Imperfect Banking Competition and the Propagation of Uncertainty Shocks*

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Abstract

Uncertainty shocks play a crucial role in driving business cycle fluctuations. This paper investigates the impact of changes in banking competition on the propagation of uncertainty shocks. Using a panel dataset of 44 countries, I show that lower banking competition amplifies the negative impact of uncertainty on output growth. I further explore this relationship through a dynamic stochastic general equilibrium model featuring imperfect banking competition and financial frictions. The model shows that lower banking competition leads to higher borrowing rates and increased risk-taking by entrepreneurs. As a result, when the number of competitors is lower, uncertainty shocks have a stronger negative impact on defaults, investment and output due to increased risk-taking.

Keywords: Financial Frictions, Financial Intermediaries, Heterogeneous Agents, Market Power, Uncertainty. (JEL E32, E44, G21, L13)

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1 Introduction

The recent conflict in Ukraine and the Covid-19 pandemic have led to a sharp increase in uncertainty.[1] When borrowers are subject to financial frictions, uncertainty shocks increase borrower defaults and lead to a contraction in the supply of loans and GDP.[2] Credit markets play a crucial role in understanding the transmission of uncertainty shocks. Structural changes in credit markets can affect how these shocks are transmitted. In this paper, I study how changes in banking competition, such as the recent fall in competition in the US banking sector, affect the propagation of uncertainty shocks.

The U.S. banking sector is highly concentrated. Since 2000, there has been a decrease in the number of commercial banks and an increase in bank asset concentration. In 2020, there were half as many commercial banks as there were in 2000 and the share of assets held by the three largest banks rose from 21% to 35%.[3]

In this paper, I provide empirical evidence on the correlation between the causal impact of uncertainty shocks on real output growth and the level of competition in the banking sector. I use disaster shocks such as natural disasters, terrorist attacks, political coups and revolutions that occurred in 44 countries between 2000Q1 and 2020Q1 as instruments for changes in first and second moments. My findings demonstrate that second moment shocks have a more severe impact on output growth when banking competition is lower.

To study the impact of banking competition on business cycle fluctuations and the effect of the recent decline in competition on the transmission of uncertainty shocks, I develop a New Keynesian business cycle model with financial frictions and imperfect competition in the banking sector. The main feature of this model is that bankers compete à la Cournot to provide loans to entrepreneurs. In this economy, there are N bankers who invest their equity and deposits in loans to entrepreneurs. Entrepreneurs own and maintain physical capital but have insufficient net worth. They borrow from bankers to buy capital goods. Bankers choose optimally their loan supply internalizing loan demand and borrower default

[1] Caldara et al. (2022), Ferrara et al. (2022) and Anayi et al. (2022) document an increase in uncertainty after the Russian invasion of Ukraine. Altig et al. (2020) and Baker et al. (2020) document an increase in uncertainty triggered by the Covid-19 pandemic.

[2] See for example Christiano et al. (2014), Caldara et al. (2016) and Alessandri and Mumtaz (2019).

[3] See Figure 8 in Appendix A.1.

probability.

Entrepreneurs face both idiosyncratic and aggregate shocks. Idiosyncratic shocks result in heterogeneous returns on entrepreneurs' capital stock. In some cases, the realized return may be insufficient to repay loans, leading to default. The cross-sectional dispersion of idiosyncratic shocks defines the level of uncertainty in the economy. As uncertainty increases, the probability of low returns and subsequent default rises. Financial frictions cause banks to respond to heightened uncertainty by reducing credit supply. This constrains entrepreneurs' ability to acquire capital and results in decreased investment and a contraction of output.

The model is developed in two stages. In the first stage, I introduce the entrepreneurial sector in a partial equilibrium framework. This allows me to present the first channel through which competition within the banking sector can influence the transmission of uncertainty shocks. Bankers in less competitive banking sectors use their higher market power to charge higher borrowing rates to borrowers. I show that as borrowing rates increase, so does risk-taking and the probability of default among entrepreneurs. Moreover, when entrepreneurs take on more risk, an increase in uncertainty leads to a larger rise in their default rate. This channel is called *risk-shifting effect*.

In the second stage, I incorporate the entrepreneurial sector in a calibrated general equilibrium model with imperfectly competitive bankers. This introduces a second channel through which bankers' market power affects their response to shocks. Specifically, as bankers' market power increases, bankers become less likely to pass shocks on to their borrowers. An uncertainty shock increases the number of non-performing loans and the monitoring costs incurred by bankers. In response to these increased costs, bankers decrease their loan supply. However, the size of this decrease is smaller for bankers with greater market power. I call this channel the *pass-through* effect.

The impact of banking competition on the transmission of uncertainty shocks is complex due to the presence of two opposing channels. To determine which channel is stronger, I calibrate the general equilibrium model to match several US credit market statistics. Then, I study the implications of changes in competition resulting from variations in the number of competitors and the rise of a few dominant bankers.

When banking competition decreases due to a reduction in the number of bankers, the risk-shifting effect causes a stronger response in the default rate of entrepreneurs following uncertainty shocks. Bankers respond by reducing their loan supply more substantially and entrepreneurs face a larger contraction in their financial resources. This results in a larger credit crunch, causing investment to fall more and leading to a greater contraction in GDP. As a result, the risk-shifting effect is stronger than the pass-through effect, and uncertainty shocks result in larger business cycle fluctuations when competition is lower. By calibrating the fall in competition to the increase in banking concentration in the US over the last 20 years, I find that a one-standard-deviation uncertainty shock implies a fall in GDP that is 0.1 percentage points larger.

To study the effects of decreased competition in banking due to the rise of dominant bankers, I assume that bankers are heterogeneous because they have varying marginal costs of providing loans. Bankers with lower intermediation costs can more easily provide loans to entrepreneurs and thus gain larger market shares. This results in a more concentrated banking sector and higher borrowing rates for entrepreneurs. In response to uncertainty shocks, smaller bankers reduce their loan offerings and increase their markups due to the pass-through effect. However, this has a limited impact on business cycle fluctuations.

Related Literature. The paper contributes to the literature on imperfect competition in the banking industry, financial frictions, uncertainty shocks and the role of banking competition in the transmission of shocks. My main contribution is connecting the literature on financial frictions, uncertainty shocks and the market structure of the banking industry. Specifically, building on the existing literature on uncertainty shocks and financial frictions, the paper examines how the impact of uncertainty shocks is affected by the market structure of the banking industry.

Imperfect Competition in the Banking Industry. This paper builds on the extensive theoretical literature on imperfect competition in the banking industry. Boyd and de Nicoló (2005) find that less competitive banking sectors charge higher borrowing rates but have riskier portfolios because borrowers optimally respond to higher borrowing rates by taking on more risk. However, Martinez-Miera and Repullo (2010) and Hakenes and Schnabel

(2011) respectively find that less competitive banking sectors have larger buffers against non-performing loans due to their larger profits and an incentive to reduce portfolio risk to protect their charter value. As a result, the relationship between banking competition and financial stability can be nonlinear.

The primary contribution of my work to this literature is the development of a DSGE model that incorporates the channel identified by Boyd and de Nicoló (2005). This channel is supported by the empirical evidence of Schaeck and Cihák (2014), Akins et al. (2016) and Berger et al. (2017). Furthermore, I introduce a novel channel, the pass-through effect.

Financial Frictions. The existing literature on financial frictions has studied the implications of such frictions on the transmission of shocks, often through the assumption of costly state verification or agency problems. Notable examples of studies that have introduced financial frictions through costly state verification frameworks include Townsend (1979), Carlstrom and Fuerst (1997), Bernanke et al. (1999), Christiano et al. (2014), Clerc et al. (2018), and Gasparini et al. (2022). On the other hand, Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) introduced financial frictions by adopting agency problems. Similarly to Kühl (2017), my paper combines the two approaches. In the model there is an agency problem because entrepreneurs can divert part of their assets after borrowing from banks. At the same time, banks have to pay a monitoring cost in order to observe the entrepreneur's realized return.

My paper contributes to the existing literature on financial frictions by introducing imperfect banking competition in the banking sector. Differently from previous studies, I assume that loans are provided to entrepreneurs by a finite amount of bankers that compete à la Cournot. This assumption implies that bankers charge a markup on the borrowing rate as observed by Corbae and D'Erasmo (2021). The introduction of imperfect banking competition in this economy creates an additional financial friction due to its impact on the borrowing rate.

Uncertainty Shocks. The literature on uncertainty shocks suggests that uncertainty shocks play an important role in driving business cycle fluctuations, as demonstrated by numerous theoretical and empirical papers such as Bloom (2009), Christiano et al. (2014), Caldara et al. (2016), Basu and Bundick (2017), Alessandri and Mumtaz (2019) and Baker et al. (forthcoming). In the literature the consensus is that uncertainty shocks have important negative effects on output.

My contribution to this literature is twofold. First, following the work of Baker et al. (forthcoming), I provide empirical evidence that uncertainty shocks have stronger negative effects on output when banking competition is lower.

Second, building on the work of Christiano et al. (2014), I contribute to the theoretical literature by developing a DSGE model that incorporates both financial frictions and imperfect banking competition. I use the model to study the implications of imperfect competition in the banking sector for the transmission of uncertainty shocks. Consistent with the empirical evidence, the model shows that uncertainty shocks have more severe contractionary effects on output when the banking sector is less competitive. The driving force is the risk-shifting effect, which makes borrowers more vulnerable when the banking sector is more concentrated. Consequently, an uncertainty shock leads to a greater increase in non-performing loans and a stronger cut in lending when the banking sector is less competitive. This further exacerbates the negative impact of uncertainty shocks.

Role of Banking Competition in the Transmission of Shocks. This paper contributes to the macro-finance literature on the transmission of shocks through the banking sector, specifically focusing on the role of banking competition.

Prior studies, including Scharfstein and Sunderam (2016), Gödl-Hanisch (2022), and Cuciniello and Signoretti (2018), investigate the implications of imperfect competition in the banking sector for the transmission of monetary policy shocks. However, these studies have produced mixed conclusions. Specifically, while Scharfstein and Sunderam (2016) find that high concentration in the U.S. banking sector leads to lower transmission of monetary policy shocks, which is consistent with their model of Cournot competition, the models of monopolistic competition in the banking sector developed by Gödl-Hanisch (2022) and Cuciniello and Signoretti (2018) suggest that monetary policy shocks have stronger effects when competition is lower. The latter finding is in line with the empirical evidence presented by Gödl-Hanisch (2022).

Other studies focused on other shocks. Jamilov and Monacelli (2021) develop a quantitative macroeconomic model with heterogeneous monopolistic financial intermediaries and study how banking competition affects the transmission of a capital quality shock. They find that credit market power decreases the impact of capital quality shocks. Villa (2020) builds a model where banks compete à la Cournout for loans and deposits, and argues that a sudden rise in the aggregate firms' default probability has stronger negative effects when banking competition is lower.

My contribution to this literature is twofold. First, I introduce a new propagation channel in this set of models, the risk-shifting effect. Second, I study the propagation mechanism of a different shock, an uncertainty shock.

Outline. The paper is structured as follows. In Section 2 I provide empirical evidence on the effect of banking competition for the propagation of uncertainty shocks. Section 3 outlines the borrower side of the model and introduces the risk-shifting effect in a partial equilibrium framework. Section 4 presents the general equilibrium model. Section 5 displays the calibration and the results of the quantitative model. In this section I show quantitatively how the level of competition affects the transmission of uncertainty shocks. Finally, Section 6 concludes.

2 Empirical Evidence

In this section I employ a panel dataset of 44 countries between 2000Q1 and 2020Q1 to empirically investigate the impact of banking competition on the transmission of uncertainty shocks. The section is structured as follows: Section 2.1 describes the data and Section 2.2 describes the regression model and the results. Appendix B provides further information on the dataset and Appendix B.1 presents the robustness tests.

2.1 Data Description

In my analysis I use data from 44 countries spanning the period 2000Q1-2020Q1.[4] For each country I collect quarterly data on real GDP growth, first and second moments of national business conditions, disaster shocks and yearly data on banking concentration. Real GDP growth is obtained from the International Financial Statistics of IMF or, if not available, from OECD. Data on first and second moments and disaster shocks are obtained from Baker et al. (forthcoming). Finally, banking concentration is obtained from the World Bank.

The measures of first and second moments of national business conditions are derived from national stock market movements. Specifically, the first moment is the stock market return of the broadest national index, while the second moment is the logarithm of the quarterly standard deviations of daily stock returns. I use the second moment measure as a proxy for uncertainty.

The disaster shocks considered in this analysis include four types of events: natural disasters, terrorist attacks, coups and revolutions. For each category, a value of one is assigned if a disaster shock has occurred. To generate the final indexes, the events are weighted by the increase in media coverage during the 15-days period following the shock compared to the 15-days period preceding the event. Media coverage is defined by the number of articles published in English-language newspapers based in the United States that mention the affected country.

Banking competition is proxied by the 3-bank asset concentration ratio which is defined as the assets of the three largest commercial banks as a share of total commercial banking assets.[5] This information is available only on annual basis. In this section the 3-bank asset concentration ratio is linearly interpolated to obtain a quarterly measure.[6] A more concentrated banking sector indicates lower banking competition.

Descriptive statistics for the dataset can be found in Table 3 of Appendix B.

[4]The countries used in this analysis are listed in Table 2 in Appendix B.

[5]As shown in Appendix B.1, the results are similar using the 5-bank asset concentration level

[6]The results hold also keeping the level of concentration constant within a year. Appendix B.1 shows the results of this robustness test.

2.2 Banking competition and the Impact of Uncertainty Shocks.

In this section I describe the regression model and I report the empirical results. In section 2.2.1 I describe the regression model and in 2.2.2 I present the results.

2.2.1 Regression Model

In order to estimate the effect of an increase in uncertainty on output growth, and to investigate how the level of banking competition affects the impact of uncertainty shocks, I estimate the following regression model

$$y_{i,t+h} = \alpha_i + \tau_t + \beta^R \tilde{R}_{i,t} + \beta^V \tilde{V}_{i,t} + \beta^C \tilde{C}_{i,t} + \beta^{RC} \tilde{R}_{i,t}\tilde{C}_{i,t} + \beta^{VC} \tilde{V}_{i,t}\tilde{C}_{i,t} + \epsilon_{i,t}.$$

where $y_{i,t+h}$ is the growth rate of real GDP from period $t-1$ to period $t+h$ for country i, α_i captures country fixed effects, τ_i captures time fixed effects, $\tilde{R}_{i,t}$ is the country demeaned measure of first moment of national business conditions, $\tilde{V}_{i,t}$ is the country demeaned measure of uncertainty and $\tilde{C}_{i,t}$ is the country demeaned 3-bank asset concentration ratio.

This model extends the one proposed by Baker et al. (forthcoming) by adding banking concentration and interactions terms between banking concentration and and first and second moments. The interactions are included to isolate the effect of concentration on the impact of first and second moment shocks. Additionally, the model controls for non-linear effects of country characteristics by demeaning the variables at the country level.

The coefficients β^V and β^{VC} measure the impact of an increase in uncertainty on real output growth. Specifically, β^V captures the impact of an uncertainty shock on output growth when banking concentration is at the country mean, while β^{VC} captures how the impact of uncertainty shocks varies with banking concentration. If β^{VC} is negative, an increase in uncertainty has a more severe negative effect on output growth when concentration is higher.

Similarly to Baker et al. (forthcoming), I instrument first and second moment variables and their interaction with concentration using disaster shocks.[7] This instrumental variable

[7]The instruments used are the disaster shocks and their interaction with with demeaned concentration.

approach allows me to study the causal impact of first and second moment shocks on output growth. Furthermore, because of the media weighting of the disaster shocks the regression gives higher weight to more important shocks.

As in Baker et al. (forthcoming) there is a potential issue with this identification strategy. The stock market level and volatility variables proxy for different channels through which disaster shocks have economic impact. The underlying exclusion restriction is that these effects impact economic activity only through shifts in the first and second moments of stock returns.

2.2.2 Results

Figure 1 shows the impact of a one-standard deviation uncertainty shock on real output growth at different levels of banking concentration. The blue line depicts the impulse response of output growth when concentration is at the country average and the blue dashed lines are the 90% confidence interval. The figure reveals a significant negative effect of an uncertainty shock on output growth.

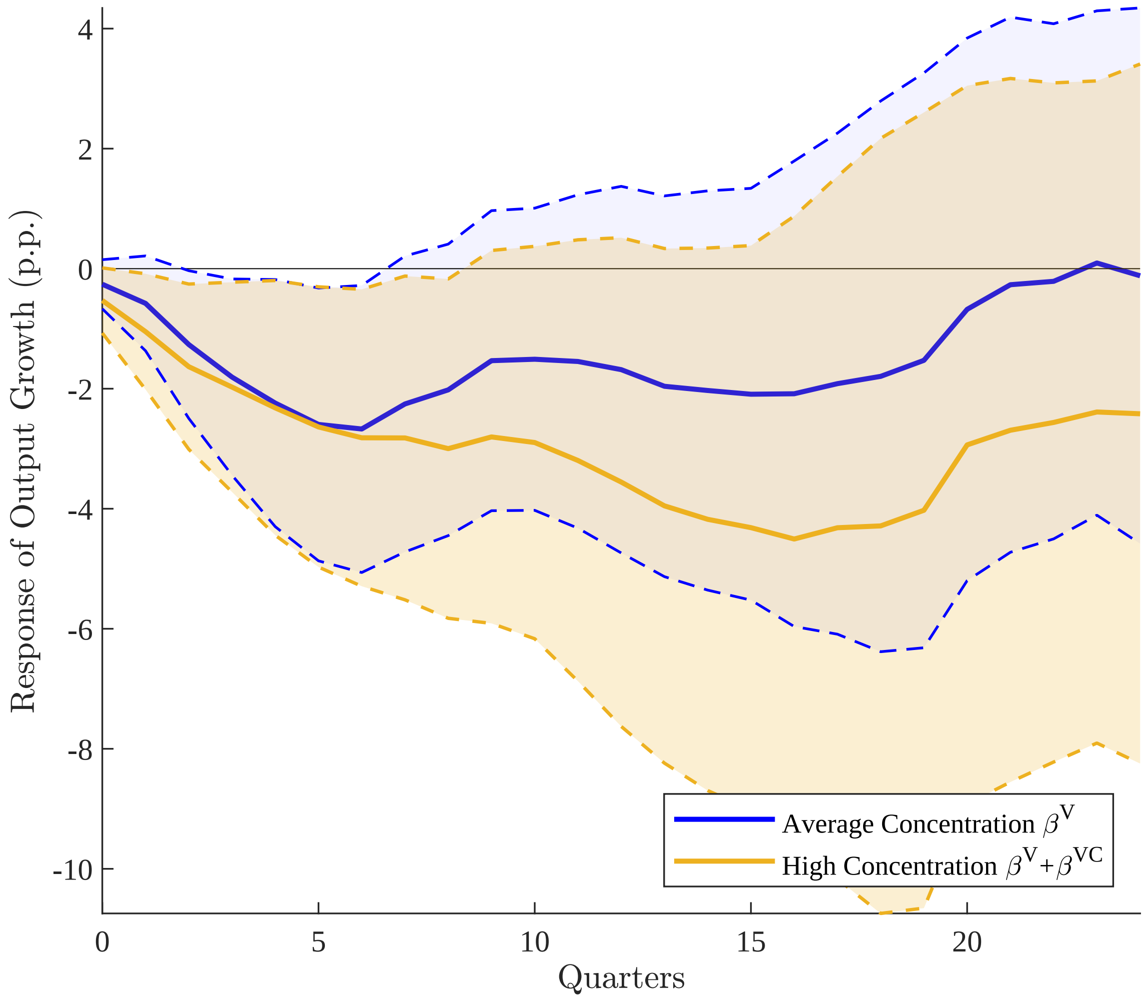
The yellow line represents the impulse response of output growth when concentration is one standard deviation above the country average and the yellow dashed lines are the 90% confidence interval.[8] In this case the fall in output growth is stronger and significant for a longer period.

