Imperfect Banking Competition and the Propagation of Uncertainty Shocks*

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Abstract

Uncertainty shocks are drivers of business cycle fluctuations. Motivated by the recent fall in banking competition, I study how the propagation of uncertainty shocks is affected by the intensity of competition in the banking sector. Analyzing a panel dataset of 44 countries, I show that an increase in uncertainty has a stronger negative impact on output growth when banking competition is lower. In order to explain this fact, I build a dynamic stochastic general equilibrium model with imperfect banking competition and financial frictions. In the model, entrepreneurs and imperfectly competitive bankers engage in a loan contract and entrepreneurs receive idiosyncratic productivity shocks. When competition in the banking sector is lower, bankers charge higher borrowing rates to entrepreneurs who optimally increase their risk of failure. Because of higher risk taking, uncertainty shocks to entrepreneurial productivity have stronger negative effects on defaults, investment and output when banking competition is lower.

Keywords: Financial intermediaries, uncertainty shocks, financial frictions, market power, heterogeneous agents. (JEL D43, D81, E32, E44, L13, L26, G21)

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1 Introduction

The recent conflict in Ukraine and the Covid-19 pandemic have resulted in a sharp increase in uncertainty.[1] When borrowers are subject to financial frictions, uncertainty shocks increase borrower defaults and lead to a contraction in the supply of loans and GDP.[2] Since credit markets are crucial for understanding the transmission of uncertainty shocks, structural changes in the credit markets have implications for the transmission of these shocks. In this paper, motivated by the recent fall in competition in the US banking sector, I study how the level of banking competition affects the propagation of uncertainty shocks.

The U.S. banking sector is highly concentrated. The number of commercial banks has fallen since the beginning of this century, while bank asset concentration has increased. In 2020, the number of commercial banks was half of what it was in 2000 and the share of assets held by the three largest banks rose from 21% in 2000 to 35% in 2020.[3]

In this paper, first, I provide empirical evidence on how the causal impact of uncertainty shocks on output growth correlate with the level of competition in the banking sector. I use exogenous shocks that occur in a panel that spans 44 countries between 2000Q1 and 2020Q1. These shocks are natural disasters, terrorist attacks, political coups and revolutions. I use these shocks to instrument for changes in the first and second moments of stock market returns to separate the effects of first and second moment shocks. I show that second moment shocks have a stronger negative impact on output growth when banking competition – proxied by banking concentration – is lower.

Second, I develop a New Keynesian business cycle model that incorporates financial frictions and imperfect competition in the banking sector. The main feature of this model is that bankers compete à la Cournot to provide loans to entrepreneurs. In the economy there are N bankers that invest their equity and their deposits in a portfolio of loans to entrepreneurs. Entrepreneurs are the agents that own and maintain the stock of physical capital. Entrepreneurs have insufficient net worth and borrow from the bankers in order to

[1]Caldara et al. (2022), Ferrara et al. (2022) and Anayi et al. (2022) document an increase in uncertainty after the Russian invasion of Ukraine. Altig et al. (2020) and Baker et al. (2020) document an increase in uncertainty triggered by the Covid-19 pandemic.

[2]See for example Christiano et al. (2014), Caldara et al. (2016) and Alessandri and Mumtaz (2019).

[3]See Figure 8 in Appendix A.1.

buy capital goods. Bankers internalize the effects of their choices on the loan demand and on the default rate of borrowers and optimally choose their credit supply.

Entrepreneurs are subject to idiosyncratic and aggregate shocks. Because of the idiosyncratic shocks, entrepreneurs obtain idiosyncratic returns on their capital stock. For some entrepreneurs the realized return is not enough to pay back their loans and these entrepreneurs declare default. The cross-sectional dispersion of the idiosyncratic shock defines the level of uncertainty in this economy. When uncertainty increases, the probability that entrepreneurs obtain a low return increases and more borrowers default. Because of financial frictions, banks react to the higher uncertainty by reducing their credit supply, therefore, entrepreneurs have less resources to buy capital. The fall in the demand of capital leads to a fall in investment and to a contraction in output.

The model is developed in two steps. First, I introduce the entrepreneurial sector in a partial equilibrium setup. In this setup, I present the first channel through which competition in the banking sector affects the transmission of uncertainty shocks. Less competitive banking sectors charge higher borrowing rates to borrowers because of higher market power. I show that the higher the borrowing rate faced by entrepreneurs, the higher the risk-taking and the probability of default. Furthermore, an increase in uncertainty implies a larger rise in the default rate of entrepreneurs when entrepreneurs take more risk. This channel is called *risk-shifting effect*.

Second, I include the entrepreneurial sector in a calibrated general equilibrium model with imperfectly competitive bankers. A second channel arises. The higher the bankers' market power, the lower the extent to which bankers pass shocks through to borrowers. In particular, an uncertainty shock increases the amount of non-performing loans and the monitoring costs incurred by bankers. When costs increase, perfectly competitive bankers decrease their loan supply by more than less competitive bankers because of the lower market power. I call this channel the *pass-through* effect.

The presence of the two opposing channels implies a non-trivial impact of banking competition on the transmission of uncertainty shocks. I calibrate the quantitative model to match several US credit market statistics, and I show that the risk-shifting effect arises also in general equilibrium. Moreover, the risk-shifting effect is stronger than the pass through effect and an uncertainty shock implies larger business cycle fluctuations when competition is lower. When competition is lower, an uncertainty shock increases the default rate of entrepreneurs by more. Bankers react by cutting their loan supply by more and entrepreneurs experience a stronger contraction in their financial resources. Because of the larger credit crunch, investment falls by more and there is a larger contraction in GDP.

The quantitative model is solved assuming homogeneous bankers. In order to disentangle the two channels, I consider an extension of the model in which I drop the assumption of homogeneous banks. In this extension, heterogeneity is driven by a different marginal cost of providing loans. Banks with a lower marginal cost provide more loans and have higher market power. First, I show that the main result is robust to this extension. Second, I show that smaller banks respond to uncertainty shocks by cutting loans by more and by increasing markups by more. In fact, smaller banks have lower market power and need to pass the rise in monitoring costs through to borrowers to a higher extent than larger banks.

Related Literature. My paper is related to four strands of literature: market structure of the banking industry, financial frictions, uncertainty shocks and macro-finance with financial intermediaries. My main contribution is to connect the literature on financial frictions, the literature on uncertainty shocks and the literature on the market structure of the banking industry. In particular, building on the literature on uncertainty shocks and financial frictions, I study how the market structure of the banking industry determines the impact of uncertainty shocks.

Market structure of the banking industry. This paper is related to the theoretical and empirical literature that studies the market structure of the banking industry and its effects on financial stability.

Several theoretical papers study the effects of banking competition on financial stability. The majority of these focuses on two-period economies. Such papers include Boyd and de Nicoló (2005) Martinez-Miera and Repullo (2010), Hakenes and Schnabel (2011) and Gasparini (2022). Boyd and de Nicoló (2005) show that less competitive banking sectors charge higher borrowing rates to borrowers implying higher default risk for borrowers. However, as shown by Martinez-Miera and Repullo (2010), less competitive banking sectors have larger buffers against non-performing loans due to their higher profits. Therefore, less competitive banking sectors have lower leverage and are less fragile.

Important empirical contributions to this literature are Berger et al. (2017), Scharfstein and Sunderam (2016), Buch et al. (2022) and Gödl-Hanisch (2022). According to Berger et al. (2017), the results of Boyd and de Nicoló (2005) and Martinez-Miera and Repullo (2010) have empirical support. Scharfstein and Sunderam (2016) indicate that monetary policy shocks are transmitted to a lesser extent to mortgage borrowers when concentration in mortgage lending is higher. Buch et al. (2022) show that the impact of monetary policy on economic activity became stronger after the U.S. interstate banking deregulation. Finally, Gödl-Hanisch (2022) shows that the pass-through of monetary policy shocks to borrowers is higher for bank branches operating in highly concentrated counties, while the pass-through of monetary policy shocks to depositors is lower.

This paper contributes to this literature in three major respects. First, this paper builds a DSGE model that features the channel of Boyd and de Nicoló (2005) and Martinez-Miera and Repullo (2010). Second, my paper introduces another mechanism, the pass-through effect, whereby competition affects the extent to which banks pass shocks through to their borrowers. This is in line with the findings of Scharfstein and Sunderam (2016) and Buch et al. (2022).

Finally, I present facts on how banking concentration affects the propagation of uncertainty shocks. I show that uncertainty shocks reduce output by more when banking concentration is higher.

Financial frictions. The existing literature on financial frictions studies the implications of financial frictions for the transmissions of shocks. These studies often introduce financial frictions assuming costly state verification or agency problems. Townsend (1979), Carlstrom and Fuerst (1997), Bernanke et al. (1999), Christiano et al. (2014), Clerc et al. (2018) and Gasparini et al. (2022), introduce financial frictions using a costly state verification framework, while Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), introduce financial frictions adopting agency problems. Similarly to Kühl, 2017 my model combines the two approaches. In my model there is an agency problem because entrepreneurs can divert part of their assets after borrowing from banks. At the same time, banks have to pay a monitoring cost in order to observe the entrepreneur's realized return.

My paper contributes to this literature introducing imperfect banking competition in the banking sector. Differently from this literature, I assume that loans are provided to entrepreneurs by a finite amount of bankers that compete à la Cournot. This assumption implies that bankers charge a markup on the borrowing rate as observed by Corbae and D'Erasmo (2021). In this economy imperfect banking competition introduces another financial friction due to its effect on the borrowing rate.

Uncertainty shocks. The literature on uncertainty shocks argues that these shocks are a driver of business cycle fluctuations. Important papers in this literature are Bloom (2009), Bachmann and Bayer (2013), Christiano et al. (2014), Fernández-Villaverde et al. (2015), Basu and Bundick (2017), Mumtaz et al. (2018), Cascaldi-Garcia and Galvao (2021) and Baker et al. (forthcoming). In the literature the consensus is that uncertainty shocks have important negative effects on output. My contribution to this literature is twofold. First, following the empirical work of Baker et al. (forthcoming) I show that uncertainty shocks reduce output by more when banking competition is higher.

Second, following the work of Christiano et al. (2014), I develop a DSGE model with financial frictions and imperfect banking competition and I study the effects of uncertainty shocks. As in the data, the uncertainty shock has stronger negative effects on output when competition is lower. Due to the risk-shifting effect, borrowers are more fragile when the banking sector is more concentrated and non-performing loans increase by more. Therefore, less competitive banking sectors cut loans by more after an uncertainty shock exacerbating its negative effects.

Macro-finance with financial intermediaries. This paper is also related to the macro-finance literature that studies the interaction between financial frictions and imperfectly competitive banking sectors. Corbae and D'Erasmo (2021), Li (2019), Jamilov and Monacelli (2021) and Villa (2020).

Corbae and D'Erasmo (2021) and Li (2019) develop models with banks that imperfectly compete for loans. My model is similar to the one of Li (2019), one important difference is that her model does not capture the risk-shifting effect.

Jamilov and Monacelli (2021) and Villa (2020) study the effects of banking competition in the transmission of shocks. Jamilov and Monacelli (2021) building a quantitative macroeconomic model with heterogeneous monopolistic financial intermediaries, study how banking competition affects the propagation of a capital quality shock. They find that credit market power decreases the impact of capital quality shocks. Villa (2020) builds a model with banks that compete à la Cournout for loans and deposits and argues that a sudden increase in the aggregate firms' default probability has stronger negative effects when banking competition is lower.

My contribution to this literature is twofold. First, I introduce a new propagation channel in this set of models, the risk-shifting effect. Second I study the propagation mechanism of a different shock, an uncertainty shock.

Outline. The remainder of the paper is structured as follows. Section 2 empirically studies how the propagation of uncertainty shocks varies with the level of banking competition. Section 3 outlines the borrower side of the model and introduces the risk-shifting effect. Section 4 outlines the quantitative model. Section 5 displays the calibration and the results of the quantitative model. In this section I show quantitatively how the level of competition affects the transmission of uncertainty shocks. Finally, Section 6 concludes.

2 Empirical Evidence

In this section I employ a panel dataset of 44 countries between 2000Q1 and 2020Q1 to empirically document how banking competition affects the propagation of uncertainty shocks. In Section 2.1 I describe the data. In Section 2.2 I describe the regression model and I report the empirical results. Further information about the data is available at Appendix B.1. Further results are available at Appendix B.2 and the robustness tests are conducted at Appendix B.3.

2.1 Data Description

In my analysis I use data from 44 countries spanning the period 2000Q1-2020Q1.[4] For each of these countries, when the data is available, I collect information about real GDP growth, first and second moments of national business conditions, disaster shocks and yearly information about banking concentration. Real GDP growth is obtained from the International Financial Statistics of IMF or, if not available, from OECD. First and second moments data are obtained from Baker et al. (forthcoming) and banking concentration is obtained from World Bank.

The measure of first and second moments of national business conditions is obtained from national stock market movements. The first moment is the stock return of the broadest national index. The second moment is the logarithm of the quarterly standard deviations of daily stock returns.

The disaster shocks cover four types of events: natural disasters, terrorist attacks, coups and revolutions. For each category it is given a value of one if a disaster shock has happened. To obtain the final indexes, the events are weighted by the increase in media coverage in the 15-days period after the shock compared to the 15-days period before the shock occurred. Media coverage is defined by the amount of English articles about the country that are published in English-language newspapers based in the United States.

Banking competition is proxied by the 3-bank asset concentration level.[5] The proxy is defined as the assets of the three largest commercial banks as a share of total commercial banking assets. Higher values of concentration indicate lower bank competition. This information is available only annually, in the results showed in this section, I linearly interpolate the 3-bank asset concentration level to obtain a quarterly measure.[6]

The descriptive statistics of the dataset are available at Table 3 in Appendix B.1.

[4]The countries used in this analysis are listed in Table 2 in Appendix B.1.
[5]As shown in Appendix B.3, the results are similar using the 5-bank asset concentration level
[6]The results hold also keeping the level of concentration constant within a year. Appendix B.3 shows the results of this robustness exercise.

2.2 Banking competition and the impact of uncertainty shocks.

In this section I describe the regression model and I report the empirical results. In section 2.2.1 I describe the regression model and in 2.2.2 I display the results.

2.2.1 Regression model

I estimate the effect of an increase in uncertainty on output growth and I study how the level of banking competition affects the impact of uncertainty shocks. In order to achieve this target I estimate the following regression model

$$y_{i,t+h} = \alpha_i + \tau_t + \beta^R \tilde{R}_{i,t} + \beta^V \tilde{V}_{i,t} + \beta^C \tilde{C}_{i,t} + \beta^{RC} \tilde{R}_{i,t}\tilde{C}_{i,t} + \beta^{VC} \tilde{V}_{i,t}\tilde{C}_{i,t} + \epsilon_{i,t},$$

where $y_{i,t+h}$ is the growth rate of real GDP from period $t-1$ to period $t+h$ for country i , α_i are country fixed effects, τ_i are time fixed effects, $\tilde{R}_{i,t}$ is the country demeaned return measure, $\tilde{V}_{i,t}$ is the country demeaned volatility measure and $\tilde{C}_{i,t}$ is the country demeaned 3-bank asset concentration level.

This model is an extension of the model proposed by Baker et al. (forthcoming). Differently from their regression model, in my regression model I add banking concentration and the interactions of banking concentration with first and second moments. The interactions are added to separate the effect of concentration on the impact of first and second moment shocks. Finally, I country demean the variables in the regression in order to control for the non-linear effects of country characteristics.

The coefficients β^V and β^{VC} give the impact of an increase in volatility on real output growth. The coefficient β^V represent the impact of an uncertainty shock on output growth when banking concentration is at the country mean. The coefficient β^{VC} estimates how banking concentration affects the impact of uncertainty shocks. If β^{VC} is negative, an increase in volatility has a stronger negative effect in output growth when concentration is higher.

