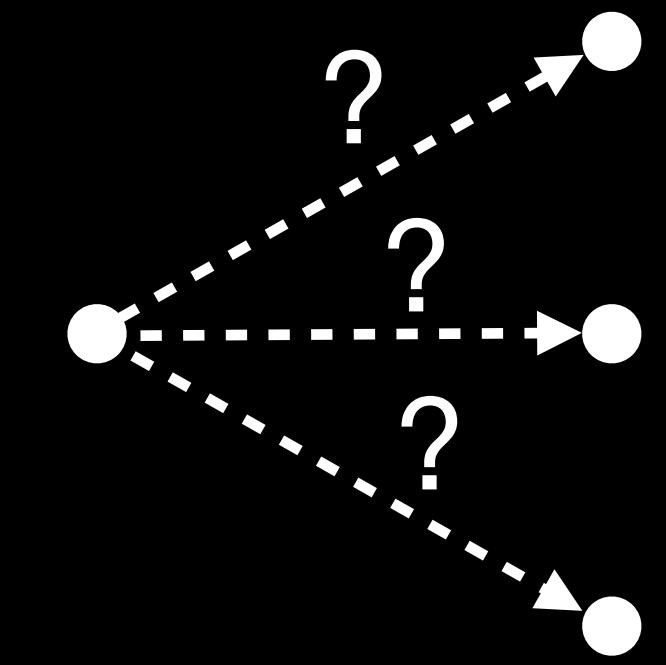
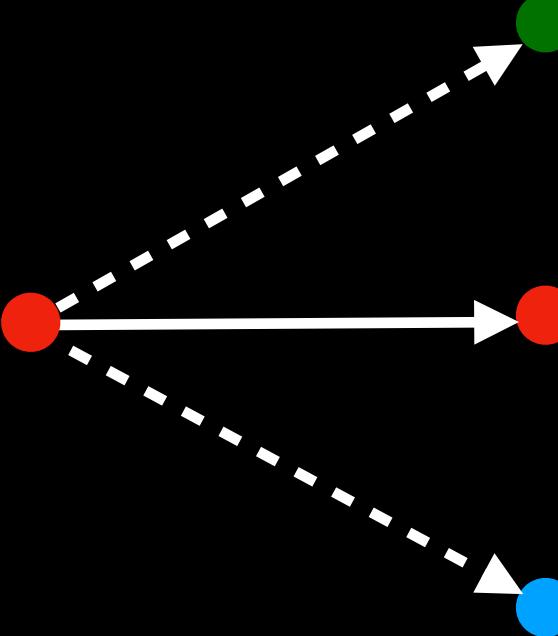
## *Preferential attachment*

(Barabási & Albert, *Science* 1999)



## *Homophily*

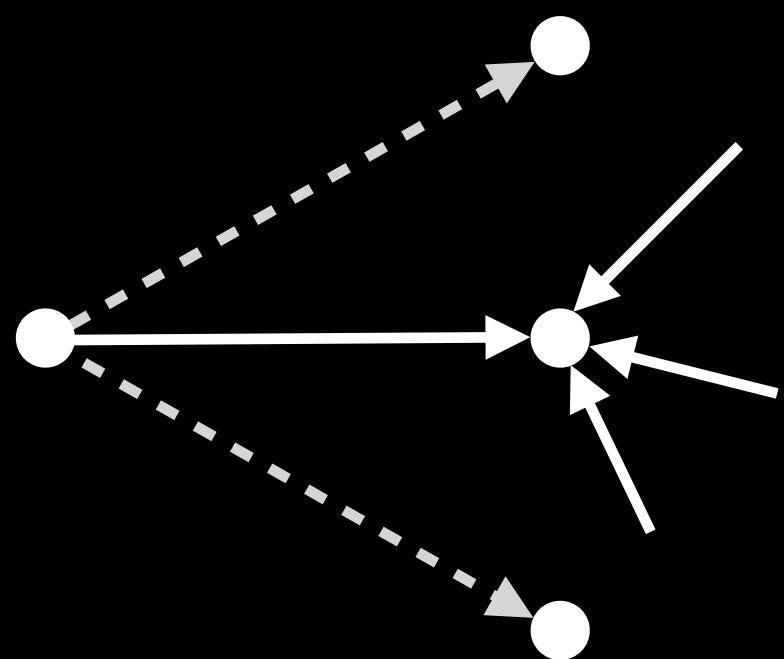
(McPherson et al., *Annual Review of Sociology* 2001)  
(Papadopoulos et al., *Nature* 2012)



# What factors drive edge formation?

## Preferential attachment

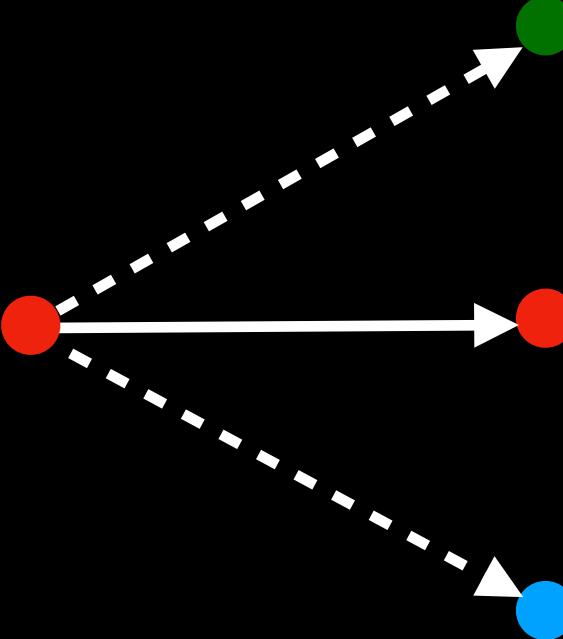
(Barabási & Albert, *Science* 1999)



## Homophily

(McPherson et al., *Annual Review of Sociology* 2001)

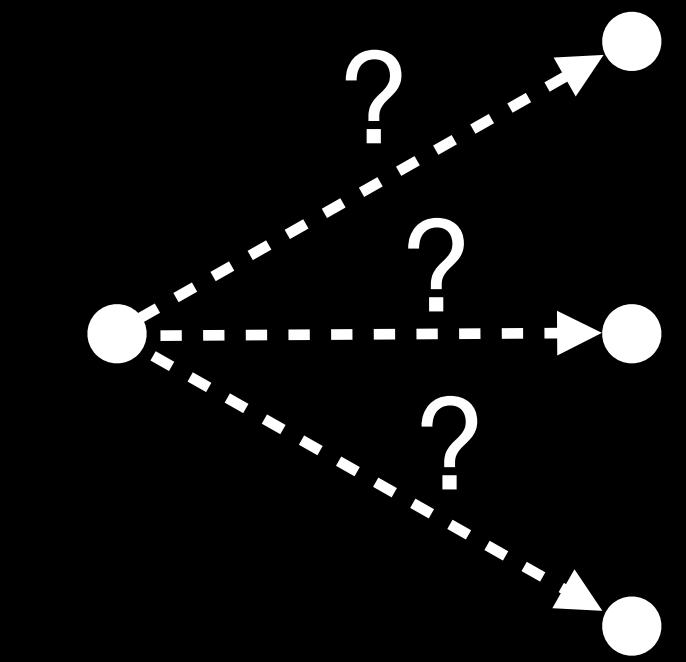
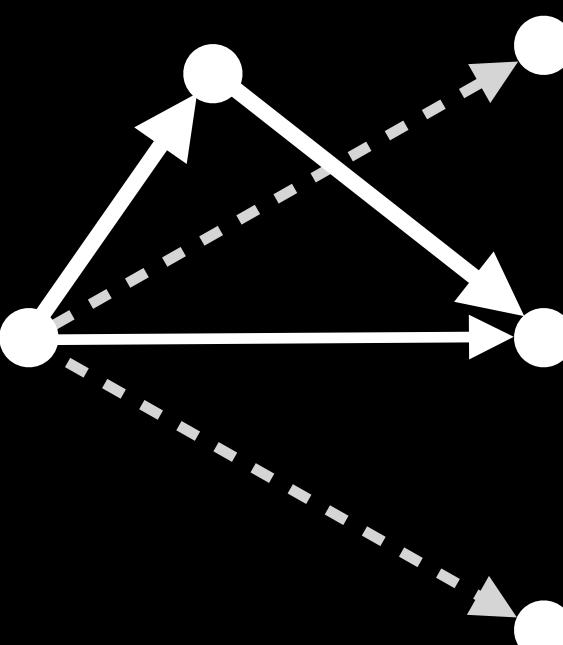
(Papadopoulos et al., *Nature* 2012)



## Triadic closure

(Rapoport, *Bulletin of Mathematical Biophysics* 1953)

(Jin et al., *Physical Review E* 2001)



# “Choosing to grow a graph”

(Overgoor et al., *SINM '19 & WWW '19*)

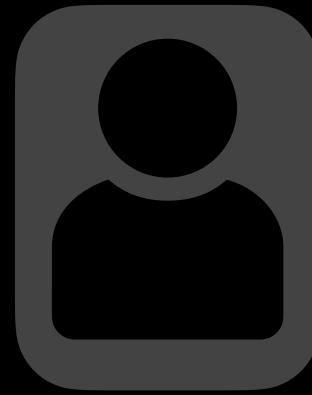
(Gupta & Porter, *arXiv* 2020)

# “Choosing to grow a graph”

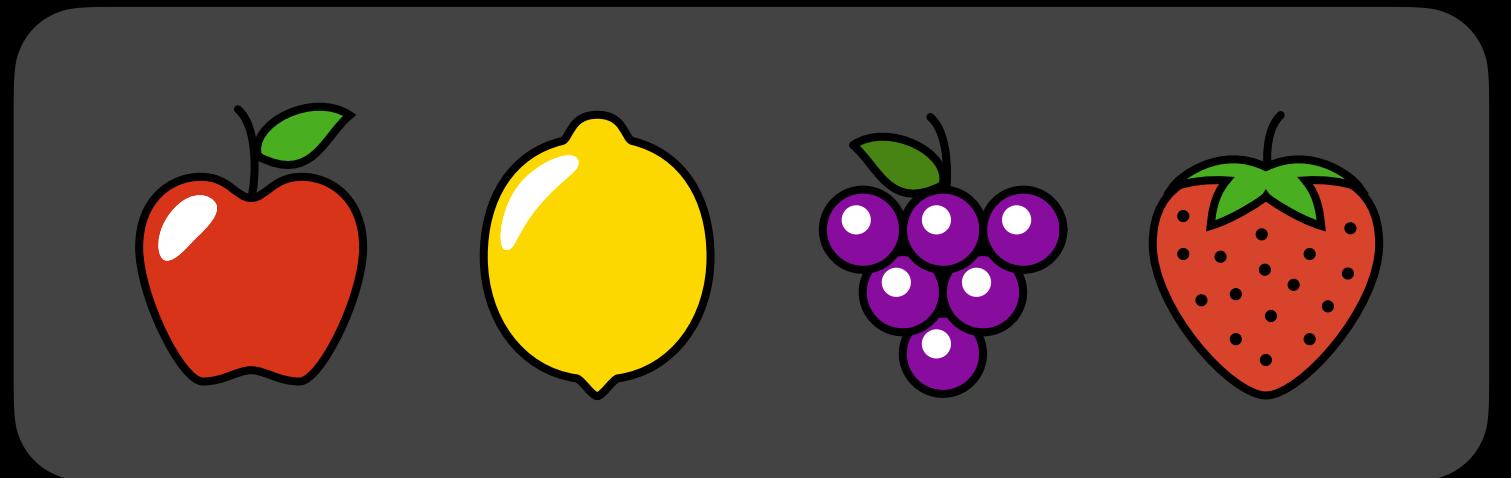
(Overgoor et al., *SINM '19 & WWW '19*)

(Gupta & Porter, *arXiv* 2020)

so far:



*chooser*



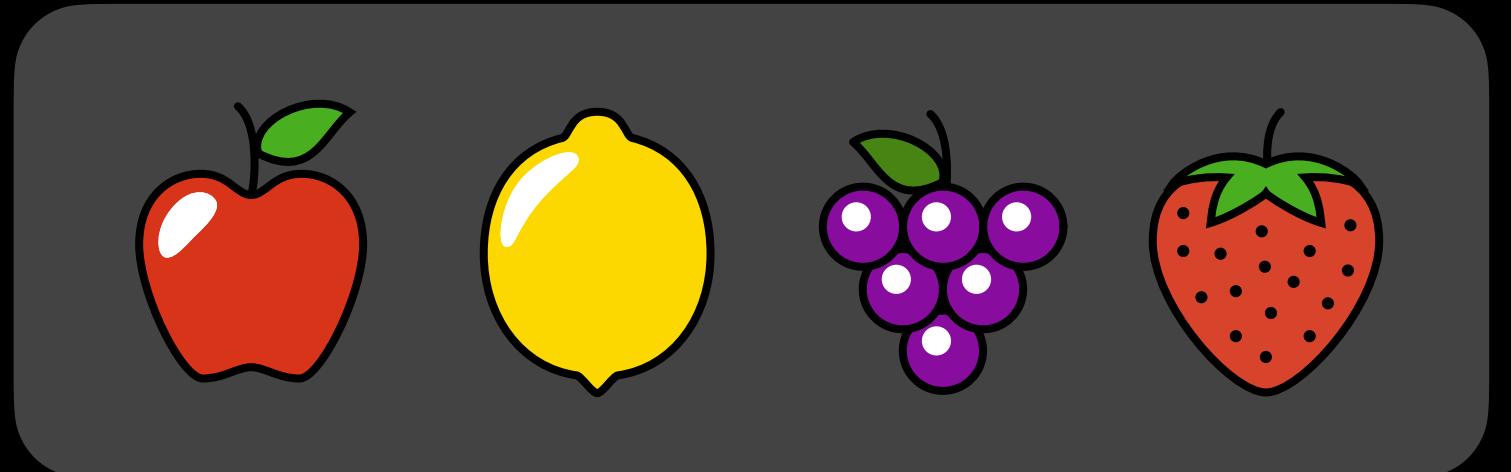
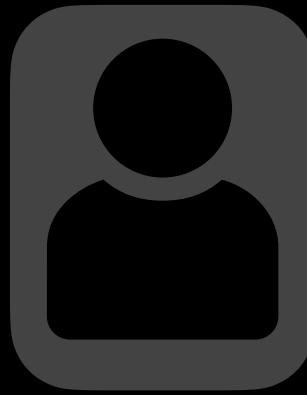
*choice set*

# “Choosing to grow a graph”

(Overgoor et al., *SINM '19 & WWW '19*)

(Gupta & Porter, *arXiv* 2020)

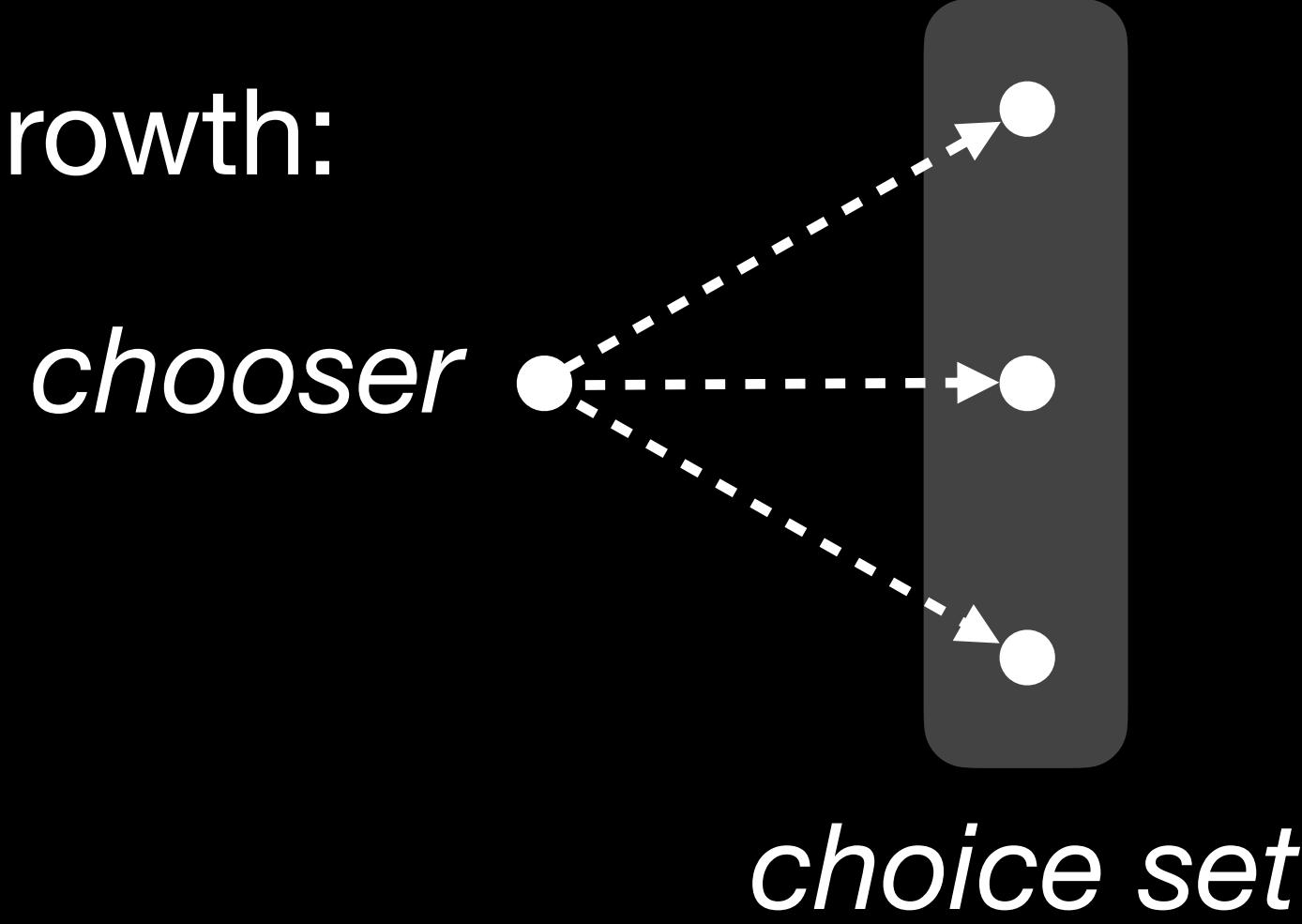
so far:



*chooser*

*choice set*

in network growth:



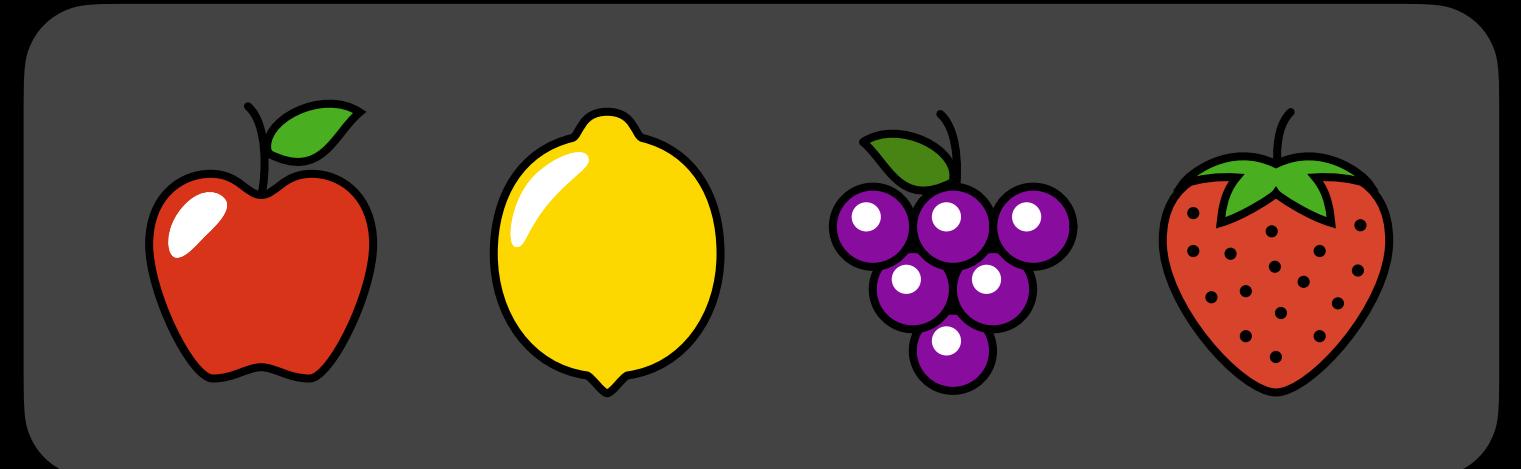
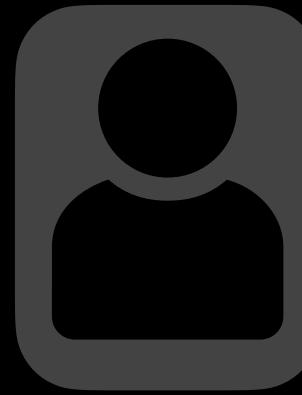
*choice set*

# “Choosing to grow a graph”

(Overgoor et al., S/INM ’19 & WWW ’19)

(Gupta & Porter, arXiv 2020)

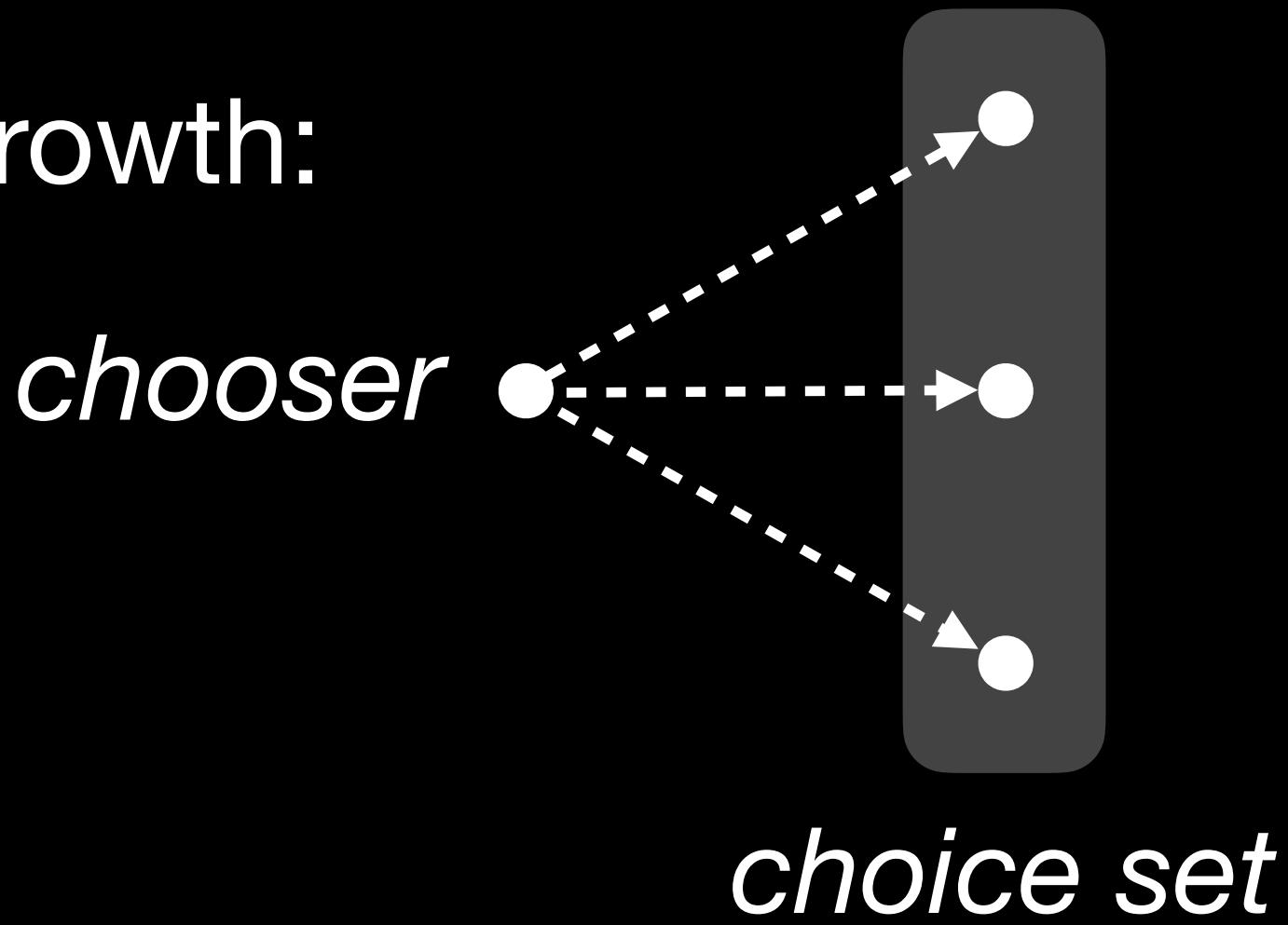
so far:



*chooser*

*choice set*

in network growth:



*choice set*

## Key usage

Timestamped edges

→ meaningful choice sets

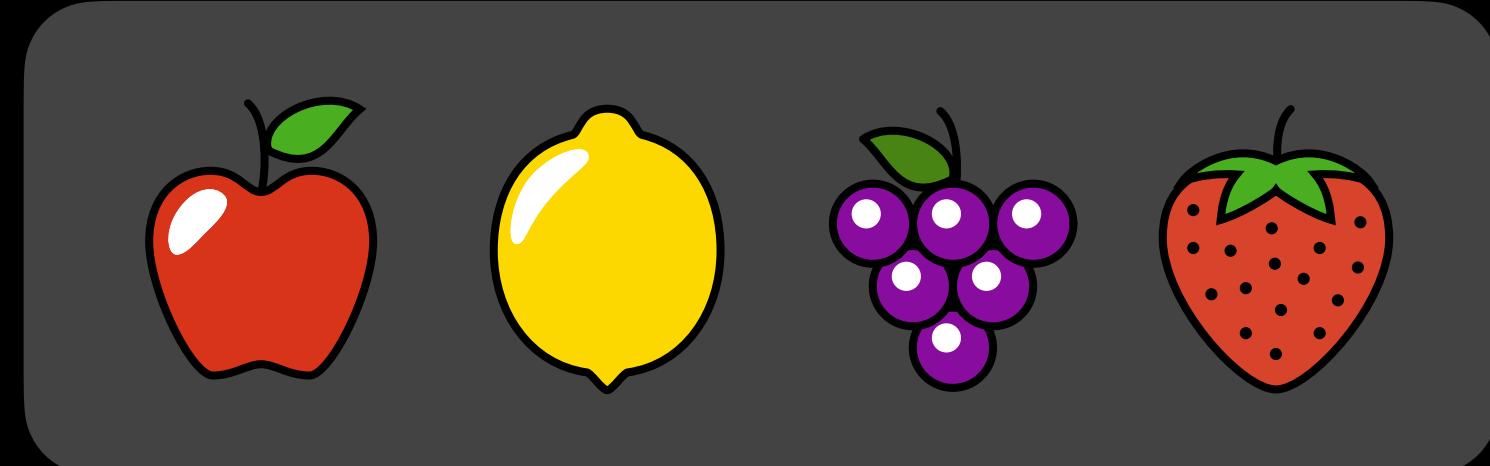
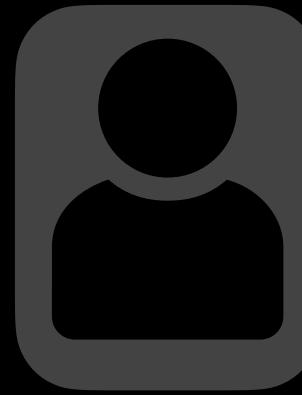
Infer relative importance of edge formation mechanisms from data

# “Choosing to grow a graph”

(Overgoor et al., S/INM ’19 & WWW ’19)

(Gupta & Porter, arXiv 2020)

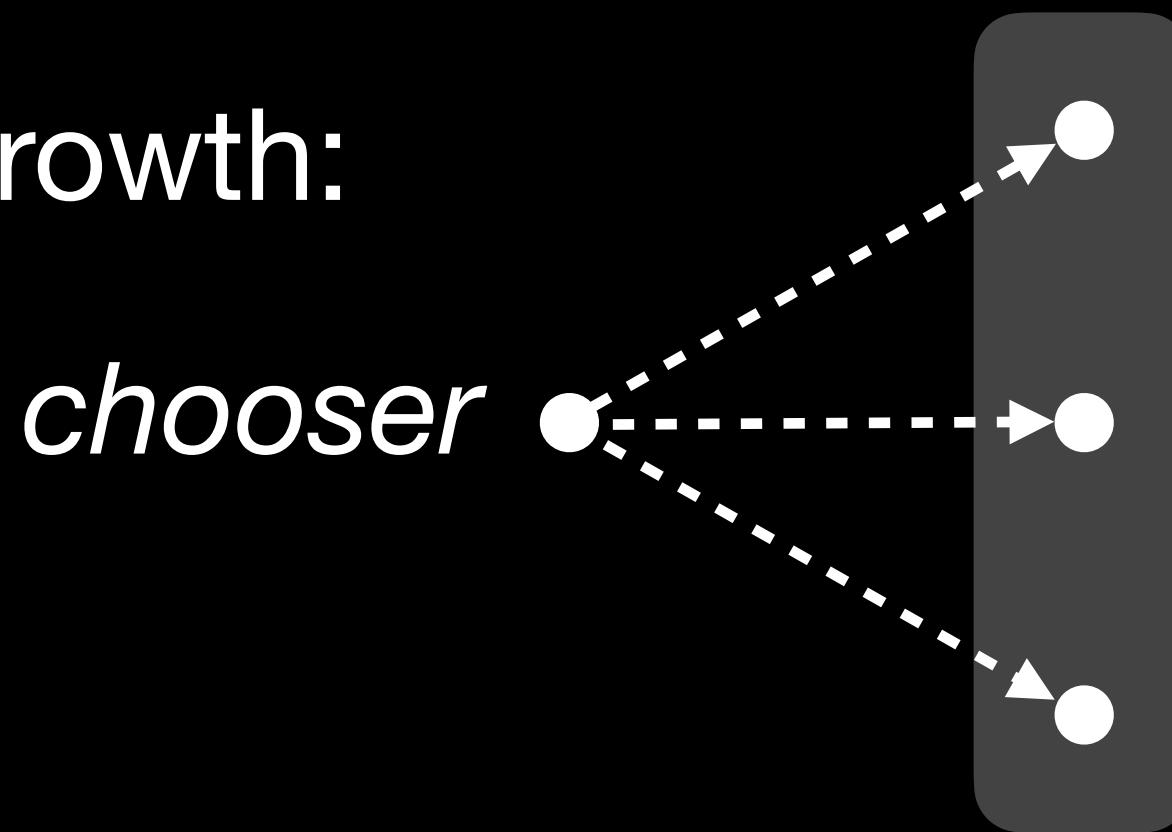
so far:



*chooser*

*choice set*

in network growth:



*choice set*

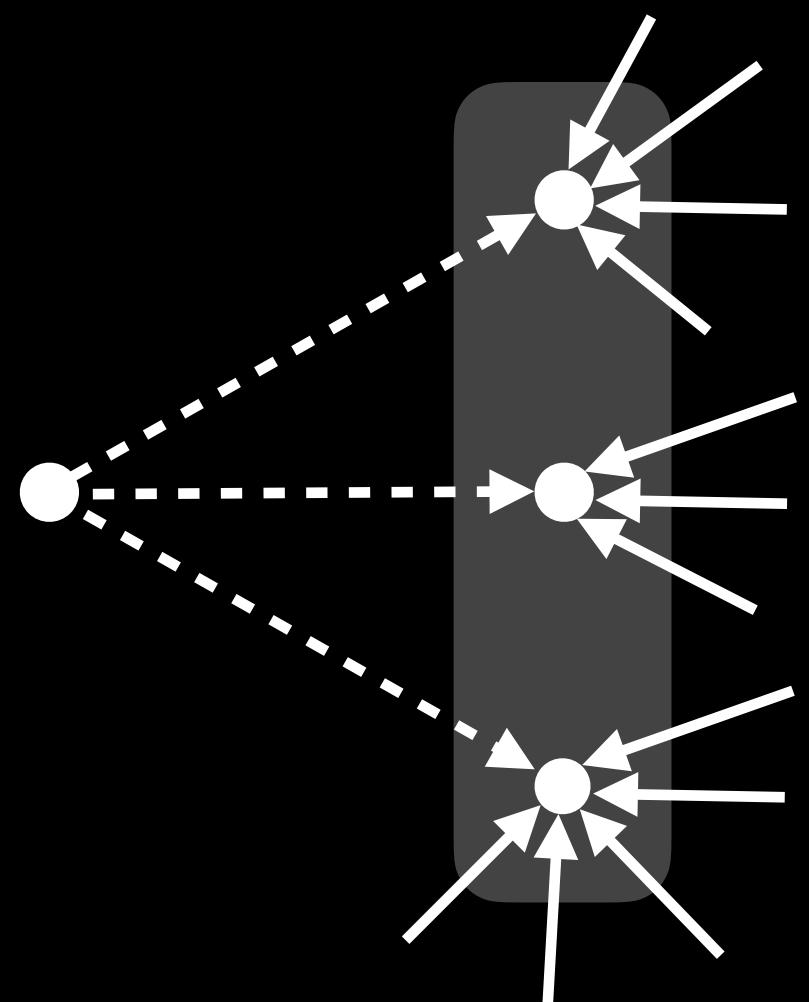
## Key usage

Timestamped edges

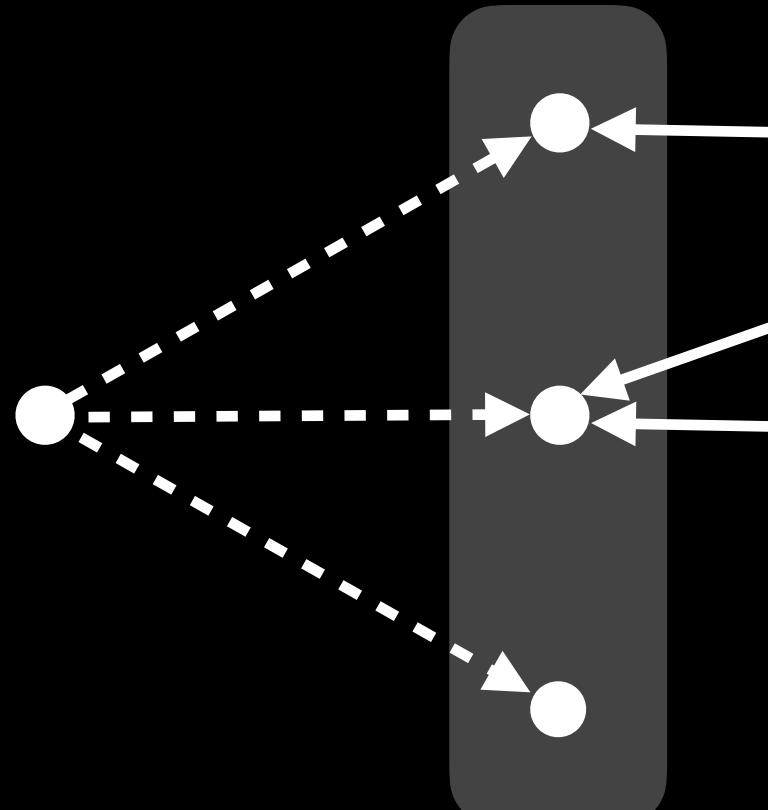
→ meaningful choice sets

Infer relative importance of edge formation mechanisms from data

feature context effects:



vs.