Figure 2 plots the difference in the output growth responses between the average banking concentration specification (blue line in Figure 1) and the high banking concentration specification (yellow line in Figure 1). The graph shows that the decline in output growth is significantly more pronounced in countries with higher banking concentration.

The results shown in this section are robust to variations in the measure of concentration. In particular, the findings hold when keeping the level of concentration constant within a year, using the 5-bank asset concentration instead of the 3-bank asset concentration and controlling for endogeneity by replacing concentration with its lag. Additionally, the results hold even after removing the year 2009 from the sample. The last robustness test shows that the findings are not driven by the impact of the global recession that occurred in that

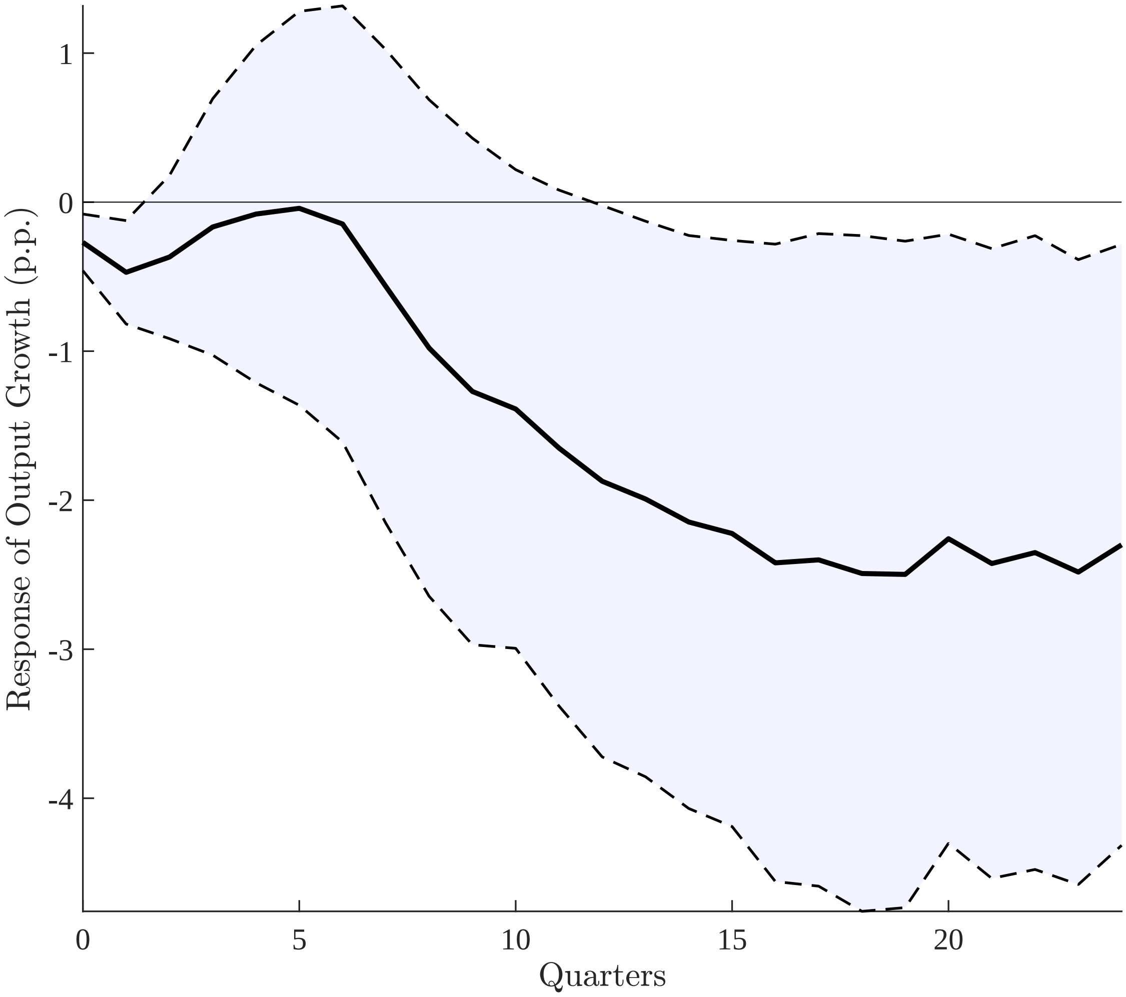
[8] A standard deviation corresponds to 10.21 percentage points.

Figure 1: Response of output growth to a one-standard deviation uncertainty shock.



Notes. The graph shows the responses of output growth to a one-standard deviation uncertainty shock for different levels of banking concentration. In the response with high banking concentration, banking concentration is 1 standard deviation higher than the country average. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1.

Figure 2: Effect of competition on output growth response to a one-standard deviation uncertainty shock.



Notes. The graph shows the difference between the two specifications plotted in Figure 1. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1.

year. The robustness tests are displayed in Appendix B.1.

In the following sections, I develop a general equilibrium model that replicates the empirical findings presented in this section. This is accomplished through a two-step process. First, I introduce the entrepreneurial sector in a partial equilibrium framework. Next, I incorporate the banking sector and integrate the credit market into a standard DSGE model. The model is calibrated to reflect the US economy and used to examine how banking competition affects business cycle fluctuations and to assess the impact of the recent decline in banking competition on the transmission of uncertainty shocks.

3 Entrepreneurial Sector and Risk-Shifting Effect

In this section I introduce and analyze the entrepreneurial sector in a partial equilibrium setup and I introduce the risk-shifting effect, which is at the heart of the results shown in the paper.

The entrepreneurial sector is modeled similarly to Clerc et al. (2018). Specifically, there is a continuum of risk-neutral entrepreneurs, each of them is indexed by $j \in (0,1)$. Each entrepreneur lives across two consecutive periods. Every entrepreneur born at time t has financial resources given by inherited wealth from the previous generation of entrepreneurs $n_t^{E,j}$ and loans b_t^j from the banking sector. Entrepreneurs use their financial resources to buy capital goods from capital good producers. The purchased capital goods are then rented out to final goods producers.

Entrepreneurs born at time t derive utility from donating a part of their final wealth in the form of dividends to the households $c_{t+1}^{E,j}$, and the rest to the next generation of entrepreneurs as retained earnings, according to the utility function $(c_{t+1}^{E,j})^{\chi^E}(n_{t+1}^{E,j})^{1-\chi^E}$. At time $t+1$, the maximization problem for the entrepreneur born at time t is

$$\max_{c_{t+1}^{E,j},n_{t+1}^{E,j}} (c_{t+1}^{E,j})^{\chi^E}(n_{t+1}^{E,j})^{1-\chi^E},$$

Subject to the budget constraint

$$c_{t+1}^{E,j} + n_{t+1}^{E,j} \leq W_{t+1}^{E,j},$$

where $W^{E,j}_{t+1}$ is the final wealth of entrepreneur j born at time t.

The first order conditions lead to the dividend payment rule $c^{E,j}_{t+1} = \chi^E W^{E,j}_{t+1}$ and the earning retention rule $n^{E,j}_{t+1} = (1-\chi^E)W^{E,j}_{t+1}$.

Future wealth is defined as

$$W^{E,j}_{t+1} = \frac{\max[\omega^j_{t+1} R^E_{t+1} q K^j_t - R^F_t b^j_t, 0]}{\Pi_{t+1}}. \tag{1}$$

Future wealth is determined by the return from renting capital to final goods producers net of borrowing costs. The borrowing costs are determined by the borrowing rate R^F_t times the amount borrowed. The return from renting capital is determined by the product of the amount of capital rented, its price q, the gross return per efficiency unit of capital R^E_{t+1} and an idiosyncratic shock ω^j_{t+1}.[9] The return from lending capital and the borrowing costs are both discounted by the gross inflation rate $\Pi_{t+1} = P_{t+1}/P_t$. I assume that R^E_{t+1} is a decreasing function in capital.

The idiosyncratic shock ω^j_{t+1} is a shock to the entrepreneur's efficiency units of capital. This shock is assumed to be independently and identically distributed across entrepreneurs and to follow a log-normal distribution with mean one and standard deviation $\sigma_t = \sigma\varsigma_t$. The cumulative distribution function and the probability density function of the idiosyncratic shock are denoted by $F(\cdot)$ and $f(\cdot)$, respectively. Uncertainty is defined as σ_t, while ς_t represents an uncertainty shock that follows an AR(1) process

$$\ln\varsigma_t = \rho\ln\varsigma_{t-1} + \varepsilon_t, \tag{2}$$

where $0 < \rho < 1$ and σ^ε_t is the standard deviation of the *iid* shock ε_t.

Entrepreneurs and bankers enter into a financial contract where the loan repayment depends on the realization of a random productivity shock. If the shock is above a default cutoff $\bar{\omega}^j_{t+1}$, the entrepreneur pays the bankers $R^F_t b^j_{t+1}$, otherwise the entrepreneur defaults. The default cutoff is given by

$$\overline{\omega}^j_{t+1} = \frac{R^F_t b^j_t}{R^E_{t+1} q K^j_t}. \tag{3}$$

[9]Note that in this section, the price of capital is assumed to be constant, while in Section 4 the price of capital will be determined by supply and demand of capital.

In contrast to Bernanke et al. (1999), the default cutoff $\overline{\omega}^j_{t+1}$ varies with the realization of the aggregate state R^E_{t+1}. The probability of default of an entrepreneur is

$$F^j_{t+1} = F(\overline{\omega}_{t+1}) = \int_0^{\overline{\omega}_{t+1}} f(\omega^j_{t+1}) \mathrm{d}\omega^j_{t+1} = \Phi\left(\frac{\log(\overline{\omega}^j_{t+1}) + 0.5\sigma^2_{t+1}}{\sigma_{t+1}}\right). \tag{4}$$

In case of default, the entrepreneur obtains nothing and the bankers must pay a monitoring cost that is discussed more in detail in Section 4.

Similarly to Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Kühl (2017), there is a moral hazard problem: at time t every entrepreneur can divert a fraction λ of available funds. To ensure that entrepreneurs do not divert funds, the following incentive constraint must hold[10]

$$\lambda \frac{qK^j_{t+1}}{\Pi_{t+1}} \leq \mathbb{E}_t(W^{E,j}_{t+1}) \tag{5}$$

Entrepreneurs born at time t maximize their expected future wealth by choosing how much capital to buy and how much to borrow from the bankers

$$\max_{K^j_t, b^j_{t+1}} \mathbb{E}_t(W^{E,j}_{t+1}),$$

subject to the resource constraint

$$qK^j_t - b^j_{t+1} = n^{E,j}_t. \tag{6}$$

and to the incentive constraint (5).[11]

Appendix C.1.1 proves that the incentive constraint is binding in an active credit market. Therefore, the loan demand and the demand for capital are implicitly determined by the incentive participation constraint

$$(1 - \Gamma(\overline{\omega}^j_{t+1}))R^E_{t+1} = \lambda, \tag{7}$$

where $\Gamma(\overline{\omega}^j_{t+1}) = \Gamma^j_{t+1}$ is the expected share of return that entrepreneurs retain after paying

[10] Note that since bankers make positive profits, the financial contract cannot be derived using bankers' zero profit condition as in Bernanke et al. (1999).

[11] Note that entrepreneurs choose their probability of default by choosing K^j_t and b^j_{t+1}.

borrowing costs. Since all entrepreneurs face the same borrowing rate and expected return, the model can be aggregated by dropping the indices j from now on.

Propositions 1 and 2 demonstrate that an increase in the borrowing rate leads to a decrease in loan demand. This results in lower entrepreneurial leverage but higher default risk due to limited liability. Entrepreneurs reduce their demand for loans and leverage as borrowing rates rise. However, limited liability limits their potential losses in case of default, so they do not reduce their leverage enough to offset the higher borrowing rate. This results in a positive relationship between the borrowing rate and both the default rate of entrepreneurs and their default cutoff.

Proposition 1. *Loan demand is a decreasing function of the loan rate.*

Proposition 2. *The default rate of the entrepreneurs and their default cutoff increase with the borrowing rate.*

The proofs of Propositions 1 and 2 are provided in Appendix C.1.2.

Furthermore, Proposition 3 shows that an increase in uncertainty has a greater impact on entrepreneurial defaults when the default cutoff is higher. As entrepreneurs take more risk with a higher default cutoff, an increase in uncertainty has a larger impact on default rates.

Proposition 3. *If* $R^E_{t+1} \leq \frac{\lambda}{1-\Gamma\left(e^{-\sigma-0.5\sigma^2}\right)}$, *an increase in uncertainty results in a larger rise in the default rate of entrepreneurs when the default cutoff is higher.*

The proof of Proposition 3 is provided in Appendix C.1.2.[12]

When the banking sector is less competitive, bankers tend to charge higher borrowing rates, leading to a higher default rate of entrepreneurs, as demonstrated in Proposition 2. Similarly to Martinez-Miera and Repullo (2010), I call the effect of lower banking competition on borrower risk-taking, the risk-shifting effect. This effect is the driving force behind why output falls more with an uncertainty shock when banking competition is lower. As uncertainty increases, the default rate of entrepreneurs rises more strongly in less competitive banking sectors, causing bankers to reduce loans even further. This reduction in loans

[12]The condition $R^E_{t+1} \leq \frac{\lambda}{1-\Gamma\left(e^{-\sigma-0.5\sigma^2}\right)}$ implies that $F(\bar{\omega}_{t+1}) \leq \Phi(-1) \approx 0.1587$. Both the conditions of Proposition 3 are satisfied in the general equilibrium model.

results in a more severe lack of resources for entrepreneurs, leading to a more significant decline in investment and output.

4 General Equilibrium

In this section, I present the remaining components of the model. The credit market builds on the partial equilibrium framework outlined in Section 3. The equilibrium price of capital is determined by the interplay between the demand and the supply of capital. Entrepreneurs purchase capital from capital good producers and rent it to intermediate goods producers. A fixed number of bankers provide loans to entrepreneurs, competing in a Cournot fashion for those loans.

The remaining parts of the model are standard. Intermediate goods producers utilize capital and labor to produce intermediate goods, which are then purchased by final goods producers who bundle them together to produce the final good. Finally, the central bank adjusts the policy rate according to a Taylor rule.

4.1 Bankers

There is a fixed number of bankers N competing in a Cournot fashion for loans. Each banker is indexed by i and lives across two consecutive periods. Bankers born at time t have equity in the form of inherited wealth from the previous generation of bankers $n_t^{F,i}$ and borrow deposits d_t^i from households. They use these resources to provide loans to entrepreneurs.

At time $t+1$ bankers derive utility by donating part of their final wealth to households in the form of dividends $c_{t+1}^{F,i}$ and by leaving the rest as retained earnings to the next generation of bankers according to the utility function $(c_{t+1}^{F,i})^{\chi^F}(n_{t+1}^{F,i})^{1-\chi^F}$. Therefore, the maximization problem of each banker at time $t+1$ is given by

$$\max_{c_{t+1}^{F,i}, n_{t+1}^{F,i}} (c_{t+1}^{F,i})^{\chi^F}(n_{t+1}^{F,i})^{1-\chi^F}$$

subject to the resource constraint

$$c_{t+1}^{F,i} + n_{t+1}^{F,i} \leq W_{t+1}^{F,i},$$

where $W_{t+1}^{F,i}$ is the final wealth of the banker i born at time t.

The first order conditions lead to the dividend payment rule

$$c_{t+1}^{F,i} = \chi^F W_{t+1}^{F,i}, \tag{8}$$

and the earning retention rule

$$n_{t+1}^{F,i} = (1 - \chi^F) W_{t+1}^{F,i}. \tag{9}$$

The future wealth of each banker is

$$W_{t+1}^{F,i} = \frac{\tilde{R}_{t+1}(b_t) b_t^i - R_t^D d_t^i - \gamma^i b_t^i}{\Pi_{t+1}}.$$

Bankers' future wealth is determined by the return they earn from lending to entrepreneurs net of deposit and intermediation costs. The return from lending is calculated as the amount of loans multiplied by the return per unit of loans $\tilde{R}_{t+1}$. The cost of deposits is given by the deposit rate R_t^D multiplied by the amount of deposits. In addition, each banker pays a per loan intermediation cost γ^i, which can vary across bankers. All of these factors are discounted by the gross inflation rate Π_{t+1}.