Similarly to Baker et al. (forthcoming), I instrument first and second moment variables and their interaction with concentration using disaster shocks.[7] This instrumental variable

[7] The instruments used are the disaster shocks and their interaction with with demeaned concentration.

approach allows me to study the causal impact of first and second moment shocks on output growth and because of the media weighting of the disaster shocks the regression gives higher weight to more important shocks.

As in Baker et al. (forthcoming) there is a potential issue with this identification strategy. The stock market level and volatility variables proxy for different channels through which disaster shocks have economic impact. There is an exclusion restriction such that disaster shocks have an economic impact only through shifts in the first and second moments of stock returns.

2.2.2 Results

Figure 1 shows the effect of an exogenous 1% increase in volatility on real output growth for different levels of banking concentration. The blue line represents the impulse response of output growth when concentration is at the country average. The blue dashed lines are the 90% confidence interval. An exogenous increase in volatility has a negative effect on output growth that is significant only around four years after the shock.

The yellow line represents the impulse response of output growth when concentration is one standard deviation above the country average.[8] The yellow dashed lines are the %90 confidence interval. In this case the fall in output is stronger and significant for a longer period.

Figure 2 shows the difference in the responses between the average concentration specification and the high concentration specification. The graph shows that the fall in output growth is significantly higher when banking concentration is higher.

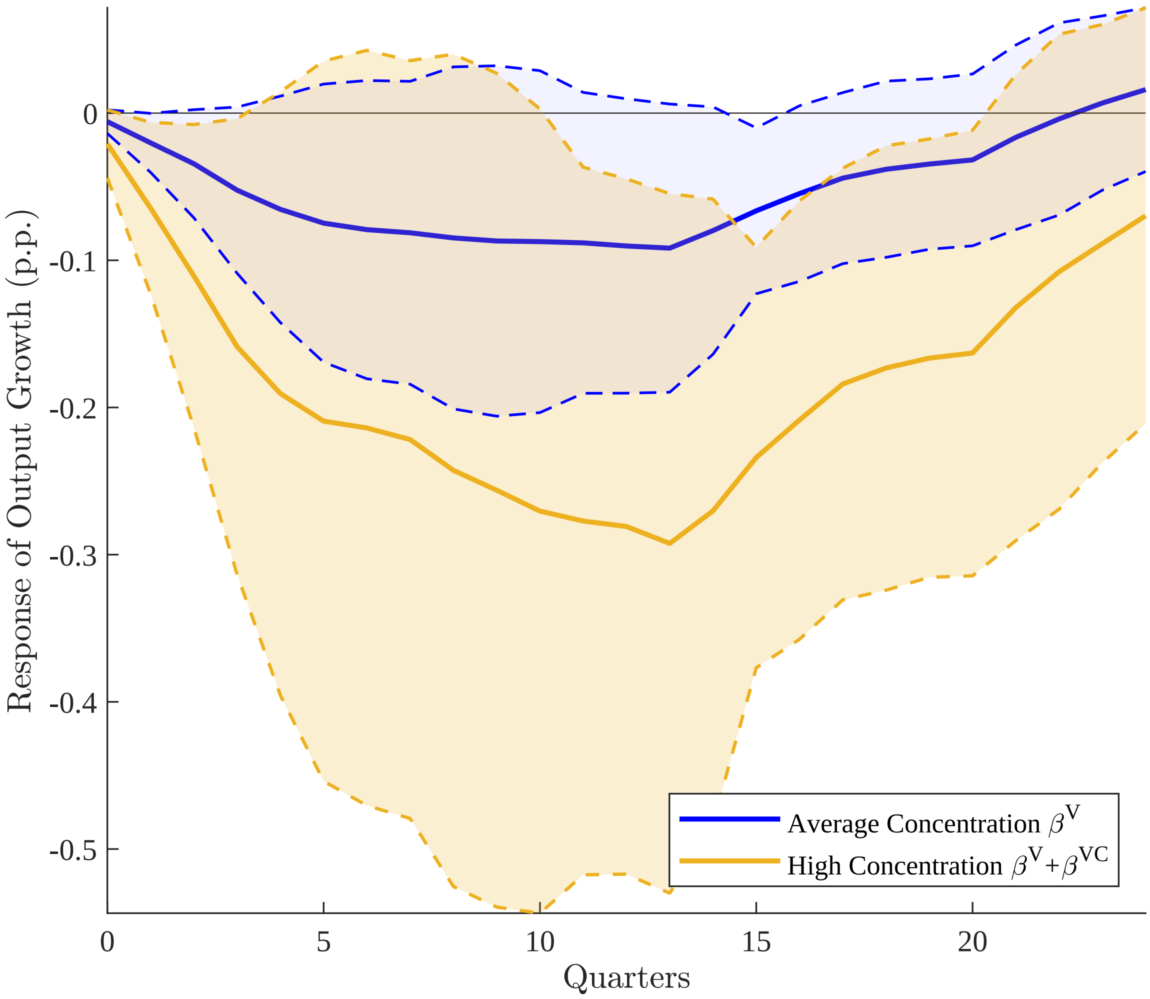
Tables 4-6 in Appendix B.2 show additional results of the baseline regressions.

The results shown in this section hold also keeping the level of concentration constant within a year and are robust to the definition of concentration. The robustness tests are conducted in Appendix B.3.

In the next sections I develop a general equilibrium model that reproduces the empirical facts showed in this section. I use this model to understand what force drives the results.

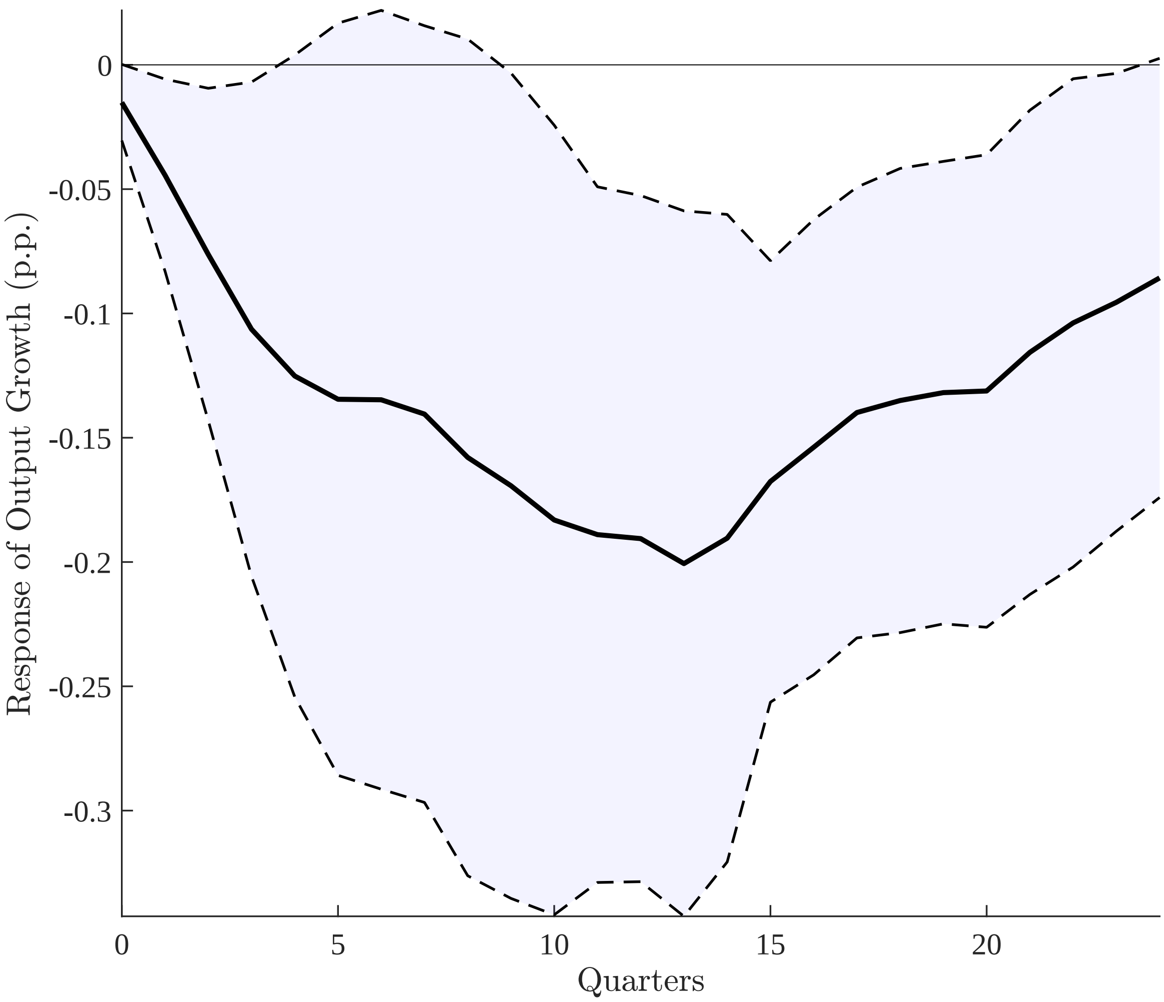
[8] A standard deviation corresponds to 19.67 percentage points.

Figure 1: Response of output growth to a 1% increase in volatility from its country average.



Notes. The graph shows the responses of output growth to an increase of volatility of 1% from its country mean for different levels of banking concentration. In the response with high banking concentration, banking concentration is 1 standard deviation larger than the country average. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1.

Figure 2: Effect of competition on the responses of output growth to a 1% increase in volatility.



Notes. The graph shows the difference between the two specifications showed in Figure 1. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1.

3 Loan demand

In this section I introduce the entrepreneurial sector in a partial equilibrium setup in order to explain the mechanism that drives the empirical results. This mechanism lies at the hearth of the general equilibrium model.

3.1 Entrepreneurs

Entrepreneurs are modelled similarly to Clerc et al. (2018). There is a continuum of risk-neutral entrepreneurs, each of them is indexed by $j \in (0,1)$. Every entrepreneur lives across two consecutive periods. Every entrepreneur born at time t has financial resources given by inherited wealth from the previous generation of entrepreneurs $n_t^{E,j}$ and loans b_t^j from the banking sector. Entrepreneurs use their financial resources to buy capital goods from capital good producers. Capital is rented to final good producers.

Entrepreneurs born at time t derive utility from donating part of their final wealth to the households $c_{t+1}^{E,j}$ in the form of dividends and the rest to the next generation of entrepreneurs as retained earnings. They derive utility according to the function $(c_{t+1}^{E,j})^{\chi^E}(n_{t+1}^{E,j})^{1-\chi^E}$. At time $t+1$, the maximization problem of the entrepreneur born at time t is

$$\max_{c_{t+1}^{E,j},n_{t+1}^{E,j}} (c_{t+1}^{E,j})^{\chi^E}(n_{t+1}^{E,j})^{1-\chi^E},$$

Subject to

$$c_{t+1}^{E,j} + n_{t+1}^{E,j} \leq W_{t+1}^{E,j},$$

where $W_{t+1}^{E,j}$ is the final wealth of the entrepreneur j born at time t.

The first order conditions lead to the dividend payment rule

$$c_{t+1}^{E,j} = \chi^E W_{t+1}^{E,j} \tag{1}$$

And the earning retention rule

$$n_{t+1}^{E,j} = (1-\chi^E) W_{t+1}^{E,j} \tag{2}$$

Future wealth is defined as

$$W_{t+1}^{E,j} = \frac{\max[\omega_{t+1}^{j} R_{t+1}^{E} q K_t^j - R_t^F b_t^j, 0]}{\Pi_{t+1}}. \tag{3}$$

Equation 3 defines the future wealth of entrepreneur j. Future wealth is given by the return from lending capital to the final goods producers minus the borrowing costs. The borrowing costs are given by the borrowing rate R_t^F times the amount borrowed. The return from renting capital is given by the amount of capital rented times its price q, times the gross return per efficiency unit of capital R_{t+1}^E and times an idiosyncratic shock ω_{t+1}^j.[9] Both the return from lending capital and the borrowing costs are discounted by the gross inflation rate $\Pi_{t+1} = P_{t+1}/P_t$. I assume that R_{t+1}^E is a decreasing function in capital.

The idiosyncratic shock ω_{t+1}^j is a shock to the entrepreneur's efficiency units of capital. This shock is assumed to be independently and identically distributed across entrepreneurs and log-normally distributed with mean one and standard deviation $\sigma_t = \sigma\varsigma_t$, which introduces time variability of firm uncertainty via an AR(1) process,

$$\ln\varsigma_t = \rho\ln\varsigma_{t-1} + \varepsilon_t, \tag{4}$$

such that $0 < \rho < 1$ and σ_t^{ε} denotes the standard deviation of the *iid* shock ε_t. The cumulative distribution function of the idiosyncratic shock is defined by

$$F_{t+1}^j = F(\overline{\omega}_{t+1}) = \int_0^{\overline{\omega}_{t+1}} f(\omega_{t+1}^j)\mathrm{d}\omega_{t+1}^j = \Phi\left(\frac{\log(\overline{\omega}_{t+1}^j) + 0.5\sigma_{t+1}^2}{\sigma_{t+1}}\right), \tag{5}$$

where $f(\cdot)$ is the respective probability density function.

The bankers enter into a financial contract with the entrepreneurs. Depending on the realization of the random productivity shock, some entrepreneurs will declare default, while others continue operating. A productivity threshold $\overline{\omega}_{t+1}^j$ is defined such that, for realizations of ω_{t+1}^j smaller than the threshold, the entrepreneur is unable to repay her loan in full, i.e. the entrepreneur declares default, while for values greater than the threshold, the entrepreneur

[9]Note that in this section I assume that the price of capital is constant, in Section 4 I will drop this assumption and the price of capital will be determined by supply and demand of capital.

will honor her contractual obligation by paying the bankers $R_t^F b_{t+1}^j$, i.e.

$$\overline{\omega}_{t+1}^j R_{t+1}^E qK_t^j = R_t^F b_t^j,$$

The default cutoff, $\bar{\omega}_{t+1}^j$, is defined as

$$\overline{\omega}_{t+1}^j = \frac{R_t^F b_t^j}{R_{t+1}^E qK_t^j}, \tag{6}$$

Unlike Bernanke et al. (1999), the cutoff productivity level $\overline{\omega}_{t+1}^j$ varies with the realization of the aggregate state R_{t+1}^E.

In case of default, the entrepreneur obtains nothing and the lender must pay a monitoring cost that is discussed more in detail in Section 4.

Similarly to Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Kühl (2017), there is a moral hazard problem: at time t the entrepreneur can divert a fraction λ of available funds. To ensure that the entrepreneur does not divert funds, the following incentive constraint must hold[10]

$$\lambda \frac{qK_{t+1}^j}{\Pi_{t+1}} \leq \mathbb{E}_t(W_{t+1}^{E,j}) \tag{7}$$

The maximization problem at period $t+1$ implies that the entrepreneur at time t chooses how much capital to buy and how much to borrow from the bankers to maximizes her expected future wealth

$$\max_{K_t^j, b_{t+1}^j} \mathbb{E}_t(W_{t+1}^{E,j}),$$

subject to the resource constraint

$$qK_t^j - b_{t+1}^j = n_t^{E,j}. \tag{8}$$

and to the incentive constraint (7).

As shown in Appendix C.1.1, the incentive constraint is binding if $R_t^F \leq R^E(n_t^E)$. The implication of this assumption is that entrepreneurial equity is scarce and because of the low borrowing rate, entrepreneurs optimally choose to borrow from bankers in order to

[10]Note that since bankers make positive profits, the financial contract cannot be derived using bankers' zero profit condition as in Bernanke et al. (1999).

produce capital. This condition is satisfied in both the partial equilibrium model and the quantitative model. Assuming that $R_t^F \leq R^E(n_t^E)$, the demand for capital and the loan demand are implicitly defined by the incentive participation constrain

$$(1 - \Gamma(\overline{\omega}_{t+1}^j))R_{t+1}^E = \lambda, \tag{9}$$

where $\Gamma(\overline{\omega}_{t+1}^j) = \Gamma_{t+1}^j$ is the expected share of return that is obtained by the entrepreneur and is not paid to the bankers in the form of borrowing costs.

