1)

a) $$Q = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \cdots \sum_{n_N=0}^{\infty} e^{-\beta \sum_1^N n_j \hbar \omega_j}$$

$$= \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \cdots e^{-\beta(n_1 \hbar \omega_1 + n_2 \hbar \omega_2 \cdots + n_N \hbar \omega_N)}$$

$$= \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \cdots e^{-\beta n_1 \hbar \omega_1} e^{-\beta n_2 \hbar \omega_2} \cdots e^{-\beta n_N \hbar \omega_N}$$

$$= \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \cdots \sum_{n_N=0}^{\infty} \prod_{j=1}^{N} e^{-\beta n_j \hbar \omega_j}$$

$$= \prod_{j=1}^{N} \sum_{n_j=0}^{\infty} e^{-\beta n_j \hbar \omega_j}$$

$$= \prod_{j=1}^{N} \sum_{n_j=0}^{\infty} \left(e^{-\beta \hbar \omega_j}\right)^{n_j}$$

$$= \prod_{j=1}^{N} \frac{1}{1 - e^{-\beta \varepsilon_j}}$$

b)

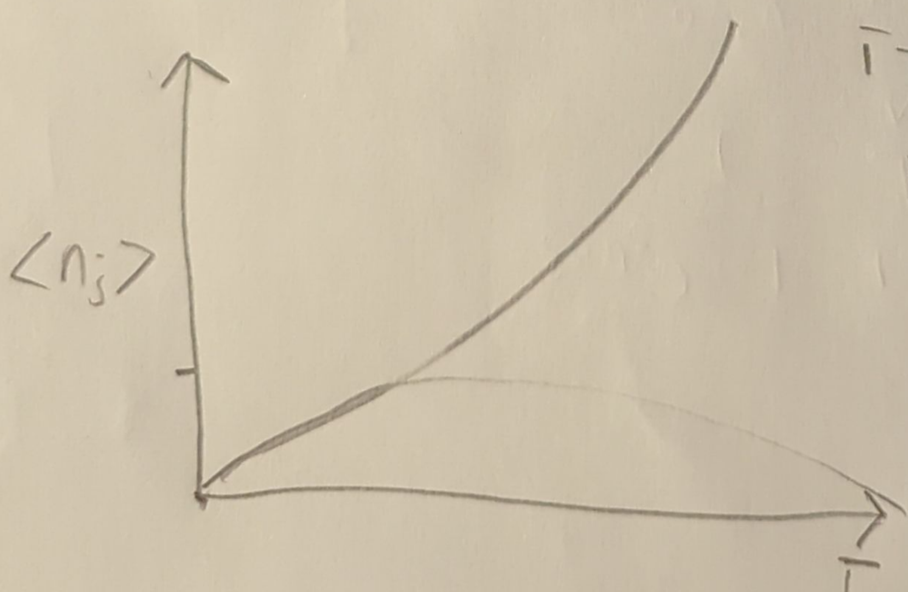
b) $\langle n_j \rangle = \dfrac{\partial \ln Q}{\partial(-\beta\epsilon_j)}$

$$\ln Q = \ln\left(\frac{1}{1-e^{-\beta\epsilon_1}}\right) + \ln\left(\frac{1}{1-e^{-\beta\epsilon_2}}\right) + \cdots \ln\left(\frac{1}{1-e^{-\beta\epsilon_j}}\right) + \cdots$$

$$\frac{\partial \ln Q}{\partial(-\beta\epsilon_j)} = -\frac{\partial}{\partial(-\beta\epsilon_j)}\left(\ln\left(1-e^{-\beta\epsilon_j}\right)\right)$$

$$= \frac{e^{-\beta\epsilon_j}}{1-e^{-\beta\epsilon_j}} \frac{(e^{\beta\epsilon_j})}{(e^{\beta\epsilon_j})}$$

$$\langle n_j \rangle = \frac{1}{e^{\beta\epsilon_j}-1}$$



$T \to 0 \quad \langle n_j \rangle \to 0$

$T \to \infty \quad \langle n_j \rangle \to \infty$