The return per unit of loans is

$$\tilde{R}_{t+1} = (1 - F_{t+1}) R_{t+1}^F + (1 - \xi) \int_0^{\overline{\omega}_{t+1}} \omega_{t+1} f(\omega_{t+1}) \mathrm{d}\omega_{t+1} \frac{R_{t+1}^E q_t K_t}{b_t}. \tag{10}$$

The first term of Equation 10 represents the return from performing loans, while the second term represents the return from non-performing loans. When a loan default bankers incur a monitoring cost ξ to observe the entrepreneur's realized return on capital. This cost is a proportion of the realized gross payoff to the entrepreneurs. Note that all bankers receive the same return from non-performing loans because they have equal seniority.

At time t, each banker chooses how much to lend to entrepreneurs and borrow from households, taking into account the decisions of other bankers. The objective is to maximize future equity

$$\max_{\{b_t^i, d_t^i\}} \frac{\tilde{R}_{t+1}(b_t)b_t^i - R_t^D d_t^i - \gamma b_t^i}{\Pi_{t+1}},$$

subject to the balance sheet constraint

$$n_t^{F,i} + d_t^i \geq b_t^i, \tag{11}$$

and the loan demand (7) due to imperfect competition. The first order condition of the maximization problem, after substituting the balance sheet constraint, is

$$\frac{\partial \tilde{R}_{t+1}}{\partial b_t} b_t^i + \tilde{R}_{t+1} - R_t^D - \gamma^i = 0.$$

It's worth noting that, due to imperfect competition, the optimal choices of each banker depend on the impact of their decisions on the return they receive from lending to entrepreneurs. The impact of bankers' decisions depend on the slope of the demand curve and on the level of competition in the banking sector.

The level of competition not only affects bankers' profits, but also the extent to which they pass shocks through to borrowers. In less competitive markets, bankers have higher profits, but pass shocks through to borrowers by a lesser extent, as profits absorb some of the impact. An uncertainty shock increases non-performing loans and monitoring costs for bankers. More competitive bankers decrease their loan supply more than less competitive bankers, due to lower market power. I define the pass-through effect the effect of banking competition on the extent to which bankers pass shocks through to borrowers.

If every banker has the same intermediation cost, the equilibrium is symmetric and the first order conditions of the bankers can be aggregated to

$$\frac{\partial \tilde{R}_{t+1}}{\partial b_t} \frac{b_t}{N} + \tilde{R}_{t+1} - R_t^D - \gamma = 0.$$

In this case, the level of competition increases with the number of bankers. As the number of bankers increases, the impact of the decisions of a single banker on the return bankers receive from lending to entrepreneurs decreases leading to a fall in the market power of bankers.

4.2 Rest of the Model

The rest of the model follows a standard New Keynesian framework. Households maximize their utility choosing consumption, labor supply deposits supply. The production sector comprises final, intermediate, and capital goods producers. Final goods producers are perfectly competitive and use intermediate goods to produce consumption bundles using a constant-elasticity-of-substitution technology. Final goods are sold to households and to capital producers. Intermediate goods producers use capital and labor to produce intermediate goods with a Cobb-Douglas technology, setting prices subject to quadratic adjustment costs. This leads to a standard New Keynesian Phillips curve. Capital goods producers buy the final good, convert it into capital, and sell it to entrepreneurs. The model is closed by a central bank that sets the policy rate following a monetary policy rule.

4.2.1 Gross Return on Capital

Differently from Section 3, the price of capital q_t is time-varying and determined by the equilibrium of demand and supply of capital.

The gross return on capital is

$$R_t^E = \frac{r_t^K + (1-\delta)\, q_t}{q_{t-1}} \Pi_t.$$

The gross return on capital is given by the sum of the real rental rate on capital r_t^K and the real capital gains net of depreciation $(1-\delta)\, q_t$, divided by the real price per unit of capital in period $t-1$. Finally, the return is expressed in nominal terms and multiplied by the inflation rate.

4.2.2 Households

Households are infinitely lived and maximize their expected lifetime utility,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln c_t - \varphi \frac{l_t^{1+\eta}}{1+\eta} \right), \tag{12}$$

where $\beta \in (0,1)$ is the discount factor, c_t is consumption, l_t is labor supply, $\varphi > 0$ is the relative weight on labor disutility and $\eta \geq 0$ is the inverse Frisch elasticity of labor supply. Households choose consumption, labor supply and deposit supply to maximize (12) subject to the of budget constraint,

$$c_t + d_t \leq w_t l_t + \frac{R_t^D d_{t-1}}{\Pi_t} + \Xi_t^K + \sum_{i=1}^{N} \chi^F W_t^{F,i} + \chi^E W_t^E + \Xi_t^P, \tag{13}$$

where w_t is the real wage, R_t^D is the gross interest rate on deposits paid in period t, Ξ_t^K and Ξ_t^P are profits earned by capital goods producers and intermediate goods producers, respectively, and $\sum_{i=1}^{N} \chi^F W_t^{F,i}$ and $\chi^E W_t^E$ are the dividends received by households from bankers and entrepreneurs respectively. The first order conditions of the optimization problem lead to a labor supply equation, $w_t = \varphi l_t^{\eta} / \Lambda_t$, and an Euler equation, $1 = \mathbb{E}_t\{\beta_{t,t+1} R_{t+1}^D / \Pi_{t+1}\}$, where $\beta_{t,t+s} = \beta^s \Lambda_{t+s}/\Lambda_t$ is the household's stochastic discount factor and $\Lambda_t = 1/c_t$ is the Lagrange multiplier on the budget constraint.

4.2.3 Final Goods Producers

Final goods producers bundle the intermediate goods Y_{it}, with $i \in (0,1)$, taking as given their price P_{it}, and sell the output Y_t at the competitive price P_t. Final goods producers choose the amount of inputs Y_{it} that maximizes profits $P_t Y_t - \int_0^1 Y_{it} P_{it} \mathrm{d}i$, subject to the production function $Y_t = (\int_0^1 Y_{it}^{(\varepsilon-1)/\varepsilon} \mathrm{d}i)^{\varepsilon/(\varepsilon-1)}$, where $\varepsilon > 1$ is the elasticity of substitution between intermediate goods. The resulting demand for intermediate good i is $Y_{it}^d = (P_{it}/P_t)^{-\varepsilon} Y_t$. The price of final output, which is interpreted as the price index, is given by $P_t = (\int_0^1 P_{it}^{1-\varepsilon} \mathrm{d}i)^{1/(1-\varepsilon)}$. In a symmetric equilibrium, the price of a variety and the price index coincide, $P_t = P_{it}$.

4.2.4 Intermediate Goods Producers

Intermediate goods producers use capital and labor to produce intermediate goods according to a Cobb-Douglas production function. Because of the assumption of constant returns to scale the production function can be aggregated. Each producer produces a differentiated good using $Y_{it} = A_t K_{it-1}^{\alpha} l_{it}^{1-\alpha}$, where $\alpha \in (0,1)$ is the capital share in production, A_t is aggregate technology, K_{it-1} is capital and l_{it} is labor. Intermediate goods producers choose the amount of inputs to maximize profits given by $P_{it}Y_{it}/P_t - r_t^K K_{it-1} - w_t l_{it}$, where the real rental rate on capital r_t^K and the real wage w_t are taken as given, subject to the technological constraint and the demand constraint. The optimization problem results in a labor demand and a capital demand that are $w_t l_{it} = (1-\alpha)s_{it}Y_{it}$ and $r_t^K K_{it-1} = \alpha s_{it} Y_{it}$, respectively, where the Lagrange multiplier on the demand constraint, s_{it}, represents real marginal costs. By combining the two demands, it is possible to obtain an expression for real marginal costs that is symmetric across producers,

$$s_t = \frac{w_t^{1-\alpha}(r_t^K)^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \frac{1}{A_t}. \tag{14}$$

Firm i sets an optimal path for its product price P_{it} to maximize the present discounted value of future profits, subject to the demand constraint and to price adjustment costs,

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta_{t,t+s} \left[\frac{P_{it+s} Y_{it+s}^d}{P_{t+s}} - \frac{\kappa_p}{2} \left(\frac{P_{it+s}}{P_{it+s-1}} - 1 \right)^2 Y_{it+s} + s_{t+s} \left(Y_{it+s} - Y_{it+s}^d \right) \right]. \tag{15}$$

Price adjustment costs are given by the second term in square brackets in (15); they depend on firm revenues and on last period's aggregate inflation rate. The parameter $\kappa_p > 0$ scales the price adjustment costs. Under symmetry, all firms produce the same amount of output, and the firm's price P_{it} equals the aggregate price level P_t, such that the price setting condition is

$$\kappa_p \Pi_t (\Pi_t - 1) = \varepsilon s_t - (\varepsilon - 1) + \kappa_p \mathbb{E}_t \left\{ \beta_{t,t+1} \Pi_{t+1} (\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \right\}. \tag{16}$$

Under symmetry across intermediate goods producers, profits (in real terms) are $\Xi_t^P = Y_t - r_t^K K_{t-1} - w_t l_t - 0.5 \cdot \kappa_p(\Pi_t - 1)^2 Y_t$.

4.2.5 Capital Goods Producers

Capital goods producers choose paths for investment I_t to maximize the expected present value of future profits given by $\mathbb{E}_t \sum_{s=0}^{\infty} \beta_{t,t+s} \left[q_{t+s} I_{t+s} - (1 + g_{t+s}) I_{t+s}\right]$. The term $g_t = 0.5 \cdot \kappa_I (I_t/I_{t-1} - 1)^2$ captures investment adjustment costs as in Christiano et al. (2014). Capital accumulation is defined as

$$I_t = K_t - (1 - \delta) K_{t-1}, \tag{17}$$

where $\delta \in (0, 1)$ is the capital depreciation rate. The maximization problem leads to the optimality condition for investment

$$1 = q_t - \frac{\kappa_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \kappa_I \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} + \mathbb{E}_t \left\{ \beta_{t,t+1} \kappa_I \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right\}. \tag{18}$$

In period t the profits of capital producers in real terms are $\Xi_t^K = q_t I_t - (1 + g_t) I_t$.

4.2.6 Central Bank

I assume the central bank sets the policy rate according to a standard Taylor rule. The monetary policy rule depends on its own lag, inflation and GDP growth. The respective feedback coefficients are τ_R, τ_Π and τ_y such that:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\tau_R} \left[\left(\frac{\Pi_t}{\Pi} \right)^{\tau_\Pi} \left(\frac{GDP_t}{GDP_{t-1}} \right)^{\tau_y} \right]^{1-\tau_R}, \tag{19}$$

where GDP is defined as output net of default costs.

Since the deposit rate is risk-free, the policy rate and the deposit rate are identical, $R_t = R_t^D$.

4.2.7 Market Clearing

The production of consumption goods must be equal to the sum of goods demanded by households, goods used for investment, resources lost when adjusting prices and investment, as well as resources lost in the recovery of funds associated with defaults and due to intermediation costs,

$$Y_t = c_t + (1 + g_t)I_t + \frac{\kappa_p}{2}\left(\Pi_t - 1\right)^2 Y_t + \mu^E G_t^E \frac{R_t^E q_{t-1} K_{t-1}}{\Pi_t} + \sum_{i=1}^{N} \gamma^i b_t^i.$$

Labor demand must equal labor supply

$$(1 - \alpha) s_t Y_t / l_t = \varphi_t l_t^\eta / \Lambda_t.$$

4.3 Symmetric Equilibrium

A symmetric equilibrium is a set of allocations $\{l_t,\, K_t,\, I_t,\, c_t,\, Y_t,\, n_t^E,\, b_t,\, n_t^F,\, d_t\}_{t=0}^{\infty}$, prices $\{q_t,\, w_t,\, r_t^K, \Pi_t, s_t\}_{t=0}^{\infty}$ and rates of return $\{R_t^F,\, R_t^E,\, R_t^D,\, \tilde{R}_t\}_{t=0}^{\infty}$ for which given the monetary policy $\{R_t\}_{t=0}^{\infty}$ and shocks to entrepreneurial uncertainty $\{\varsigma_t\}_{t=0}^{\infty}$

- Entrepreneurs maximize expected future wealth,
- Producers and bankers maximize profits,
- Households maximize utility,
- All markets clear.

5 Results

This section presents the calibration of the general equilibrium model and discusses how the transmission of an uncertainty shock in this economy changes with the level of banking competition. First, I show the implications of a change in competition due to a change in the number of competitors. Second, I examine the implications of a change in competition due to the rise of a few dominant bankers.

5.1 Calibration

Table 1 presents the parameter values used for calibrating the model to the period 2010Q1-2019Q4. The discount factor β is chosen to match the average yearly Federal Funds Effective Rate of 0.6%, while the capital share in production α and the depreciation rate of capital are the same as in Christiano et al. (2014). The fraction of resources lost due to entrepreneur defaults ξ matches the charge-off rate on business loans. The dividend payout ratios of entrepreneurs χ^E and bankers χ^F are selected to match the leverage of non-financial corporate business and the ratio of banker equity over assets, respectively. The proportion of assets that can be diverted by entrepreneurs λ is chosen to match the ratio between non-financial corporate business loans and GDP. The intermediation cost γ matches the average markup of bankers used by Jamilov and Monacelli (2021). The number of bankers N is chosen such that the 3-bank asset concentration ratio is 33%, which is close to the data (35.15%). The autocorrelation of the uncertainty shock ρ, and the inverse Frish labor elasticity η are obtained from Christiano et al. (2014). The parameter that determines the substitutability between intermediate goods ϵ is taken from Christensen and Dib (2008) to match a markup of 1.2. The price adjustment cost is taken from Smets and Wouters (2007) and the investment adjustment cost is from Carlstrom et al. (2014). The weight on labor disutility is chosen to normalize labor supply to 1. For the coefficients of the Taylor rule, the conventional Taylor rule parameters are used as in Gertler and Karadi (2011). The smoothing parameter is set to 0.8, the coefficient of the Taylor rule for inflation is 1.5 and the coefficient for GDP growth is 0.5/4. In the following a period corresponds to a quarter.

5.2 Implications of a Reduction in the Number of Bankers

In this section, I examine how the transmission of uncertainty shocks is affected by changes in banking competition resulting from variations in the number of bankers. Specifically, I consider an uncertainty shock that does not cause bankers' equity to become negative, which would lead to defaults. Figure 3 illustrates the responses of important variables in the model to a one-standard deviation uncertainty shock for different levels of competition.

Consider first the light blue solid line, which corresponds to the baseline level of compe-

Table 1: Calibration of the baseline model

Variable	Meaning	Value	Target
β	Discount factor	0.9985	FED Funds Rate
α	Capital share in production	0.4	Christiano et al. (2014)
δ	Depreciation rate capital	0.025	Christiano et al. (2014)
ξ	Entrepreneur bankruptcy cost	0.3519	Charge-Off Rate Business Loans
σ	Steady-state uncertainty	0.2541	Delinquency Rate Business loans
χ^E	Dividend payout entrepreneurs	0.0812	Non-financial Corporate Business Leverage
χ^F	Dividend payout bankers	0.3466	Banker equity ratio = 12%
λ	Proportion divertible assets entrepreneurs	0.8110	Non-financial Corporate Business Loans/GDP
γ	Banker intermediation cost	0.0431	Markup bankers
N	Number of bankers	9	3-Bank asset concentration
ρ	Autocorrelation risk shock	0.97	Christiano et al. (2014)
η	Inverse Frisch labor elasticity	1	Christiano et al. (2014)
ε	Substitutability between goods	6	Christensen and Dib (2008)
κ_p	Price adjustment cost	20	Smets and Wouters (2007)
κ_I	Investment adjustment cost	2.43	Carlstrom et al. (2014)
φ	Weight on labor disutility	0.5718	Labor supply = $l = 1$
τ_R	Coeff. TR for lag policy rate	0.8	Gertler and Karadi (2011)
τ_Π	Coeff. TR for inflation	1.5	Gertler and Karadi (2011)
τ_y	Coeff. TR for GDP	0.5/4	Gertler and Karadi (2011)

Notes. The table describes the calibration of the baseline model.

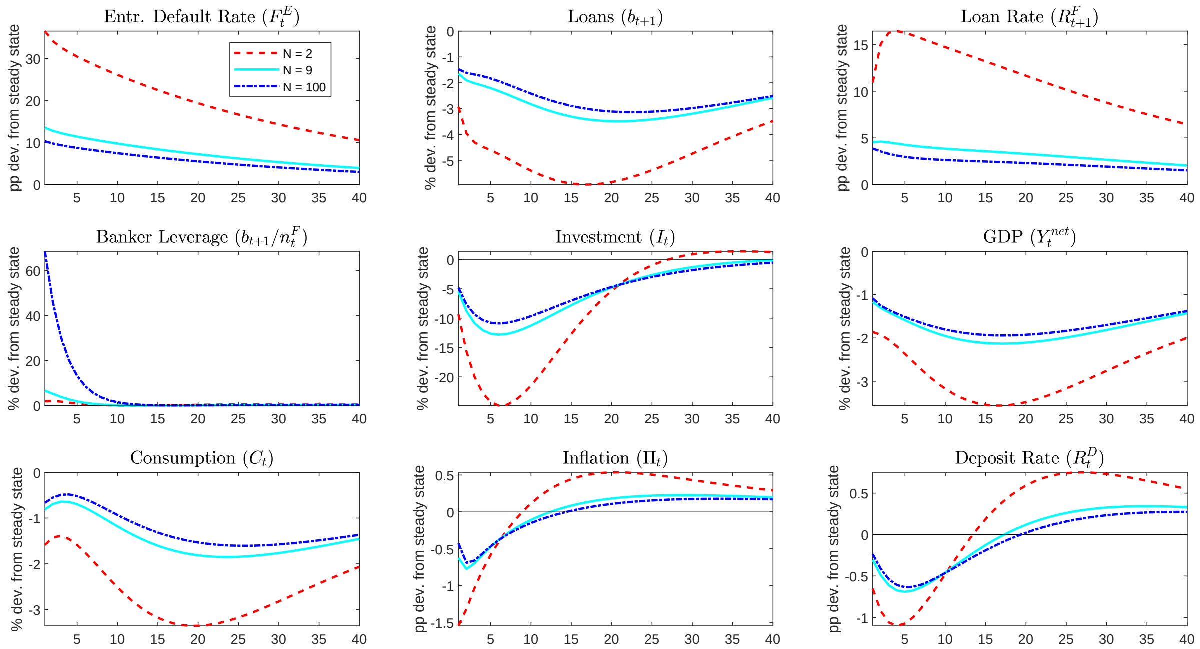
tition. An uncertainty shock raises the default rate of entrepreneurs by increasing the share of entrepreneurs with productivity below the default cutoff. This increases credit risk, leading bankers to reduce loan supply and increase the loan rate. The higher loan rate further increases the default rate of entrepreneurs. Due to the spike in defaults, bankers equity falls and their leverage increases. With reduced loan supply, entrepreneurs have less resources to buy capital and investment falls. Due to the fall in investment, also GDP decreases and inflation falls: we observe a demand-driven downturn as in Christiano et al. (2014). Finally, the central bank reacts to the falls in output and inflation by cutting the policy rate.