Note that each entrepreneur faces the same return R_{t+1}^E and that the borrowing rate R_t^F is the same across entrepreneurs. Aggregation of the model over entrepreneurs is therefore trivial. Indices j can therefore be dropped from now on.

Propositions 1 and 2 suggest that when the borrowing rate increases, borrowers decrease their demand for loans. Because of the fall in demand, entrepreneurial leverage decreases but entrepreneurial default increases. The reason is that the fall in leverage does not compensate the rise in interest rate and the default cutoff (6) increases with the borrowing rate.

Proposition 1. *Loan demand is a decreasing function of the loan rate.*

Proposition 2. *The default rate of the entrepreneurs increases with the borrowing rate.*

The proofs of Propositions 1 and 2 is included in Appendix C.1.2.

As proved by Proposition 3 an increase in uncertainty has a stronger effect on entrepreneurial defaults when the default cutoff is higher. If entrepreneurs have a higher default cutoff they take more risk and are are more fragile to shocks. Therefore, because of higher borrower fragility an increase in uncertainty implies a stronger rise in defaults when the default cutoff is higher.

Proposition 3. *If $\sigma_{t+1} \leq 1$ and R_{t+1}^F is low enough, such that the default rate of the entrepreneurs F is lower than 15%, an increase in uncertainty implies a larger rise in the default rate of entrepreneurs when the default cutoff is higher.*

The proof of Proposition 3 is included in Appendix C.1.2.

Proposition 3 introduces the risk-shifting effect. The risk-shifting effect is the effect of lower banking competition on borrower stability. Less competitive banking sectors charge higher borrowing rates. Because of Proposition 2, when the banking sector is less competitive, the default rate of entrepreneurs is higher and an increase in uncertainty implies a larger rise in defaults. This is the force of the model that explains why output falls by more with an uncertainty shock when banking competition is lower. In fact, when competition is lower, an uncertainty shock increases borrower defaults by more. Bankers respond to the larger wave of defaults by cutting loans by more in less competitive markets. Due to the larger fall in loans in less competitive markets, entrepreneurs face a more severe lack of resources, investment falls by more and output falls by more.

4 General Equilibrium

In this section I describe the rest of the model. The credit market of this model is an extension of the partial equilibrium setup described in Section 3. In this section the price of capital is given by the equilibrium between the demand of capital and the supply of capital. Entrepreneurs buy capital from capital good producers and rent it to intermediate goods producers. An exogenous number of bankers provide loans to entrepreneurs. I assume that bankers compete á la Cournot for loans.

The rest of the model is standard. Intermediate goods producers use capital and labor to produce the intermediate goods. Final good producers buy the intermediate goods and bundle them together to produce the final good. Finally there is a monetary policy rule by which the central bank adjusts the policy rate.

4.1 Bankers

In the economy there is an exogenous number of bankers N that compete á la Cournot for loans. Each banker is indexed by i. Similarly to entrepreneurs, every banker lives across two consecutive periods. Every banker born at time t has financial resources given by inherited wealth from the previous generation of bankers $n_t^{F,i}$ and deposits d_t^i from households. Bankers use their financial resources in order to provide loans to entrepreneurs. Capital is rented to final good producers.

Bankers born at time t derive utility from donating part of their final wealth to the households $c_{t+1}^{F,i}$ in the form of dividends and the rest to the next generation of bankers as retained earnings. They derive utility according to the function $(c_{t+1}^{F,i})^{\chi^F}(n_{t+1}^{F,i})^{1-\chi^F}$. Therefore, at time $t+1$ the maximization problem of the banker born at time t is

$$\max_{c_{t+1}^{F,i},n_{t+1}^{F,i}} (c_{t+1}^{F,i})^{\chi^F}(n_{t+1}^{F,i})^{1-\chi^F},$$

Subject to

$$c_{t+1}^{F,i} + n_{t+1}^{F,i} \leq W_{t+1}^{F,i},$$

where $W_{t+1}^{F,i}$ is the final wealth of the banker i born at time t.

The first order conditions lead to the dividend payment rule

$$c_{t+1}^{F,i} = \chi^F W_{t+1}^{F,i} \tag{10}$$

And the earning retention rule

$$n_{t+1}^{F,i} = (1-\chi^F) W_{t+1}^{F,i} \tag{11}$$

The final wealth of every banker is

$$W_{t+1}^{F,i} = \frac{\tilde{R}_{t+1}(b_t)b_t^i - R_t^D d_t^i - \gamma^i b_t^i}{\Pi_{t+1}},$$

Equation 4.1 defines the future wealth of banker i. Future wealth is given by the return from lending to the borrowers minus the cost of deposits minus intermediation costs. The return from lending is given by the amount of loans times the return per unit of loans $\tilde{R}_{t+1}$. The cost of deposits is given by the deposit rate R_t^D times the amount of deposits. Finally, bankers have to pay a per loan intermediation cost γ^i that is banker specific. The return from lending, the cost of deposits and the intermediation costs are discounted by the gross inflation rate Π_{t+1}.

The return bankers obtain on their assets is

$$\tilde{R}_{t+1} = (1-F_{t+1})R_{t+1}^F + (1-\xi)\int_0^{\overline{\omega}_{t+1}} \omega_{t+1} f(\omega_{t+1}) \mathrm{d}\omega_{t+1} \frac{R_{t+1}^E q_t K_t}{b_t}. \tag{12}$$

The first term of Equation 12 represents the return from performing loans while the second term is the return bankers obtain from non-performing loans. When a loan default bankers have to pay a monitoring cost ξ in order to observe the entrepreneur's realized return on capital. This cost is a proportion ξ of the realized gross payoff to the entrepreneurs and is incurred in the case of default. Note that all the bankers obtain the same return from non-performing loans because they have the same seniority.

The banker maximization problem at period $t+1$ implies that every banker at time t, taking as given the decisions of the other bankers, chooses how much loans to give and how much to borrow from households in order to maximizes her expected future wealth

$$\max_{\{b_t^i, d_t^i\}} \frac{\tilde{R}_{t+1}(b_t)b_t^i - R_t^D d_t^i - \gamma b_t^i}{\Pi_{t+1}}.$$

The maximization problem is subject to the banker's balance sheet

$$n_t^{F,i} + d_t^i \geq b_t^i. \tag{13}$$

Furthermore, because of imperfect competition, the maximization problem is subject to the loan demand (9).

After substituting the balance sheet constraint, the first order condition of the maximization problem is

$$\frac{\partial \tilde{R}_{t+1}}{\partial b_t} b_t^i + \tilde{R}_{t+1} - R_t^D - \gamma^i = 0$$

Note that because of imperfect competition, bankers' optimal choices depend on the impact of their decisions on the return they obtain from lending to entrepreneurs.

4.2 Rest of the model

The remainder of the model is a standard New Keynesian setup. Households choose their optimal consumption and labor supply within the period, and their optimal deposits across periods. Within the production sector, we distinguish between final goods producers, intermediate goods producers, and capital goods producers. Final goods producers are perfectly competitive. They create consumption bundles by combining intermediate goods using a constant-elasticity-of-substitution technology and sell them to the household sector and to capital producers. Intermediate goods producers use capital and labor to produce, with a Cobb-Douglas technology, the goods used as inputs by the final goods producers. They set prices subject to quadratic adjustment costs, which introduces a New Keynesian Phillips curve in our model. Capital goods producers buy the final good and convert it to capital, which they sell to the entrepreneurs. Finally, the model is closed by a central bank that chooses the policy rate following a monetary policy rule.

4.2.1 Entrepreneurs

The entrepreneurial sector is the general equilibrium version of the entrepreneurial sector described in Section 3. In particular, the price of capital q_t is time variant and it is determined by the equilibrium of entrepreneurial demand of capital and the supply of capital of capital good producers.

The gross return on capital is

$$R_t^E = \frac{r_t^K + (1-\delta)\, q_t}{q_{t-1}} \Pi_t.$$

The gross return on capital is given by the real rental rate on capital r_t^K plus the real capital gains net of depreciation $(1-\delta)\, q_t$ divided by the real price per unit of capital in period $t-1$. Finally the return is expressed in nominal terms and is multiplied by the inflation rate.

4.2.2 Households

Households are infinitely lived and have expected lifetime utility,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln c_t - \varphi \frac{l_t^{1+\eta}}{1+\eta} \right), \tag{14}$$

where $\beta \in (0,1)$ is the subjective discount factor, c_t is consumption, l_t is labor supply, $\varphi > 0$ is the relative weight on labor disutility and $\eta \geq 0$ is the inverse Frisch elasticity of labor supply. The household chooses paths for c_t, l_t and deposits d_t to maximize (14) subject to a sequence of budget constraints,

$$c_t + d_t + t_t \leq w_t l_t + \frac{R_t^D d_{t-1}}{\Pi_t} + \Xi_t^K + \sum_{i=1}^{N} \chi^F W_t^{F,i} + \chi^E W_t^E + \Xi_t^P, \tag{15}$$

where t_t are lump sum taxes (in terms of the final consumption good), w_t is the real wage, R_t^D is the gross interest rate on deposits paid in period t, Ξ_t^K and Ξ_t^P are capital goods producers' and intermediate goods producers' profits, respectively, which are redistributed to households in a lump sum fashion. The household's first order optimality conditions can be simplified to a labor supply equation, $w_t = \varphi l_t^\eta / \Lambda_t$, and a consumption Euler equation, $1 = \mathbb{E}_t\{\beta_{t,t+1} R_{t+1}^D / \Pi_{t+1}\}$, where $\beta_{t,t+s} = \beta^s \Lambda_{t+s}/\Lambda_t$ is the household's stochastic discount factor between t and $t+s$ and the Lagrange multiplier on the budget constraint (15), $\Lambda_t = 1/c_t$, captures the shadow value of household wealth in real terms.

4.2.3 Final goods producers

A final goods firm bundles the differentiated industry goods Y_{it}, with $i \in (0,1)$, taking as given their price P_{it}, and sells the output Y_t at the competitive price P_t. The optimization problem of the final goods firm is to choose the amount of inputs Y_{it} that maximizes profits $P_t Y_t - \int_0^1 Y_{it} P_{it} \mathrm{d}i$, subject to the production function $Y_t = (\int_0^1 Y_{it}^{(\varepsilon-1)/\varepsilon} \mathrm{d}i)^{\varepsilon/(\varepsilon-1)}$, where $\varepsilon > 1$ is the elasticity of substitution between intermediate goods. The resulting demand for intermediate good i is $Y_{it}^d = (P_{it}/P_t)^{-\varepsilon} Y_t$. The price of final output, which we interpret as the price index, is given by $P_t = (\int_0^1 P_{it}^{1-\varepsilon} \mathrm{d}i)^{1/(1-\varepsilon)}$. In a symmetric equilibrium, the price of a variety and the price index coincide, $P_t = P_{it}$.

4.2.4 Intermediate goods producers

Firms use capital and labor to produce intermediate goods according to a Cobb-Douglas production function. The assumption of constant returns to scale allows us to write the production function as an aggregate relationship. Each individual firm produces a differentiated good using $Y_{it} = A_t K_{it-1}^\alpha l_{it}^{1-\alpha}$, where $\alpha \in (0,1)$ is the capital share in production, A_t is aggregate technology, K_{it-1} is capital and l_{it} is labor. Intermediate goods firms choose factor inputs to maximize per-period profits given by $P_{it} Y_{it}/P_t - r_t^K K_{it-1} - w_t l_{it}$, where the real rental rate on capital r_t^K and the real wage w_t are taken as given, subject to the technological constraint and the demand constraint. The resulting demands for capital and labor are $w_t l_{it} = (1-\alpha)s_{it}Y_{it}$ and $r_t^K K_{it-1} = \alpha s_{it} Y_{it}$, respectively, where the Lagrange multiplier on the demand constraint, s_{it}, represents real marginal costs. By combining the two factor demands, we obtain an expression showing that real marginal costs are symmetric across producers,

$$s_t = \frac{w_t^{1-\alpha}(r_t^K)^\alpha}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\frac{1}{A_t}. \tag{16}$$

Firm i sets an optimal path for its product price P_{it} to maximize the present discounted value of future profits, subject to the demand constraint and to price adjustment costs,

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta_{t,t+s} \left[\frac{P_{it+s}Y_{it+s}^d}{P_{t+s}} - \frac{\kappa_p}{2}\left(\frac{P_{it+s}}{P_{it+s-1}} - 1\right)^2 Y_{it+s} + s_{t+s}\left(Y_{it+s} - Y_{it+s}^d\right) \right]. \tag{17}$$

Price adjustment costs are given by the second term in square brackets in (17); they depend on firm revenues and on last period's aggregate inflation rate. The parameter $\kappa_p > 0$ scales the price adjustment costs. Under symmetry, all firms produce the same amount of output, and the firm's price P_{it} equals the aggregate price level P_t, such that the price setting condition is

$$\kappa_p \Pi_t(\Pi_t - 1) = \varepsilon s_t - (\varepsilon - 1) + \kappa_p \mathbb{E}_t \left\{ \beta_{t,t+1}\Pi_{t+1}(\Pi_{t+1} - 1)\frac{Y_{t+1}}{Y_t} \right\}. \tag{18}$$

In (18), perfectly flexible prices are given by $\kappa_p \to 0$. Under symmetry across intermediate goods producers, profits (in real terms) are thus $\Xi_t^P = Y_t - r_t^K K_{t-1} - w_t l_t - 0.5 \cdot \kappa_p(\Pi_t - 1)^2 Y_t$.

4.2.5 Capital goods production

The representative capital-producing firm chooses a path for investment I_t to maximize profits given by $\mathbb{E}_t \sum_{s=0}^{\infty} \beta_{t,t+s}\left[q_{t+s}I_{t+s} - (1+g_{t+s})I_{t+s}\right]$. The term $g_t = 0.5 \cdot \kappa_I (I_t/I_{t-1} - 1)^2$ captures investment adjustment costs as in Christiano et al. (2014). Capital accumulation is defined as:

$$I_t = K_t - (1-\delta)K_{t-1}, \tag{19}$$

where $\delta \in (0,1)$ is the capital depreciation rate. The optimality condition for investment is given by:

$$1 = q_t - \frac{\kappa_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \kappa_I \left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}} + \mathbb{E}_t\left\{\beta_{t,t+1}\kappa_I\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right\}. \quad (20)$$

Capital producers' period-t profits, in real terms, are $\Xi_t^K = q_t I_t - (1+g_t)I_t$.

4.2.6 Monetary policy

I consider a monetary policy rule by which the central bank may adjust the policy rate in response to its own lag and inflation. The respective feedback coefficients are τ_R and τ_Π such that:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\tau_R}\left[\left(\frac{\Pi_t}{\Pi}\right)^{\tau_\Pi}\right]^{1-\tau_R}. \quad (21)$$

Since the deposit rate is risk-free, the policy rate and the deposit rate are identical, $R_t = R_t^D$.

4.2.7 Market clearing

Consumption goods produced must equal goods demanded by households, goods used for investment, resources lost when adjusting prices and investment, as well as resources lost in the recovery of funds associated with entrepreneur and due to intermediation costs,

$$Y_t = c_t + (1+g_t)I_t + \frac{\kappa_p}{2}\left(\Pi_t - 1\right)^2 Y_t + \mu^E G_t^E \frac{R_t^E q_{t-1}K_{t-1}}{\Pi_t} + \sum_{i=1}^{N}\gamma^i b_t^i. \quad (22)$$

Firms' labor demand must equal labor supply, $(1-\alpha)s_t Y_t / l_t = \varphi_t l_t^\eta / \Lambda_t$.

