MC 3

a) For the transition matrix $\underline{\underline{M}}$, the probability of going from state i to j is given by M_{ij}

$$\underline{\underline{M}} = \begin{pmatrix} 0.1 & 0.9 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.3 & 0.7 \\ 0 & 0 & 0.6 & 0.4 \end{pmatrix}$$

In this matrix 2 subsystems exist ~~stat~~ (1, 2) and (3, 4)
One can never transition from state 1/2 to state 3/4 and vice versa ⇒ won't sample state space effectively

Self transition probabilities are non-zero, there is a chance of remaining in the same state ⇒ Also limits effective sampling of the states

Detailed balance is not observed ⇒ $M_{ij} \neq M_{ji}$

b) Detailed balance for a transition is given by

$p_i T_{ij} = p_j T_{ji}$ where p_x is the probability of being in state x and T_{nm} is the probability of a transition from state n to m

Consider super-Markov

transition from i to j via several small transitions

$(i \to k,\ k \to l,\ l \to m \dots,\ z \to j)$

Consider $\rho_i \pi^n_{ij}$ where $\pi^n = \pi_{ik}\pi_{kl}\cdots\pi_{zj}$

$$\Rightarrow \rho_i \pi^n_{ij} = \rho_i\left(\sum_k \pi^{n-1}_{kj}\,\pi_{ik}\right) \qquad \frac{\rho_k \pi_{ki}}{\rho_i}$$

use detailed ballance for individual steps

$$= \cancel{\rho_i}\left(\sum_k \pi^{n-1}_{kj}\,\frac{\rho_k}{\cancel{\rho_i}}\,\pi_{ki}\right)$$

$$= \rho_k \sum_k \pi^{n-1}_{kj}\,\pi_{ki}$$

$$= \rho_k \sum_k \sum_l \pi^{n-2}_{lj}\,\pi_{kl}\,\pi_{ki}$$

$$= \rho_l \sum_k \sum_l \pi^{n-2}_{lj}\,\pi_{lk}\,\pi_{ki}$$

Repeat for all intervening states

$$\Rightarrow\ = \rho_j \sum_a \cdots \sum_z \pi_{ja} \cdots \pi_{ki} = \rho_j \pi^n_{ji}$$

sum over all states