Consider now the red dashed and the blue dot-dashed lines which corresponds to models with 2 and 100 bankers, respectively. The former represents a highly concentrated banking sector, while the latter represents a nearly perfectly competitive one.

As discussed in Section 3, less competitive banking sectors experience higher default rates among entrepreneurs and greater borrower risk-taking.[13] Therefore, after an uncertainty shock, the default rate for entrepreneurs increases more in economies with less competitive banking sectors. Despite the lower pass-through, bankers in less competitive sectors reduce loan supply more due to the stronger rise in credit risk. However, the larger losses incurred

[13] This is consistent with the empirical evidence of Berger et al. (2017)

Figure 3: Impulse responses to an uncertainty shock varying the number of bankers.



Notes. The graph shows how several variables in the model respond to a one-standard deviation uncertainty shock for different levels of competition. The light blue solid lines represent the baseline model's impulse responses, while the red dashed lines show the impulse responses of an economy with a highly concentrated banking sector. The blue dot-dashed lines display the impulse responses of an economy with a banking sector that is nearly perfectly competitive.

by less competitive banking sectors due to the stronger rise in entrepreneurial defaults lead to a smaller fall in equity and smaller increase in leverage because less competitive bankers have larger equity buffers. The stronger rise in entrepreneurial defaults and the stronger fall in loan supply leads to a stronger fall in investment and GDP when competition is lower. Finally, because of the stronger recession, consumption, inflation and the deposit rate also fall by more.

5.3 Implications of the Recent Fall in Banking Competition

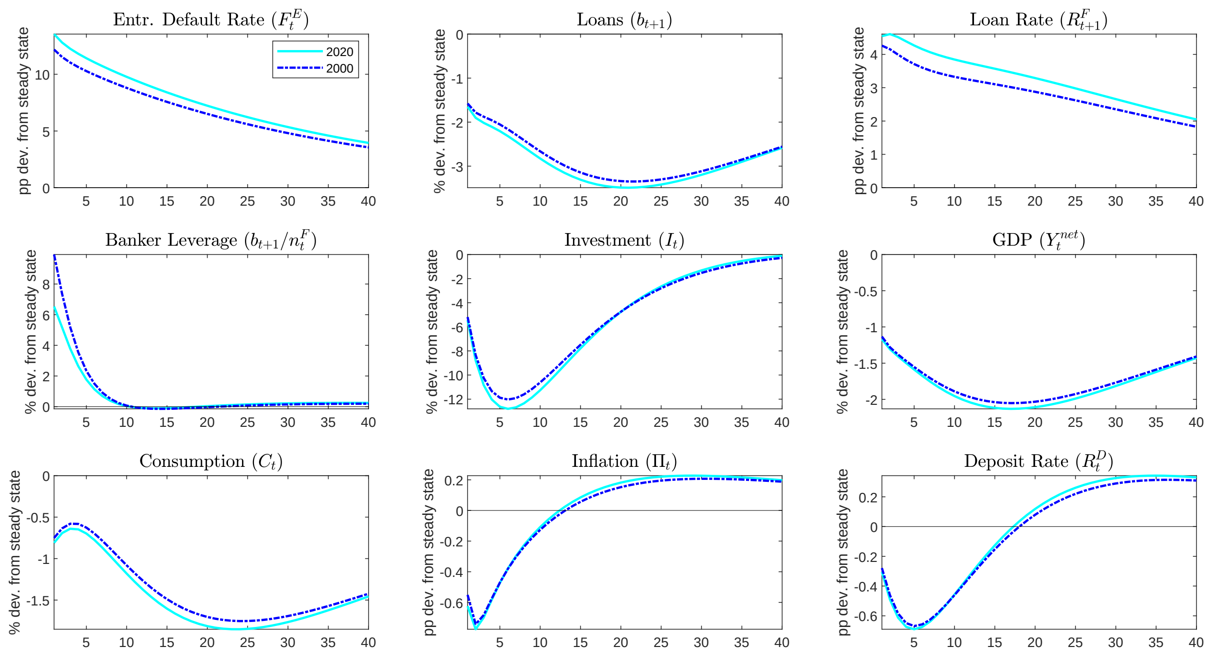
In this section I quantify the business cycle implications of the recent decline in banking competition in the United States. The number of commercial banks in the US has decreased by approximately 50% between 2000 and 2020, mainly due to bank mergers.[14] This consolidation trend may have contributed to the increase in concentration and reduction in banking competition identified by Corbae and D'Erasmo (2021). To understand the implications of

[14] Labonte and Scott (2021) provides an analysis of this trend.

this trend, I use the model to analyze how the responses to uncertainty shocks have changed over the past two decades.

Specifically, in Figure 4, I compare the baseline impulse responses (light blue dashed line) with the impulse responses of a variant of the model in which I set the number of bankers to match the share of assets held by the three largest banks in 2000 (blue dot-dashed lines). The figure shows that after the recent fall in banking competition, the effects of uncertainty shocks on the US economy are stronger. Specifically, a standard deviation uncertainty shock implies an increase in the default rate of entrepreneurs that is more than a percentage point higher at its peak and a decrease in GDP that is 0.1 percentage stronger.

Figure 4: Comparison impulse responses: Impact of recent fall in banking competition.



Notes. The graph shows how several variables in the model respond to a one-standard deviation uncertainty shock for different levels of competition. The light blue solid lines represent the impulse responses of the baseline model, while the blue dot-dashed lines display the impulse responses of an economy in which the number of banks is set to match the share of assets held by the three largest banks in the US in 2000.

5.4 Rise of Dominant Bankers

In this section, I explore the implications of a reduction in competition resulting from the concentration of market share among a few bankers. The concentration of market share arises from differences in productivity among bankers. Specifically, I assume that every banker inherits an intermediation cost γ^i when it is born and passes it on to the next generation of bankers in the following period. Bankers that face lower intermediation costs find it easier to provide loans to entrepreneurs and, consequently, obtain larger market shares. As a result of the more concentrated banking sector, entrepreneurs face higher borrowing rates.

To generate a distribution of bankers with a few large and many small bankers, as in Li (2019), I assume that the first bankers draw γ^i from a reverse bounded Pareto distribution. Further details on the distribution can be found in Appendix D.

The reverse bounded Pareto distribution is characterized by three parameters: the lower and upper bounds of the distribution and the shape parameter. Consistent with Li (2019), I set the shape parameter to 0.1. The lower and upper bounds are calibrated such that the sum of the resources lost due to intermediation costs is equal to the baseline model, and the standard deviation in markups is equal to σ^γ.[15] These assumptions imply that as σ^γ increases, concentration in the banking sector increases, while competition decreases. In fact, the higher σ^γ, the larger the differences in productivity across banks.

The model is solved 1000 times each time drawing a distribution of γ^i for the first generations of bankers. After solving the model, I average across replications the steady-state variables and the impulse responses.

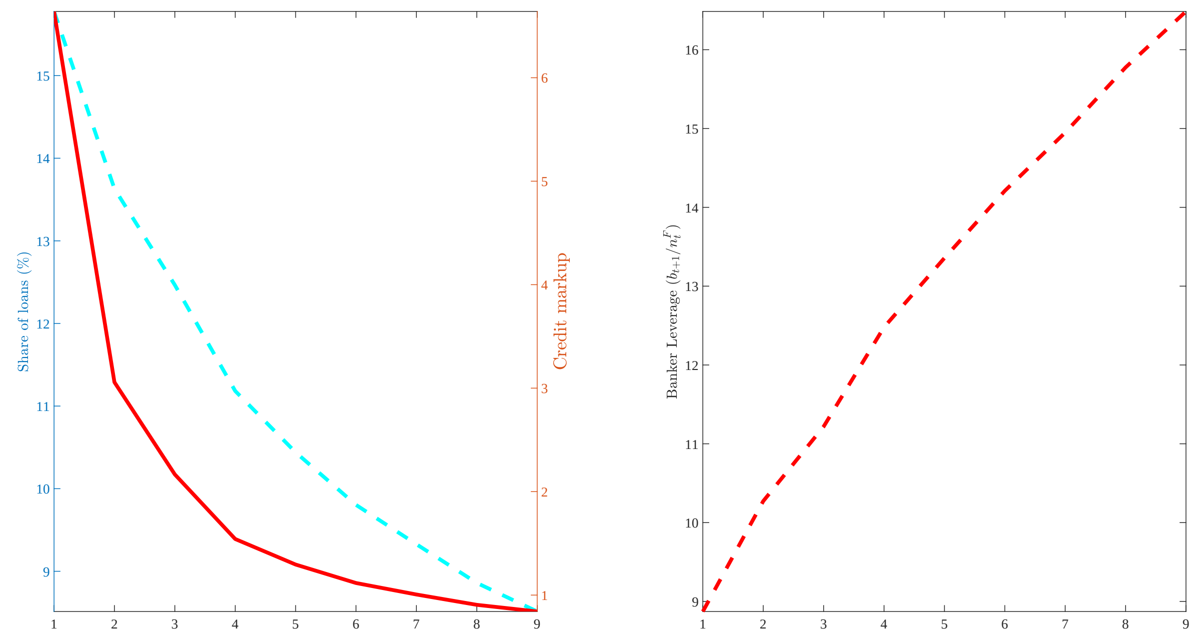
Figure 5 shows the steady-state effects of banker heterogeneity on banker variables. In this figure $\sigma^\gamma = 0.7$. The left panel plots the average share of assets (light blue dashed line) and the average markup (red solid line) of the nine bankers in the model. Bankers are sorted from the most productive to the least productive.

The left panel of Figure 5 shows that because of higher productivity, bankers with a low intermediation cost have a larger market share than their less productive counterparts. As a result, they can charge higher markups, obtain more profits and accumulate more equity. As a result of these differences, smaller bankers tend to have higher leverage, as depicted in the right panel of Figure 5.

Figure 6 displays the average impulse responses of the model with heterogeneous bankers for various levels of σ^γ. The results indicate that the rise of a few dominant bankers does

[15] Credit markup is defined as $\frac{(1-F^E)(R^F-1)}{R^D-1+\gamma^i} - 1$ as in Corbae and D'Erasmo (2021)

Figure 5: Steady-state bankers variables



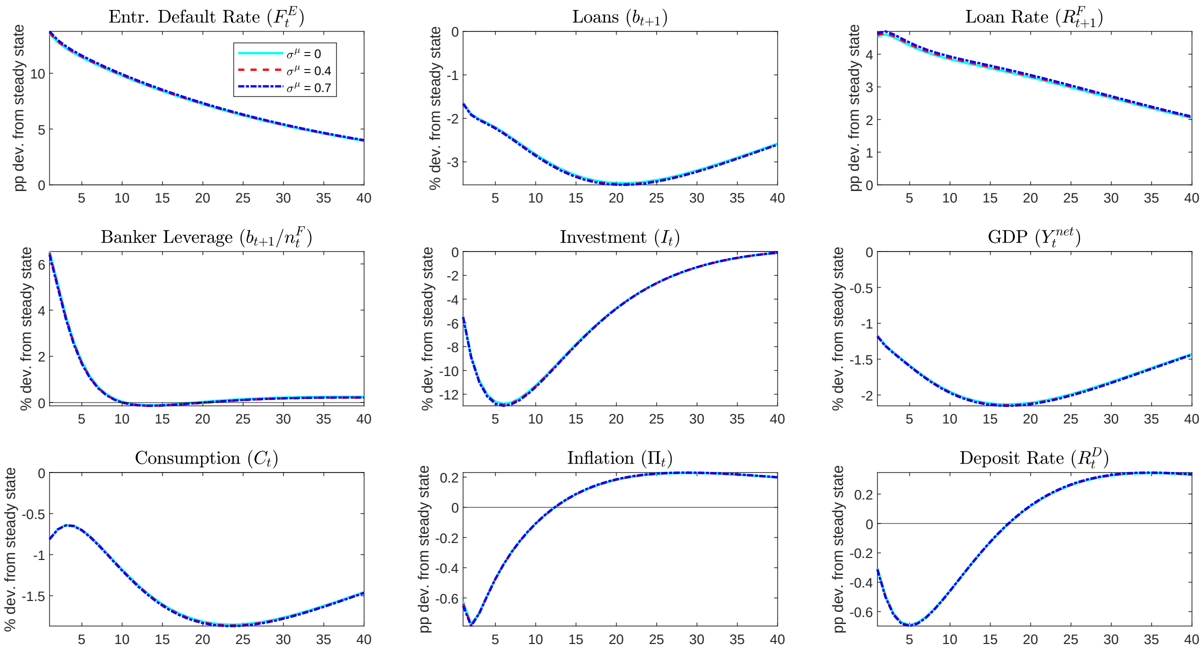
Notes. The graph shows the model generated steady-state distributions of bankers when the standard deviation of the distribution of markups is chosen to be 0.7. The figures are obtained by simulating the economy 1000 times and averaging across repetitions. The x-axis displays each of the 9 bankers, sorted from the most (lowest γ) to the least (highest γ) productive. The left figure shows how both the market share and the credit markup vary with banker productivity, while the right figure shows how banker leverage changes with productivity.

not substantially impact the responses of aggregate variables since they are similar to the responses of the baseline model. This suggests that the empirical findings are not driven by the rise of a few dominant banks.

Figure 7 shows how bankers' responses to an uncertainty shock vary according to their asset size. The red dashed lines represent the impulse responses of the most productive banker, while the light blue solid lines display the impulse responses of the median banker. The blue dot-dashed lines correspond to the impulse responses of the least productive banker. In this figure, $\sigma^{\gamma} = 0.7$.

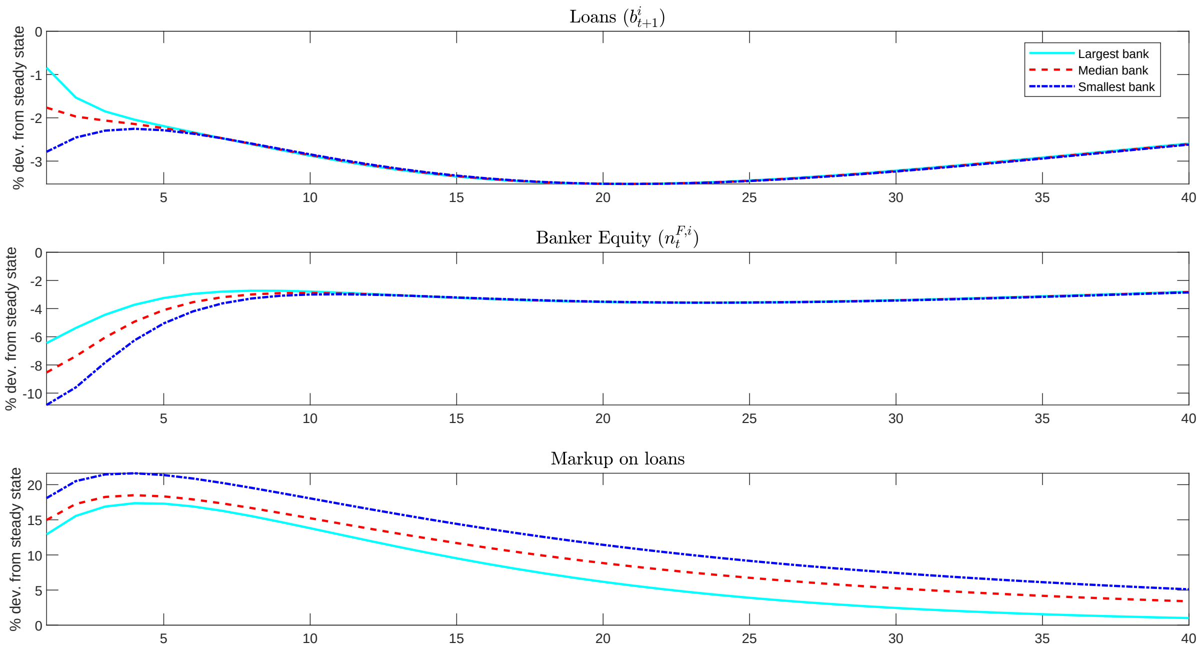
The impulse responses reveal that smaller bankers are more severely affected by uncertainty shocks. They reduce their loan supply to a greater extent and experience a more substantial decline in equity compared to larger bankers. Due to their lower market power, smaller bankers have a higher pass-through of shocks to borrowers and need to transfer more of the shock onto them. As a result, their markup increases by more.

Figure 6: Impulse response functions to an uncertainty shock - Heterogeneous bankers



Notes. The graph illustrates the responses of several variables in the model to a one-standard-deviation uncertainty shock at different levels of σ^γ. The light blue solid lines represent the impulse responses of the baseline model, while the red dashed lines display the impulse responses of an economy where σ^γ is 0.4. The blue dot-dashed lines correspond to the impulse responses of an economy where σ^γ is 0.7.

Figure 7: Impulse response functions - Bankers variables



Notes. The graph illustrates the responses of several banker variables in the model to a one-standard-deviation uncertainty shock when σ^γ is 0.7. The red dashed lines represent the impulse responses of the most productive banker, while the light blue solid lines display the impulse responses of the banker with median productivity. The blue dot-dashed lines correspond to the impulse responses of the least productive banker.

6 Conclusion

The literature on uncertainty argues that uncertainty shocks play a crucial role in driving business cycles. In light of the recent decline in banking competition, I study how lower competition in the banking sector affects the propagation of uncertainty shocks.