4.3 Symmetric equilibrium

A symmetric equilibrium is a set of allocations $\{l_t,\, K_t,\, I_t,\, C_t,\, Y_t,\, n_t^E,\, b_t,\, n_t^F,\, d_t\}_{t=0}^{\infty}$, prices $\{q_t,\, w_t,\, r_t^K, \Pi_t, s_t\}_{t=0}^{\infty}$ and rates of return $\{R_t^F,\, R_t^E,\, R_t^D,\, \tilde{R}_t\}_{t=0}^{\infty}$ for which given the monetary policy $\{R_t\}_{t=0}^{\infty}$ and shocks to entrepreneurial uncertainty $\{\varsigma_t\}_{t=0}^{\infty}$

- Entrepreneurs maximize expected future wealth
- Firms maximize profits,

- Banks maximize profits
- Households maximize utility
- All markets clear.

5 Results

This section presents the calibration of the model and discusses the results of the quantitative model with homogeneous bankers. Finally, I discuss the implications of heterogeneity in the banking sector.

5.1 Calibration

Table 1 display the value chosen for the parameters of the model. I calibrate the model to the period 2010Q1-2019Q4.

I choose the discount factor β to target the average yearly Federal Funds Effective Rate of 0.6%. The capital share in production α and the depreciation rate of capital are the same of Christiano et al. (2014). The fraction of resources lost because of entrepreneur defaults ξ is set to match the charge-off rate on business loans. The dividend payout of entrepreneurs χ^E and bankers χ^F are chosen to match the leverage of non-financial corporate business and a ratio of banker equity over assets of 8% respectively. The proportion of assets that can be diverted by entrepreneurs λ is chosen to match the ratio between non-financial corporate business loans and GDP. The intermediation cost γ is chosen to match the average markup of bankers used by Jamilov and Monacelli (2021). The number of bankers N is chosen to have a 3-bank asset concentration level of 33% that is close to the data (35.15%). The size of the uncertainty shock σ^ϵ, the autocorrelation of the uncertainty shock ρ and the inverse Frish labor elasticity η are obtained from Christiano et al. (2014). The parameter that determines the substitutability between intermediate goods ϵ is taken from Christensen and Dib (2008) to match a markup of 1.2. The price adjustment cost is from Smets and Wouters (2007) and the investment adjustment cost is from Carlstrom et al. (2014). I choose the weight on labor disutility to normalize labor supply to 1. For the coefficients of the Taylor rule, I use the conventional Taylor rule parameters as in Gertler and Karadi (2011). The coefficient of the Taylor rule for inflation is 1.5 and the smoothing parameter is set to 0.8.

Table 1: Calibration of the baseline model

Variable	Meaning	Value	Target
β	Discount factor	0.9985	FED Funds Rate
α	Capital share in production	0.4	Christiano et al. (2014)
δ	Depreciation rate capital	0.025	Christiano et al. (2014)
ξ	Entrepreneur bankruptcy cost	0.3588	Charge-Off Rate Business Loans
σ	Steady state uncertainty	0.3230	Delinquency Rate Business loans
χ^E	Dividend payout entrepreneurs	0.3644	Non-financial Corporate Business Leverage
χ^F	Dividend payout bankers	0.4429	Banker equity ratio = 8%
λ	Proportion divertible assets entrepreneurs	0.8110	Non-financial Corporate Business Loans/GDP
γ	Banker intermediation cost	0.0431	Markup Banks
N	Number of bankers	9	3-Bank asset concentration
ρ	Autocorrelation uncertainty shock	0.97	Christiano et al. (2014)
η	Inverse Frisch labor elasticity	1	Christiano et al. (2014)
ε	Substitutability between goods	6	Christensen and Dib (2008)
κ_p	Price adjustment cost	20	Smets and Wouters (2007)
κ_I	Investment adjustment cost	2.43	Carlstrom et al. (2014)
φ	Weight on labor disutility	0.562	Labor supply = $l = 1$
τ_Π	Coeff. TR for inflation	1.5	Gertler and Karadi (2011)
τ_R	Coeff. TR for lag policy rate	0.8	Gertler and Karadi (2011)

Notes. The table describes the calibration of the baseline model.

5.2 Dynamic implications of banking competition

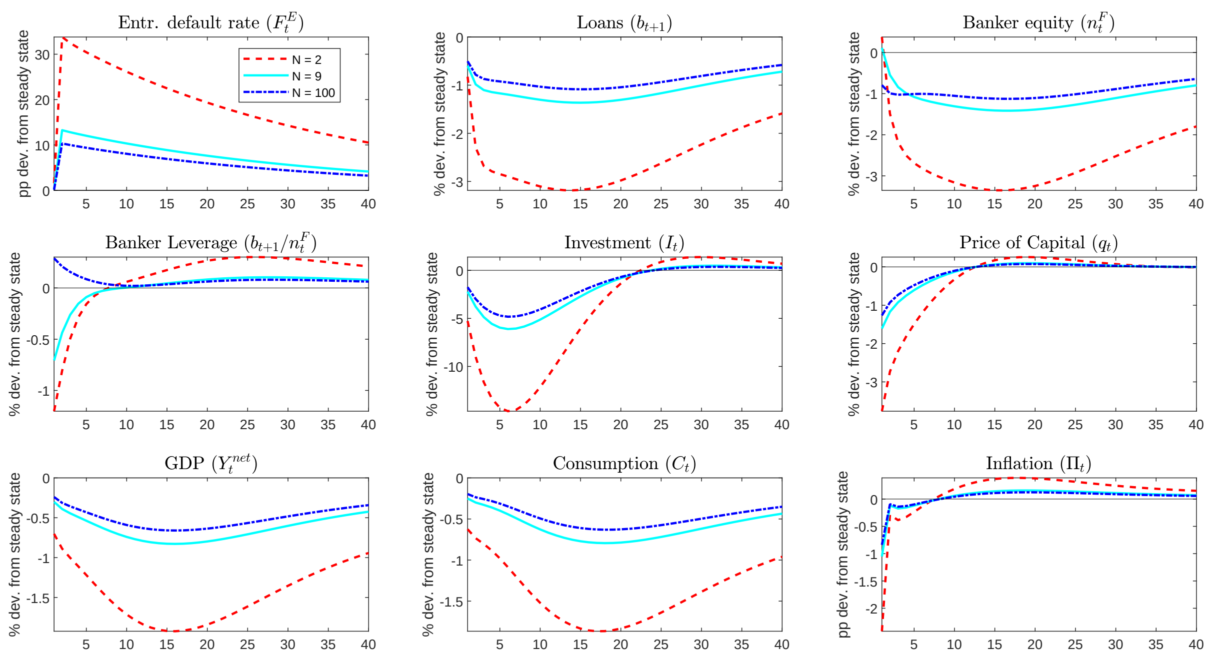
In this section I discuss the effects of banking competition competition on the transmission of uncertainty shocks. In particular, the uncertainty shock I study is a one-period ahead anticipated increase in uncertainty. I assume that at time $t1$ the economy is in the stationary equilibrium and at time t the agents know that uncertainty will increase in period $t + 1$.[11] Figure 3 describes the evolution of various variables of the model to an unanticipated one-standard deviation uncertainty shock for different levels of competition.

Consider first the light blue solid line, the one that corresponds to the baseline level of competition. In this case the number of bankers is 9. An uncertainty-shock increases the default rate of entrepreneurs. At time $t + 1$, the entrepreneurial default rate increases as a direct effect of the shock and then it slowly goes back to steady state. At time t, the entrepreneurial default rate increases because of the fall in the price of capital that decreases the return earned by the entrepreneurs. Because of the increase in entrepreneurial defaults,

[11] This assumption allows me to abstract from bank default since banks have time to adjust their portfolio of loans before the risk of their portfolio increases.

credit risk increases and bankers react decreasing their loan supply. Due to the spike in defaults, bankers equity falls. Because of the higher risk, bankers decrease the amount of loans they supply by more than the fall in banker equity and banker leverage falls. Because of the fall in loans, entrepreneurs have less resources to buy capital and investment falls as the price of capital. Due to the fall in investment, also GDP decreases. Finally, also inflation falls: we observe a demand-driven downturn as in Christiano et al. (2014).

Figure 3: Impulse responses to an uncertainty shock varying bank competition.



Notes. The graph shows the responses of several variables of the model to a one-standard deviation uncertainty shock for different levels of competition. The light blue solid lines represents the impulse responses of the baseline model, the red dashed lines correspond to the impulse responses of an economy with a very concentrated banking sector and the blue dot-dashed lines display the impulse responses of an economy with a banking sector that is close to be perfectly competitive.

Let us now turn to the dynamic responses given by the red dashed lines and by the blue dot-dashed lines. The red lines are originated by a model where the number of bankers is equal to 2 and there is lower banking competition. The blue lines are originated by a model where the number of bankers is equal to 100 and there is a higher level of banking competition.

As I showed in Section 3 less competitive banking sectors are characterized by a higher default rate of entrepreneurs and higher borrower fragility. Since borrowers are more fragile, the default rate of entrepreneurs increases by more after an uncertainty shock when banking competition is lower. Because of the stronger rise in credit risk, bankers cut their loan supply by more when competition is lower. This happens despite the fact that less competitive bankers have a lower pass-through. The stronger rise in the default rate of entrepreneurs implies larger losses for less competitive banking sectors and bankers equity falls by more when competition is lower. Banker leverage falls by more in less competitive banking sector because of the strong cut in loan supply that is larger than the fall in banker equity.[12] Due to the stronger rise in entrepreneurial defaults, investment and GDP fall by a larger extend when competition is lower. Finally, because of the stronger recession, consumption and inflation fall by more.

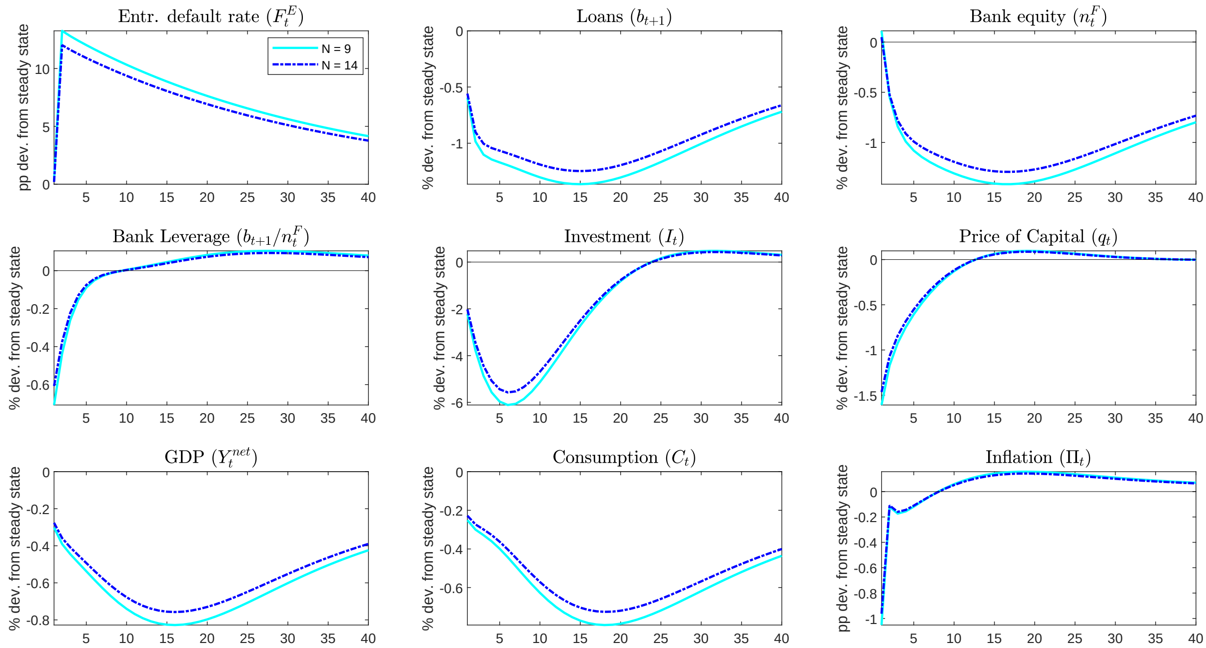
5.3 Implications of the recent fall in banking competition

In this section I study the business cycle implications of the recent fall in banking competition. Furthermore, I analyze the policy implications of the recent fall in banking competition for a central bank with objective of stabilize inflation.

Figure 4 compares the baseline impulse responses (light blue dashed line) with the impulse responses of a variant of the model in which the number of banks is set to match the share of assets held by the three largest banks in 2000 (blue dot-dashed lines). The figure implies that the US economy in the period between 2010 and 2019 would be less affected by an uncertainty shock if its level of banking concentration would be as the one observed in 2000. A standard deviation uncertainty shock implies that the rise in the default rate of entrepreneurs would be more than a percentage point lower at its peak if the level of banking concentration was the one observed in 2000. Similarly, the fall in GDP would be 9% smaller if banking concentration was the one observed at the beginning of the century. Finally, a higher level of competition would imply a fall in inflation that is about 0.1 percentage points lower compared to the baseline model.

[12]Note that the cut in loan supply in a highly competitive banking sector is lower than the fall in equity and bank leverage increases after an uncertainty shock.

Figure 4: Impulse responses baseline model and model with bank concentration observed in 2000.



Notes. The graph shows the responses of several variables of the model to a one-standard deviation uncertainty shock for different levels of competition. The light blue solid lines represents the impulse responses of the baseline model and the blue dot-dashed lines display the impulse responses of an economy in which the number of banks is set to match the share of assets held by the three largest banks in the US in 2000.

5.4 What is the effect of banker heterogeneity?

In this section I investigate the effects of introducing a heterogeneous banking sector. I introduce heterogeneity in the banking sector assuming that every banker has a different intermediation cost γ^i. In particular, I assume that every banker draws a realization of γ^i when it is born and passes it to the next generation. Because of different intermediation costs, bankers provide different amount of loans and accumulate equity at a different speed.

As in Li (2019), I assume that bankers draw γ^i from a reverse bounded Pareto distribution in order to produce a distribution of bankers with few large bankers and many small bankers. More details on the distribution can be found in Appendix D.

The reverse bounded Pareto distribution is characterized by three parameters: the lower and upper bounds of the distribution and the shape parameter. I assume that the shape parameter is equal to 0.1 as in Li (2019) and I calibrate the lower and upper bounds such that the mean of the distribution of γ^i is equal to 0.0431 as in the baseline calibration and the standard deviation of the distribution of γ^i is equal to σ^γ. The assumptions imply that heterogeneity in the banking sector is an increasing function of σ^γ. In fact, the higher σ^γ, the larger the differences in productivity across banks.

In the rest of the section, in order to show the steady state and dynamic effects of heterogeneity, I solve the model 1000 times each time drawing a realization of γ^i for every generation of bankers. In every repetition, the bankers are sorted by their γ^i, from the banker that has the lowest draw (the most productive) to the one with the largest draw (the least productive). After solving the model I take the averages of the steady state variables and impulse responses across replications.