# Choosing to close triangles

*Triadic closure* offers small choice sets

- tractable inference
- varied choice sets

# Choosing to close triangles

*Triadic closure* offers small choice sets  
→ tractable inference  
→ varied choice sets

**Our data**  
Timestamped edges  
(including repeats)

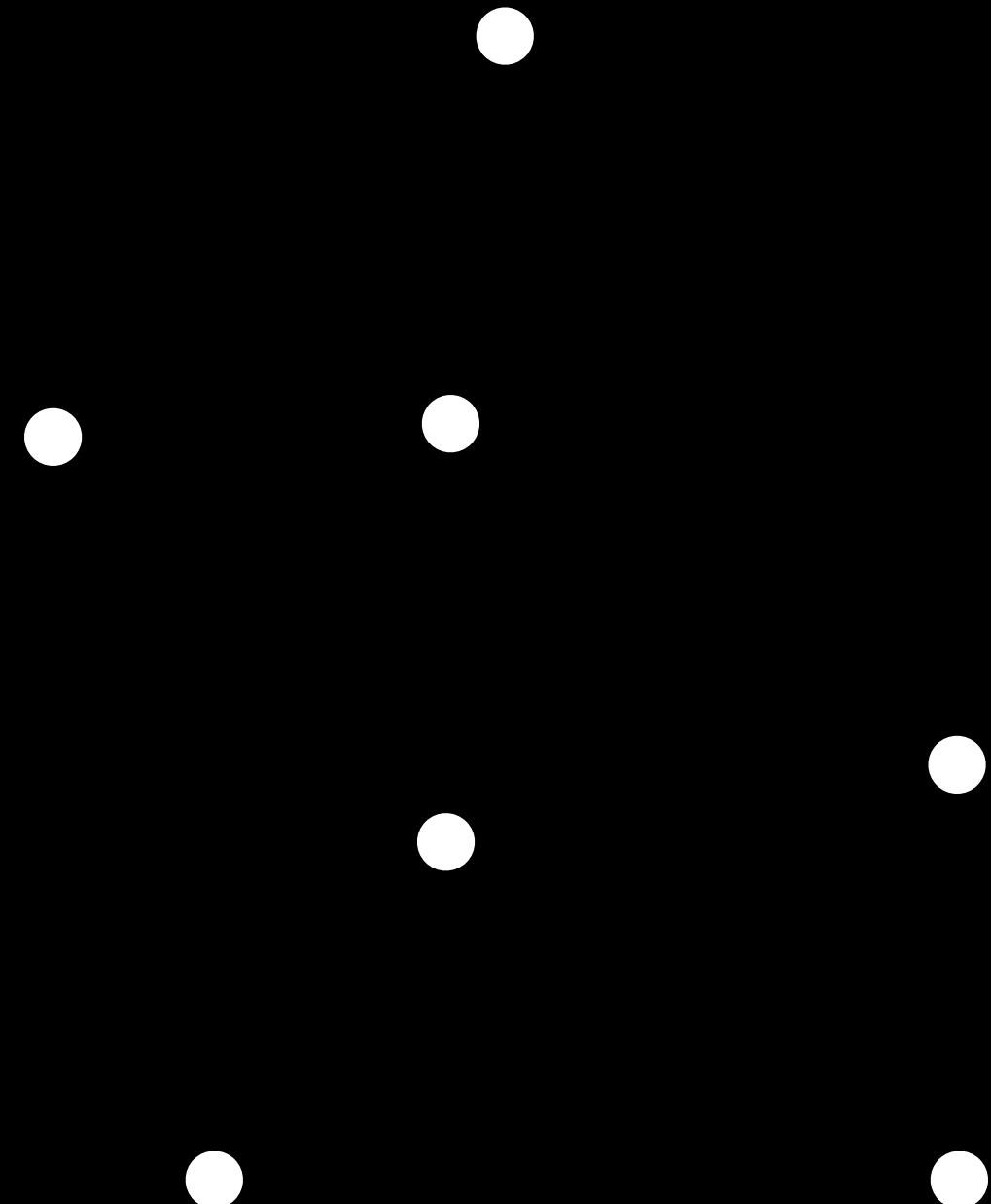
# Choosing to close triangles

*Triadic closure* offers small choice sets

- tractable inference
- varied choice sets

**Our data**

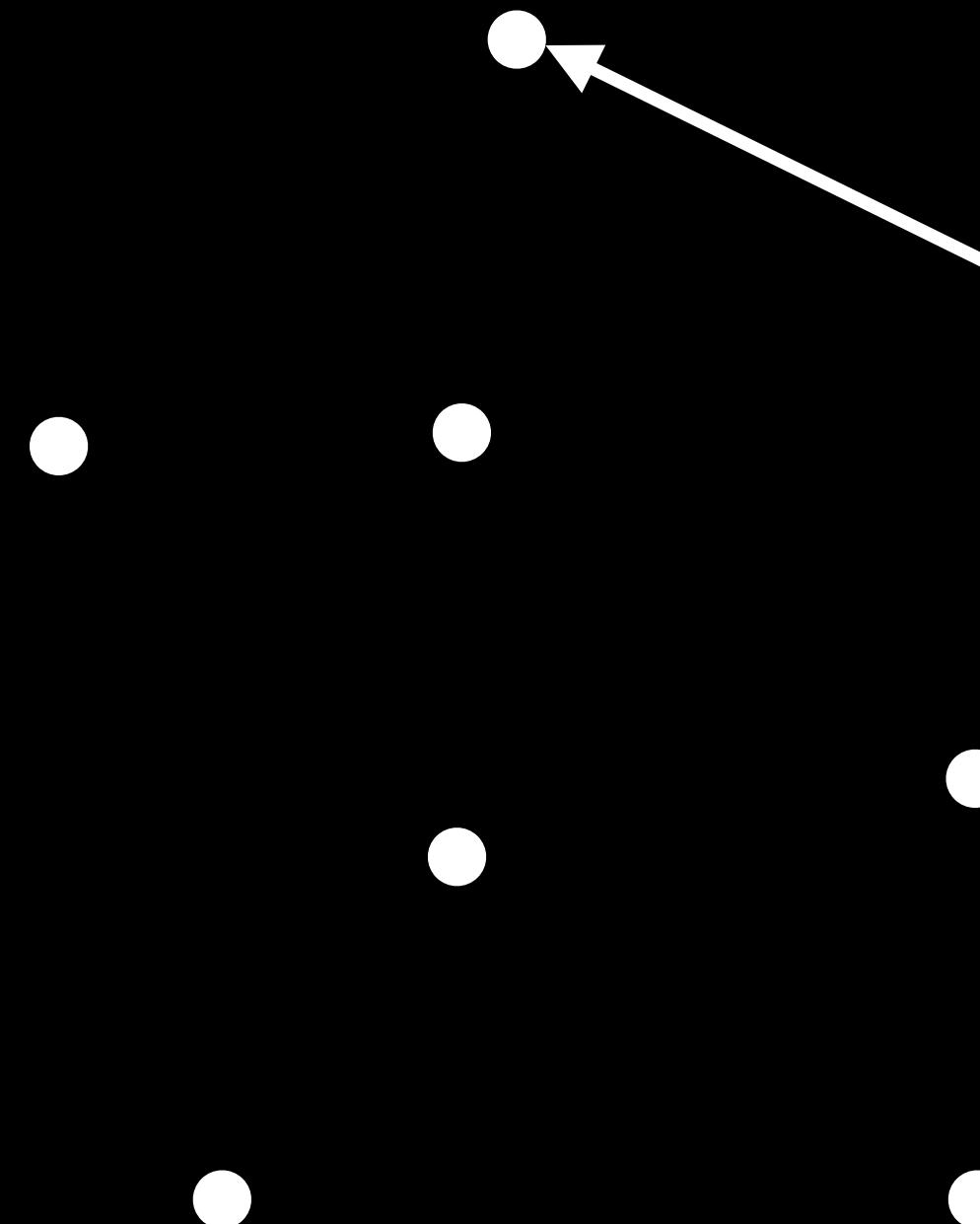
Timestamped edges  
(including repeats)



# Choosing to close triangles

*Triadic closure* offers small choice sets  
→ tractable inference  
→ varied choice sets

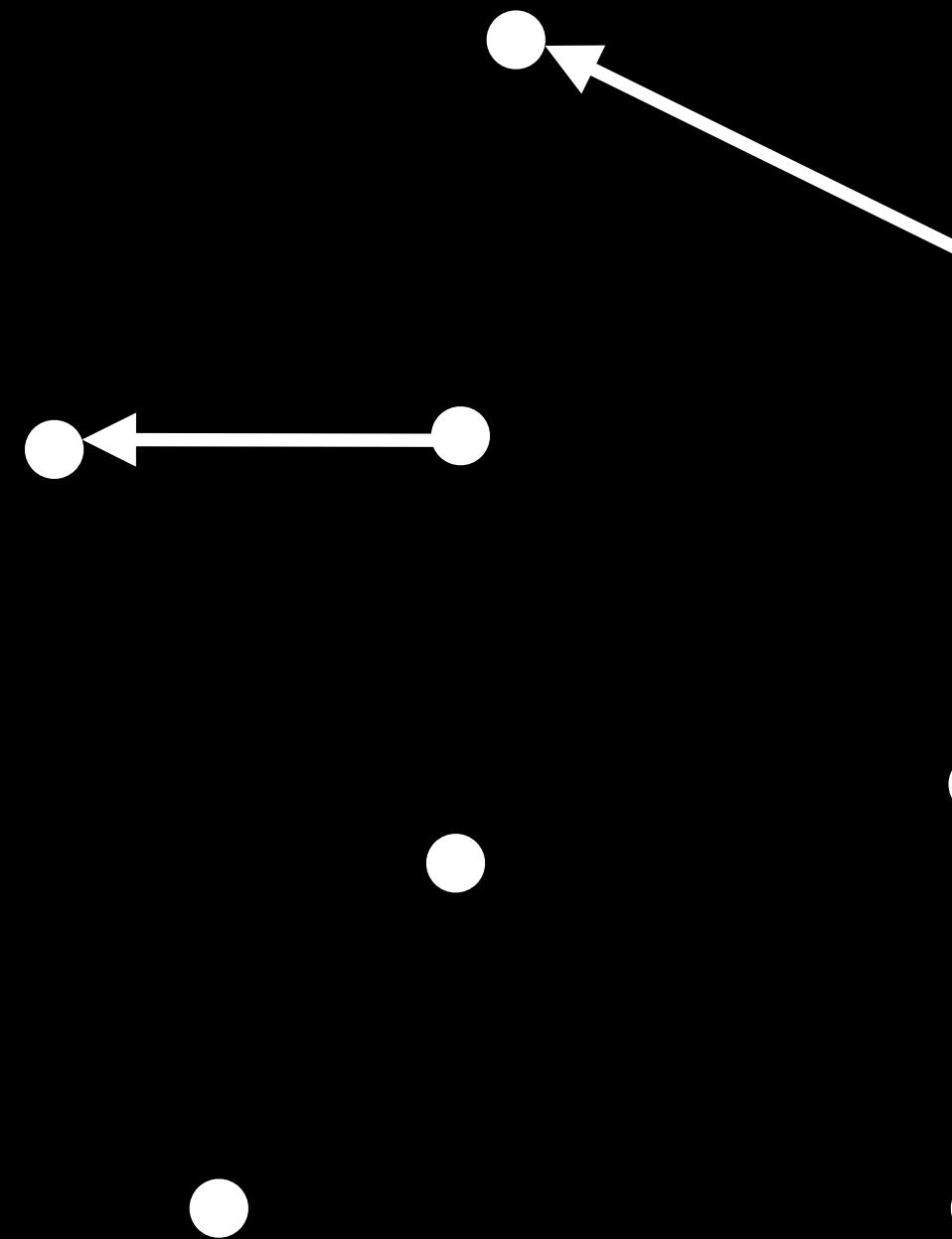
**Our data**  
Timestamped edges  
(including repeats)



# Choosing to close triangles

*Triadic closure* offers small choice sets  
→ tractable inference  
→ varied choice sets

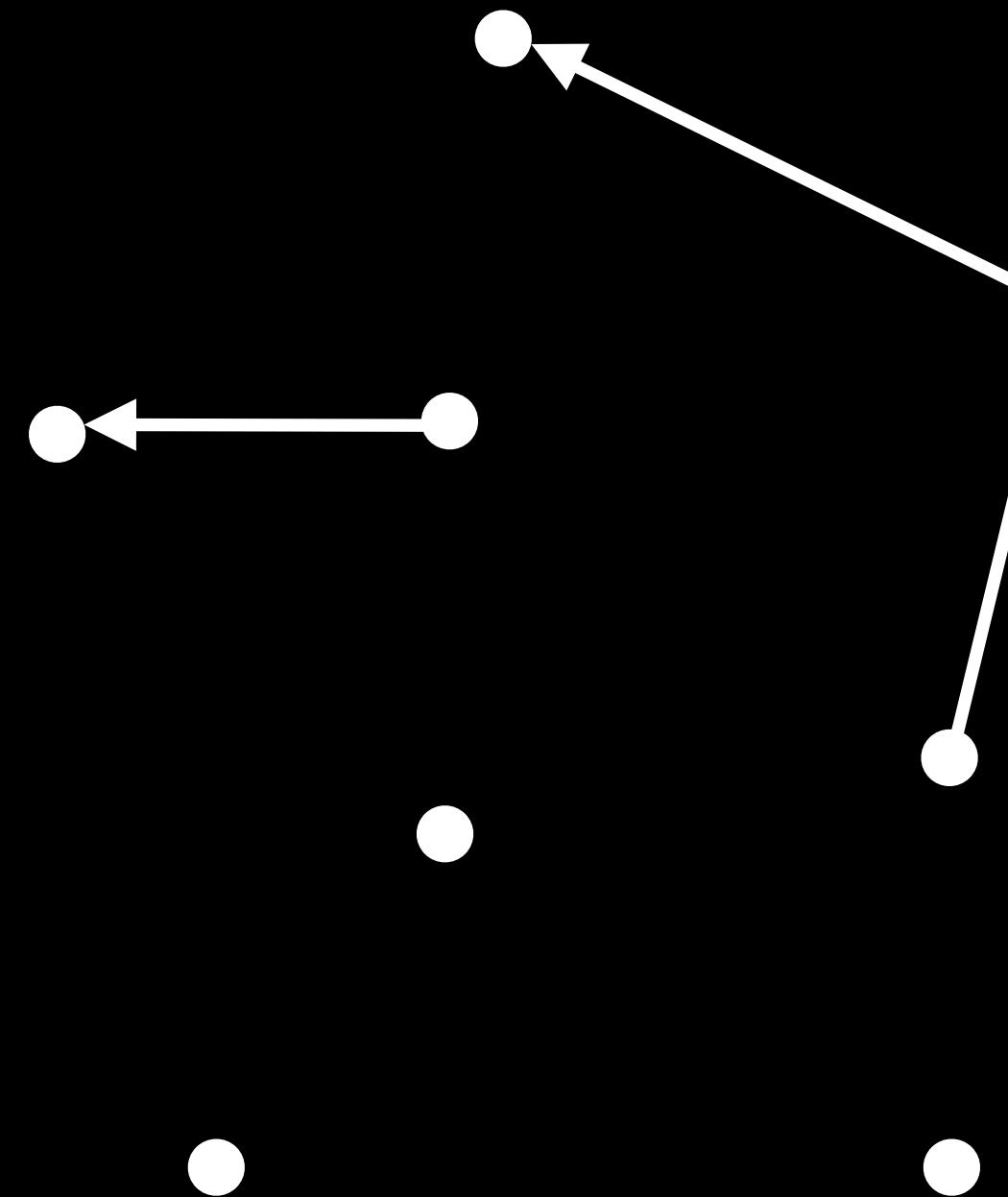
**Our data**  
Timestamped edges  
(including repeats)