Empirically, I find a negative correlation between the impact of uncertainty shocks on real output growth and banking sector competition. I construct a calibrated New Keynesian dynamic stochastic general equilibrium model that incorporates financial frictions and imperfect competition in the banking sector to capture this result.

Banking competition can change due to mergers that reduce the number of competitors or an increase in market share concentration among a few bankers. With fewer competitors, bankers charge higher borrowing rates to entrepreneurs due to reduced competition. This increases borrowers' risk-taking due to limited liability - a channel known as the risk-shifting effect.

An uncertainty shock increases entrepreneurial defaults to a greater extent in less competitive banking sectors due to increased risk-taking by entrepreneurs. This leads to a stronger increase in credit risk and a stronger reduction in bankers' loan supply. As a result, investment and output fall more after an uncertainty shock in economies with less competitive banking sectors.

I also explore the implications of a reduction in competition resulting from the concentration of market share among a few bankers. I assume that the concentration of market share arises from differences in productivity among bankers. My results show that larger bankers have more market power and charge higher markups. They are also less affected by uncertainty shocks due to their higher market power, which results in a lower pass-through of shocks to borrowers.

However, the rise of a few dominant bankers does not substantially impact the responses of aggregate variables. The responses of an economy with dominant bankers are similar to those of the baseline model. This suggests that the empirical findings are driven by a reduction in the number of competitors rather than by the rise of a few dominant banks.

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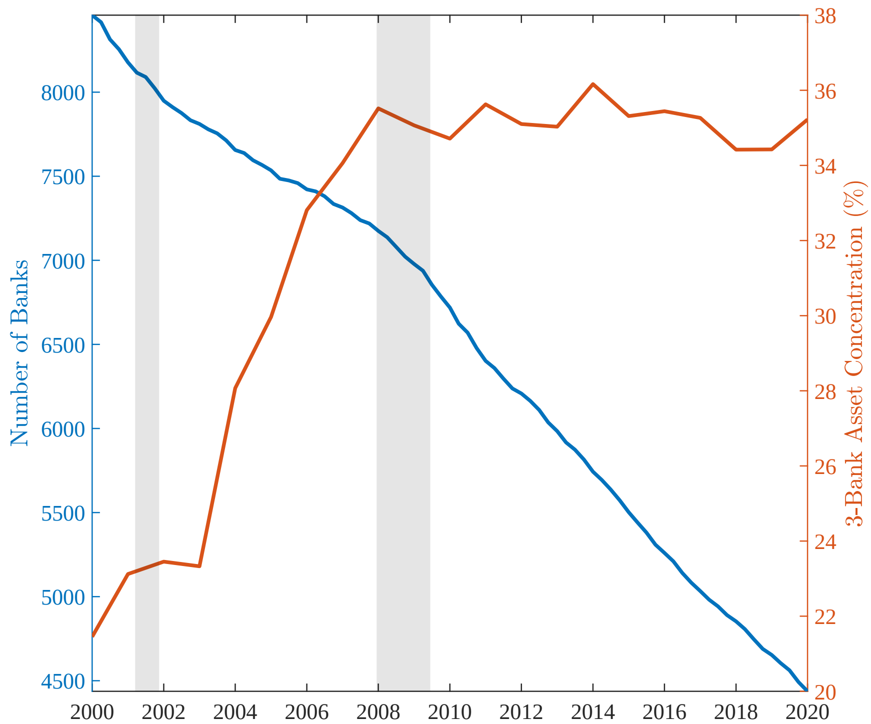
A Additional Data

A.1 Evolution of Banking Competition

Figure 8 shows the evolution of the number of commercial banks and the 3-Bank asset concentration for the United States. The number of banks is retrieved from FRED, the 3-Bank asset concentration is obtained from World Bank. The number of banks has been decreasing since 2000, while the 3-Bank asset concentration has been increasing. This suggests that banking competition has been falling in recent years.

Figure 8: Number of banks and bankers concentration



Notes. Sample period: January 2000 to January 2020. The number of banks is measured as the number of commercial banks (FRED). Bank concentration is measured as the assets of three largest commercial banks as a share of total commercial banking assets (World Bank).

B Additional Information Empirical Evidence

In this section, I provide additional details about the dataset used in the empirical analysis.

The disaster shocks are obtained from Baker et al. (forthcoming) and are available for 59 countries from 1970Q1 to 2020Q1. However, information regarding banking concentration is only available from 2000 limiting the sample period to 2000Q1 to 2020Q1.

The reduction in the sample period means that some countries did not experience any shock during the period from 2000Q1 to 2020Q1. As a result, these countries are dropped from the analysis. Additionally, countries with GDP data available only at a yearly frequency are also dropped. Finally, observations with a concentration level equal to 100% are removed from the sample.

Table 2 lists the countries used in the analysis.

Table 2: Countries

Asia & Pacific	Europe & North America	LatAm & Caribbean	MENA	SSAF
Australia	Austria	Brazil	Israel	South Africa
China	Belgium	Chile	Turkey	
India	Canada	Colombia		
Indonesia	Czech Republic	Ecuador		
Japan	Denmark	Mexico		
New Zealand	Finland			
Philippines	France			
Russian Federation	Germany			
Singapore	Greece			
South Korea	Hungary			
Thailand	Ireland			
	Italy			
	Luxembourg			
	Netherlands			
	Norway			
	Poland			
	Portugal			
	Romania			
	Serbia			
	Spain			
	Sweden			
	Switzerland			
	Ukraine			
	United Kingdom			
	United States			

Notes. List of the countries used in the empirical analysis.

Table 3 presents the descriptive statistics of the dataset. It is worth noting that compared to Baker et al. (forthcoming), the number of observations is smaller and fewer disaster shocks are available due to sample availability.

The disaster shocks are defined as follows:

Natural Disasters: Extreme weather events such as, droughts, earthquakes, insect infes-

Table 3: Descriptive statistics

	Obs.	Mean	Median	St.dev.	min	max
Real GDP growth	3104	0.53	0.64	1.75	-24.16	24.25
GDP	3104	13.46	13.03	2.60	8.96	21.75
3-Bank Concentration (interpolated)	2953	65.77	66.70	20.01	21.45	99.99
3-Bank Concentration	3091	66.05	67.27	20.35	21.45	100.00
5-Bank Concentration (interpolated)	2863	77.26	81.19	17.80	28.12	100.00
Return	3064	0.00	0.01	0.06	-0.32	0.30
Volatility	3063	-4.48	-4.48	0.44	-5.96	-2.99
Nat. Disasters	3093	0.02	0.00	0.09	0.00	1.99
Coups	3093	0.00	0.00	0.01	0.00	0.33
Revolutions	3093	0.00	0.00	0.01	0.00	0.27
Terror attacks	3093	0.00	0.00	0.05	0.00	1.22

Notes. Descriptive statistics of the dataset used in the empirical analysis. The sample period is 2000Q1-2020Q1.

tations, pandemics, floods, extreme temperatures, avalanches, landslides, storms, volcanoes, fires, and hurricanes.

Terrorist Attacks: Bombings and other non-state-sponsored attacks.

Coups: Military action which results in the seizure of executive authority taken by an opposition group from within the government.

Revolutions: A violent uprising or revolution seeking to replace the government or substantially change the governance of a given region.

To construct the disaster shock variables, for each category, country, and quarter, the shock variable is set to 1 if there was at least one disaster shock of that category in that quarter. The weights of these shocks are determined by the increase in media coverage 15 days after the event compared to 15 days before the event.

The increase in media coverage is defined as the percentage increase in the number of articles related to the event that were published in English-language newspapers based in the United States, comparing the 15-day period after the event to the 15-day period before the event.

B.1 Robustness Tests

This section presents the results of the robustness tests. The first test is reported in B.1.1, where banking concentration is kept constant within each year instead of being interpolated.

The second test is reported in B.1.2 where the 5-Bank asset concentration ratio is used as a proxy for the level of banking competition.

B.1.1 Constant Concentration within each Year

In this section, I present the results of the first robustness test, where banking concentration is kept constant within each year instead of interpolating the level of concentration.

Figure 9 displays the effect of an exogenous one-standard-deviation increase in uncertainty on real output growth for two different levels of banking concentration. The blue line indicates the impulse response of output growth when banking concentration is at the country average, while the blue dashed lines represent the 90% confidence intervals. The figure shows that an exogenous increase in uncertainty has no significant negative impact on output growth.

The yellow line represents the impulse response of output growth when banking concentration is one standard deviation above the country average. The yellow dashed lines represent the 90% confidence interval. In this case, the fall in output growth is stronger and significant.

Figure 10 depicts the difference in output growth responses between the average concentration and the high concentration specifications. It shows that the decline in output growth is significantly stronger when banking concentration is higher.

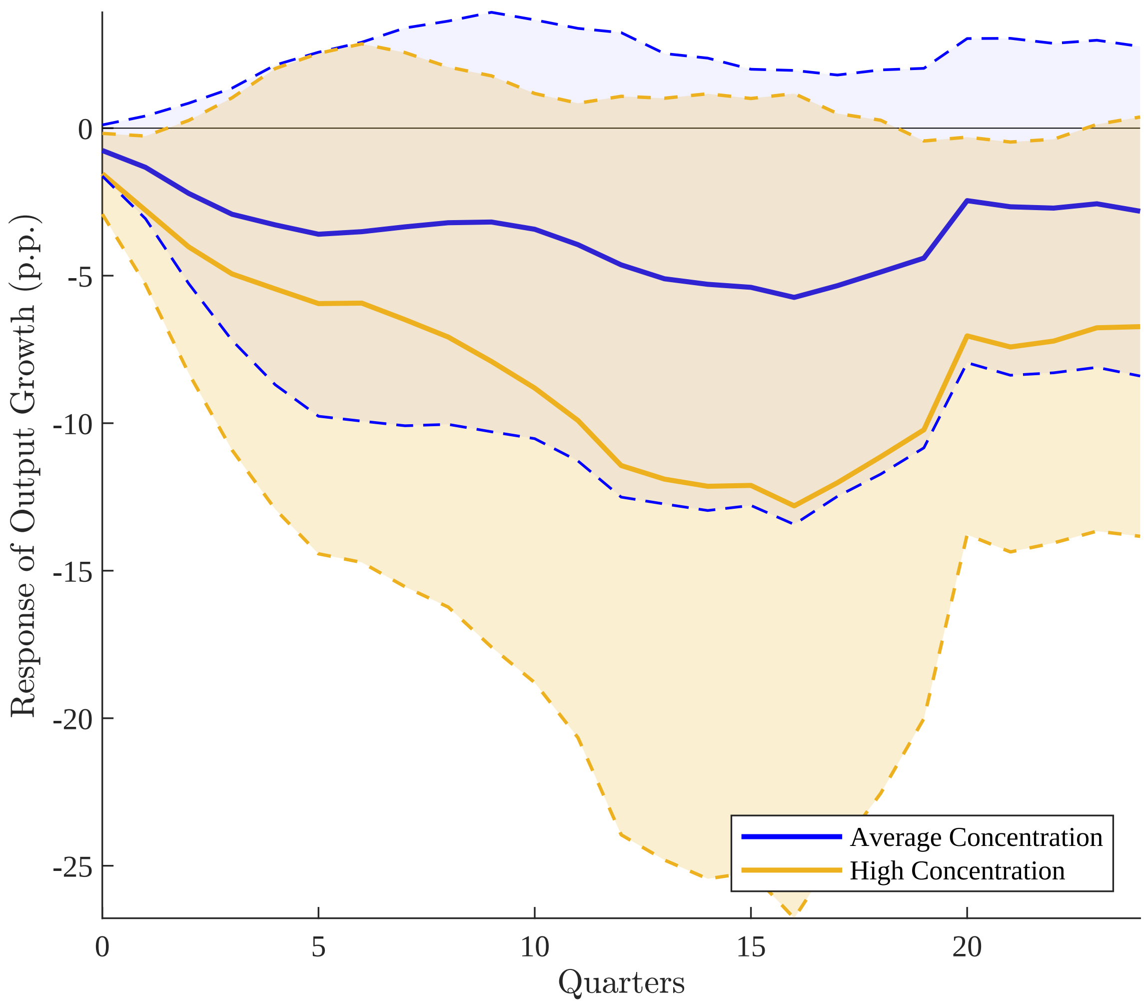
B.1.2 5-Bank Asset Concentration

In this section, I present the results of the second robustness test, where the level of banking competition is measured by the 5-Bank asset concentration ratio, the share of assets held by the five largest banks.

Figure 11 shows the effect of an exogenous one-standard-deviation increase in uncertainty on output growth for different levels of banking concentration. The blue line indicates the impulse response of output growth when banking concentration is at the country average, while the blue dashed lines represent the 90% confidence intervals. The figure shows that an exogenous increase in uncertainty has no significant negative impact on output growth.

The yellow line represents the impulse response of output growth when concentration

Figure 9: Response of output growth to a one-standard-deviation uncertainty shock



Notes. The graph shows the responses of output growth to a one-standard-deviation uncertainty shock for different levels of banking concentration. In the response with high banking concentration, banking concentration is 1 standard deviation higher than the country average. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1. The level of banking concentration is constant within each year.

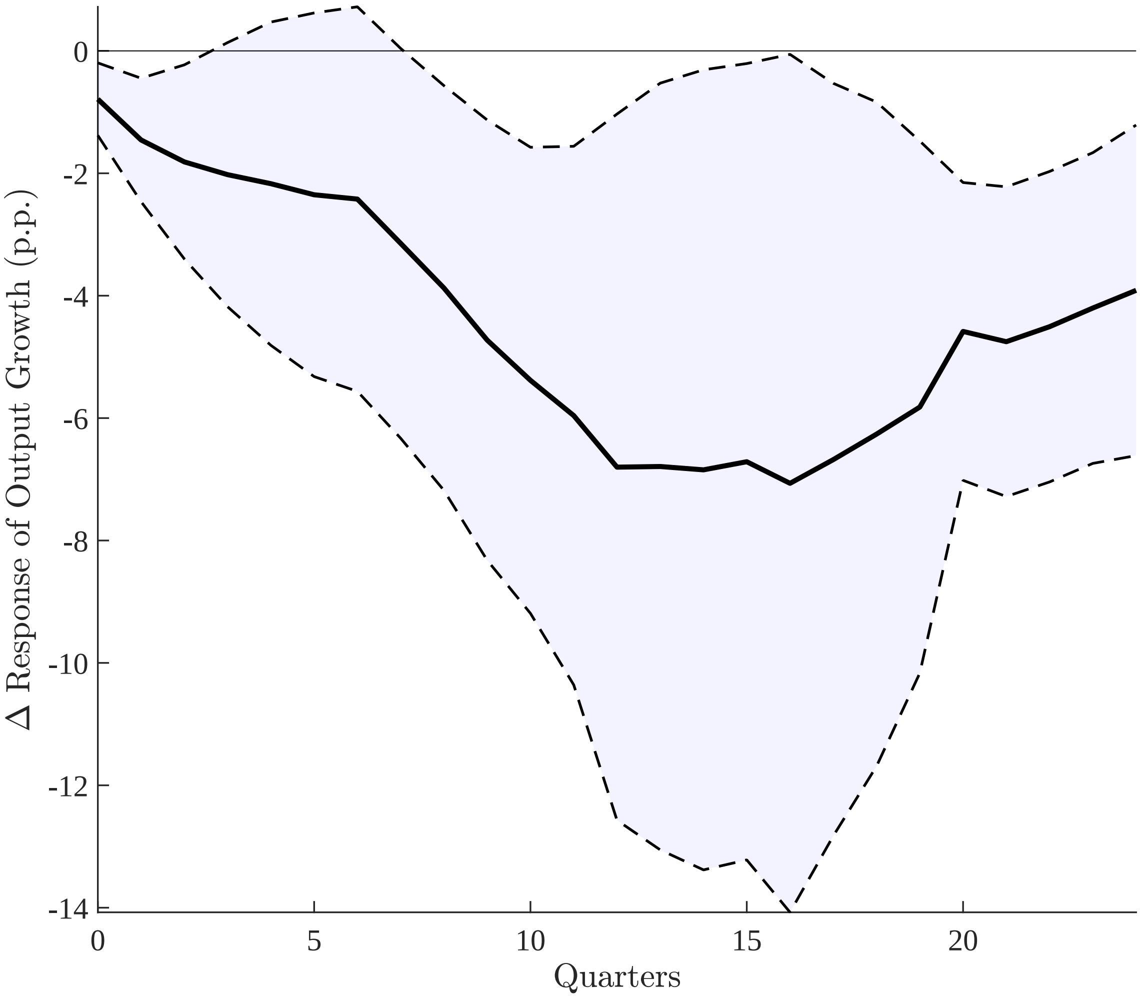
is one standard deviation above the country average. The yellow dashed lines are the 90% confidence interval. In this case, the fall in output is stronger.

Figure 12 depicts the difference in output growth responses between the average concentration and the high concentration specifications. It shows that the decline in output growth is significantly stronger when banking concentration is higher.

B.1.3 Lagged Concentration

In this section, I present the results of the third robustness test, where the level of banking competition is measured by the 3-Bank asset concentration ratio lagged by one quarter. The

Figure 10: Effect of competition on output growth response to a one-standard-deviation uncertainty shock.