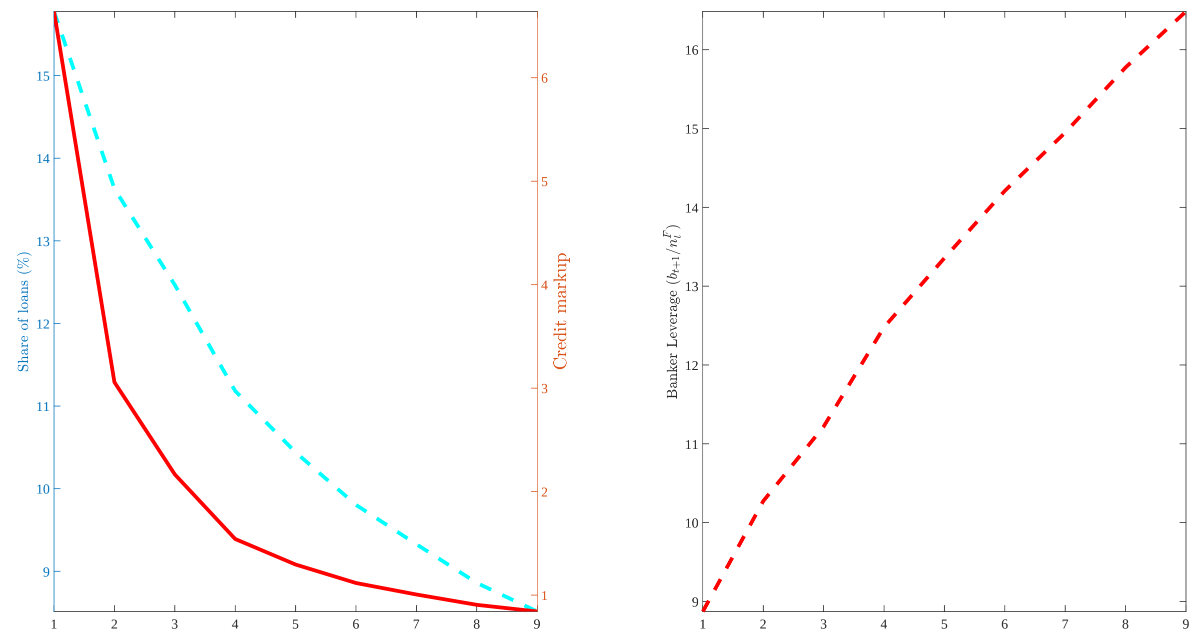
Figure 5 shows the steady state effects of banker heterogeneity on banker variables. In this figure $\sigma^\gamma = 0.015$. The left panel plots the average share of assets (light blue dashed line) and the average markup (red solid line) of the nine bankers in the model. Bankers are sorted from the most productive to the least productive.

As the left panel of Figure 5 shows, the bankers with a low intermediation cost have a larger share of the credit market than those with a high intermediation cost. Because of a lower intermediation cost, more productive bankers decide to supply more loans than less productive bankers. However, since more productive bankers know that they are more productive than the other bankers, they obtain a larger market power and they charge a higher markup.[13] Due to the larger market power and the lower intermediation cost larger bankers make higher profits than smaller bankers and accumulate more equity. Equity accumulation explains why smaller bankers have a higher leverage as shown in the right panel of Figure 5.

Figure 6 shows the average impulse responses of the model with heterogeneous bankers for different levels of σ^γ. As it can be seen, banking heterogeneity does not have a meaningful impact on the responses of aggregate variables. Their responses are similar to the responses of the model with homogeneous bankers. The main differences are on the responses of banker equity and banker leverage. The response of banker equity is stronger when there is bank heterogeneity. However, the effect of banker heterogeneity on the responses is non-linear. This result is due to the different responses of large and small banks and by their

[13]Credit markup is defined as $\frac{(1-F^E)(R^F-1)}{R^D-1+\gamma^i} - 1$ as in Corbae and D'Erasmo (2021).

Figure 5: Steady state bankers variables

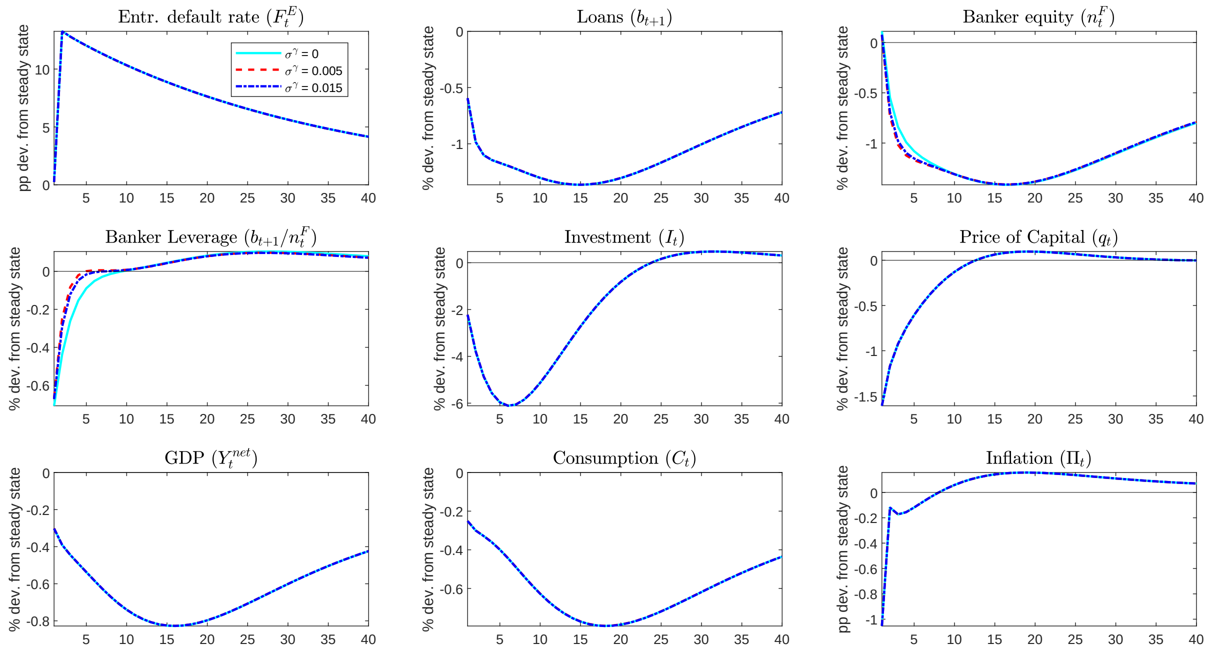


Notes. The graph shows the model generated steady state distributions of bankers when the amount of bankers and the mean of the distribution of γ are set to match the values of the baseline calibration. The standard deviation of the distribution of γ is chosen to be 0.015. The figures are obtained simulating the economy 1000 times and taking the average across repetitions. The x-axis shows each of the 9 banks sorted from the most productive (lowest γ) to the least productive (highest γ). The left figure shows how the share of loans and the credit markup vary with bank productivity. The right figure shows how bank leverage changes with bank productivity.

relative share of loans. As shown by Figure 7, equity of smaller bankers respond by more to uncertainty shocks, however their relative size changes with σ^γ. When σ^γ is small, smaller bankers hold a large share of loans and the response of their equity to uncertainty shocks has a larger importance on the response of aggregate equity, however, when σ^γ is large, the share of assets held by small banks is small and the response of equity of the larger bankers becomes more important. The effect of σ^γ on the response of aggregate banker equity determines also how the response of bankers leverage changes with σ^γ.

Figure 7 shows how bankers respond to the uncertainty shock depending on their asset size. In this figure, $\sigma^\gamma = 0.015$. The impact of an uncertainty shock is stronger on smaller bankers. Smaller bankers cut their loan supply by more and experience a stronger fall in equity. In fact, smaller bankers are characterized by a higher pass-through of shocks to borrowers. Smaller banks have a lower market power and have to pass the shock through

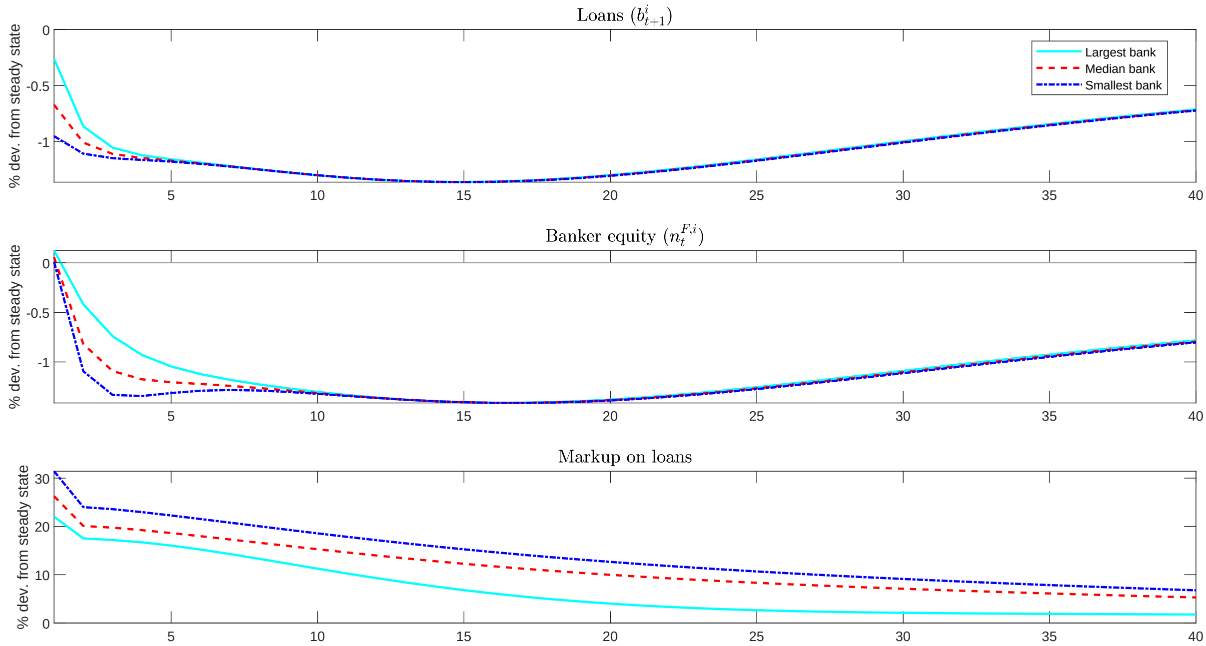
Figure 6: Impulse response functions to an uncertainty shock - Heterogeneous bankers



Notes. The graph shows the responses of several variables of the model to a one-standard deviation uncertainty shock for different levels of σ^γ. The light blue solid lines represents the impulse responses of the baseline model, the red dashed lines display the impulse responses of an economy in which σ^γ is 0.005 and the the blue dot-dashed lines correspond to the impulse responses of an economy in which σ^γ is 0.015.

to borrowers to a larger extent. This implies that their markup increases by more.

Figure 7: Impulse response functions - Bankers variables



Notes. The graph shows the responses of several banker variables of the model to a one-standard deviation uncertainty shock when σ^γ is 0.005. The red dashed lines represents the impulse responses of the most productive banker, the light blue solid lines display the impulse responses of the banker with median productivity and the the blue dot-dashed lines correspond to the impulse responses of the least productive banker.

6 Conclusion

According to empirical and theoretical literature on uncertainty, uncertainty shocks are drivers of the business cycle. Motivated by the recent fall in banking competition I study the effects of lower competition in the banking sector on the propagation of uncertainty shocks.

I empirically document that an increase in uncertainty leads to a stronger fall in output growth when banking competition is lower. In order to explain this fact I build a calibrated New Keynesian dynamic stochastic general equilibrium model that incorporates financial frictions and imperfect competition in the banking sector. The model can explain the empirical result. When banking competition is lower, bankers charge higher borrowing rates to entrepreneurs. Entrepreneurial risk-taking increases with the borrowing rate amplifying the negative effects of uncertainty shocks on entrepreneurs. Since entrepreneurs take more risk when banking competition is lower, an uncertainty shock increases entrepreneurial defaults by more. The larger rise in borrower defaults leads to a stronger increase in credit risk and a stronger reduction in bankers loan supply. Due to the larger fall in loans, investment and output fall by more after an uncertainty shock in less competitive banking sectors. Lower banking competition exacerbates the negative effects of uncertainty shocks.

The result holds also in an economy with heterogeneous bankers. The response of aggregate variables to an uncertainty shock with heterogeneous bankers is similar to the case with homogeneous bankers. However, with heterogeneous bankers, smaller bankers are more affected by the uncertainty shock because of the higher pass-through.

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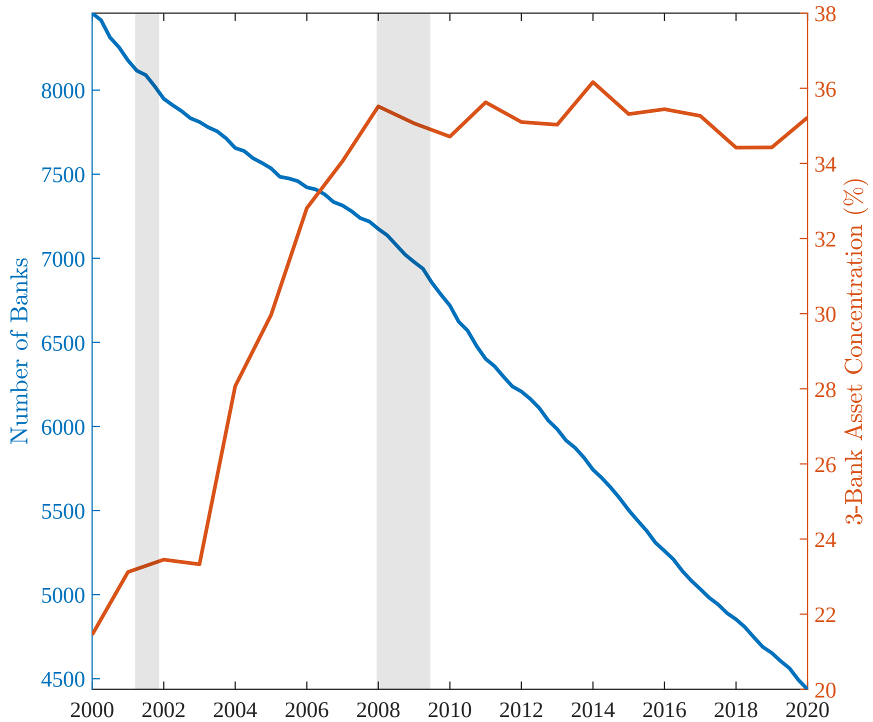
A Additional Data

A.1 Evolution of Banking Competition

Figure 8 shows the evolution of the number of commercial banks and the 3-Bank asset concentration for United States. The number of banks is retrieved from FRED, the 3-Bank asset concentration is obtained from World Bank. The number of banks has been decreasing since 2000, while the 3-Bank asset concentration has been increasing. This suggests that banking competition has been falling in the recent years.

Figure 8: Number of banks and bankers concentration



Notes. Sample period: January 2000 to January 2020. The number of banks is measured as the number of commercial banks (FRED). Bank concentration is measured as the assets of three largest commercial banks as a share of total commercial banking assets (World Bank).

B Additional results empirical motivation

B.1 Data Description

In this subsection I give further information about the dataset used in the empirical analysis. The disaster shocks are obtained from Baker et al. (forthcoming) and are available for 59 countries from 1970Q1 to 2020Q1. However, information about banking concentration is available only starting from 2000. Therefore, the sample period reduces to the period from 2000Q1 to 2020Q1.

The reduction in the sample period implies that for some countries no shocks occurred between 2000Q1 and 2020Q1. I drop these countries.

Finally, I drop countries whose GDP is available only at a yearly frequency. Table 2 lists the countries used in the analysis.

Table 2: Countries

Asia & Pacific	Europe & North America	LatAm & Caribbean	MENA	SSAF
Australia	Austria	Brazil	Israel	South Africa
China	Belgium	Chile	Turkey	
India	Canada	Colombia		
Indonesia	Czech Republic	Ecuador		
Japan	Denmark	Mexico		
New Zealand	Finland			
Philippines	France			
Russian Federation	Germany			
Singapore	Greece			
South Korea	Hungary			
Thailand	Ireland			
	Italy			
	Luxembourg			
	Netherlands			
	Norway			
	Poland			
	Portugal			
	Romania			
	Serbia			
	Spain			
	Sweden			
	Switzerland			
	Ukraine			
	United Kingdom			
	United States			

Notes. List of the countries used in the empirical analysis.

Table 3 shows the descriptive statistics of the dataset. Compared to Baker et al. (forthcoming), due to sample availability, the number of observations is lower and less disaster shocks are available.