# Choosing to close triangles

*Triadic closure* offers small choice sets  
→ tractable inference  
→ varied choice sets

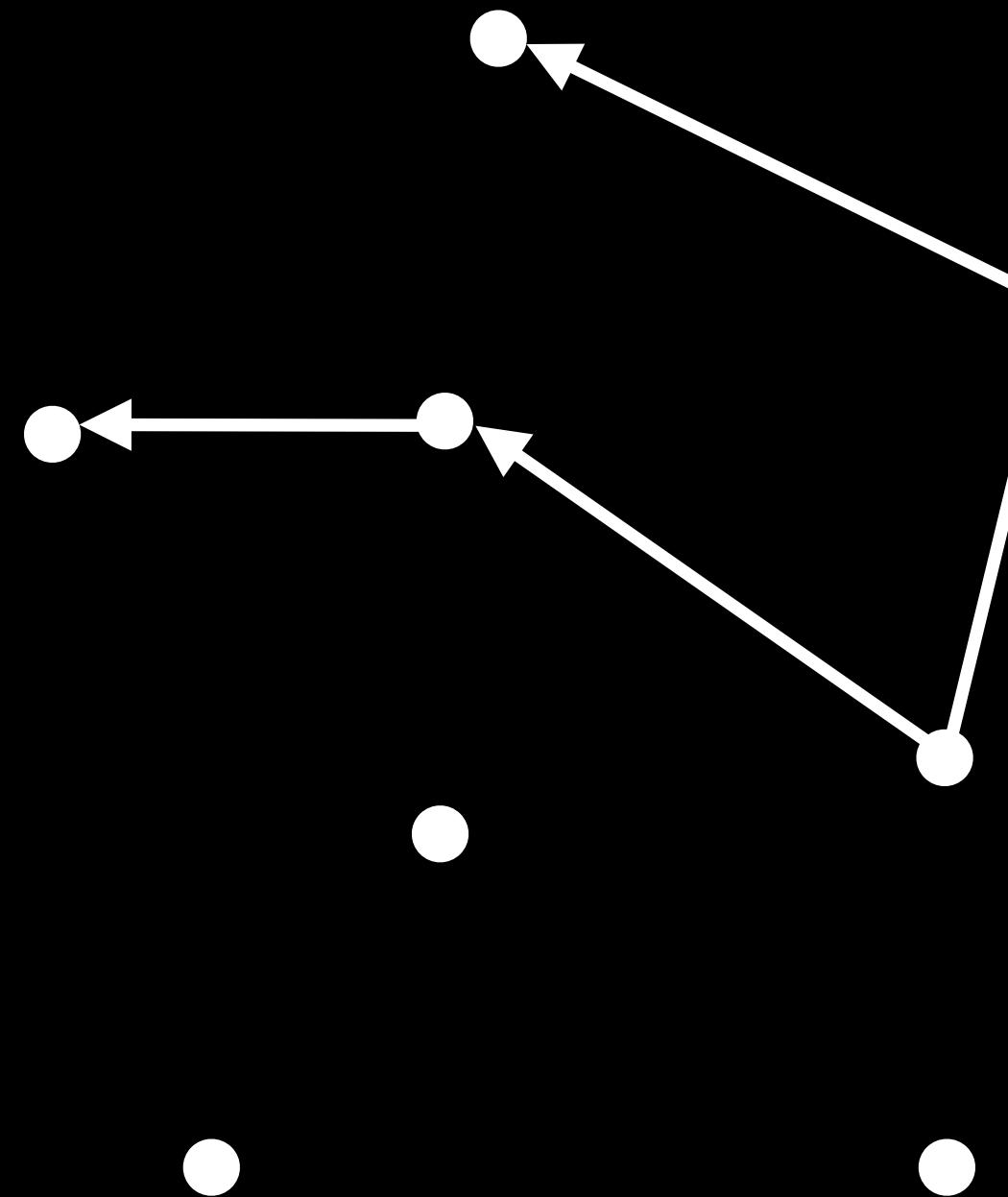
**Our data**  
Timestamped edges  
(including repeats)



# Choosing to close triangles

*Triadic closure* offers small choice sets  
→ tractable inference  
→ varied choice sets

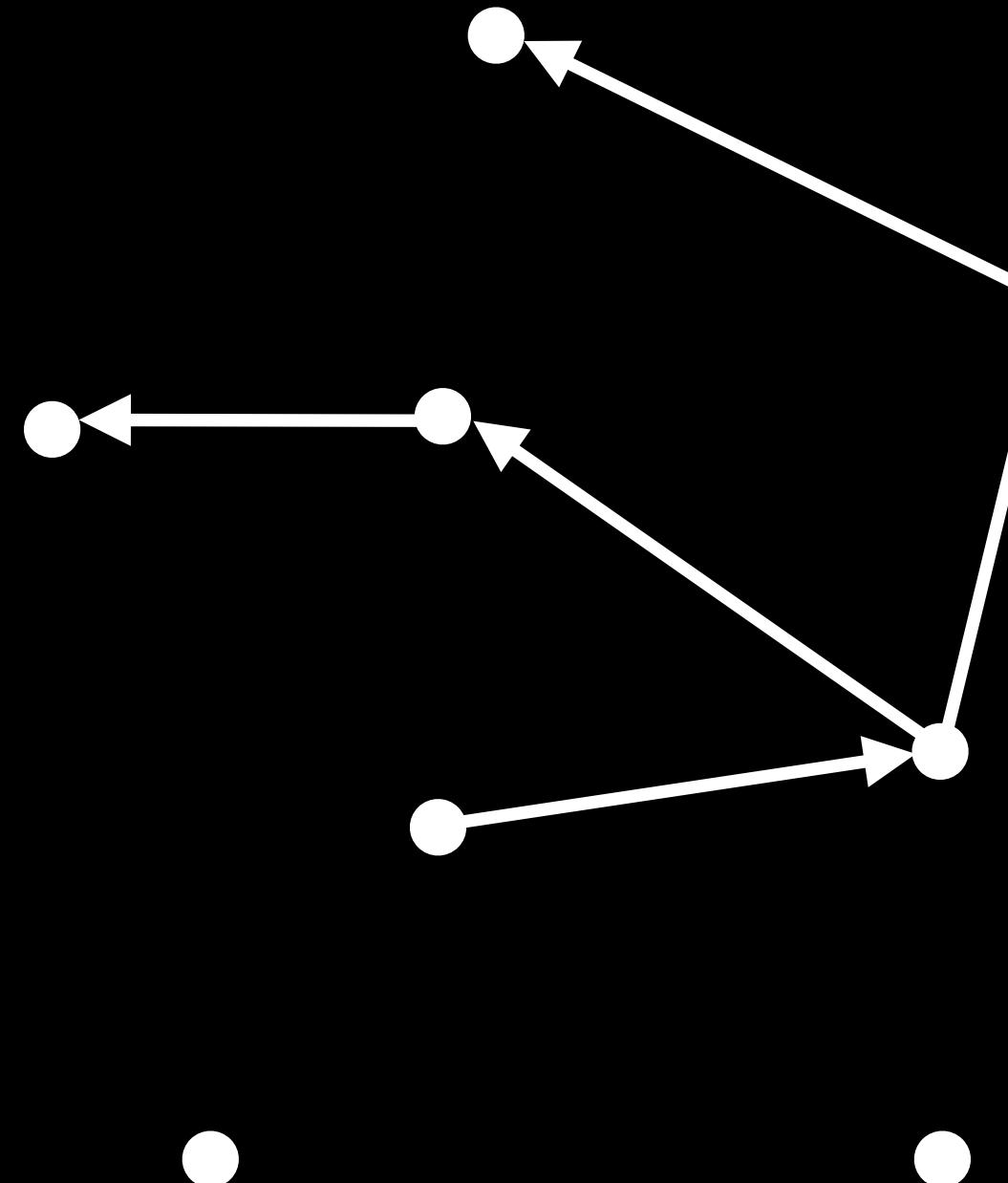
**Our data**  
Timestamped edges  
(including repeats)



# Choosing to close triangles

*Triadic closure* offers small choice sets  
→ tractable inference  
→ varied choice sets

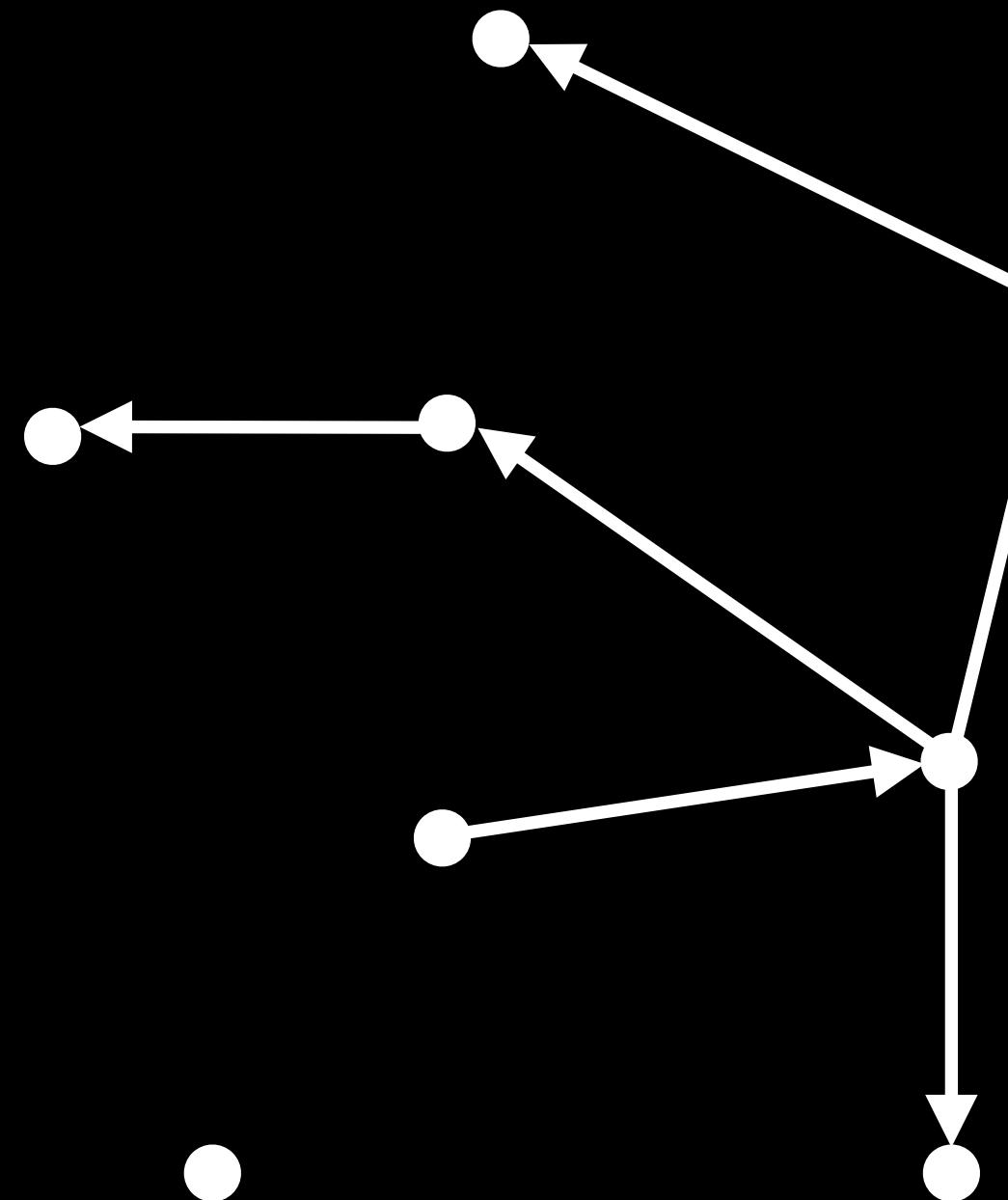
**Our data**  
Timestamped edges  
(including repeats)



# Choosing to close triangles

*Triadic closure* offers small choice sets  
→ tractable inference  
→ varied choice sets

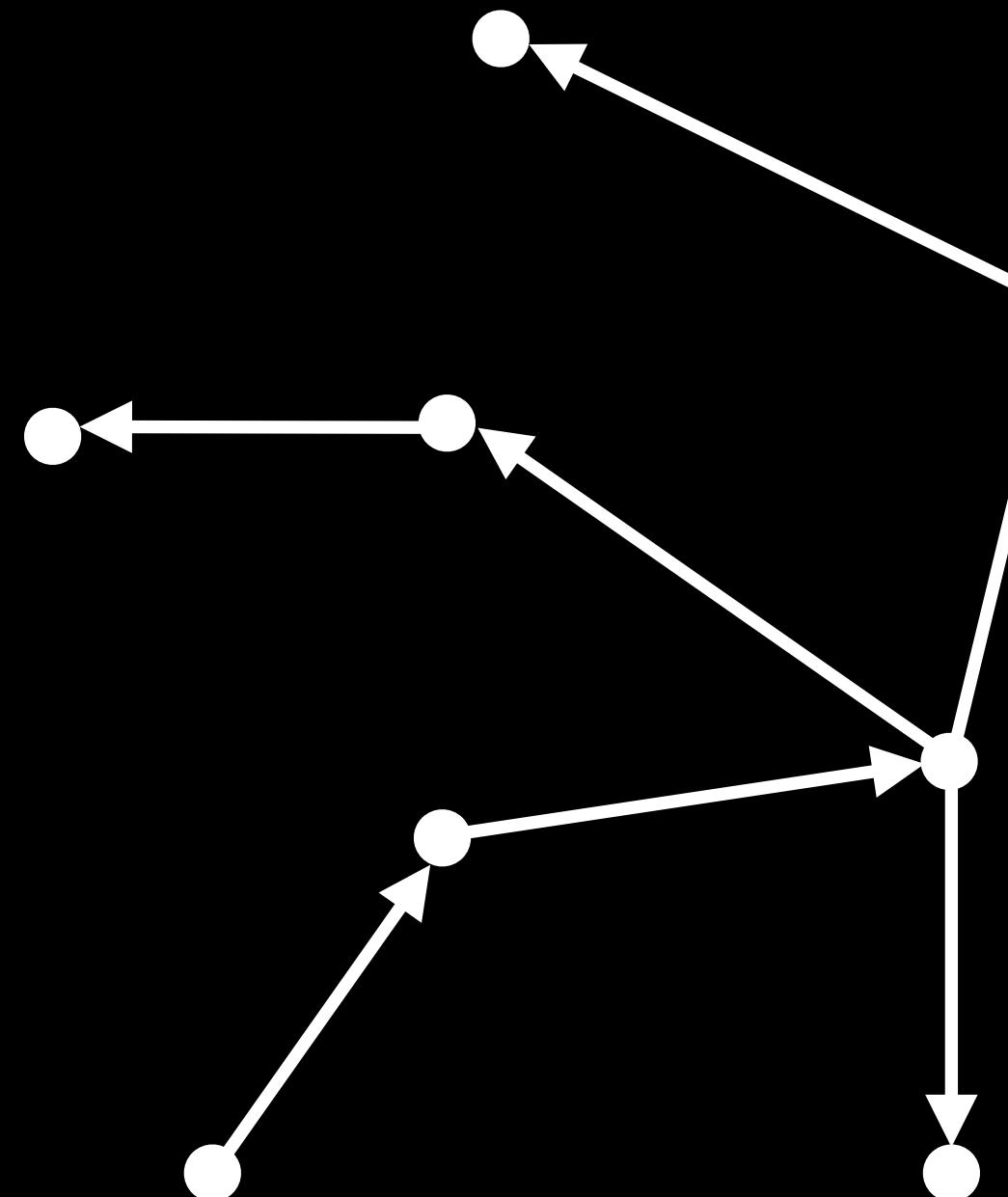
**Our data**  
Timestamped edges  
(including repeats)



# Choosing to close triangles

*Triadic closure* offers small choice sets  
→ tractable inference  
→ varied choice sets