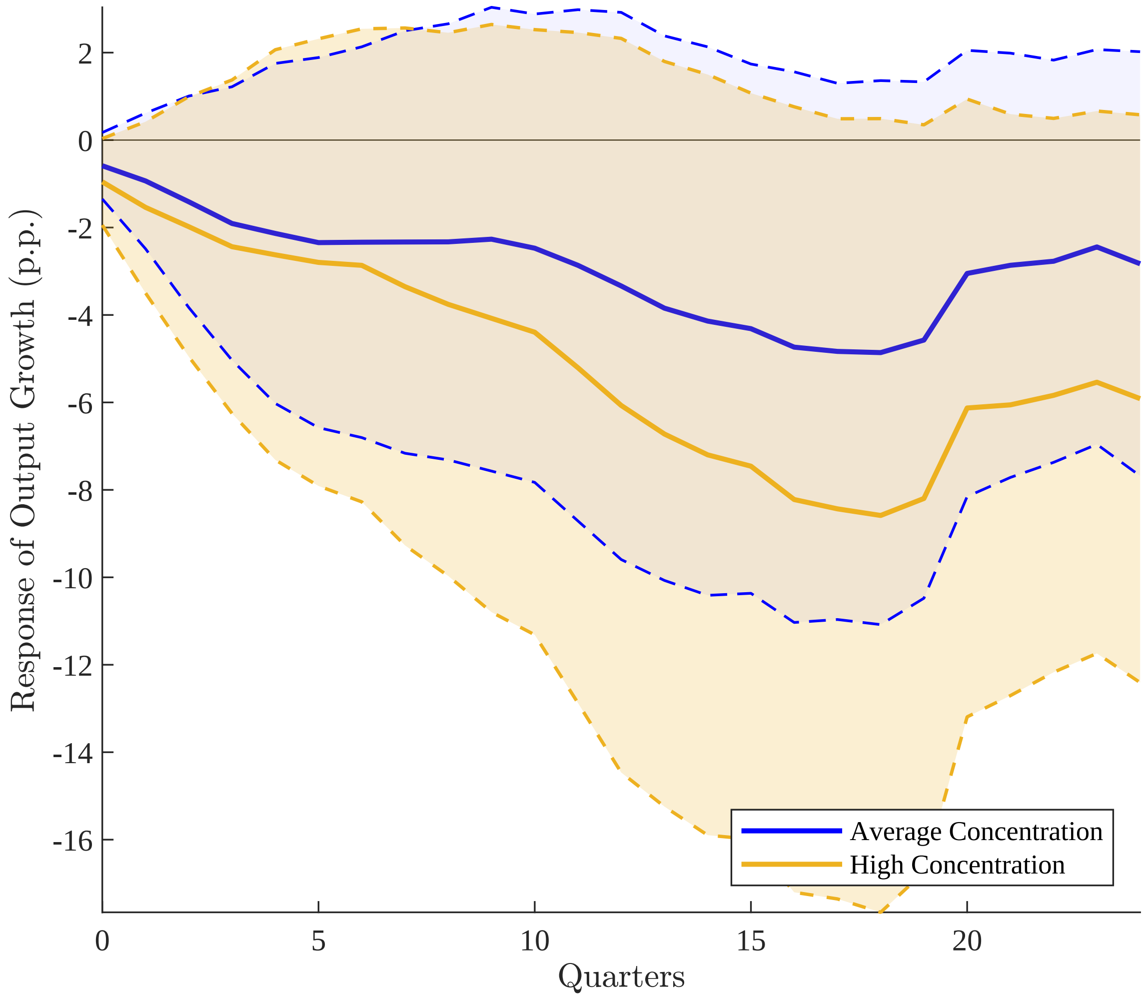
Notes. The graph shows the difference between the two specifications plotted in Figure 9. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1.

measure of concentration is lagged by one quarter to control for the possible endogeneity of concentration.

Figure 13 shows the effect of an exogenous one-standard-deviation increase in uncertainty on output growth for different levels of banking concentration. The blue line indicates the impulse response of output growth when banking concentration is at the country average, while the blue dashed lines represent the 90% confidence intervals. The figure shows that an exogenous increase in uncertainty has no significant negative impact on output growth.

The yellow line represents the impulse response of output growth when concentration is one standard deviation above the country average. The yellow dashed lines are the 90% confidence interval. In this case, the fall in output is stronger.

Figure 11: Response of output growth to a one-standard-deviation uncertainty shock



Notes. The graph shows the responses of output growth to a one-standard-deviation uncertainty shock for different levels of banking concentration. In the response with high banking concentration, banking concentration is 1 standard deviation higher than the country average. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1. The level of banking competition is measured by the 5-Bank asset concentration ratio.

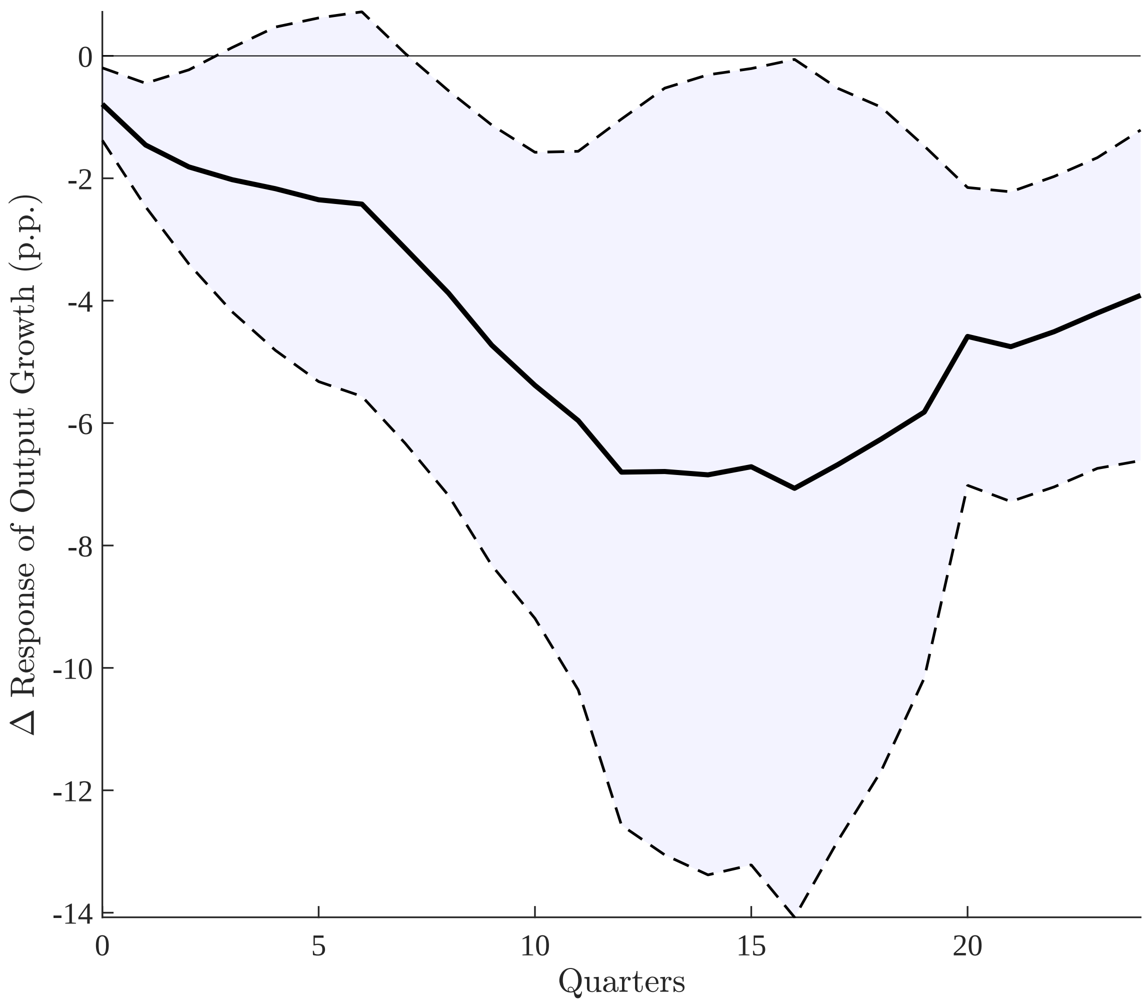
Figure 14 depicts the difference in output growth responses between the average concentration and the high concentration specifications. It shows that the decline in output growth is significantly stronger when banking concentration is higher.

B.1.4 Impact of the 2009 Global Recession

In this section, I present the results of the fourth robustness test. In this test, I exclude the year 2009 from the sample. This test shows that the findings are not driven by the impact of the global recession that occurred in that year.

Figure 15 shows the effect of an exogenous one-standard-deviation increase in uncertainty

Figure 12: Effect of competition on output growth response to a one-standard-deviation uncertainty shock.



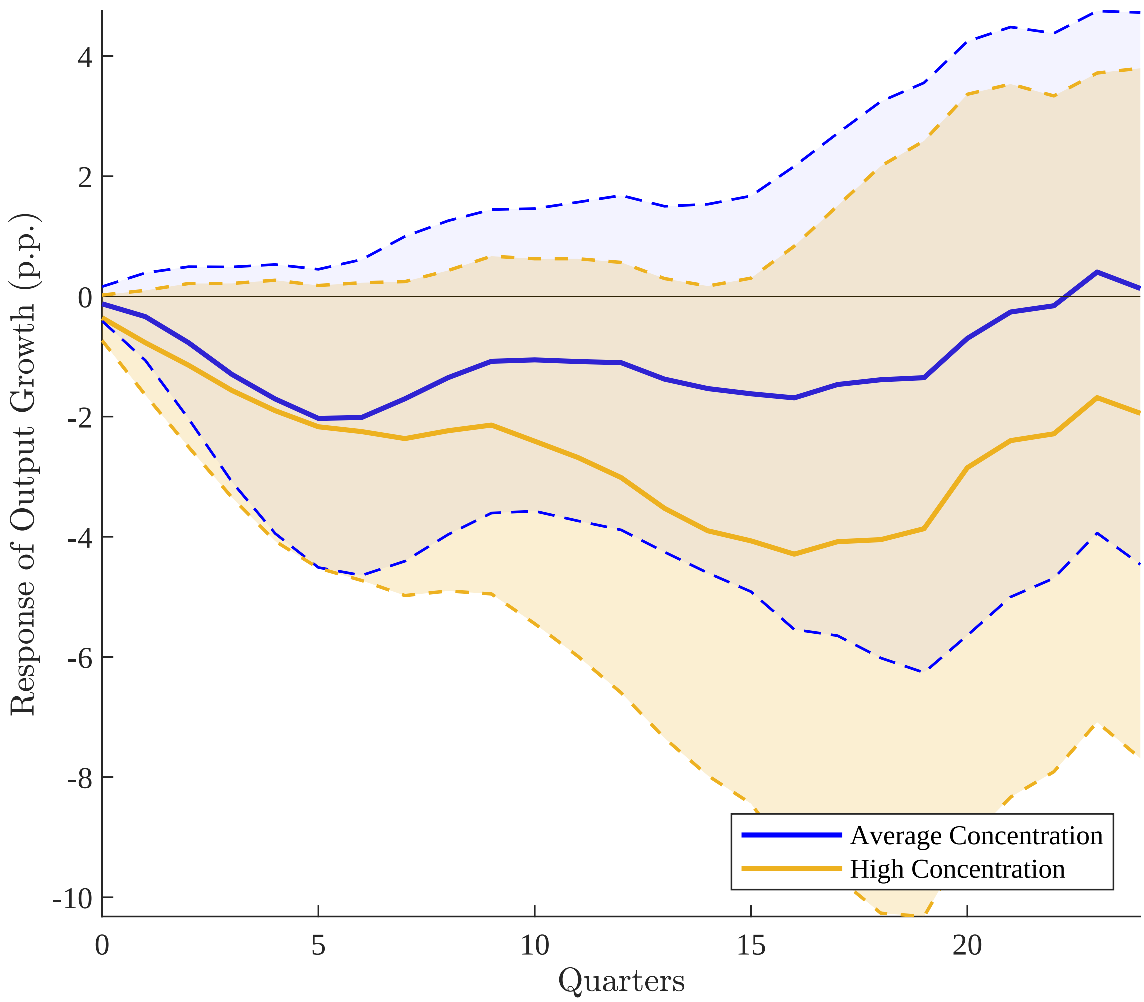
Notes. The graph shows the difference between the two specifications plotted in Figure 11. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1.

on output growth for different levels of banking concentration. The blue line indicates the impulse response of output growth when banking concentration is at the country average, while the blue dashed lines represent the 90% confidence intervals. The figure shows that an exogenous increase in uncertainty has no significant negative impact on output growth.

The yellow line represents the impulse response of output growth when concentration is one standard deviation above the country average. The yellow dashed lines are the 90% confidence interval. In this case, the fall in output is stronger.

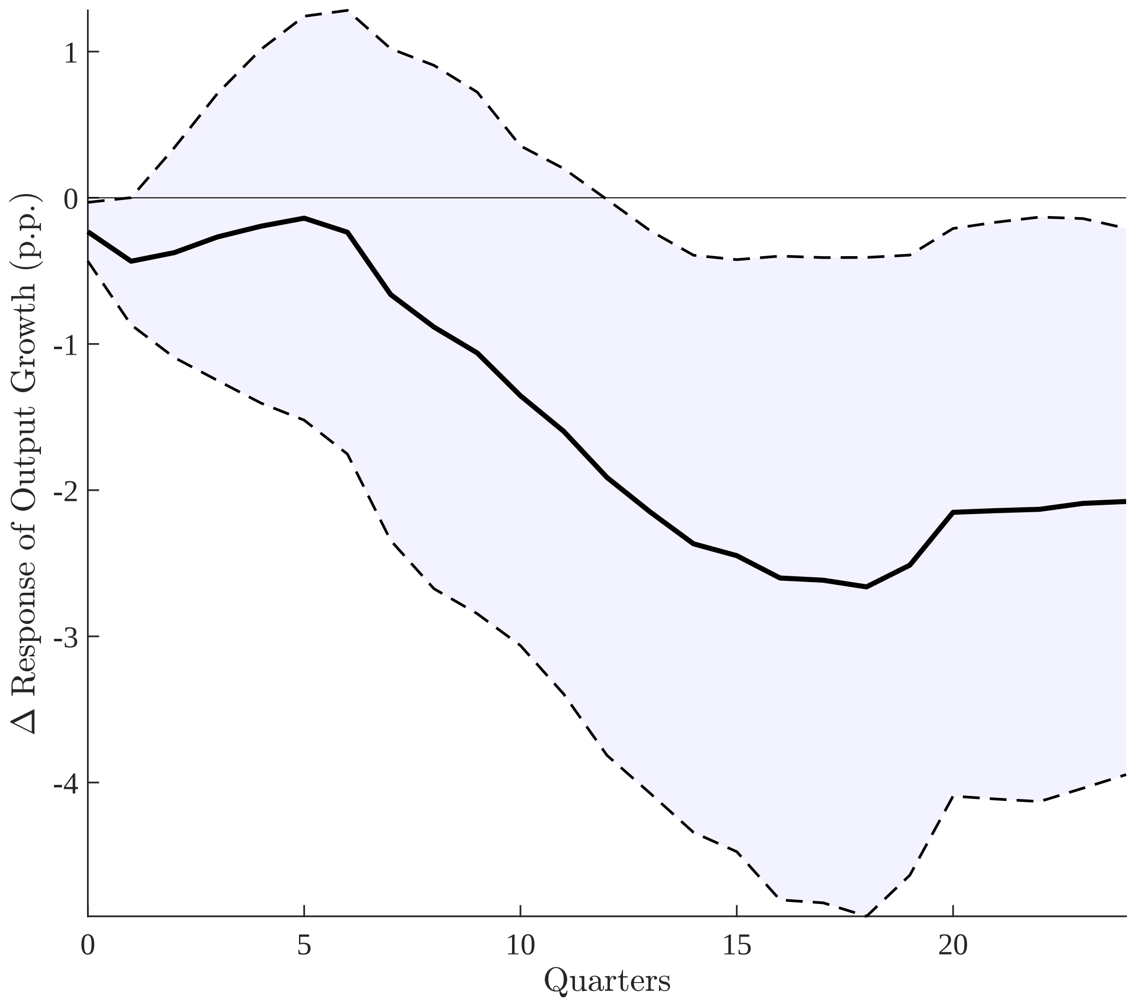
Figure 16 depicts the difference in output growth responses between the average concentration and the high concentration specifications. It shows that the decline in output growth is significantly stronger when banking concentration is higher.

Figure 13: Response of output growth to a one-standard-deviation uncertainty shock.



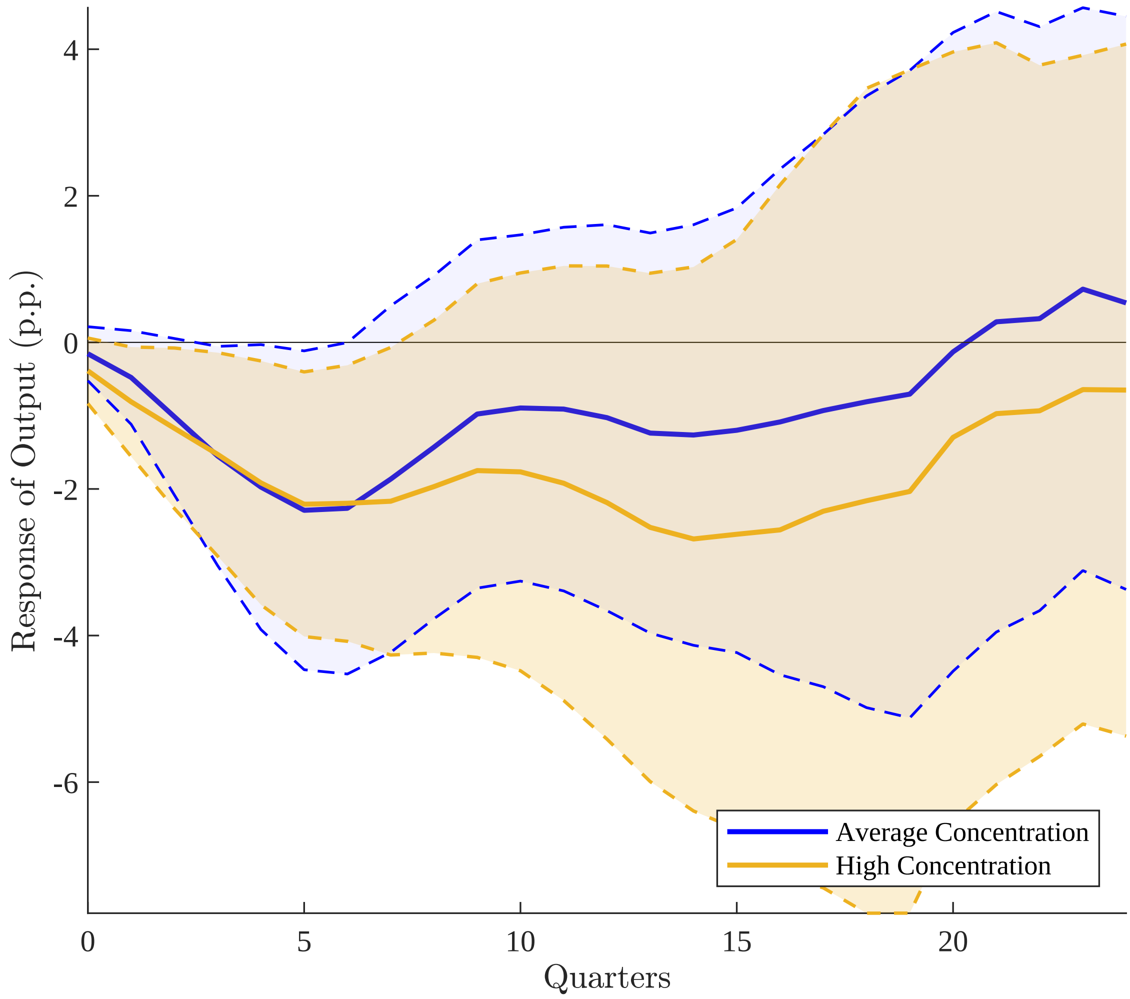
Notes. The graph shows the responses of output growth to a one-standard-deviation uncertainty shock for different levels of banking concentration. In the response with high banking concentration, banking concentration is 1 standard deviation higher than the country average. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1. The level of banking competition is measured by the 3-Bank asset concentration ratio lagged by one quarter.