The disaster shocks are defined as follows:

<u>Natural Disasters</u>: Extreme weather events such as, droughts, earthquakes, insect infes-

Table 3: Descriptive statistics

	Obs.	Mean	Median	St.dev.	min	max
Real GDP growth	2880	0.65	0.67	1.16	-6.48	9.38
3-Bank Concentration (interpolated)	2825	64.89	65.74	19.67	21.45	99.94
3-Bank Concentration	2870	65.13	66.36	19.96	21.45	99.94
5-Bank Concentration (interpolated)	2838	77.32	81.27	17.81	28.12	100.00
Return	2880	0.03	0.14	1.07	-5.35	4.96
Volatility	2880	-14.12	-14.13	1.40	-18.77	-9.42
Nat. Disasters	2880	0.02	0.00	0.08	0.00	1.44
Coups	2880	0.00	0.00	0.01	0.00	0.33
Revolutions	2880	0.00	0.00	0.01	0.00	0.27
Terror attacks	2880	0.00	0.00	0.05	0.00	1.22

Notes. Descriptive statistics of the dataset used in the empirical analysis. The sample period is 2000Q1-2020Q1.

tations, pandemics, floods, extreme temperatures, avalanches, landslides, storms, volcanoes, fires, and hurricanes.

Terrorist Attacks: Bombings and other non-state-sponsored attacks.

Coups: Military action which results in the seizure of executive authority taken by an opposition group from within the government.

Revolutions: A violent uprising or revolution seeking to replace the government or substantially change the governance of a given region.

For every category by country and quarter, the disaster shock variables are set to one if at least a disaster shock of the specified category has occurred. Finally, the shocks are weighted by the increase in media coverage 15 days after the event compared to 15 days before the event.

The increase in media coverage is defined as the percentage increase in the number of articles written in the 15 days after the event compared to the 15 days before the event. The articles are limited to those that are published in English-language newspapers based in the United States.

B.2 Baseline regression results

Tables 4-6 show the results of the baseline specification described in Section 2.2.2. The level of banking concentration is divided by its standard deviation. The coefficients for banking concentration and its interaction with first and second moments correspond to the effect of a standard deviation increase in competition.

The coefficient on the interaction between concentration and volatility is negative implying that an increase in uncertainty has a stronger negative impact on real GDP growth when concentration is higher.

Table 4: Baseline regression results horizons 0-8

	$h = 0$	$h = 1$	$h = 2$	$h = 3$	$h = 4$	$h = 5$	$h = 6$	$h = 7$	$h = 8$
Return_t	-0.211	-1.224	-2.781	-5.201*	-7.320**	-8.783**	-9.339**	-9.794**	-10.265**
	(0.409)	(1.048)	(1.749)	(2.664)	(3.532)	(4.156)	(4.362)	(4.403)	(4.806)
Volatility_t	-0.590	-2.067	-3.519	-5.326	-6.640	-7.590	-8.035	-8.261	-8.638
	(0.504)	(1.260)	(2.302)	(3.521)	(4.784)	(5.876)	(6.295)	(6.387)	(7.222)
$\text{Conc}_t \times \text{Ret}_t$	0.003	0.001	0.008	0.040	0.070	0.102	0.120	0.134	0.137
	(0.023)	(0.056)	(0.091)	(0.137)	(0.183)	(0.218)	(0.236)	(0.248)	(0.276)
$\text{Conc}_t \times \text{Vol}_t$	-0.088	-0.259*	-0.446*	-0.619*	-0.728	-0.784	-0.785	-0.820	-0.923
	(0.055)	(0.139)	(0.241)	(0.356)	(0.462)	(0.542)	(0.561)	(0.560)	(0.605)
Concentration_t	0.006	0.019	0.034	0.055	0.064	0.074	0.077	0.077	0.090
	(0.009)	(0.025)	(0.044)	(0.064)	(0.077)	(0.085)	(0.088)	(0.088)	(0.098)
Observations	2825	2825	2825	2825	2808	2793	2778	2738	2693
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
F-test R_t	96.175	96.175	96.175	96.175	94.656	96.823	100.387	101.526	103.684
F-test V_t	59.968	59.968	59.968	59.968	65.346	67.983	67.762	65.499	63.353
F-test $\text{C}_t \times \text{R}_t$	103.755	103.755	103.755	103.755	101.058	122.712	134.113	180.440	188.139
F-test $\text{C}_t \times \text{V}_t$	26.539	26.539	26.539	26.539	28.166	32.233	35.529	39.476	40.896
Hansen p-value	0.639	0.786	0.810	0.832	0.776	0.740	0.654	0.614	0.586

Notes: The dependent variable is GDP growth at horizon h. The concentration series is scaled to have unit standard deviation over the regression sample. Robust standard errors clustered by country in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

B.3 Robustness tests

This subsection shows the results of the robustness tests. Section B.3.1 shows the results of the first robustness test. In this test I keep the level of banking concentration constant within a year instead of interpolating the level of concentration.

Section B.3.2 shows the result of the second robustness test. In this test the level of banking competition is proxied by the 5-Bank asset concentration level.

B.3.1 Constant concentration within a year

In this section I show the results of the first robustness test. Here I keep the level of banking concentration constant within a year instead of interpolating the level of concentration.

Table 5: Baseline regression results horizons 9-16

	$h=9$	$h=10$	$h=11$	$h=12$	$h=13$	$h=14$	$h=15$	$h=16$
Return$_t$	-9.960**	-9.697**	-8.776**	-8.361***	-8.458***	-8.259***	-6.539***	-6.750***
	(4.811)	(4.683)	(3.851)	(3.138)	(2.608)	(2.181)	(2.423)	(1.920)
Volatility$_t$	-8.874	-8.939	-9.033	-9.257	-9.430	-8.248	-6.888*	-5.703
	(7.407)	(7.229)	(6.359)	(6.219)	(6.108)	(5.277)	(3.550)	(3.765)
Conc$_t$ × Ret $_t$	0.130	0.114	0.088	0.034	-0.017	-0.005	0.230	0.127
	(0.283)	(0.286)	(0.287)	(0.278)	(0.278)	(0.251)	(0.353)	(0.226)
Conc$_t$ × Vol $_t$	-0.991*	-1.071*	-1.105**	-1.115**	-1.175**	-1.117**	-0.983***	-0.902***
	(0.598)	(0.573)	(0.504)	(0.497)	(0.512)	(0.472)	(0.321)	(0.333)
Concentration$_t$	0.101	0.117	0.123	0.093	0.077	0.063	0.072	0.082
	(0.105)	(0.115)	(0.114)	(0.107)	(0.108)	(0.105)	(0.103)	(0.094)
Observations	2650	2607	2564	2522	2482	2442	2402	2362
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
F-test R$_t$	104.851	108.661	117.782	128.437	138.894	151.171	158.120	161.612
F-test V$_t$	61.008	60.228	64.003	67.662	71.068	72.427	30.747	29.774
F-test C$_t$ × R$_t$	182.842	177.607	157.701	135.295	127.059	111.826	103.393	93.042
F-test C$_t$ × V$_t$	40.915	38.646	36.948	37.148	34.548	29.246	23.346	19.699
Hansen p-value	0.613	0.611	0.670	0.705	0.742	0.771	0.828	0.830

Notes: The dependent variable is GDP growth at horizon h. The concentration series is scaled to have unit standard deviation over the regression sample. Robust standard errors clustered by country in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

Figure 9 shows the effect of an exogenous 1% increase in volatility on real output growth for different levels of banking concentration. The blue line represents the impulse response of output growth when concentration is at the country average. The blue dashed lines are the 90% confidence interval. An exogenous increase in volatility has a not significant negative effect on output growth.

The yellow line represents the impulse response of output growth when concentration is one standard deviation above the country average. The yellow dashed lines are the %90 confidence interval. In this case the fall in output is stronger and significant for a some periods.

Figure 12 shows the difference in the responses between the average concentration specification and the high concentration specification. The graph shows that the fall in output growth is significantly higher when banking concentration is higher.

Tables 7-9 show the results of the specification described in this section. The level of banking concentration is divided by its standard deviation. The coefficients for banking

Table 6: Baseline regression results horizons 17-24

	$h = 17$	$h = 18$	$h = 19$	$h = 20$	$h = 21$	$h = 22$	$h = 23$	$h = 24$
Return_t	-6.316***	-6.030***	-5.253***	-4.604**	-3.242	-2.249	-1.764	-1.023
	(1.880)	(2.055)	(2.036)	(2.067)	(2.483)	(2.550)	(2.402)	(2.095)
Volatility_t	-4.625	-4.002	-3.637	-3.346	-1.707	-0.378	0.746	1.701
	(3.670)	(3.778)	(3.671)	(3.714)	(3.951)	(4.053)	(3.624)	(3.370)
$\text{Conc}_t \times \text{Ret}_t$	0.059	-0.029	-0.058	-0.082	-0.077	-0.079	-0.082	-0.086
	(0.164)	(0.155)	(0.145)	(0.147)	(0.145)	(0.144)	(0.137)	(0.136)
$\text{Conc}_t \times \text{Vol}_t$	-0.819**	-0.790**	-0.771**	-0.766**	-0.669*	-0.596*	-0.545*	-0.486
	(0.329)	(0.339)	(0.338)	(0.346)	(0.352)	(0.352)	(0.327)	(0.311)
Concentration_t	0.085	0.085	0.087	0.085	0.067	0.053	0.048	0.050
	(0.086)	(0.082)	(0.080)	(0.081)	(0.079)	(0.078)	(0.079)	(0.079)
Observations	2322	2282	2242	2201	2160	2119	2078	2036
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
F-test R_t	166.383	170.238	171.061	169.984	166.121	160.836	148.557	137.309
F-test V_t	30.091	34.002	27.210	21.700	17.505	15.050	14.888	14.978
F-test $\text{C}_t \times \text{R}_t$	87.732	83.959	79.414	78.957	73.341	63.265	38.334	36.179
F-test $\text{C}_t \times \text{V}_t$	15.875	11.324	8.577	7.614	7.482	8.200	9.443	10.200
Hansen p-value	0.811	0.792	0.785	0.784	0.696	0.579	0.528	0.517

Notes: The dependent variable is GDP growth at horizon h. The concentration series is scaled to have unit standard deviation over the regression sample. Robust standard errors clustered by country in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

concentration and its interaction with first and second moments correspond to the effect of a standard deviation increase in competition.

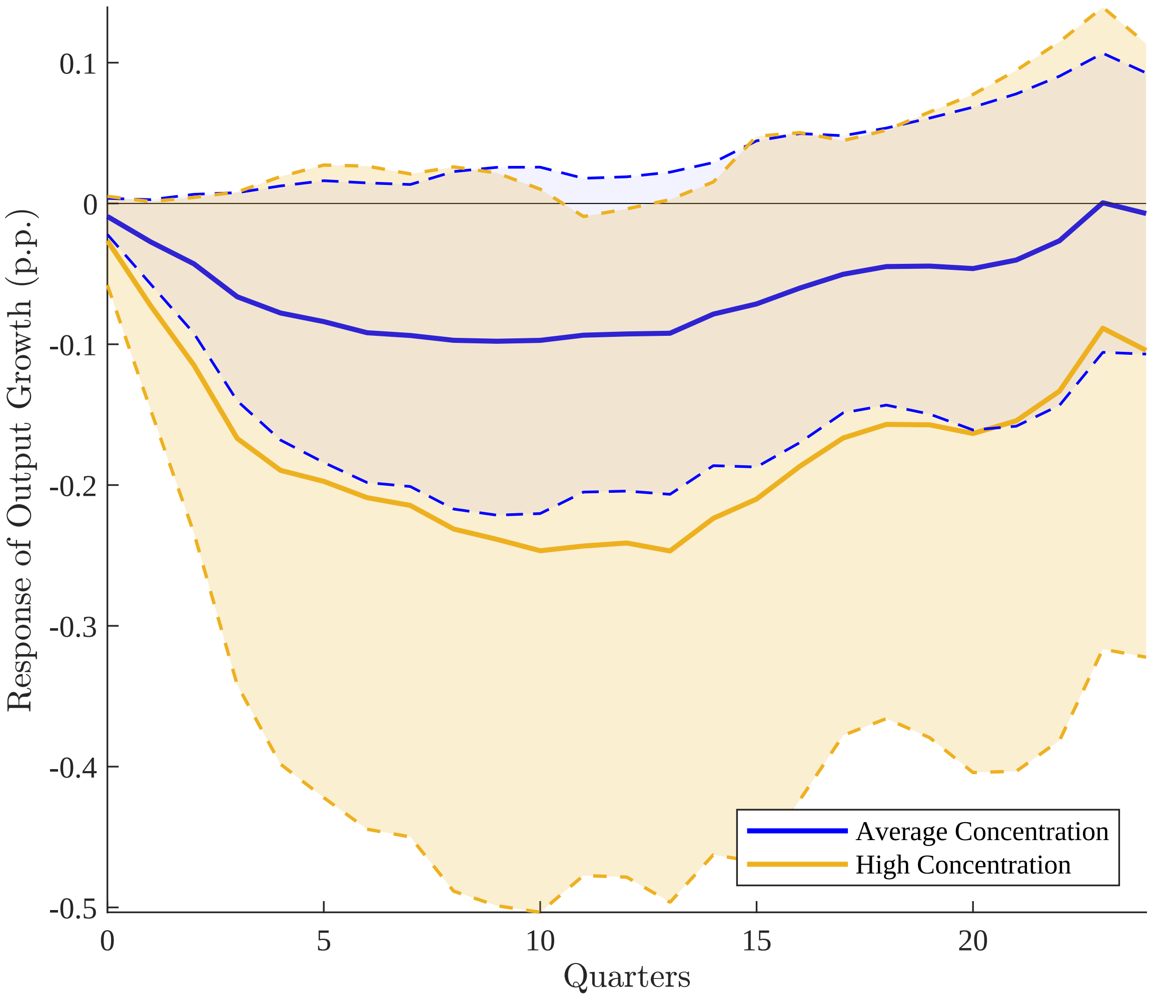
The coefficient on the interaction between concentration and volatility is negative implying that an increase in uncertainty has a stronger negative impact on real GDP growth when concentration is higher.

B.3.2 5-Bank asset concentration

In this section I show the results of the second robustness test. Here I proxy the level of banking competition with the 5-Bank asset concentration level, the share of assets held by the five largest banks.

Figure 11 shows the effect of an exogenous 1% increase in volatility on real output growth for different levels of banking concentration. The blue line represents the impulse response of output growth when concentration is at the country average. The blue dashed lines are the 90% confidence interval. An exogenous increase in volatility has a not significant negative

Figure 9: Response of output growth with constant concentration within a year.



Notes. The graph shows the responses of output growth to an increase of volatility of 1% from its country mean for different levels of banking concentration. In the response with high banking concentration, banking concentration is 1 standard deviation larger than the country average. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1. The level of banking concentration is constant within a year.