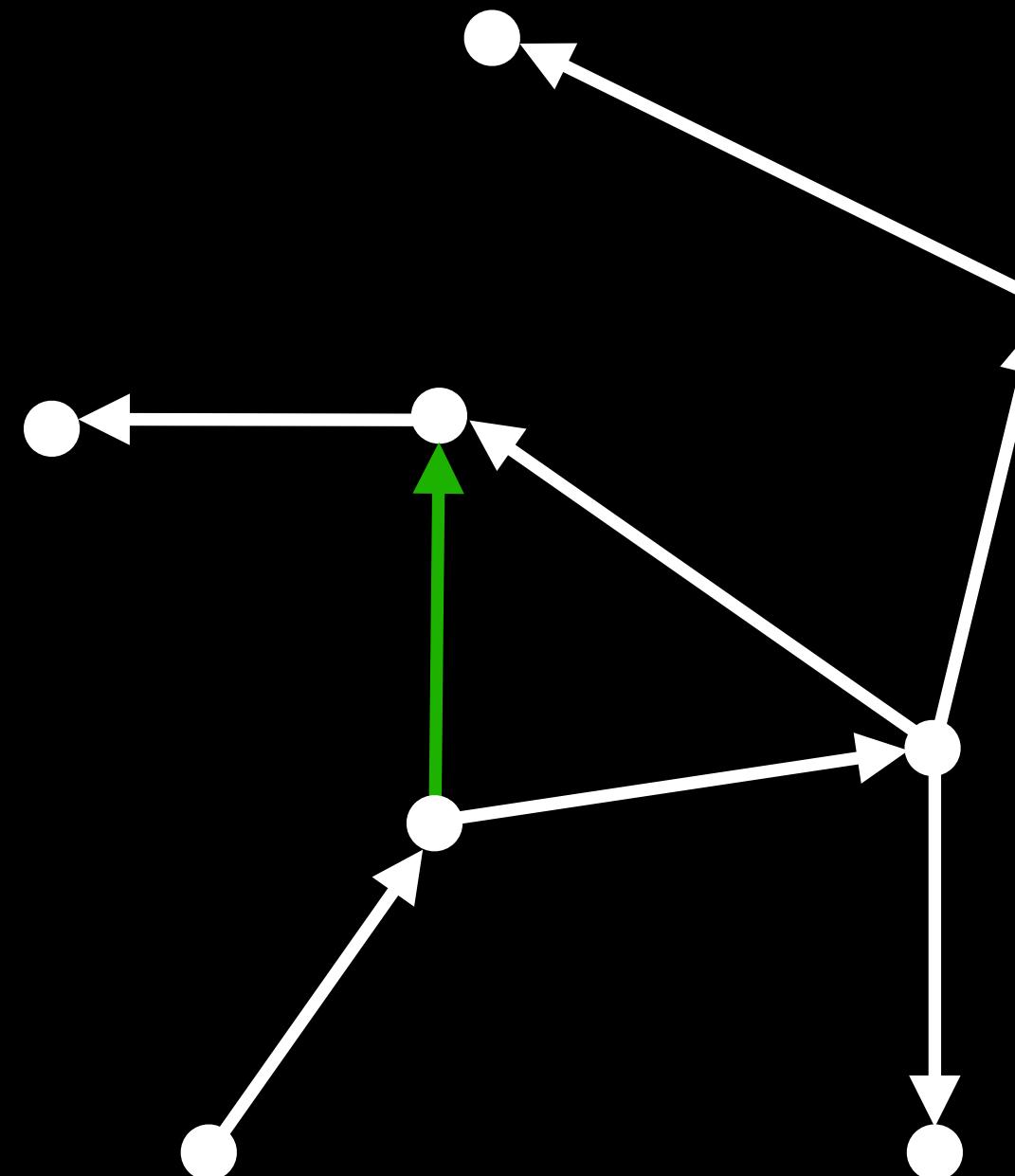
**Our data**  
Timestamped edges  
(including repeats)



# Choosing to close triangles

*Triadic closure* offers small choice sets  
→ tractable inference  
→ varied choice sets

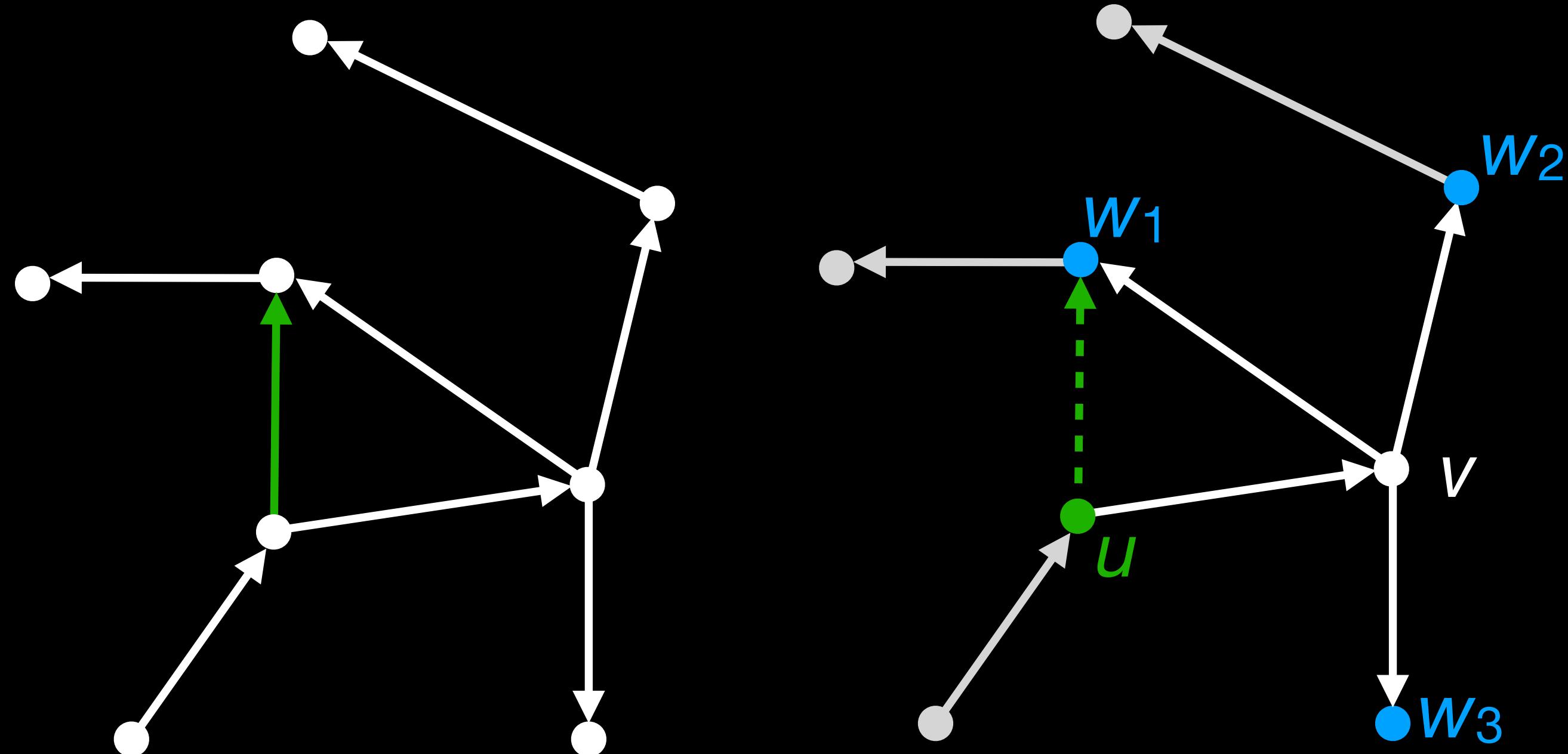
**Our data**  
Timestamped edges  
(including repeats)



# Choosing to close triangles

*Triadic closure* offers small choice sets  
→ tractable inference  
→ varied choice sets

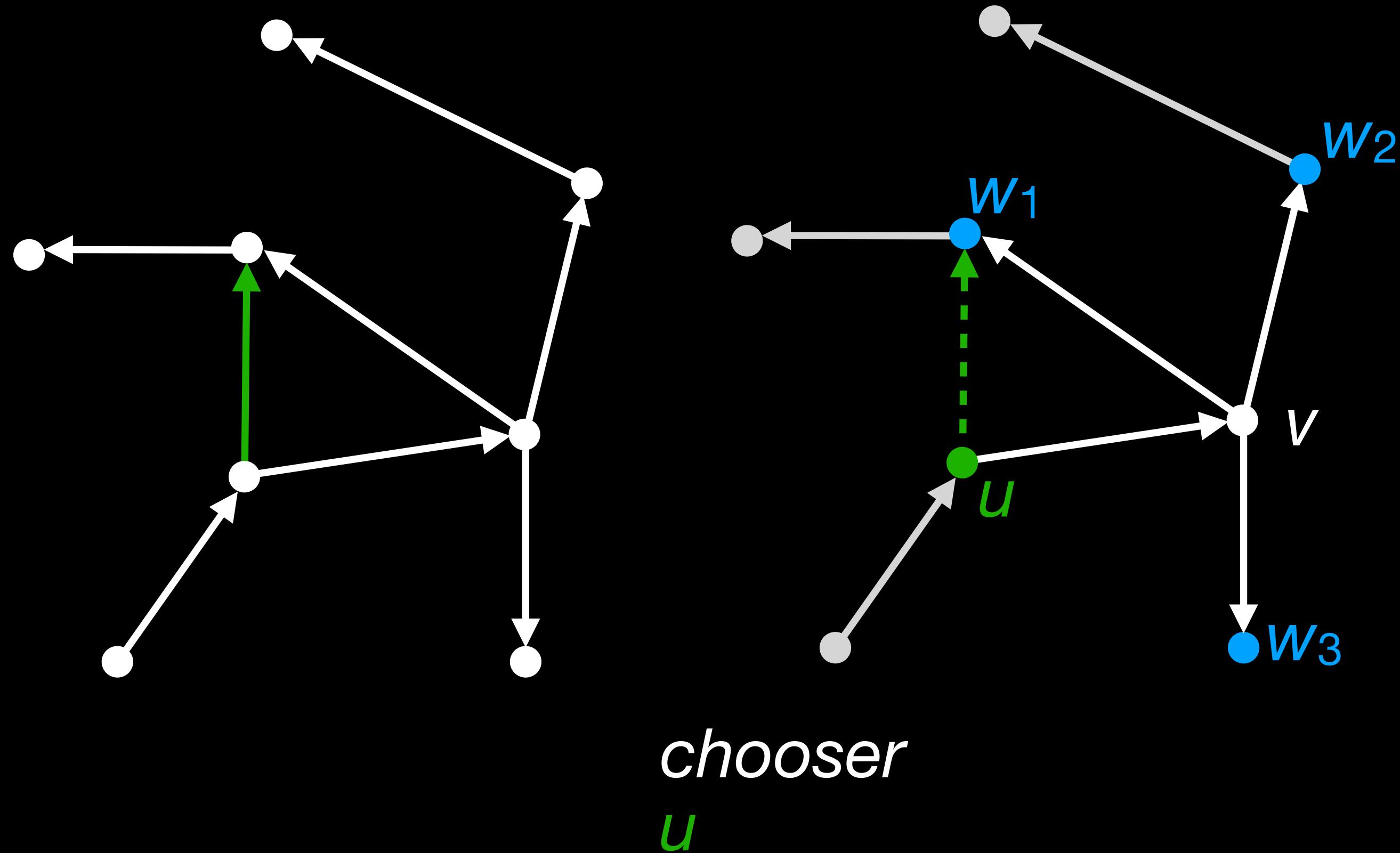
**Our data**  
Timestamped edges  
(including repeats)



# Choosing to close triangles

*Triadic closure* offers small choice sets  
→ tractable inference  
→ varied choice sets

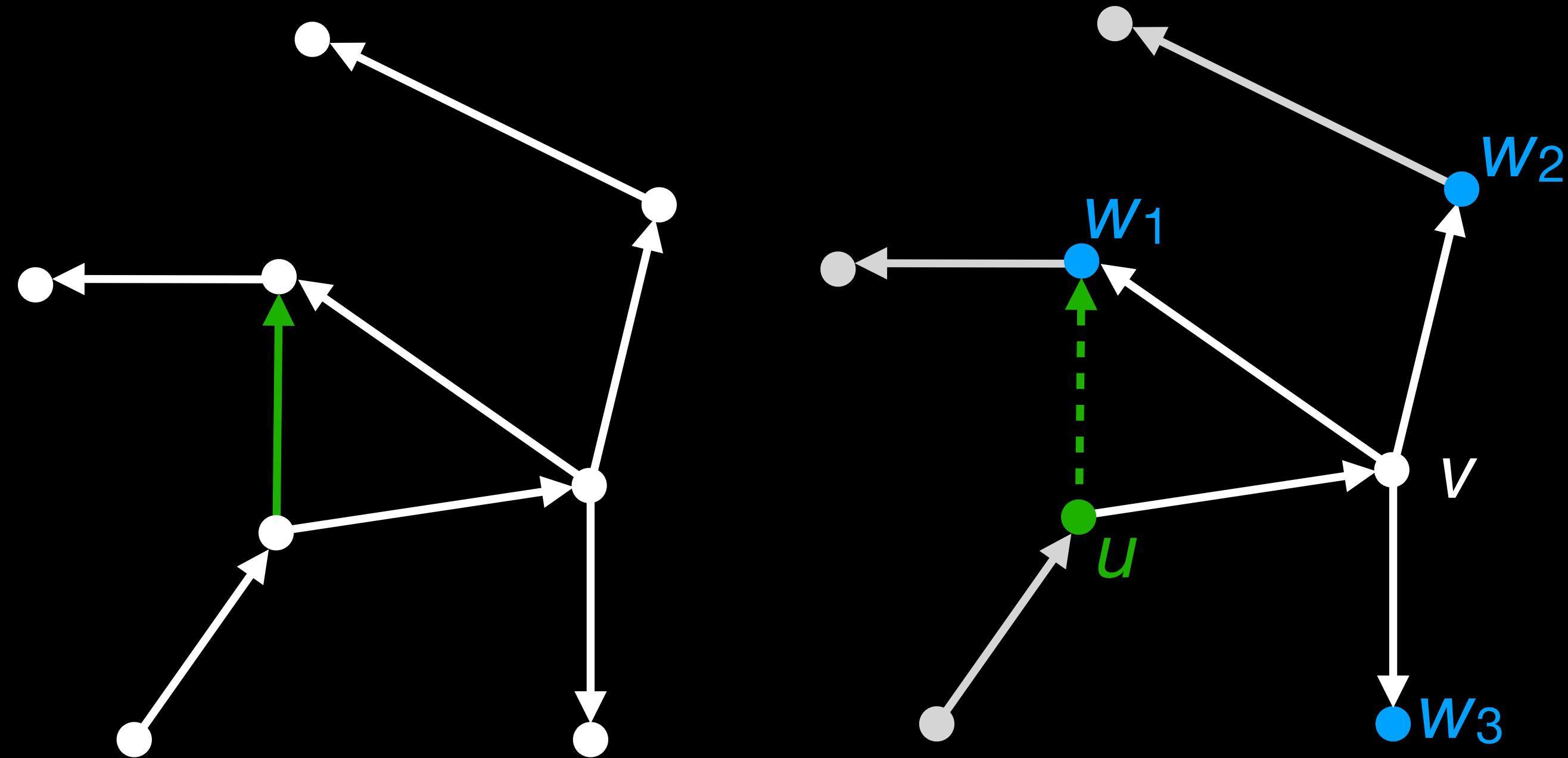
**Our data**  
Timestamped edges  
(including repeats)



# Choosing to close triangles

*Triadic closure* offers small choice sets  
→ tractable inference  
→ varied choice sets