Figure 14: Effect of competition on output growth response to a one-standard-deviation uncertainty shock.



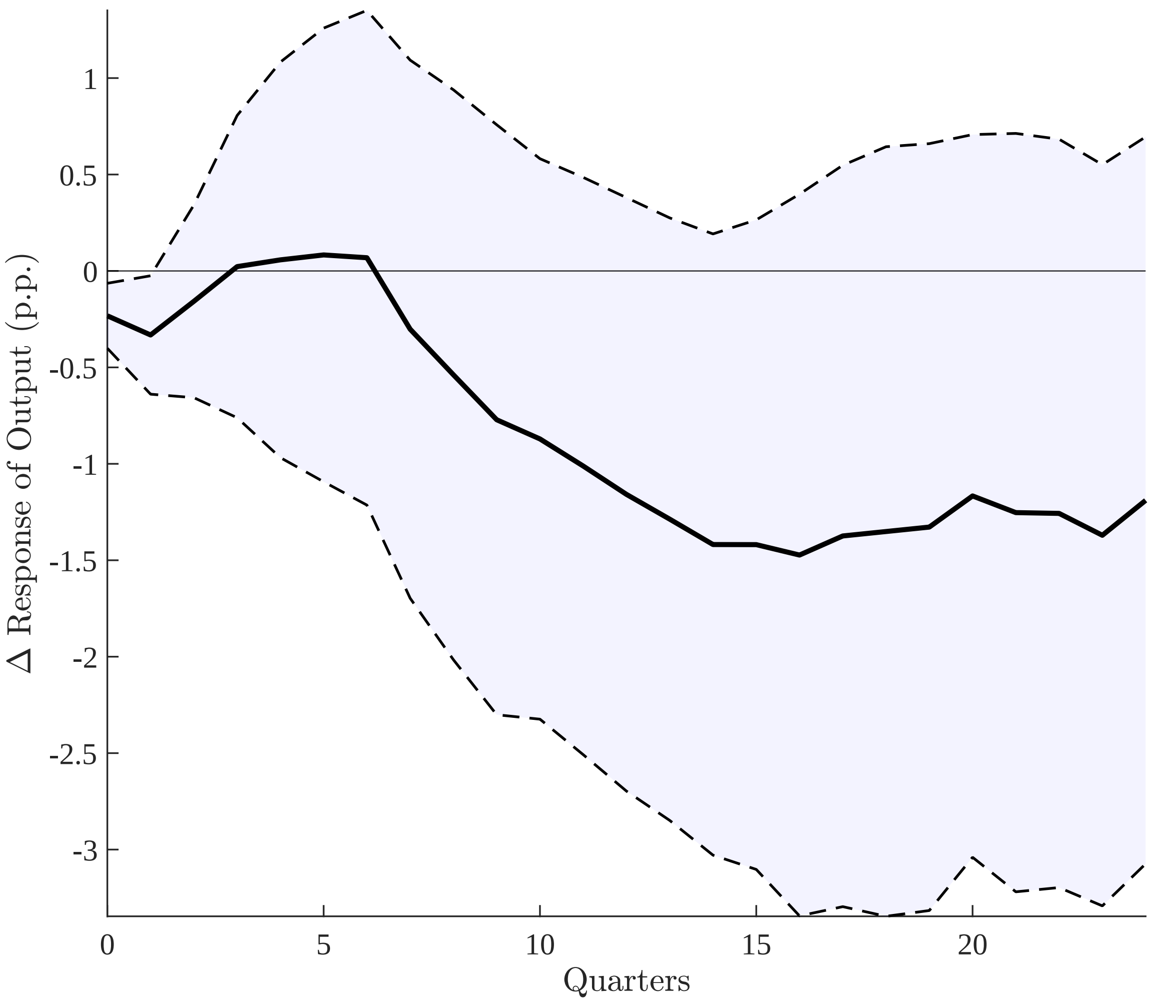
Notes. The graph shows the difference between the two specifications plotted in Figure 13. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1.

Figure 15: Response of output growth to a one-standard-deviation uncertainty shock.



Notes. The graph shows the responses of output growth to a one-standard-deviation uncertainty shock for different levels of banking concentration. In the response with high banking concentration, banking concentration is 1 standard deviation higher than the country average. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1. The year 2009 is removed from the sample.

Figure 16: Effect of competition on output growth response to a one-standard-deviation uncertainty shock.



Notes. The graph shows the difference between the two specifications plotted in Figure 15. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1.

C Entrepreneurial Optimization Problem

C.1 Loan Demand and its Properties

In this section, I derive and describe the properties of the loan demand function. Specifically, in Section C.1.1, I derive the loan demand function, and in Section C.1.2, I outline its properties.

C.1.1 Derivation of Loan Demand

In this section, I derive the loan demand function.

After substituting the resource constraint of the entrepreneurs (6), the Lagrangian of the maximization of the entrepreneurs is

$$\mathcal{L}(K_t^j, \Lambda_t) = \mathbb{E}_t(W_{t+1}^{E,j}) + \Lambda \left(\lambda \frac{qK_t^j}{\Pi_{t+1}} - \mathbb{E}_t(W_{t+1}^{E,j}) \right) \tag{20}$$

It is possible to rewrite the expected future wealth as:

$$\begin{aligned} \mathbb{E}_t(W^{E,j})_{t+1} &= \left(\int_{\overline{\omega}_{t+1}^j}^{\infty} \omega_{t+1}^j R_{t+1}^E q K_t^j f(\omega_{t+1}^j) \mathrm{d}\omega_{t+1}^j - (1 - F(\overline{\omega}_{t+1}) R_t^F b_t^j \right) \frac{1}{\Pi_{t+1}} \\ &= \left(\int_{\overline{\omega}_{t+1}^j}^{\infty} \omega_{t+1}^j R_{t+1}^E q K_t^j f(\omega_{t+1}^j) \mathrm{d}\omega_{t+1}^j - \int_{\overline{\omega}_{t+1}^j}^{\infty} R_t^F b_t^j f(\omega_{t+1}^j) \mathrm{d}\omega_{t+1}^j \right) \frac{1}{\Pi_{t+1}}, \end{aligned} \tag{21}$$

where $f(\cdot)$ and $F(\cdot)$ are the probability density function and cumulative distribution function, respectively, of the distribution of ω_{t+1}.

Using the definition of the default cutoff (3), (21) can be simplified as

$$\int_{\overline{\omega}_{t+1}^j}^{\infty} (\omega_{t+1}^j - \overline{\omega}_{t+1}^j) R_{t+1}^E q K_t^j f(\omega_{t+1}^j) \mathrm{d}\omega_{t+1}^j. \tag{22}$$

I can rewrite the term $\int_{\overline{\omega}_{t+1}^j}^{\infty}(\omega_{t+1}^j - \overline{\omega}_{t+1}^j)f(\omega_{t+1}^j)d\omega_{t+1}^j$ as

$$\begin{aligned}
\int_{\overline{\omega}_{t+1}^j}^{\infty}(\omega_{t+1}^j - \overline{\omega}_{t+1}^j)f(\omega_{t+1}^j)\mathrm{d}\omega_{t+1}^j &= \int_{\overline{\omega}_{t+1}^j}^{\infty}\omega_{t+1}^j f(\omega_{t+1}^j)\mathrm{d}\omega_{t+1}^j - \overline{\omega}_{t+1}^j\int_{\overline{\omega}_{t+1}^j}^{\infty}f(\omega_{t+1}^j)\mathrm{d}\omega_{t+1}^j \\
&= 1 - \left(\underbrace{\int_0^{\overline{\omega}_{t+1}^j}\omega_{t+1}^j f(\omega_{t+1}^j)\mathrm{d}\omega_{t+1}^j}_{\equiv G(\overline{\omega}_{t+1}^j)} + \overline{\omega}_{t+1}^j\int_{\overline{\omega}_{t+1}^j}^{\infty}f(\omega_{t+1}^j)\mathrm{d}\omega_{t+1}^j\right) \\
&= 1 - \underbrace{\left(G(\overline{\omega}_{t+1}^j) + (1 - F(\overline{\omega}_{t+1}^j))\overline{\omega}_{t+1}^j\right)}_{\equiv\Gamma(\overline{\omega}_{t+1}^j)\geq 0}. \qquad (23)
\end{aligned}$$

We can express $\Gamma(\overline{\omega}_{t+1}^j)$ as

$$\begin{aligned}
\Gamma(\overline{\omega}_{t+1}^j) &= \int_0^{\overline{\omega}_{t+1}^j}\omega_{t+1}^j f(\omega_{t+1}^j)\mathrm{d}\omega_{t+1}^j + \overline{\omega}_{t+1}^j\int_{\overline{\omega}_{t+1}^j}^{\infty}f(\omega_{t+1}^j)\mathrm{d}\omega_{t+1}^j \\
&= \overline{\omega}_{t+1}^j F(\overline{\omega}_{t+1}^j) - \int_0^{\overline{\omega}_{t+1}^j}F(\omega_{t+1}^j)\mathrm{d}\omega_{t+1}^j + \overline{\omega}_{t+1}^j(1 - F(\overline{\omega}_{t+1}^j)) \\
&= \overline{\omega}_{t+1}^j - \int_0^{\overline{\omega}_{t+1}^j}F(\omega_{t+1}^j)\mathrm{d}\omega_{t+1}^j \qquad (24)
\end{aligned}$$

Combining (22) and (23), expected future wealth can be written as

$$\mathbb{E}_t(W_{t+1}^{E,j}) = \frac{(1 - \Gamma(\overline{\omega}_{t+1}^j))R_{t+1}^E qK_t^j}{\Pi_{t+1}} \qquad (25)$$

Substituting in the Lagrangian of the maximization problem of the entrepreneurs, we have

$$\mathcal{L}(K_t^j, \Lambda_t) = \frac{(1 - \Gamma(\overline{\omega}_{t+1}^j))R_{t+1}^E qK_t^j}{\Pi_{t+1}} + \Lambda_t\left(\lambda\frac{qK_t^j}{\Pi_{t+1}} - \frac{(1 - \Gamma(\overline{\omega}_{t+1}^j))R_{t+1}^E qK_t^j}{\Pi_{t+1}}\right)$$

The first-order conditions of the Lagrangian are

$$\frac{\partial\mathcal{L}}{\partial K_t^j} = \frac{\partial\mathbb{E}_t(W_{t+1}^{E,j})}{\partial K_t^j} + \Lambda_t\left(\lambda\frac{q}{\Pi_{t+1}} - \frac{\partial\mathbb{E}_t(W_{t+1}^{E,j})}{\partial K_t^j}\right) = 0 \qquad (26)$$

$$\frac{\partial\mathcal{L}}{\partial\Lambda_t^j} = \frac{qK_t^j\lambda}{\Pi_{t+1}} - \mathbb{E}_t(W_{t+1}^{E,j}) = 0 \qquad (27)$$

The first derivative of expected future wealth with respect to capital is [16]

$$\frac{\partial \mathbb{E}_t(W^{E,j}_{t+1})}{\partial K^j_t} = \left((1-\Gamma(\overline{\omega}^j_{t+1}))R^E_{t+1}q - \Gamma'(\overline{\omega}^j_{t+1})\frac{\partial \overline{\omega}^j_{t+1}}{\partial K^j_t}R^E_{t+1}qK^j_t \right)\frac{1}{\Pi_{t+1}}. \tag{28}$$

We first focus on the second term of (28). We can obtain the expression for $\Gamma'(\overline{\omega}^j_{t+1})$ by differentiating (23). This yields

$$\Gamma'(\overline{\omega}^j_{t+1}) = G'(\overline{\omega}^j_{t+1}) - f(\overline{\omega}^j_{t+1})\overline{\omega}^j_{t+1} + (1 - F(\overline{\omega}^j_{t+1})), \tag{29}$$

where $G'(\overline{\omega}^j_{t+1})$ is obtained by differentiating the definition of $G(\overline{\omega}^j_{t+1})$ included in (23),

$$G'(\overline{\omega}^j_{t+1}) = \frac{\partial \int_0^{\overline{\omega}^j_{t+1}} \omega^j_{t+1} f(\omega^j_{t+1})\mathrm{d}\omega^j_{t+1}}{\partial \overline{\omega}^j_{t+1}} = \overline{\omega}^j_{t+1} f(\overline{\omega}^j_{t+1}). \tag{30}$$

Combining (30) and (29), $\Gamma'(\overline{\omega}^j_{t+1})$ can be simplified to

$$\Gamma'(\overline{\omega}^j_{t+1}) = 1 - F(\overline{\omega}^j_{t+1}) \geq 0. \tag{31}$$

Using the definition of the default cutoff (3) and the resource constraint of the entrepreneur (6), we can derive an expression for the term $\frac{\partial \overline{\omega}^j_{t+1}}{\partial K^j_t}$

$$\frac{\partial \overline{\omega}^j_{t+1}}{\partial K^j_t} = \frac{R^F_t}{R^E_t q}\frac{qK^j_t - b^j_t}{K^2_t} = \frac{R^F_t n^{E,j}_t}{R^E_t q K^2_t} \geq 0 \tag{32}$$

Substituting (31) and (32) into (28) we obtain

$$\frac{\partial \mathbb{E}_t(W^{E,j}_{t+1})}{\partial K^j_t} = \left((1-\Gamma(\overline{\omega}^j_{t+1}))R^E_t q - (1 - F^E(\overline{\omega}^j_{t+1}))\frac{R^F_t n^{E,j}_t}{K_t} \right)\frac{1}{\Pi_{t+1}}$$

[16] Although R^E_{t+1} is a function of capital, I assume entrepreneurs treat the return on capital as exogenous.

Using the definition of $\Gamma(\overline{\omega}^j_{t+1})$ derived in (23) and the definition of the default cutoff (3)

$$\begin{aligned}
\frac{\partial \mathbb{E}(W^{E,j}_{t+1})}{\partial K^j_t} &= \left(R^E_{t+1} q - (1 - F(\overline{\omega}^j_{t+1})) R^E_{t+1} q \overline{\omega}^j_{t+1} - R^E_{t+1} q G(\overline{\omega}^j_{t+1}) - (1 - F^E(\overline{\omega}^j_{t+1})) \frac{R^F_t n^{E,j}_t}{K^j_t} \right) \frac{1}{\Pi_{t+1}} \\
&= \left(R^E_{t+1} q - (1 - F^E(\overline{\omega}^j_{t+1})) R^F_t \frac{b^j_t + n^{E,j}_t}{K^j_t} - R^E_{t+1} q G(\overline{\omega}^j_{t+1}) \right) \frac{1}{\Pi_{t+1}} \\
&= \left(R^E_{t+1}(1 - G(\overline{\omega}^j_{t+1})) - R^F_t (1 - F^E(\overline{\omega}^j_{t+1})) \right) \frac{q}{\Pi_{t+1}} \qquad (33)
\end{aligned}$$

Substituting in the first order condition of the Lagrangian with respect to capital (26)

$$0 = R^E_{t+1}(1 - G(\overline{\omega}^j_{t+1})) - R^F_t (1 - F^E(\overline{\omega}^j_{t+1})) + \Lambda_t(\lambda - R^E_{t+1}(1 - G(\overline{\omega}^j_{t+1})) + R^F_t (1 - F^E(\overline{\omega}^j_{t+1}))) \qquad (34)$$

Implying that

$$\begin{aligned}
\Lambda_t &= \frac{R^E_{t+1}(1 - G(\overline{\omega}^j_{t+1})) - R^F_t (1 - F^E(\overline{\omega}^j_{t+1}))}{R^E_{t+1}(1 - G(\overline{\omega}^j_{t+1})) - R^F_t (1 - F^E(\overline{\omega}^j_{t+1})) - \lambda} \\
&= 1 + \frac{\lambda}{R^E_{t+1}(1 - G(\overline{\omega}^j_{t+1})) - R^F_t (1 - F^E(\overline{\omega}^j_{t+1})) - \lambda} \qquad (35)
\end{aligned}$$

The incentive constraint binds when the Lagrange multiplier Λ_t is positive. This occurs when the expected return earned by the entrepreneurs $R^E_{t+1}(1 - G(\overline{\omega}^j_{t+1}))$ exceeds the expected cost of borrowing $R^F_t(1 - F^E(\overline{\omega}^j_{t+1}))$. In this case, entrepreneurial equity is scarce, and entrepreneurs find it optimal to borrow from bankers, as indicated by a positive (33). This implies that, in an active credit market, the loan demand is implicitly defined by the incentive participation constraint

$$(1 - \Gamma(\overline{\omega}^j_{t+1})) R^E_{t+1} = \lambda$$

C.1.2 Properties of the Loan Demand

This section presents the properties of the loan demand function that was derived in Section C.1.1.