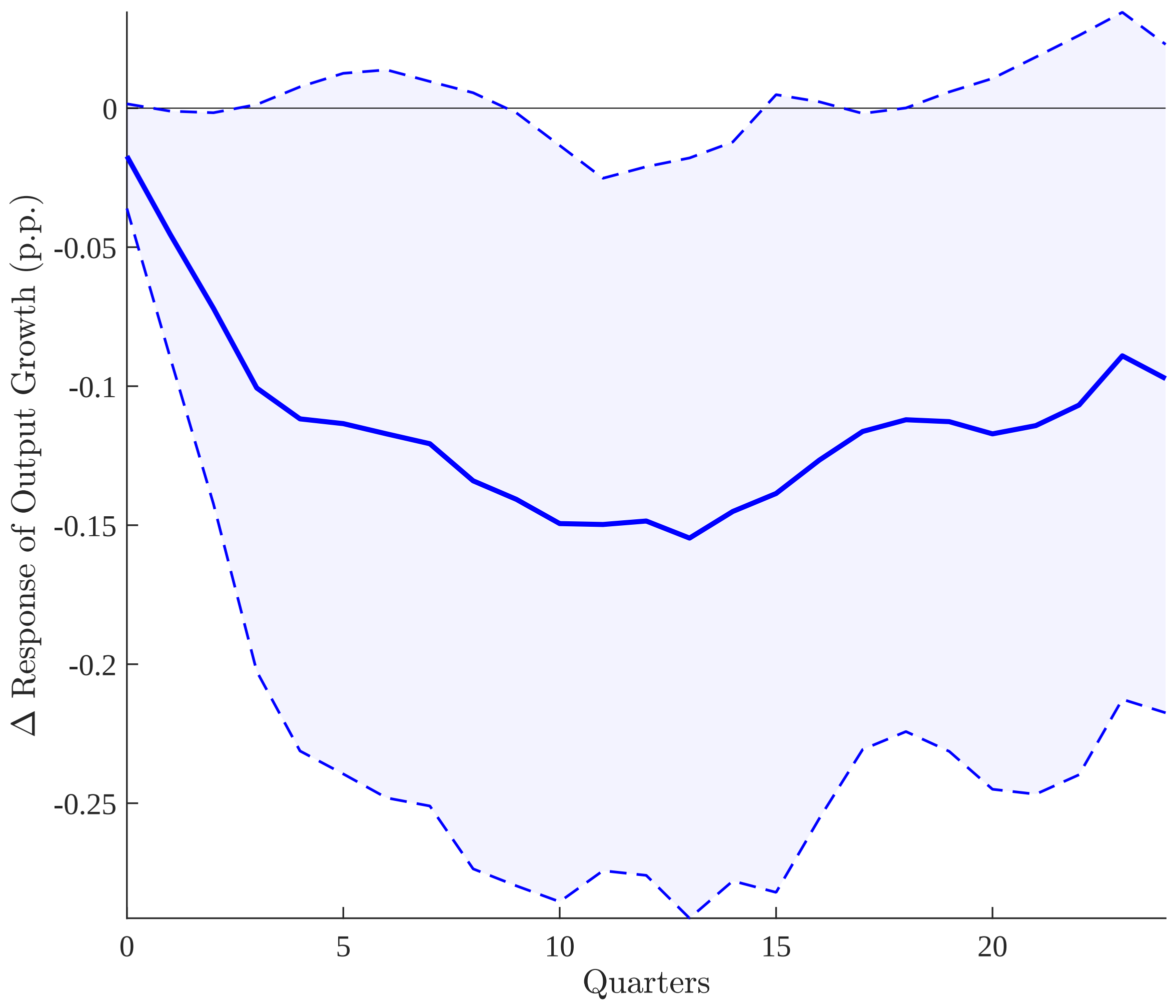
effect on output growth.

The yellow line represents the impulse response of output growth when concentration is one standard deviation above the country average. The yellow dashed lines are the %90 confidence interval. In this case the fall in output is stronger and significant for a some periods.

Figure 12 shows the difference in the responses between the average concentration specification and the high concentration specification. The graph shows that the fall in output growth is significantly higher when banking concentration is higher.

Tables 10-12 show the results of the specification described in this section. The level

Figure 10: Effect of concentration with constant concentration within a year.



Notes. The graph shows the difference between the two specifications showed in Figure 9. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1.

of banking concentration is divided by its standard deviation. The coefficients for banking concentration and its interaction with first and second moments correspond to the effect of a standard deviation increase in competition.

The coefficient on the interaction between concentration and volatility is negative implying that an increase in uncertainty has a stronger negative impact on real GDP growth when concentration is higher.

Table 7: Robustness regression with constant concentration within a year, horizons 0-8

	$h=0$	$h=1$	$h=2$	$h=3$	$h=4$	$h=5$	$h=6$	$h=7$	$h=8$
Return_t	-0.488	-1.791	-3.556*	-6.355**	-8.453**	-9.760**	-10.436***	-10.858***	-11.371**
	(0.494)	(1.167)	(1.897)	(2.844)	(3.526)	(3.879)	(4.028)	(4.027)	(4.515)
Volatility_t	-0.919	-2.728	-4.289	-6.626	-7.780	-8.391	-9.181	-9.378	-9.719
	(0.780)	(1.824)	(3.003)	(4.496)	(5.487)	(6.085)	(6.469)	(6.521)	(7.286)
$\text{Conc}_t \times \text{Ret}_t$	-0.125	-0.269	-0.139	0.233	0.772	1.433	1.608	1.920	1.982
	(0.473)	(1.070)	(1.653)	(2.334)	(2.872)	(3.171)	(3.420)	(3.569)	(3.961)
$\text{Conc}_t \times \text{Vol}_t$	-1.727	-4.538*	-7.199*	-10.064	-11.177	-11.347	-11.715	-12.070	-13.404
	(1.144)	(2.693)	(4.277)	(6.197)	(7.262)	(7.661)	(7.960)	(7.922)	(8.486)
Concentration_t	0.041	0.155	0.303	0.539	0.629	0.812	0.914	0.904	1.019
	(0.182)	(0.470)	(0.749)	(1.087)	(1.270)	(1.351)	(1.436)	(1.455)	(1.591)
Observations	2825	2825	2825	2825	2808	2793	2778	2738	2693
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
F-test R_t	99.422	99.422	99.422	99.422	97.623	98.511	100.529	101.009	103.919
F-test V_t	63.611	63.611	63.611	63.611	67.621	69.976	69.387	67.266	64.366
F-test $\text{C}_t \times \text{R}_t$	31.149	31.149	31.149	31.149	29.377	33.542	36.544	45.740	45.915
F-test $\text{C}_t \times \text{V}_t$	37.407	37.407	37.407	37.407	42.314	47.031	50.792	57.492	59.812
Hansen p-value	0.898	0.922	0.920	0.913	0.800	0.702	0.682	0.674	0.610

Notes: The dependent variable is GDP growth at horizon h. The concentration series is scaled to have unit standard deviation over the regression sample. The sample period is 2000Q1-2020Q1. The level of banking concentration is constant within a year. Robust standard errors clustered by country in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

Table 8: Robustness regression with constant concentration within a year, horizons 9-16

	$h = 9$	$h = 10$	$h = 11$	$h = 12$	$h = 13$	$h = 14$	$h = 15$	$h = 16$
Return_t	-10.882**	-10.527**	-9.486**	-8.885***	-8.816***	-8.636***	-8.112***	-7.835***
	(4.614)	(4.614)	(3.912)	(3.357)	(2.932)	(2.597)	(2.514)	(2.434)
Volatility_t	-9.786	-9.723	-9.356	-9.267	-9.217	-7.858	-7.134	-6.009
	(7.510)	(7.479)	(6.775)	(6.786)	(6.955)	(6.545)	(7.042)	(6.672)
$\text{Conc}_t \times \text{Ret}_t$	1.851	1.650	1.452	0.899	0.462	0.924	3.335	2.011
	(4.092)	(4.208)	(4.161)	(4.163)	(4.404)	(4.310)	(4.470)	(3.560)
$\text{Conc}_t \times \text{Vol}_t$	-14.071*	-14.943*	-14.975**	-14.850*	-15.464*	-14.506*	-13.860	-12.659
	(8.452)	(8.269)	(7.571)	(7.748)	(8.313)	(8.080)	(8.722)	(7.834)
Concentration_t	1.086	1.165	1.111	0.596	0.376	0.241	0.415	0.572
	(1.643)	(1.742)	(1.722)	(1.602)	(1.615)	(1.570)	(1.639)	(1.456)
Observations	2650	2607	2564	2522	2482	2442	2402	2362
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
F-test R_t	107.079	111.158	119.986	129.943	139.572	150.852	157.388	160.031
F-test V_t	60.639	59.345	63.776	68.991	75.253	78.452	33.948	33.818
F-test $\text{C}_t \times \text{R}_t$	44.162	42.954	37.349	30.983	27.382	23.133	20.760	18.620
F-test $\text{C}_t \times \text{V}_t$	57.614	52.154	49.522	57.719	59.120	56.861	45.117	42.719
Hansen p-value	0.580	0.568	0.583	0.597	0.638	0.659	0.730	0.684

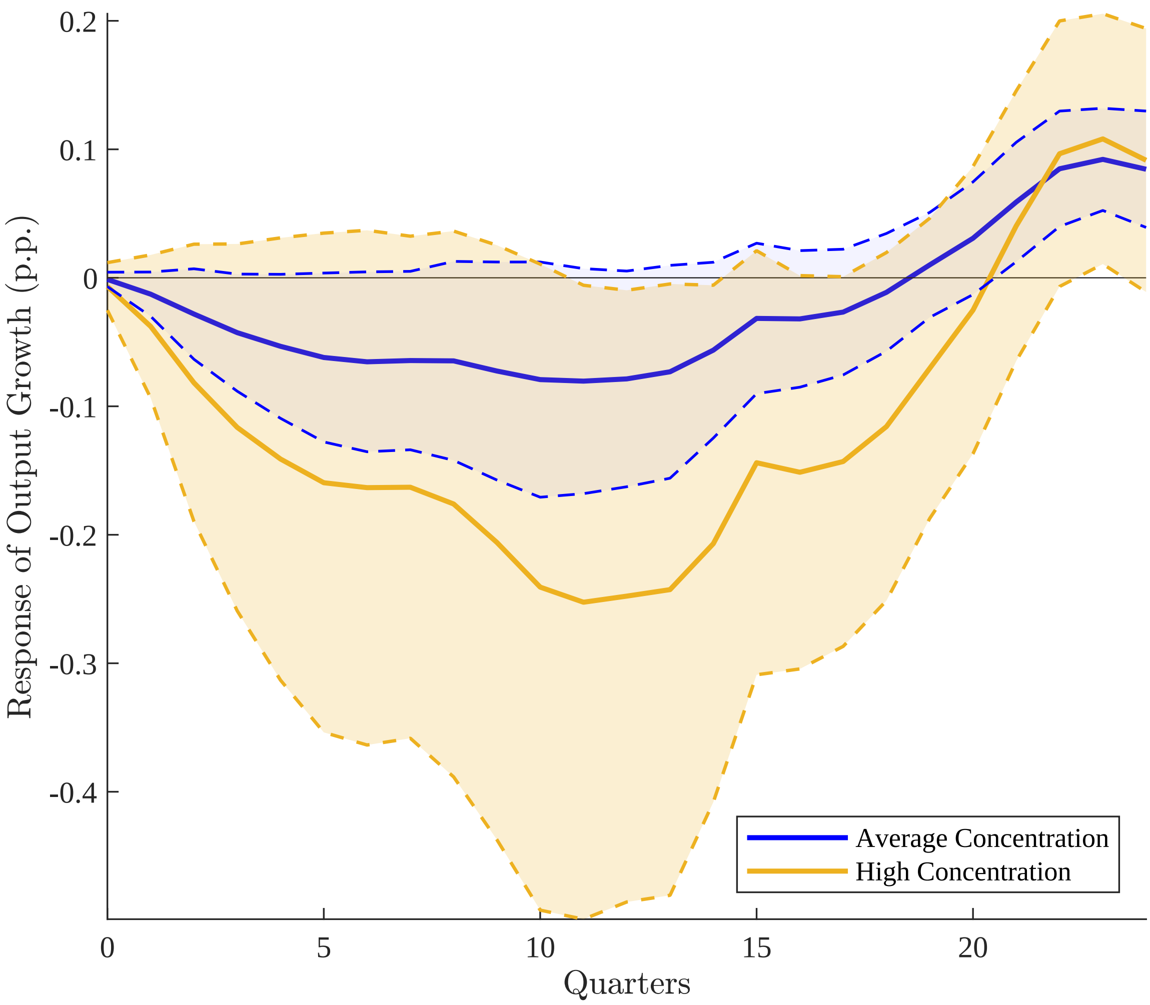
Notes: The dependent variable is GDP growth at horizon h. The concentration series is scaled to have unit standard deviation over the regression sample. The sample period is 2000Q1-2020Q1. The level of banking concentration is constant within a year. Robust standard errors clustered by country in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

Table 9: Robustness regression with constant concentration within a year, horizons 17-24

	$h = 17$	$h = 18$	$h = 19$	$h = 20$	$h = 21$	$h = 22$	$h = 23$	$h = 24$
Return$_t$	-6.990***	-6.309***	-5.522**	-4.970**	-3.847	-3.025	-2.449	-1.973
	(2.237)	(2.186)	(2.193)	(2.317)	(2.709)	(2.913)	(2.915)	(2.721)
Volatility$_t$	-5.029	-4.482	-4.448	-4.624	-4.018	-2.645	0.048	-0.712
	(5.984)	(5.982)	(6.392)	(6.969)	(7.175)	(7.106)	(6.457)	(6.071)
Conc$_t$ × Ret $_t$	1.031	-0.203	-0.839	-1.487	-1.997	-2.099	-1.516	-2.382
	(3.171)	(3.364)	(3.453)	(3.652)	(3.688)	(3.581)	(3.277)	(3.227)
Conc$_t$ × Vol $_t$	-11.629*	-11.208	-11.275	-11.715	-11.419	-10.680	-8.908	-9.727
	(6.954)	(6.818)	(7.210)	(7.772)	(8.059)	(8.084)	(7.512)	(7.308)
Concentration$_t$	0.687	0.721	0.771	0.738	0.487	0.298	0.197	0.304
	(1.321)	(1.252)	(1.270)	(1.331)	(1.365)	(1.421)	(1.477)	(1.583)
Observations	2322	2282	2242	2201	2160	2119	2078	2036
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
F-test R$_t$	162.634	166.875	166.520	165.894	161.106	156.505	144.784	135.830
F-test V$_t$	32.113	30.392	22.750	17.252	13.376	11.735	11.561	11.851
F-test C$_t$ × R$_t$	17.416	16.441	15.167	13.924	11.558	8.854	6.656	7.068
F-test C$_t$ × V$_t$	37.876	31.128	24.741	21.491	18.499	17.266	16.851	16.586
Hansen p-value	0.679	0.690	0.762	0.826	0.825	0.832	0.688	0.761

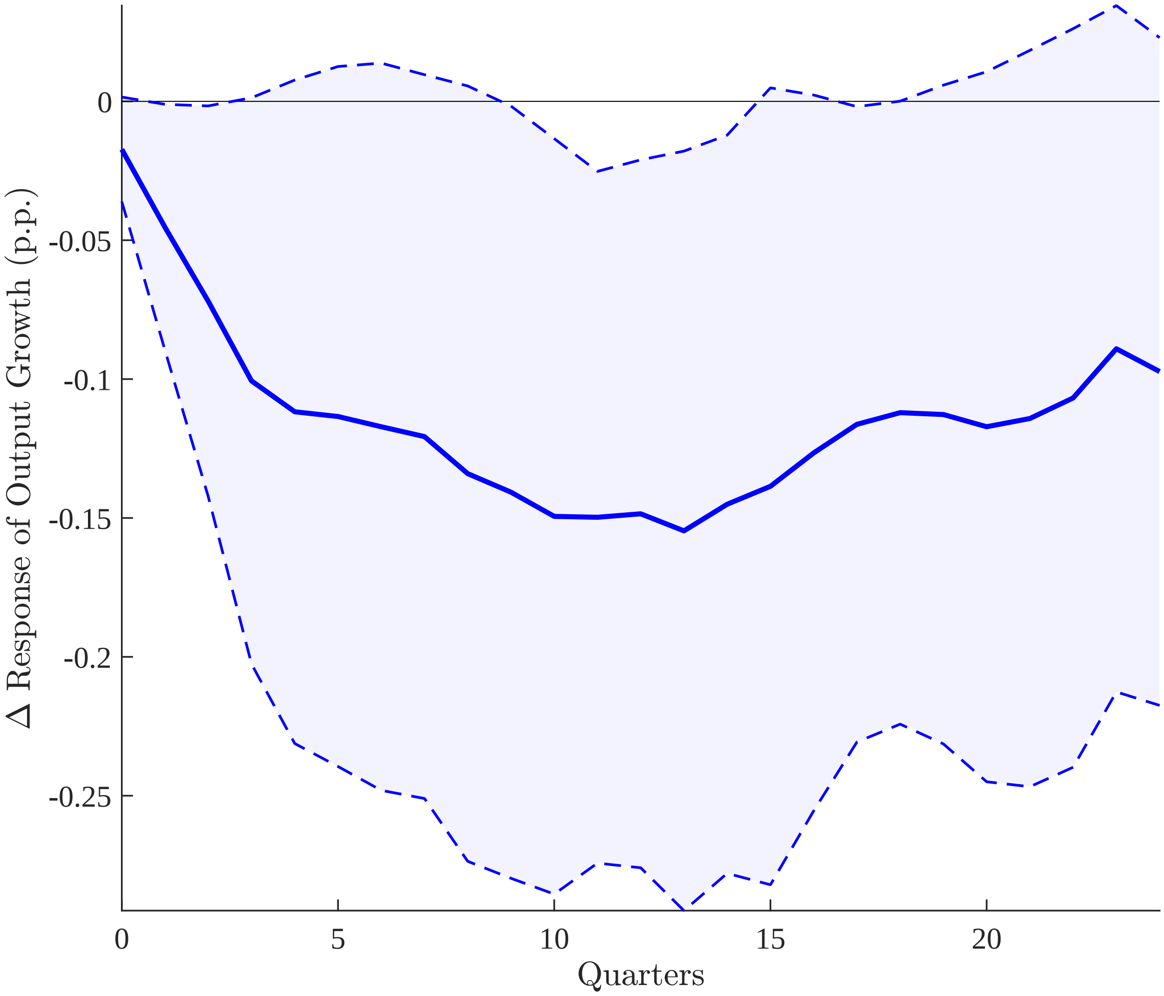
Notes: The dependent variable is GDP growth at horizon h. The concentration series is scaled to have unit standard deviation over the regression sample. The sample period is 2000Q1-2020Q1. The level of banking concentration is constant within a year. Robust standard errors clustered by country in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

Figure 11: Response of output growth with 5-Bank asset concentration.