**Our data**  
Timestamped edges  
(including repeats)

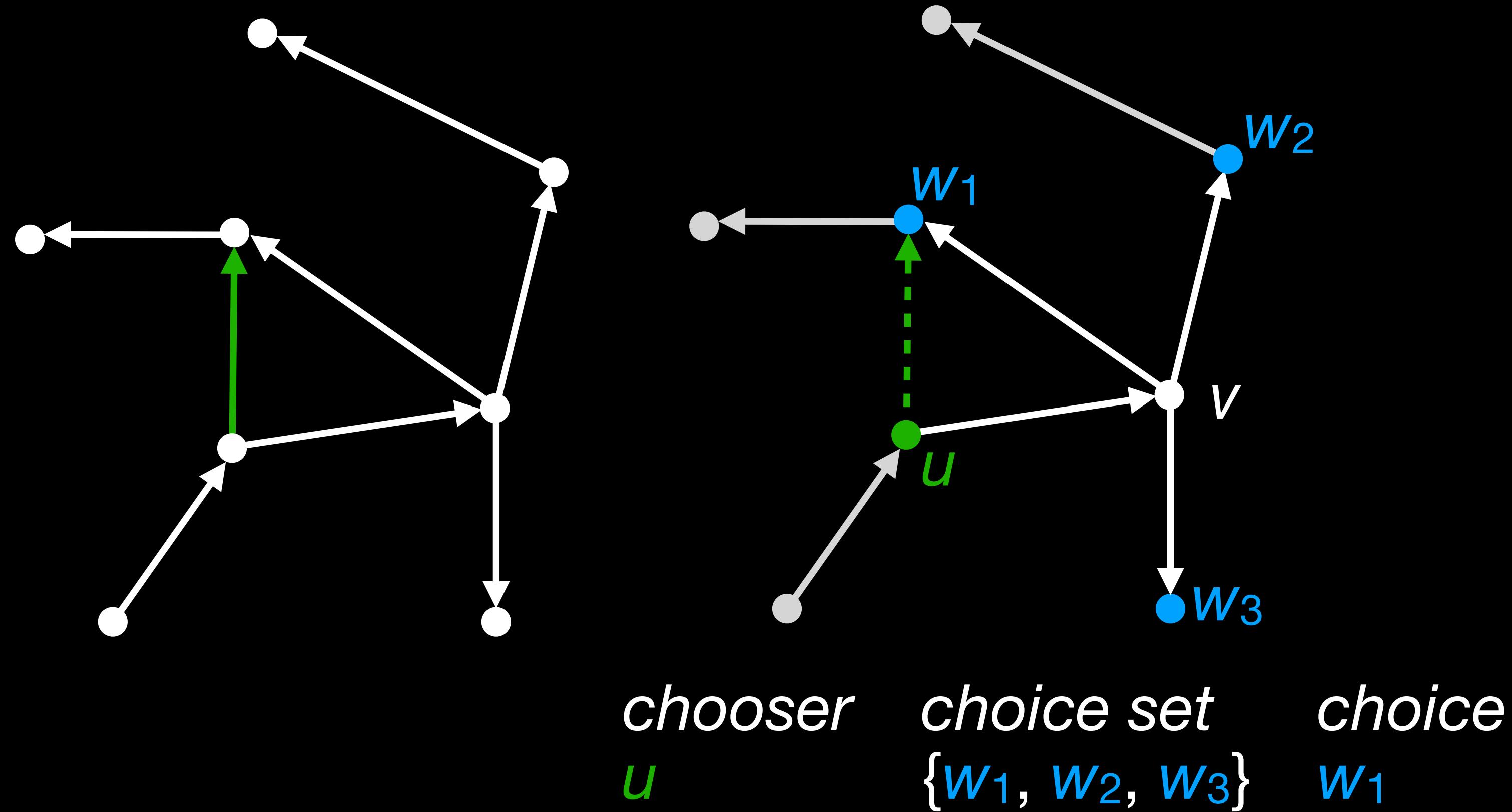


*chooser*    *choice set*  
 $u$              $\{w_1, w_2, w_3\}$

# Choosing to close triangles

*Triadic closure* offers small choice sets  
→ tractable inference  
→ varied choice sets

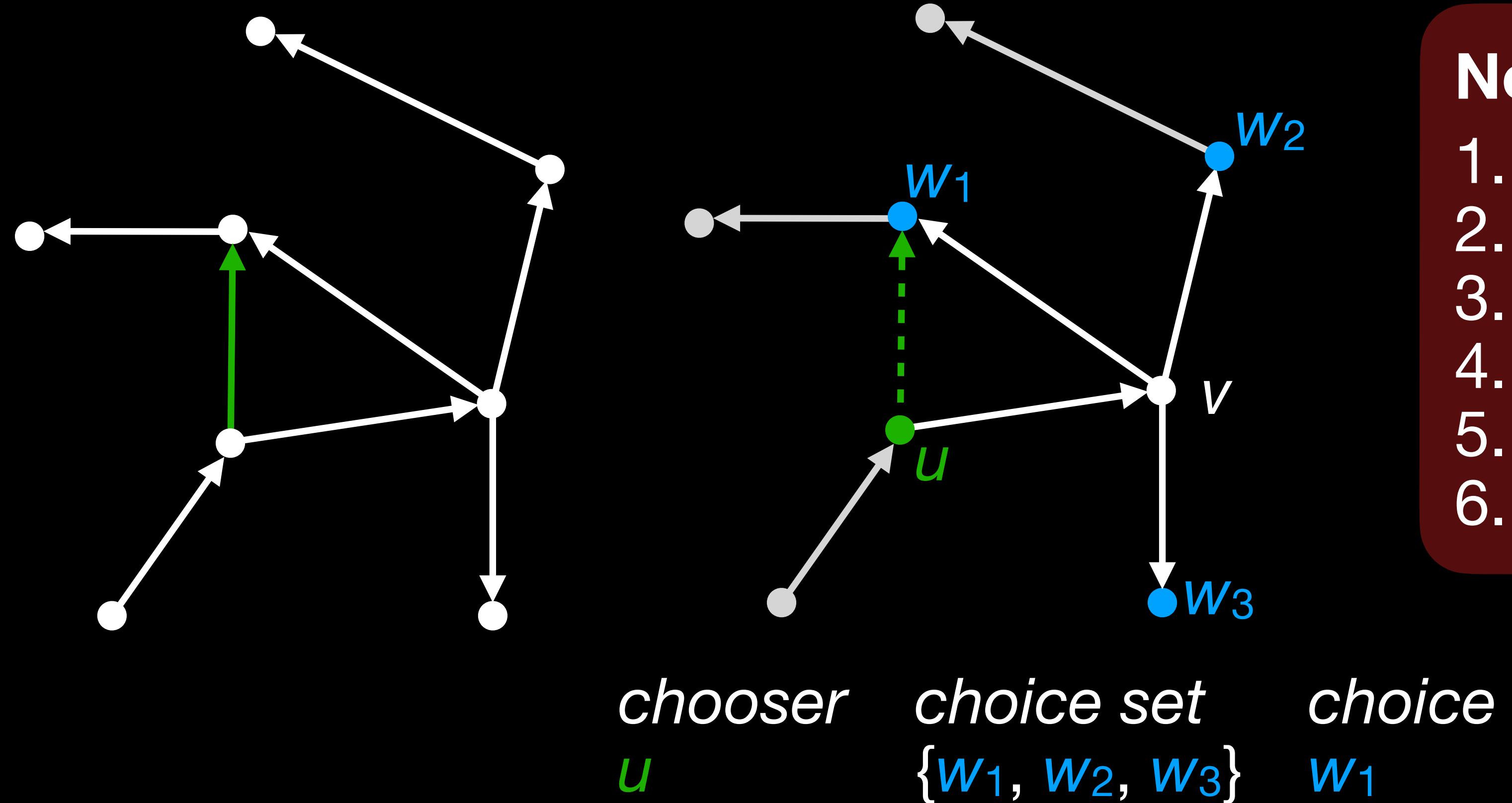
**Our data**  
Timestamped edges  
(including repeats)



# Choosing to close triangles

*Triadic closure* offers small choice sets  
→ tractable inference  
→ varied choice sets

**Our data**  
Timestamped edges  
(including repeats)



- Node features**
1. in-degree of  $w$
  2. # shared neighbors of  $u, w$
  3. weight of edge  $w \rightarrow u$
  4. time since last edge into  $w$
  5. time since last edge out of  $w$
  6. time since last  $w \rightarrow u$  edge

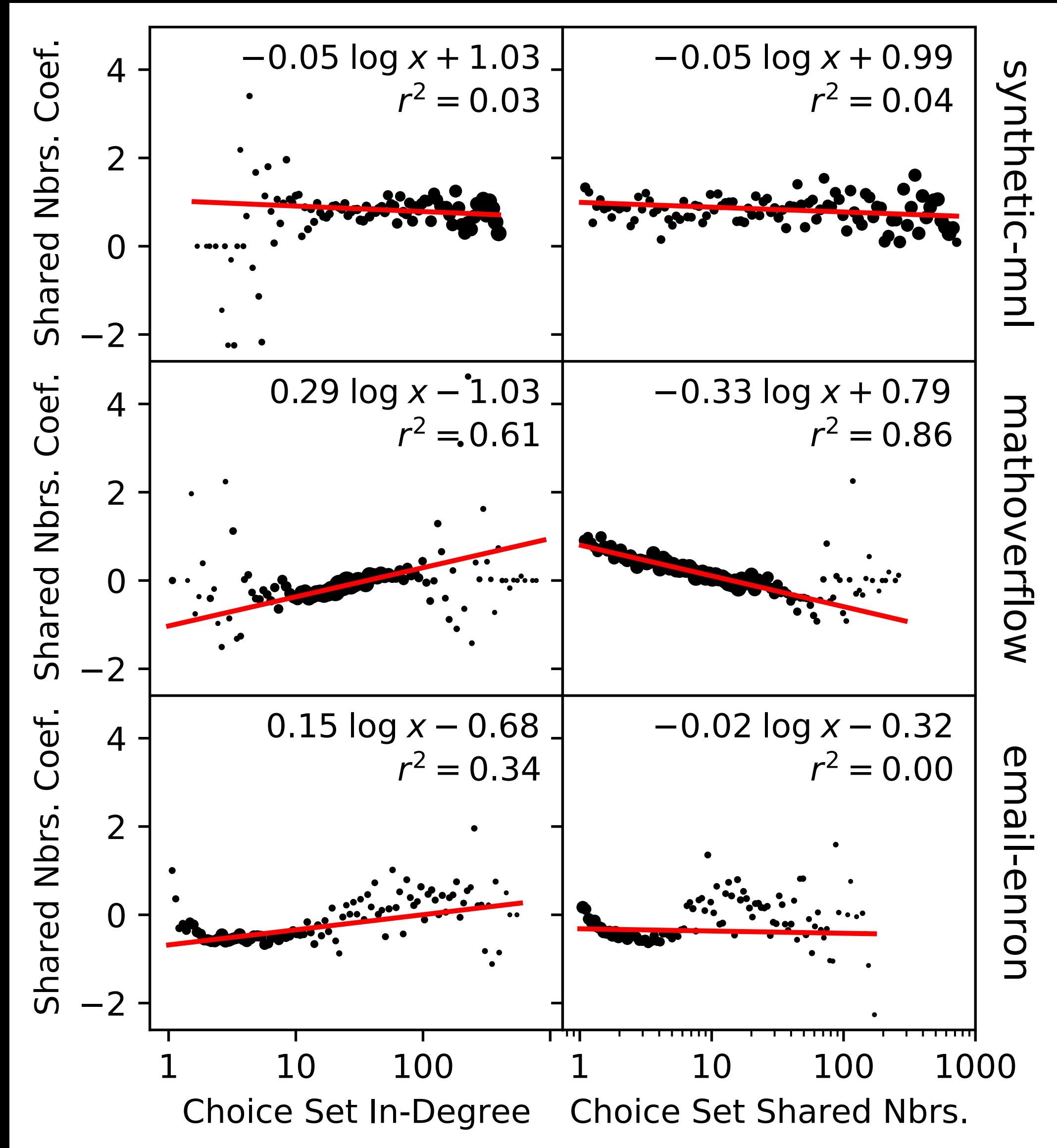
# Context matters in triadic closure

# Context matters in triadic closure

**Datasets**

- email-enron
- email-eu
- email-w3c
- wiki-talk
- reddit-hyperlink
- bitcoin-alpha
- bitcoin-otc
- mathoverflow
- college-msg
- facebook-wall
- sms-a
- sms-b
- sms-c

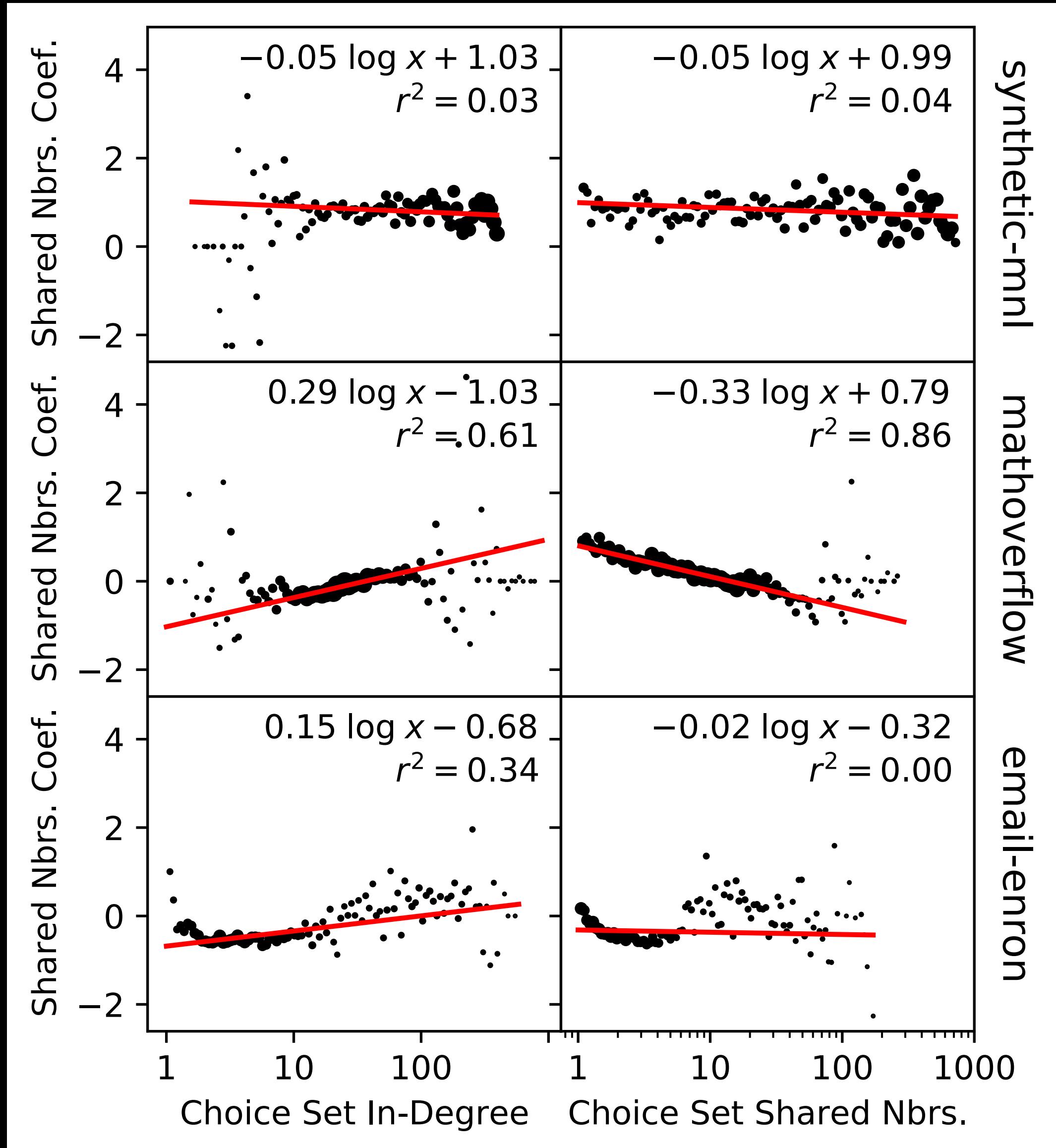
# Context matters in triadic closure



synthetic-mnl    mathoverflow    email-enron

- Datasets**
- email-enron
  - email-eu
  - email-w3c
  - wiki-talk
  - reddit-hyperlink
  - bitcoin-alpha
  - bitcoin-otc
  - mathoverflow
  - college-msg
  - facebook-wall
  - sms-a
  - sms-b
  - sms-c

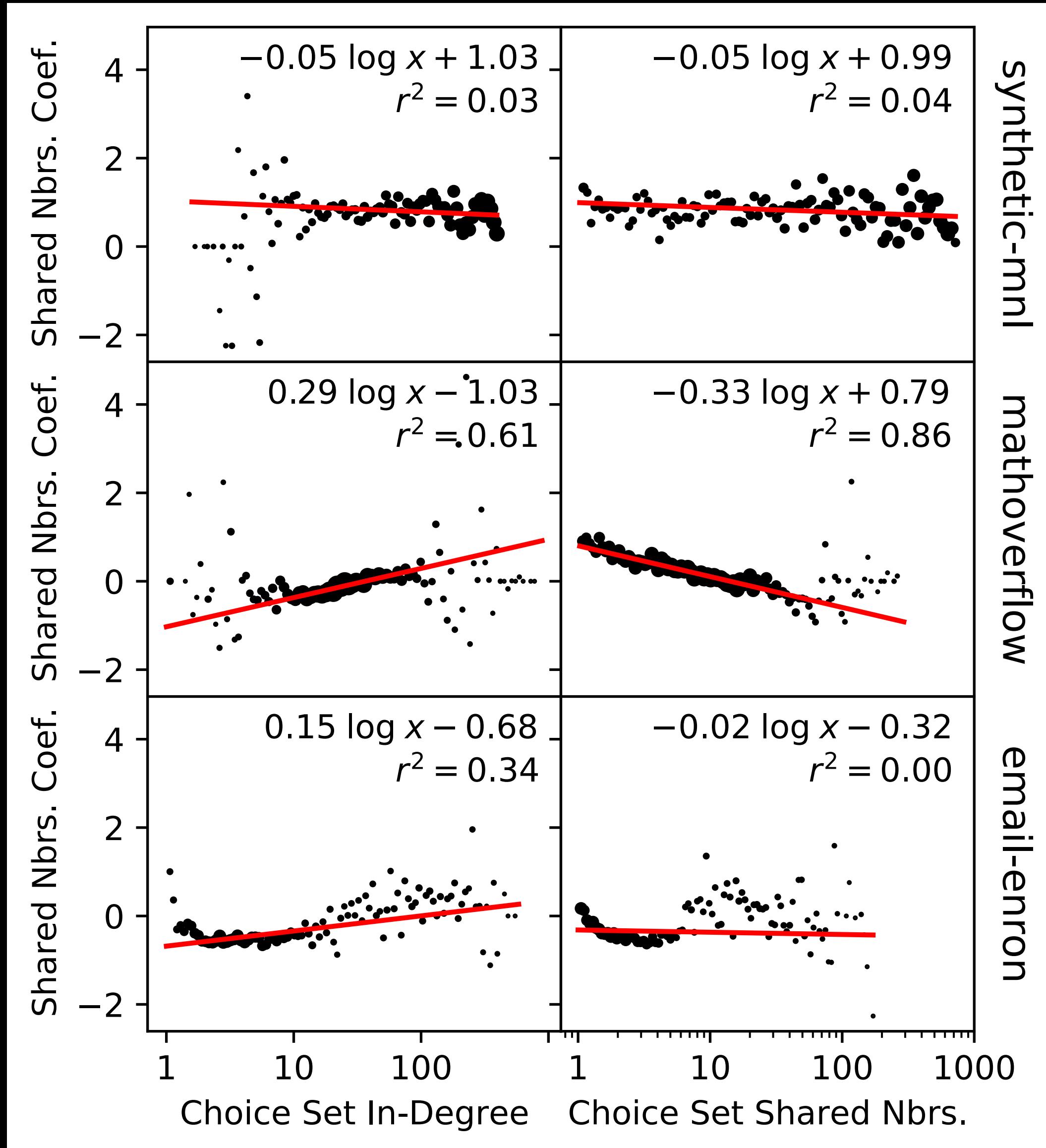
# Context matters in triadic closure



Synthetic data,  
no context effects

- Datasets**
- email-enron
  - email-eu
  - email-w3c
  - wiki-talk
  - reddit-hyperlink
  - bitcoin-alpha
  - bitcoin-otc
  - mathoverflow
  - college-msg
  - facebook-wall
  - sms-a
  - sms-b
  - sms-c