Proof of Proposition 1

Proposition 1 states that loan demand decreases as the borrowing rate increases. Let $\mathcal{I}$ be defined as

$$\mathcal{I}_t \equiv (1 - \Gamma(\overline{\omega}^j_{t+1}))R^E_{t+1} - \lambda$$

The derivative of the loan demand with respect to the borrowing rate is given by

$$\frac{\mathrm{d}b_t}{\mathrm{d}R^F_t} = -\frac{\frac{\partial \mathcal{I}_t}{\partial R^F_t}}{\frac{\partial \mathcal{I}_t}{\partial b_t}} \tag{36}$$

The numerator of (36) can be expressed as

$$\frac{\partial \mathcal{I}_t}{\partial R^F_t} = -R^E_{t+1}\Gamma'(\overline{\omega}_{t+1})\frac{\partial \overline{\omega}_{t+1}}{\partial R^F_t} \tag{37}$$

From the definition of the default cutoff (3), the term $\frac{\partial \overline{\omega}_{t+1}}{\partial R^F_t}$ is equal to

$$\frac{\partial \overline{\omega}_{t+1}}{\partial R^F_t} = \frac{b_t}{R^E_{t+1}qK_t} \tag{38}$$

Substituting the expressions for $\Gamma'(\overline{\omega}_{t+1})$ and $\frac{\partial \overline{\omega}_{t+1}}{\partial R^F_t}$ derived in (31) and (38) respectively, into (37)

$$\begin{aligned}\frac{\partial \mathcal{I}_t}{\partial R^F_t} &= -R^E_{t+1}(1 - F(\overline{\omega}_{t+1}))\frac{b_t}{R^E_{t+1}qK_t} \\ &= -(1 - F(\overline{\omega}_{t+1}))\frac{b_t}{qK_t} \leq 0\end{aligned} \tag{39}$$

The denominator of (36) can be expressed as

$$\frac{\partial \mathcal{I}_t}{\partial b_t} = -R^E_{t+1}\Gamma'(\overline{\omega}_{t+1})\frac{\partial \overline{\omega}_{t+1}}{\partial b_t} + (1 - \Gamma(\overline{\omega}_{t+1}))R^{E\prime}_{t+1} \tag{40}$$

The term $\frac{\partial \overline{\omega}_{t+1}}{\partial b_t}$ is equal to

$$\frac{\partial \overline{\omega}_{t+1}}{\partial b_t} = \frac{R^F_t}{q}\frac{R^E_{t+1}qK_t - (R^{E\prime}_{t+1}K_t + R^E_{t+1})b_t}{(R^E_{t+1}K_t)^2} = \frac{R^F_t}{q}\frac{R^E_{t+1}n^E_t - R^{E\prime}_{t+1}K_t}{(R^E_{t+1}K_t)^2} \geq 0 \tag{41}$$

Substituting the expressions for $\Gamma'(\overline{\omega}_{t+1})$ and $\frac{\partial \overline{\omega}_{t+1}}{\partial b_t}$ derived in (31) and (41) respectively, into (40) derived

$$\frac{\partial \mathcal{I}_t}{\partial b_t} = -R^E_{t+1}(1 - F(\overline{\omega}_{t+1}))\frac{R^F_t}{q}\frac{R^E_{t+1}n^E_t - R^{E\prime}_{t+1}K_t}{(R^E_{t+1}K_t)^2} + (1 - \Gamma(\overline{\omega}_{t+1}))R^{E\prime}_{t+1} \leq 0 \tag{42}$$

Substituting (39) and (42) in (36)

$$\frac{\mathrm{d}b_t}{\mathrm{d}R^F_t} = -\frac{(1 - F(\overline{\omega}_{t+1}))\frac{b_t}{qK_t}}{R^E_{t+1}(1 - F(\overline{\omega}_{t+1}))\frac{R^F_t}{q}\frac{R^E_{t+1}n^E_t - R^{E\prime}_{t+1}K_t}{(R^E_{t+1}K_t)^2} - (1 - \Gamma(\overline{\omega}_{t+1}))R^{E\prime}_{t+1}} \leq 0$$

Since $\frac{\mathrm{d}b_t}{\mathrm{d}R^F_t} \leq 0$, loan demand is a decreasing function of the borrowing rate.

Proof of Proposition 2

Proposition 2 states that the default rate of entrepreneurs rises with the borrowing rate. This is because the default rate $F(\overline{\omega}_{t+1})$, is an increasing function of $\overline{\omega}_{t+1}$. This can be seen by taking the derivative of the default rate (4) with respect to the default threshold

$$F'(\overline{\omega}_{t+1}) = f(\overline{\omega}_{t+1}) = \frac{1}{\overline{\omega}_{t+1}\sigma_t}\phi\left(\frac{\log(\overline{\omega}_{t+1}) + 0.5\sigma^2_t}{\sigma_t}\right) \geq 0, \tag{43}$$

where $\phi(\cdot)$ is the p.d.f. of the standard normal distribution.

In order to show that the entrepreneurial default rate increases with the borrowing rate, it is necessary to show that the default threshold $\overline{\omega}_{t+1}$ increases with the borrowing rate. Using the loan demand function (7), the default threshold can be expressed as

$$\Gamma(\overline{\omega}_{t+1}) = 1 - \frac{\lambda}{R^E_{t+1}}$$

Inverting $\Gamma(\overline{\omega}_{t+1})$

$$\overline{\omega}_{t+1} = \Gamma^{-1}\left(1 - \frac{\lambda}{R^E_{t+1}}\right) \tag{44}$$

The derivative of (44) can be expressed as

$$\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}R^F_t} = \frac{1}{1 - F\left(1 - \frac{\lambda}{R^E_{t+1}}\right)}\frac{1}{(R^E_{t+1})^2}R^{E\prime}_{t+1}\frac{1}{q}\frac{\mathrm{d}b_t}{\mathrm{d}R^F_t} \geq 0 \tag{45}$$

Since the default rate increases with the default threshold that is increasing in the loan rate, the default rate increases with the loan rate.

Proof of Proposition 3

Proposition 3 states that if $R^E_{t+1} \leq \frac{\lambda}{1-\Gamma\left(e^{-\sigma-0.5\sigma^2}\right)}$, a rise in uncertainty leads to a stronger rise in the default rate of the entrepreneurs when the default cutoff is higher.

The effect of an increase of the default cutoff on the default rate is given by the derivative of (4) with respect to $\overline{\omega}_{t+1}$

$$F'(\overline{\omega}_{t+1}) = \frac{1}{\overline{\omega}_{t+1}\sigma_{t+1}}\phi\left(\frac{\log(\overline{\omega}_{t+1}) + 0.5\sigma^2_{t+1}}{\sigma_{t+1}}\right). \tag{46}$$

In order to show that an increase in uncertainty has a stronger effect on the default rate when the default cutoff is higher, $\frac{\mathrm{d}F'_{t+1}}{\mathrm{d}\sigma_{t+1}}$ must to be positive

$$\begin{aligned}\frac{\mathrm{d}F'_{t+1}}{\mathrm{d}\sigma_{t+1}} = &- \frac{\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}\sigma_{t+1} + \overline{\omega}_{t+1}}{\overline{\omega}^2_{t+1}\sigma^2_{t+1}}\phi\left(\frac{\log(\overline{\omega}_{t+1}) + 0.5\sigma^2_{t+1}}{\sigma_{t+1}}\right) \\ &+ \phi'\left(\frac{\log(\overline{\omega}_{t+1}) + 0.5\sigma^2_{t+1}}{\sigma_{t+1}}\right)\frac{1}{\overline{\omega}_{t+1}\sigma_{t+1}}\frac{\frac{1}{\overline{\omega}_{t+1}}\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}\sigma_{t+1} + 0.5\sigma^2_{t+1} - \log(\overline{\omega}_{t+1})}{\sigma^2_{t+1}}\end{aligned} \tag{47}$$

The term $\phi'(x)$ can be written as

$$\phi'(x) = -\frac{1}{\sqrt{2\pi}}xe^{-0.5x^2} \tag{48}$$

The term $\phi(x)$ can be written as

$$\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-0.5x^2} \tag{49}$$

Substituting (48) and (49) into (47)

$$\begin{aligned}\frac{\mathrm{d}F'_{t+1}}{\mathrm{d}\sigma_{t+1}} =& \frac{1}{\sqrt{2\pi}}e^{-0.5x^2}\frac{1}{\overline{\omega}\sigma^2}\Bigg(- \frac{\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}\sigma_{t+1} + \overline{\omega}_{t+1}}{\overline{\omega}_{t+1}} \\ &+ \frac{\log(\overline{\omega}_{t+1}) + 0.5\sigma^2_{t+1}}{\sigma_{t+1}}\frac{\log(\overline{\omega}_{t+1}) - 0.5\sigma^2_{t+1} - \frac{1}{\overline{\omega}_{t+1}}\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}\sigma_{t+1}}{\sigma_{t+1}}\Bigg)\end{aligned} \tag{50}$$

Equation 50 can be written as

$$\frac{\mathrm{d}F'_{t+1}}{\mathrm{d}\sigma_{t+1}} = \frac{1}{\sqrt{2\pi}} e^{-0.5x^2} \frac{1}{\overline{\omega}\sigma^2} \Bigg(- \frac{\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}\sigma_{t+1}}{\overline{\omega}_{t+1}} - \frac{\log(\overline{\omega}_{t+1}) + 0.5\sigma_{t+1}^2}{\overline{\omega}_{t+1}} \frac{1}{\overline{\omega}_{t+1}} \frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}$$
$$- 1 + \frac{\log(\overline{\omega}_{t+1}) + 0.5\sigma_{t+1}^2}{\sigma_t} \frac{\log(\overline{\omega}_{t+1}) - 0.5\sigma_{t+1}^2}{\sigma_t} \Bigg) \tag{51}$$

Equation 51 is positive when $R^E_{t+1} \leq \frac{\lambda}{1-\Gamma\left(e^{-\sigma-0.5\sigma^2}\right)}$. In order to see that this assumption is sufficient, note that

$$-\frac{\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}\sigma_{t+1}}{\overline{\omega}_{t+1}} + \frac{1}{\overline{\omega}_{t+1}} \frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}, \tag{52}$$

is positive if $\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}} \geq 0$. The effect of uncertainty on the default cutoff is

$$\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}} = \frac{R^F_t}{q} \frac{\mathrm{d}b_t}{\mathrm{d}\sigma_{t+1}} \frac{R^E_{t+1} q K_t - R^E b - R^{E\prime}_{t+1} K_t b_t}{(R^E_{t+1} K_t)^2} = \frac{R^F_t}{q} \frac{\mathrm{d}b_t}{\mathrm{d}\sigma_{t+1}} \frac{R^E_{t+1} n^E - R^{E\prime}_{t+1} K_t b_t}{(R^E_{t+1} K_t)^2}.$$

The effect of uncertainty on the default cutoff is positive if $\frac{\mathrm{d}b_t}{\mathrm{d}\sigma_{t+1}} \geq 0$. The effect of uncertainty on loan demand is

$$\frac{\mathrm{d}b_t}{\mathrm{d}\sigma_{t+1}} = -\frac{\frac{\partial \mathcal{I}_t}{\partial \sigma_{t+1}}}{\frac{\partial \mathcal{I}_t}{\partial b_t}}. \tag{53}$$

The numerator of (53) can be expressed as

$$\frac{\partial \mathcal{I}_t}{\partial \sigma_{t+1}} = -\frac{\partial \Gamma(\overline{\omega}_{t+1})}{\partial \sigma_{t+1}} R^E_{t+1}. \tag{54}$$

Substituting the definition of $\Gamma(\overline{\omega}_{t+1})$, the term $\frac{\partial \Gamma(\overline{\omega}_{t+1})}{\partial \sigma_{t+1}}$ in (54) can be expressed as

$$\begin{aligned}
\frac{\partial \Gamma(\overline{\omega}_{t+1})}{\partial \sigma_{t+1}} &= - \frac{\partial F(\overline{\omega}_{t+1})}{\partial \sigma_{t+1}} \overline{\omega}_{t+1} + \frac{\partial G(\overline{\omega}_{t+1})}{\partial \sigma_{t+1}} \\
&= - F'(\overline{\omega}_{t+1}) \overline{\omega}_{t+1}^2 \frac{0.5\sigma_{t+1}^2 - \log(\overline{\omega}_{t+1})}{\sigma_{t+1}} + F'(\overline{\omega}_{t+1}) \overline{\omega}_{t+1}^2 \frac{-0.5\sigma_{t+1}^2 - \log(\overline{\omega}_{t+1})}{\sigma_{t+1}} \\
&= - F'(\overline{\omega}_{t+1}) \overline{\omega}_{t+1}^2 \sigma_{t+1} \leq 0.
\end{aligned}$$

Substituting the last expression into (54)

$$\frac{\partial \mathcal{I}_t}{\partial \sigma_{t+1}} = F'(\overline{\omega}_{t+1}) \overline{\omega}_{t+1}^2 \sigma_{t+1} R^E_{t+1} \tag{55}$$

Substituting (55) and (42) into (53)

$$\frac{\mathrm{d}b_t}{\mathrm{d}\sigma_{t+1}} = \frac{F'(\overline{\omega}_{t+1})\overline{\omega}_{t+1}^2\sigma_{t+1}R_{t+1}^E}{R_{t+1}^E(1-F(\overline{\omega}_{t+1}))\frac{R_t^F}{q}\frac{R_{t+1}^E n_t^E - R_{t+1}^{E\prime}K_t}{(R_{t+1}^E K_t)^2} - (1-\Gamma(\overline{\omega}_{t+1}))R_{t+1}^{E\prime}} \geq 0$$

Therefore, $\left(-\frac{\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}\sigma_{t+1}}{\overline{\omega}_{t+1}} + \frac{1}{\overline{\omega}_{t+1}}\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}\right) \geq 0$.

The assumption $R_{t+1}^E \leq \frac{\lambda}{1-\Gamma\left(e^{-\sigma_{t+1}-0.5\sigma_{t+1}^2}\right)}$ implies that

$$\begin{aligned}
R_{t+1}^E &\leq \frac{\lambda}{1-\Gamma\left(e^{-\sigma_{t+1}-0.5\sigma_{t+1}^2}\right)} \\
1 - \frac{\lambda}{R_{t+1}^E} &\leq \Gamma\left(e^{-\sigma_{t+1}-0.5\sigma_{t+1}^2}\right) \\
\Gamma^{-1}\left(1 - \frac{\lambda}{R_{t+1}^E}\right) &\leq e^{-\sigma_{t+1}-0.5\sigma_{t+1}^2} \\
\log(\overline{\omega}_{t+1}) &\leq -\sigma_{t+1} - 0.5\sigma_{t+1}^2 \\
\frac{\log(\overline{\omega}_{t+1}) + 0.5\sigma_{t+1}^2}{\sigma_{t+1}} &\leq -1
\end{aligned} \tag{56}$$

Because of (56) and since $-\frac{\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}\sigma_{t+1}}{\overline{\omega}_{t+1}} + \frac{1}{\overline{\omega}_{t+1}}\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}} \geq 0$

$$-\frac{\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}\sigma_{t+1}}{\overline{\omega}_{t+1}} - \frac{\log(\overline{\omega}_{t+1}) + 0.5\sigma_{t+1}^2}{\overline{\omega}_{t+1}}\frac{1}{\overline{\omega}_{t+1}}\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}} \geq 0. \tag{57}$$

Moreover, because of (56)

$$-1 + \frac{\log(\overline{\omega}_{t+1}) + 0.5\sigma_{t+1}^2}{\sigma_t}\frac{\log(\overline{\omega}_{t+1}) - 0.5\sigma_{t+1}^2}{\sigma_t} \geq 0 \tag{58}$$

Finally, because of (57) and (58), (47) is positive and a rise in uncertainty leads to a stronger rise in the default rate of the entrepreneurs when the default cutoff is higher.

D Reverse Bounded Pareto Distribution

Suppose that γ follows a Pareto distribution with scale parameter $a > 0$ and support $\gamma \in [\gamma_s, \infty)$. Its p.d.f. and its c.d.f. are

$$f_\gamma(\gamma) = \frac{a\gamma_s^a}{\gamma^{a+1}}$$

$$F_\gamma(\gamma) = 1 - \left(\frac{\gamma_s}{\gamma}\right)^a$$

A bounded Pareto distribution is a distribution obtained from restricting the domain of the Pareto distribution. Let S and H be the lower bound and the upper bounds of the bounded Pareto distribution. The resulting p.d.f. and c.d.f. are

$$f_{\gamma B}(\gamma) = \frac{f_\gamma(\gamma)}{F_\gamma(H) - F_\gamma(S)} = \frac{\frac{a\gamma_s^a}{\gamma^{a+1}}}{1 - \left(\frac{\gamma_s}{H}\right)^a - \left[1 - \left(\frac{\gamma_s}{S}\right)^a\right]} = \frac{aS^a\gamma^{-a-1}}{1 - \left(\frac{S}{H}\right)^a}$$

$$F_{\gamma B}(\gamma) = \frac{F_\gamma(\gamma) - F_\gamma(S)}{F_\gamma(H) - F_\gamma(S)} = \frac{1 - \left(\frac{\gamma_s}{\gamma}\right)^a - \left[1 - \left(\frac{\gamma_s}{S}\right)^a\right]}{1 - \left(\frac{\gamma_s}{H}\right)^a - \left[1 - \left(\frac{\gamma_s}{S}\right)^a\right]} = \frac{1 - S^a\gamma^{-a}}{1 - \left(\frac{S}{H}\right)^a}$$

This distribution is characterized by a positive skewness and a long right tail. A market share distribution that features many small bankers and a few large bankers can be obtained by flipping the distribution around the y-axis and shifting it to the right by $S + H$. This leads to a reverse bounded Pareto distribution whose domain is (S, H). The p.d.f. and the c.d.f. of this distribution are

$$f_{\gamma BR}(\gamma) \equiv f_{\gamma B}(-\gamma + H + S) = \frac{aS^a(-\gamma + H + S)^{-a-1}}{1 - \left(\frac{S}{H}\right)^a}$$

$$F_{\gamma BR}(\gamma) = \int_S^\gamma \frac{aS^a(-\gamma + H + S)^{-a-1}}{1 - \left(\frac{S}{H}\right)} d\gamma = \frac{S^a(-\gamma + H + S)^{-a} - S^a H^{-a}}{1 - \left(\frac{S}{H}\right)^a}$$