# Context matters in triadic closure

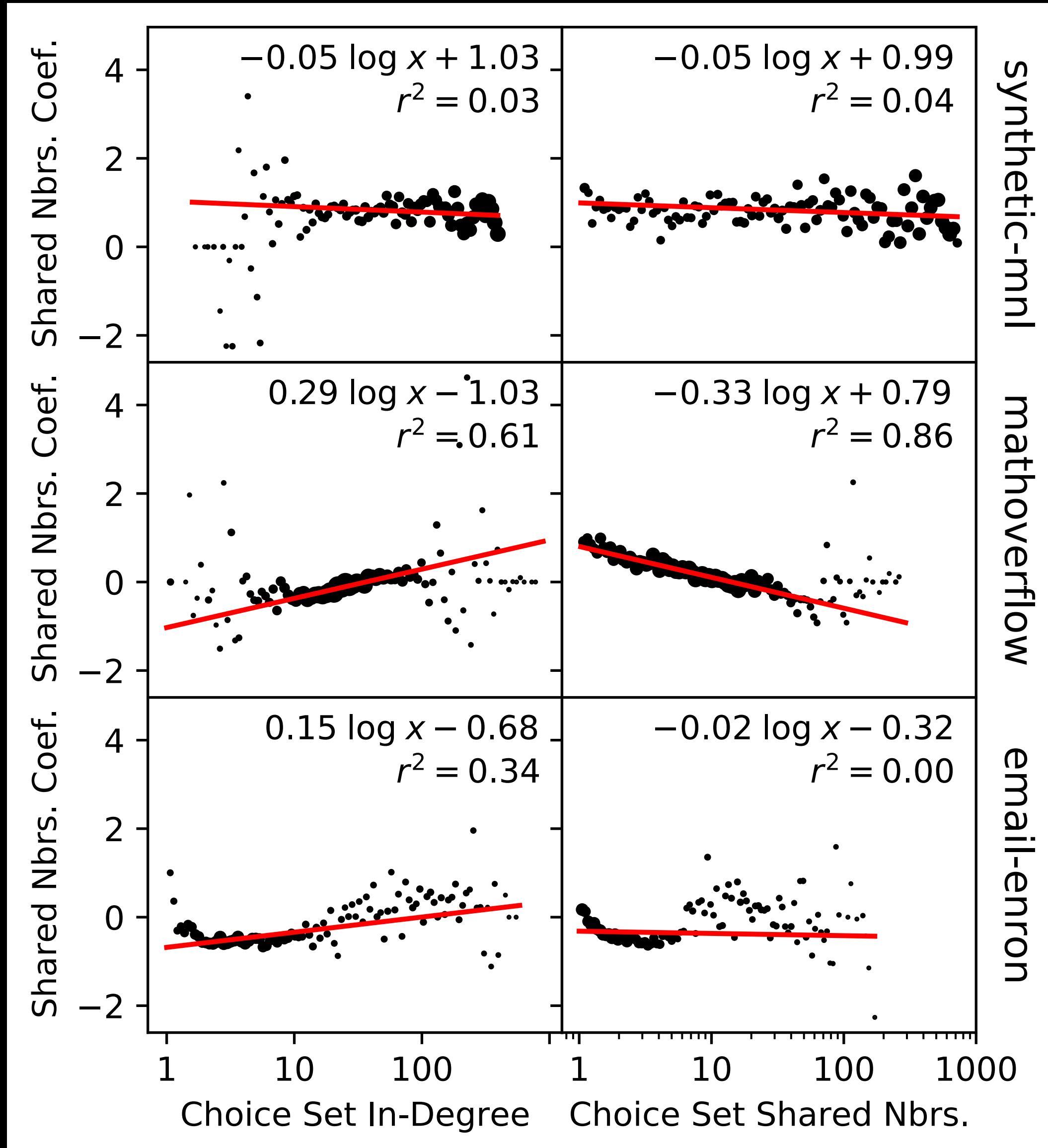


Synthetic data,  
no context effects

Commenting network,  
linear context effects

- Datasets**
- email-enron
  - email-eu
  - email-w3c
  - wiki-talk
  - reddit-hyperlink
  - bitcoin-alpha
  - bitcoin-otc
  - mathoverflow
  - college-msg
  - facebook-wall
  - sms-a
  - sms-b
  - sms-c

# Context matters in triadic closure



Synthetic data,  
no context effects

Commenting network,  
linear context effects

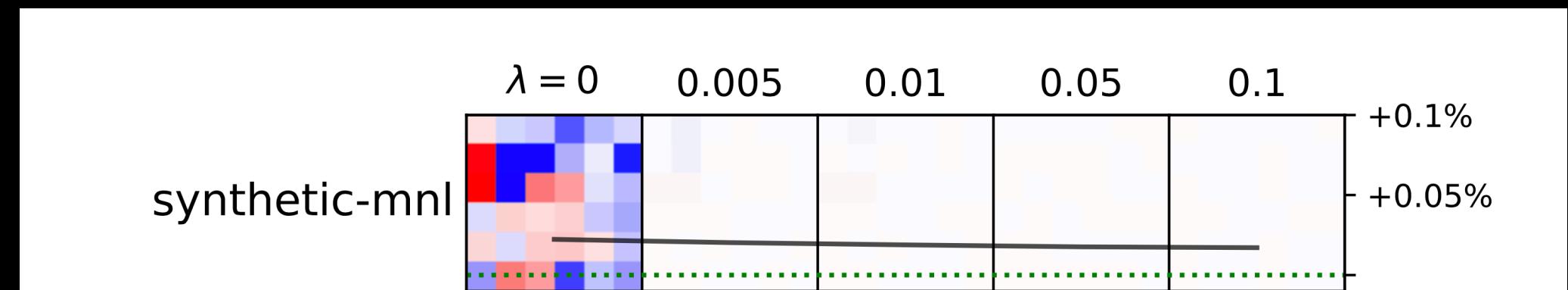
Email network,  
nonlinear context effects?

## Datasets

- email-enron
- email-eu
- email-w3c
- wiki-talk
- reddit-hyperlink
- bitcoin-alpha
- bitcoin-otc
- mathoverflow
- college-msg
- facebook-wall
- sms-a
- sms-b
- sms-c

# LCL reveals interpretable feature context effects

# LCL reveals interpretable feature context effects

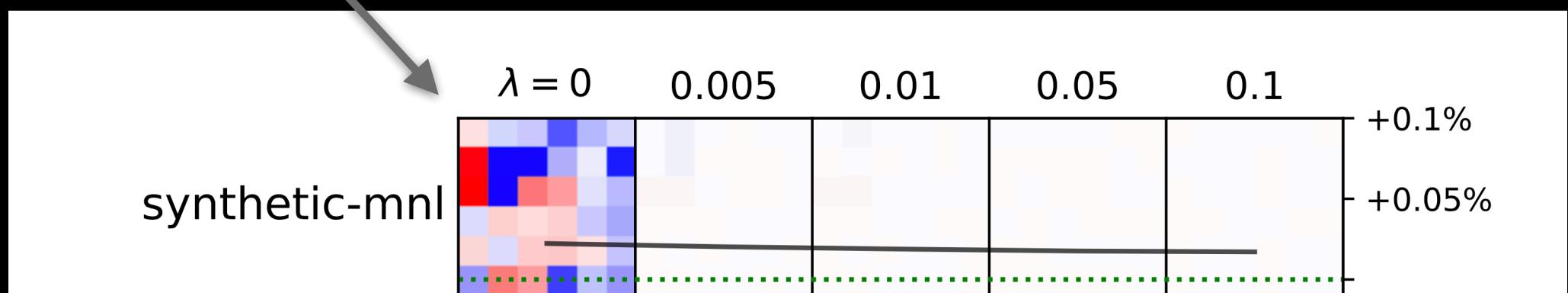


# LCL reveals interpretable feature context effects

context effect matrix  $A$

red: +, blue: -, white: 0

(column acts on row)



**Node features**  
(left-right, top-bottom)

1. in-degree
2. shared neighbors
3. reciprocal weight
4. send recency
5. receive recency
6. reciprocal recency

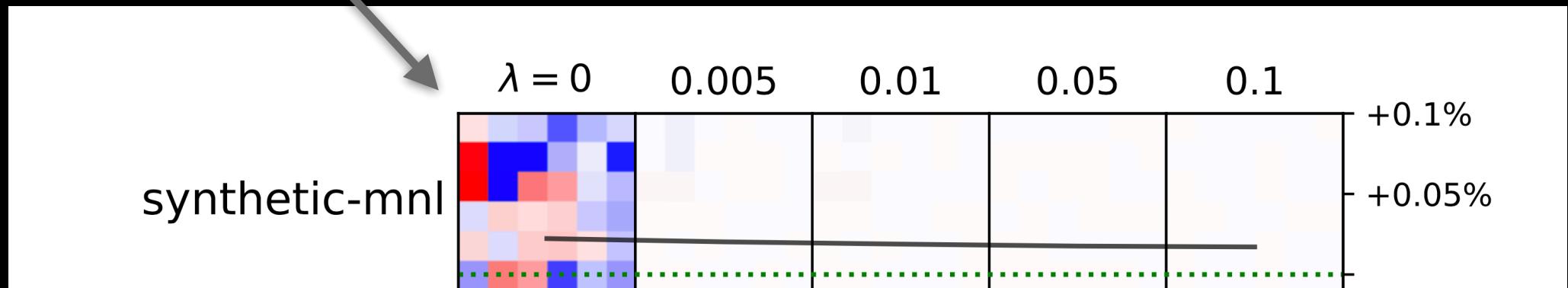
# LCL reveals interpretable feature context effects

context effect matrix  $A$

red: +, blue: -, white: 0

(column acts on row)

increasing  $L_1$  regularization on  $A$



**Node features**  
(left-right, top-bottom)

1. in-degree
2. shared neighbors
3. reciprocal weight
4. send recency
5. receive recency
6. reciprocal recency

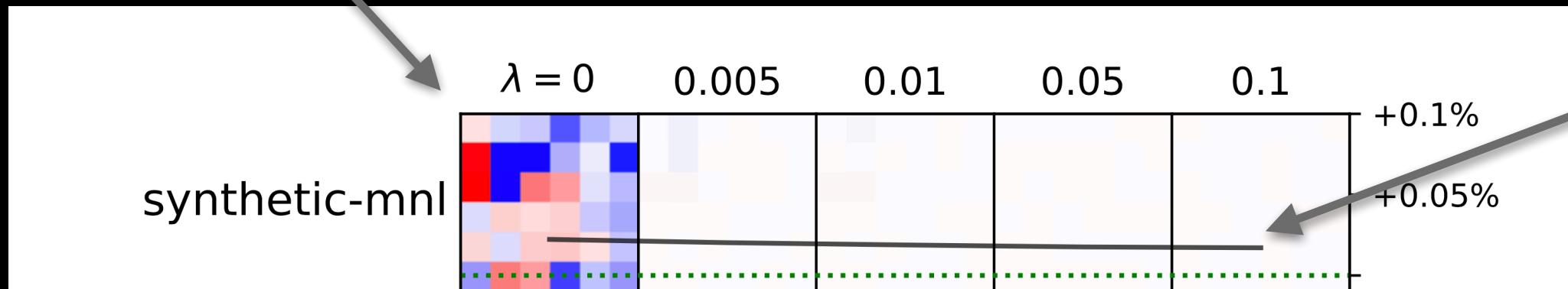
# LCL reveals interpretable feature context effects

context effect matrix  $A$

red: +, blue: -, white: 0

(column acts on row)

increasing  $L_1$  regularization on  $A$



**Node features**  
(left-right, top-bottom)

1. in-degree
2. shared neighbors
3. reciprocal weight
4. send recency
5. receive recency
6. reciprocal recency

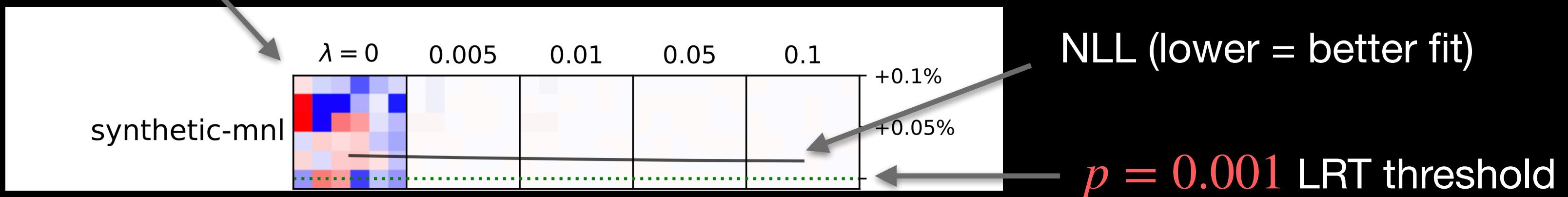
# LCL reveals interpretable feature context effects

context effect matrix  $A$

red: +, blue: -, white: 0

(column acts on row)

increasing  $L_1$  regularization on  $A$



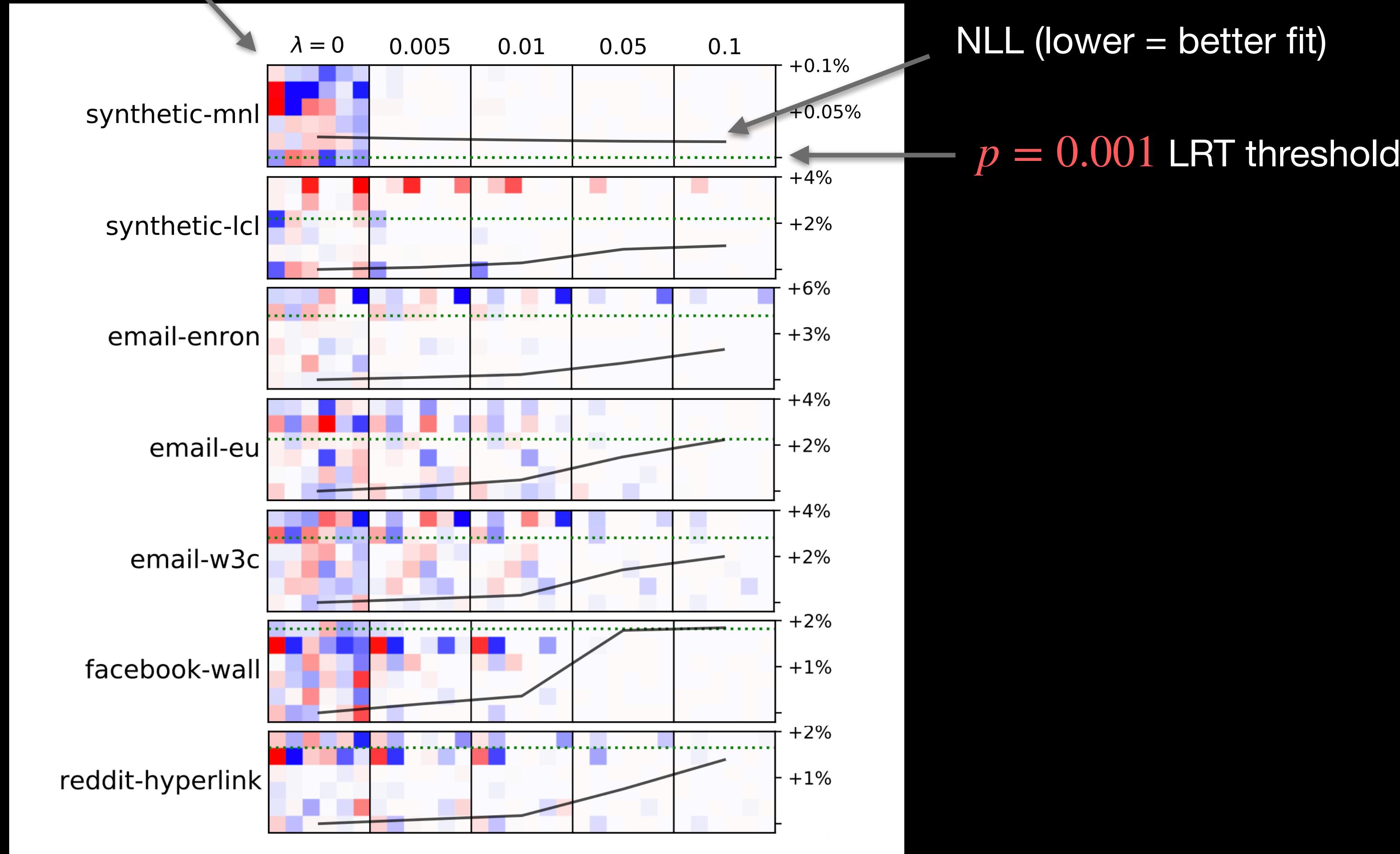
**Node features**  
(left-right, top-bottom)

1. in-degree
2. shared neighbors
3. reciprocal weight
4. send recency
5. receive recency
6. reciprocal recency

# LCL reveals interpretable feature context effects

context effect matrix  $A$   
red: +, blue: -, white: 0  
(column acts on row)

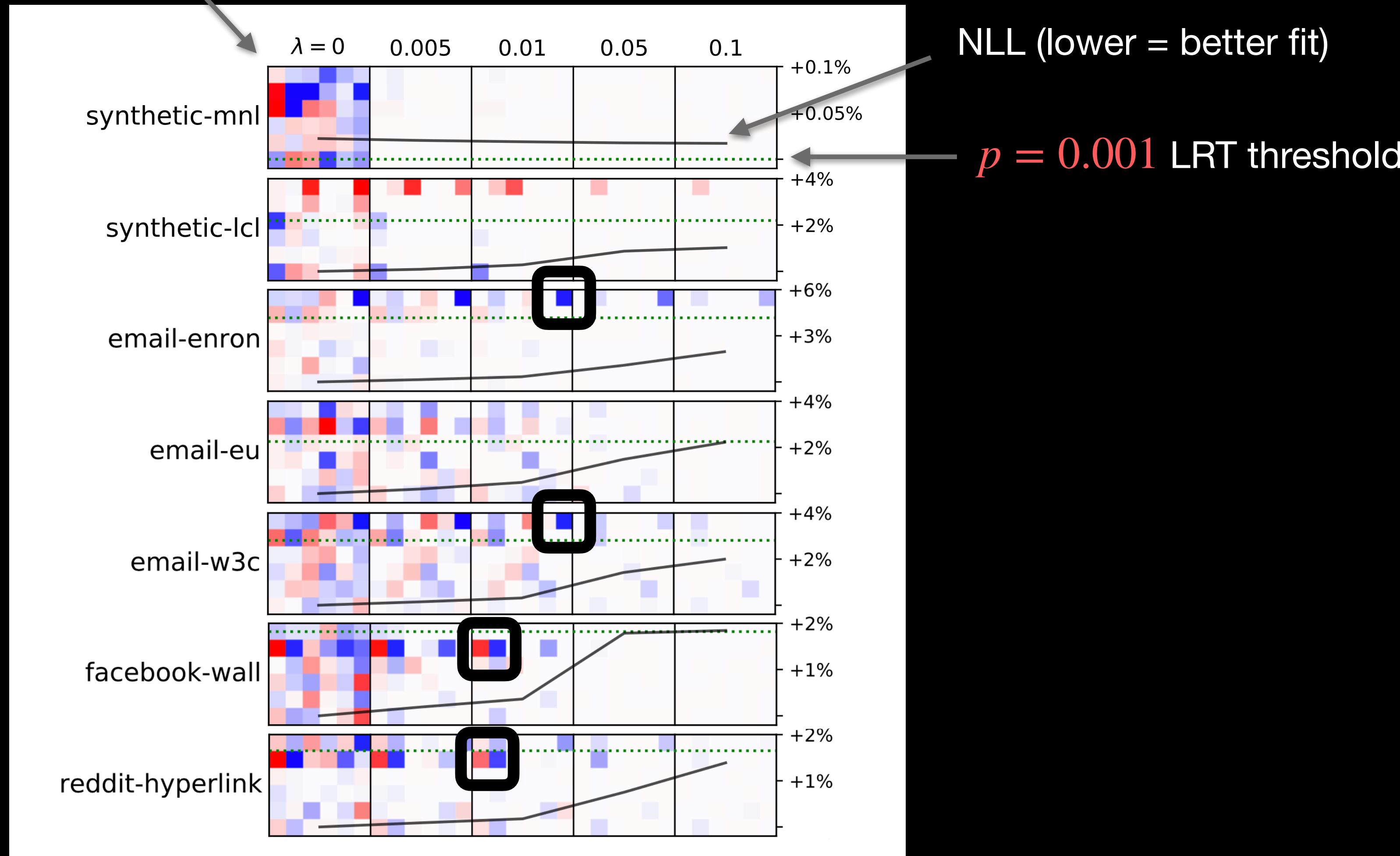
increasing  $L_1$  regularization on  $A$



# LCL reveals interpretable feature context effects

context effect matrix  $A$   
red: +, blue: -, white: 0  
(column acts on row)

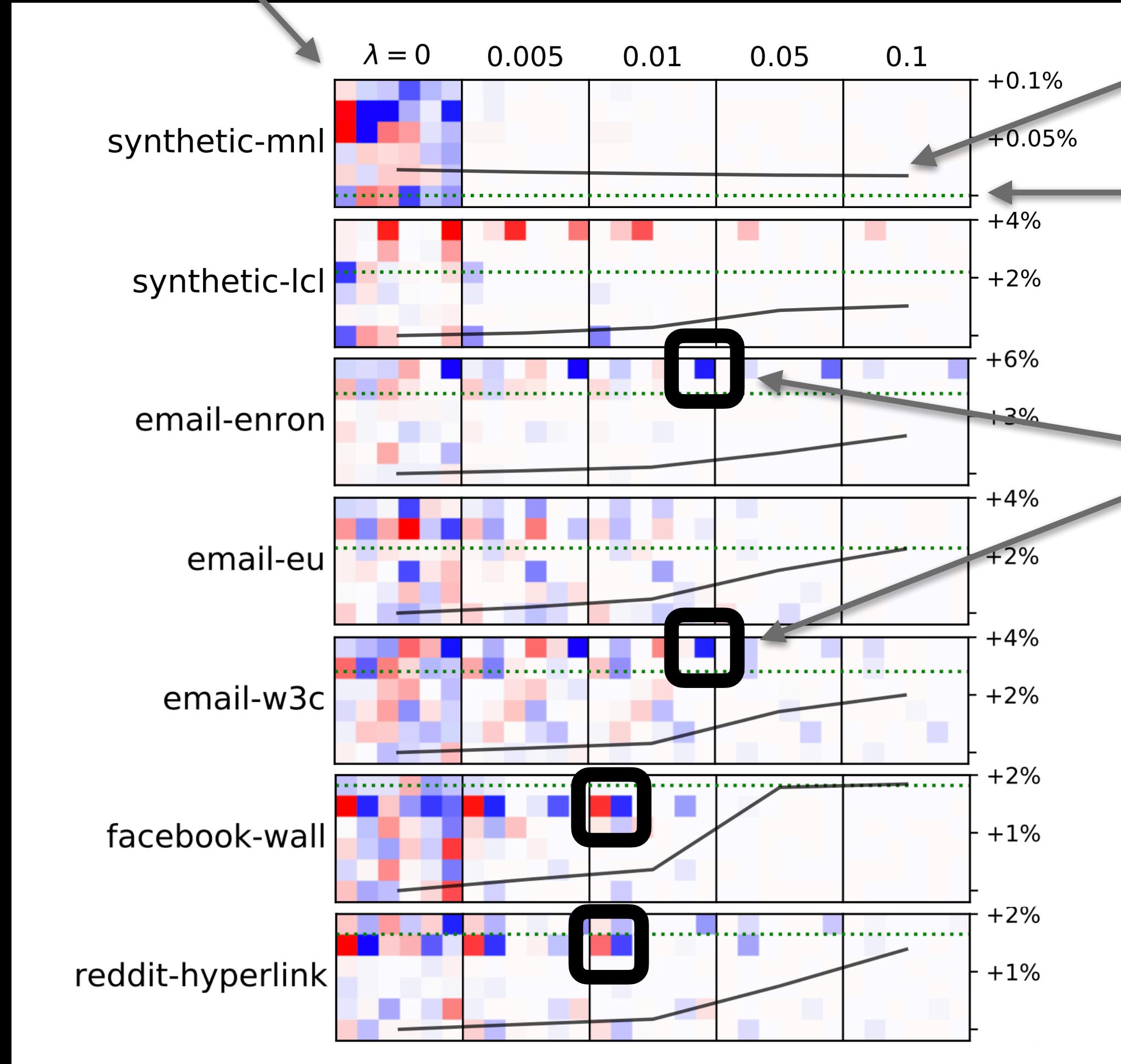
increasing  $L_1$  regularization on  $A$



# LCL reveals interpretable feature context effects

context effect matrix  $A$   
red: +, blue: -, white: 0  
(column acts on row)

increasing  $L_1$  regularization on  $A$



## Node features (left-right, top-bottom)

1. in-degree
2. shared neighbors
3. reciprocal weight
4. send recency
5. receive recency
6. reciprocal recency

NLL (lower = better fit)

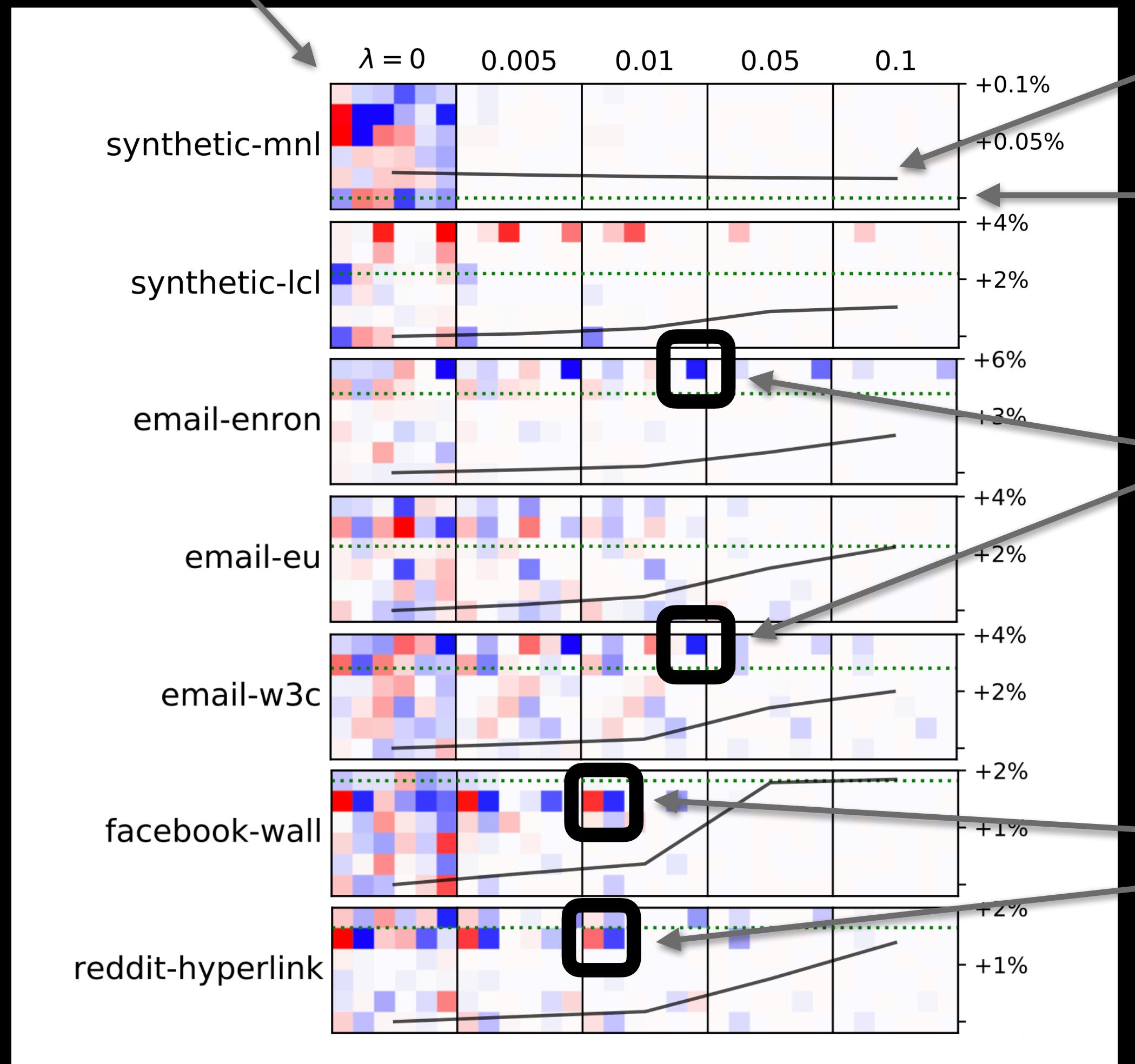
$p = 0.001$  LRT threshold

*“cluttered inbox”*  
high choice set reciprocal recency  
→ in-degree less important

# LCL reveals interpretable feature context effects

context effect matrix  $A$   
red: +, blue: -, white: 0  
(column acts on row)

increasing  $L_1$  regularization on  $A$



## Node features (left-right, top-bottom)

1. in-degree
2. shared neighbors
3. reciprocal weight
4. send recency
5. receive recency
6. reciprocal recency

*“cluttered inbox”*  
high choice set reciprocal recency  
→ in-degree less important

red: *“cocktail party introduction”*  
high choice set in-degree  
→ shared neighbors more important

blue: *“familiarity saturation”*  
high choice set shared neighbors  
→ shared neighbors less important

# Concluding thoughts

Code: [bit.ly/lcl-code](https://bit.ly/lcl-code)  
Data: [bit.ly/lcl-data](https://bit.ly/lcl-data)  
Slides: [bit.ly/lcl-kdd-slides](https://bit.ly/lcl-kdd-slides)

## Key takeaways

*Feature context effects* extend item-level effects

LCL offers an interpretable and tractable way to reveal them

## Future work

Non-linear context effects

Negative sampling

Discovering relational effects

## Causal context effects?

See our other KDD '21 paper:

“Choice Set Confounding in Discrete Choice”

**Submit to our NeurIPS '21 workshop!**  
[bit.ly/WHMD2021](https://bit.ly/WHMD2021)

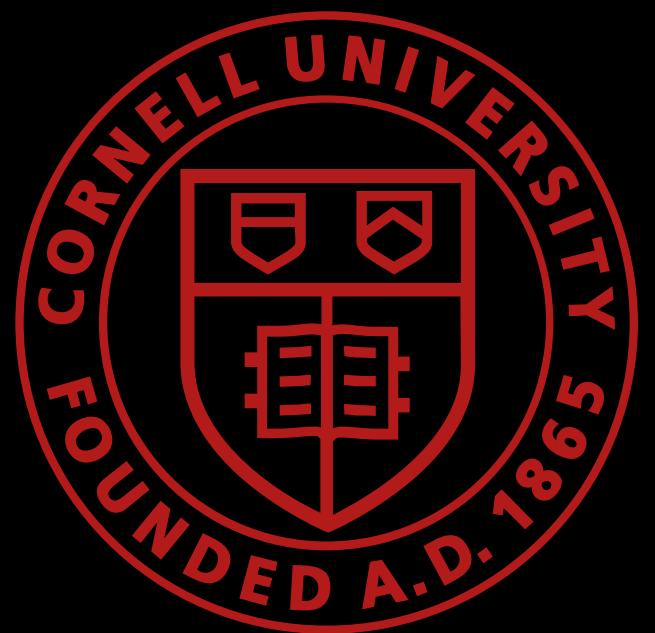
## Thank you!

More questions or ideas?

Email me: [kt@cs.cornell.edu](mailto:kt@cs.cornell.edu)



@kiran\_tomlinson



## Acknowledgments

Funding from NSF, ARO

Thanks to Johan Ugander, Jan Overgoor, and Sophia Franco