Notes. The graph shows the responses of output growth to an increase of volatility of 1% from its country mean for different levels of banking concentration. In the response with high banking concentration, banking concentration is 1 standard deviation larger than the country average. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1. The level of banking competition is proxied by the 5-Bank asset concentration level.

Figure 12: Effect of concentration with 5-Bank asset concentration.



Notes. The graph shows the difference between the two specifications showed in Figure 11. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1.

Table 10: Robustness regression with 5-Bank asset concentration, horizons 0-8

	$h = 0$	$h = 1$	$h = 2$	$h = 3$	$h = 4$	$h = 5$	$h = 6$	$h = 7$	$h = 8$
Return$_t$	0.248	-0.081	-0.992	-2.592	-4.179	-5.410	-5.929	-6.167*	-6.412
	(0.312)	(0.903)	(1.630)	(2.338)	(3.016)	(3.496)	(3.678)	(3.665)	(4.029)
Volatility$_t$	-0.125	-1.274	-2.820	-4.268	-5.332	-6.201	-6.538	-6.440	-6.466
	(0.341)	(1.048)	(2.141)	(2.771)	(3.406)	(3.995)	(4.256)	(4.221)	(4.707)
Conc$_t$ × Ret $_t$	0.336	0.602	0.941	2.044	3.001	3.804	4.261	4.751	5.030
	(0.224)	(0.703)	(1.323)	(1.802)	(2.361)	(2.845)	(3.110)	(3.270)	(3.594)
Conc$_t$ × Vol $_t$	-0.545	-2.510	-5.351	-7.374	-8.771	-9.748	-9.794	-9.859	-11.137
	(0.804)	(2.375)	(4.490)	(5.979)	(7.122)	(7.889)	(8.014)	(7.765)	(8.324)
Concentration$_t$	0.067	0.339	0.751	1.213	1.507	1.763	1.833	1.811	2.009
	(0.130)	(0.405)	(0.802)	(1.099)	(1.296)	(1.436)	(1.473)	(1.435)	(1.576)
Observations	2825	2825	2825	2825	2808	2793	2778	2738	2693
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
F-test R$_t$	74.651	74.651	74.651	74.651	74.130	76.048	78.428	79.498	82.366
F-test V$_t$	33.380	33.380	33.380	33.380	33.834	33.584	33.242	32.530	31.683
F-test C$_t$ × R$_t$	174.078	174.078	174.078	174.078	169.520	222.390	243.895	347.655	352.019
F-test C$_t$ × V$_t$	11.061	11.061	11.061	11.061	11.270	12.071	12.694	13.148	13.554
Hansen p-value	0.448	0.435	0.634	0.729	0.766	0.785	0.749	0.705	0.671

Notes: The dependent variable is GDP growth at horizon h. The concentration series is scaled to have unit standard deviation over the regression sample. The sample period is 2000Q1-2020Q1. The level of banking competition is proxied by the 5-Bank asset concentration level. Robust standard errors clustered by country in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

Table 11: Robustness regression with 5-Bank asset concentration, horizons 9-16

	$h=9$	$h=10$	$h=11$	$h=12$	$h=13$	$h=14$	$h=15$	$h=16$
Return_t	-6.551	-6.917	-6.432	-6.123	-6.175*	-5.883*	-4.316	-4.912*
	(4.336)	(4.667)	(4.404)	(3.903)	(3.553)	(3.014)	(2.858)	(2.557)
Volatility_t	-7.255	-7.923	-8.041	-7.868	-7.314	-5.631	-3.160	-3.198
	(5.158)	(5.565)	(5.324)	(5.103)	(5.035)	(4.160)	(3.559)	(3.231)
$\text{Conc}_t \times \text{Ret}_t$	4.567	3.783	3.022	2.333	1.864	2.428	5.401	3.658
	(3.919)	(4.334)	(4.665)	(4.728)	(4.929)	(4.557)	(4.727)	(3.844)
$\text{Conc}_t \times \text{Vol}_t$	-13.343	-16.151*	-17.208*	-16.906*	-16.954*	-15.062*	-11.232*	-11.936*
	(9.017)	(9.815)	(9.751)	(9.437)	(9.498)	(8.127)	(6.552)	(6.146)
Concentration_t	2.184	2.499	2.514	2.054	1.832	1.559	1.403	1.652
	(1.792)	(2.131)	(2.208)	(2.154)	(2.162)	(1.997)	(1.752)	(1.760)
Observations	2650	2607	2564	2522	2482	2442	2402	2362
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
F-test R_t	83.225	86.131	95.615	104.955	113.661	125.678	131.811	135.859
F-test V_t	31.001	30.383	31.459	30.688	29.690	29.537	18.610	17.710
F-test $\text{C}_t \times \text{R}_t$	333.908	299.988	247.157	195.576	187.235	162.944	151.174	144.227
F-test $\text{C}_t \times \text{V}_t$	13.494	12.432	11.307	10.862	10.162	9.608	9.317	9.183
Hansen p-value	0.689	0.641	0.615	0.584	0.555	0.541	0.514	0.560

Notes: The dependent variable is GDP growth at horizon h. The concentration series is scaled to have unit standard deviation over the regression sample. The sample period is 2000Q1-2020Q1. The level of banking competition is proxied by the 5-Bank asset concentration level. Robust standard errors clustered by country in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

Table 12: Robustness regression with 5-Bank asset concentration, horizons 17-24

	$h = 17$	$h = 18$	$h = 19$	$h = 20$	$h = 21$	$h = 22$	$h = 23$	$h = 24$
Return_t	-4.923**	-4.781**	-3.771*	-2.671	-0.773	0.666	0.980	1.083
	(2.360)	(2.295)	(2.018)	(1.779)	(1.845)	(1.827)	(1.685)	(1.353)
Volatility_t	-2.668	-1.131	1.005	3.079	5.901**	8.486***	9.218***	8.450***
	(2.974)	(2.785)	(2.484)	(2.665)	(2.826)	(2.731)	(2.418)	(2.754)
$\text{Conc}_t \times \text{Ret}_t$	2.373	1.442	1.736	2.267	3.098	3.763*	3.463*	2.486
	(3.077)	(2.484)	(2.015)	(2.010)	(1.914)	(1.998)	(1.993)	(2.068)
$\text{Conc}_t \times \text{Vol}_t$	-11.631**	-10.449*	-8.051*	-5.594	-1.856	1.171	1.594	0.698
	(5.842)	(5.537)	(4.778)	(4.407)	(3.906)	(3.893)	(3.836)	(3.849)
Concentration_t	1.747	1.621	1.285	0.890	0.308	-0.107	-0.125	0.076
	(1.710)	(1.690)	(1.595)	(1.498)	(1.392)	(1.407)	(1.476)	(1.487)
Observations	2322	2282	2242	2201	2160	2119	2078	2036
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
F-test R_t	141.450	143.970	145.772	141.010	134.398	126.703	114.612	106.669
F-test V_t	17.028	16.905	14.386	12.394	10.934	10.036	10.582	10.847
F-test $\text{C}_t \times \text{R}_t$	144.385	161.205	192.533	212.247	210.686	181.930	128.583	95.921
F-test $\text{C}_t \times \text{V}_t$	9.192	8.686	8.788	9.804	11.254	12.628	14.146	14.926
Hansen p-value	0.553	0.502	0.400	0.327	0.264	0.239	0.232	0.238

Notes: The dependent variable is GDP growth at horizon h. The concentration series is scaled to have unit standard deviation over the regression sample. The sample period is 2000Q1-2020Q1. The level of banking competition is proxied by the 5-Bank asset concentration level. Robust standard errors clustered by country in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

C Entrepreneurial Problem

C.1 Loan Demand and its Properties

In this section I derive the loan demand and I show some properties of the loan demand. In particular, in Section C.1.1 I derive the loan demand function and in Section C.1.2 I describe the properties of the loan demand.

C.1.1 Derivation of Loan Demand

In this section I derive the loan demand function.

After substituting the resource constraint of the entrepreneurs (8), the Lagrangian of the maximization of the entrepreneurs is

$$\mathcal{L}(K_t^j, \Lambda_t) = \mathbb{E}_t(W_{t+1}^{E,j}) + \Lambda \left(\frac{qK_t^j \lambda}{\Pi_{t+1}} - \mathbb{E}_t(W_{t+1}^{E,j}) \right) \tag{23}$$

First, let me rewrite the expected future wealth as

$$\begin{aligned}
\mathbb{E}_t(W^{E,j})_{t+1} &= \left(\int_{\overline{\omega}_{t+1}^j}^{\infty} \omega_{t+1}^j R_{t+1}^E qK_t^j f(\omega_{t+1}^j) \mathrm{d}\omega_{t+1}^j - (1 - F(\overline{\omega}_{t+1}) R_t^F b_t^j \right) \frac{1}{\Pi_{t+1}} \\
&= \left(\int_{\overline{\omega}_{t+1}^j}^{\infty} \omega_{t+1}^j R_{t+1}^E qK_t^j f(\omega_{t+1}^j) \mathrm{d}\omega_{t+1}^j - \int_{\overline{\omega}_{t+1}^j}^{\infty} R_t^F b_t^j f(\omega_{t+1}^j) \mathrm{d}\omega_{t+1}^j \right) \frac{1}{\Pi_{t+1}}, \qquad (24)
\end{aligned}$$

where $f(\cdot)$ and $F(\cdot)$ are the p.d.f. and the c.d.f. of the distribution of ω_{t+1}. Using the definition of the default cutoff (6), (24) can be written as

$$\int_{\overline{\omega}_{t+1}^j}^{\infty} (\omega_{t+1}^j - \overline{\omega}_{t+1}^j) R_{t+1}^E qK_t^j f(\omega_{t+1}^j) \mathrm{d}\omega_{t+1}^j. \tag{25}$$

the term $\int_{\overline{\omega}^j_{t+1}}^{\infty}(\omega^j_{t+1}-\overline{\omega}^j_{t+1})f(\omega^j_{t+1})d\omega^j_{t+1}$ can be rewritten as

$$\begin{aligned}
\int_{\overline{\omega}^j_{t+1}}^{\infty}(\omega^j_{t+1}-\overline{\omega}^j_{t+1})f(\omega^j_{t+1})\mathrm{d}\omega^j_{t+1} &= \int_{\overline{\omega}^j_{t+1}}^{\infty}\omega^j_{t+1}f(\omega^j_{t+1})\mathrm{d}\omega^j_{t+1}-\overline{\omega}^j_{t+1}\int_{\overline{\omega}^j_{t+1}}^{\infty}f(\omega^j_{t+1})\mathrm{d}\omega^j_{t+1} \\
&= 1-\left(\underbrace{\int_0^{\overline{\omega}^j_{t+1}}\omega^j_{t+1}f(\omega^j_{t+1})\mathrm{d}\omega^j_{t+1}}_{\equiv G(\overline{\omega}^j_{t+1})}+\overline{\omega}^j_{t+1}\int_{\overline{\omega}^j_{t+1}}^{\infty}f(\omega^j_{t+1})\mathrm{d}\omega^j_{t+1}\right) \\
&= 1-\underbrace{\left(G(\overline{\omega}^j_{t+1})+(1-F(\overline{\omega}^j_{t+1}))\overline{\omega}^j_{t+1}\right)}_{\equiv\Gamma(\overline{\omega}^j_{t+1})\geq 0}. \qquad (26)
\end{aligned}$$

Note also that I can express $\Gamma(\overline{\omega}^j_{t+1})$ as

$$\begin{aligned}
\Gamma(\overline{\omega}^j_{t+1}) &= \int_0^{\overline{\omega}^j_{t+1}}\omega^j_{t+1}f(\omega^j_{t+1})\mathrm{d}\omega^j_{t+1}+\overline{\omega}^j_{t+1}\int_{\overline{\omega}^j_{t+1}}^{\infty}f(\omega^j_{t+1})\mathrm{d}\omega^j_{t+1} \\
&=\overline{\omega}^j_{t+1}F(\overline{\omega}^j_{t+1})-\int_0^{\overline{\omega}^j_{t+1}}F(\omega^j_{t+1})\mathrm{d}\omega^j_{t+1}+\overline{\omega}^j_{t+1}(1-F(\overline{\omega}^j_{t+1})) \\
&=\overline{\omega}^j_{t+1}-\int_0^{\overline{\omega}^j_{t+1}}F(\omega^j_{t+1})\mathrm{d}\omega^j_{t+1} \qquad (27)
\end{aligned}$$

Combining (25) and (26), expected future wealth can be written as

$$\mathbb{E}_t(W^{E,j}_{t+1})=\frac{(1-\Gamma(\overline{\omega}^j_{t+1}))R^E_{t+1}qK^j_t}{\Pi_{t+1}} \qquad (28)$$

Substituting in the Lagrangian of the maximization problem of the entrepreneurs

$$\mathcal{L}(K^j_t,\Lambda_t)=\frac{(1-\Gamma(\overline{\omega}^j_{t+1}))R^E_{t+1}qK^j_t}{\Pi_{t+1}}+\Lambda_t\left(\frac{qK^j_t\lambda}{\Pi_{t+1}}-\frac{(1-\Gamma(\overline{\omega}^j_{t+1}))R^E_{t+1}qK^j_t}{\Pi_{t+1}}\right)$$

The first order conditions of the Lagrangian are

$$\frac{\partial\mathcal{L}}{\partial K^j_t}=\frac{\partial\mathbb{E}_t(W^{E,j}_{t+1})}{\partial K^j_t}+\Lambda_t\left(\frac{q\lambda}{\Pi_{t+1}}-\frac{\partial\mathbb{E}_t(W^{E,j}_{t+1})}{\partial K^j_t}\right)=0 \qquad (29)$$

$$\frac{\partial\mathcal{L}}{\partial\Lambda^j_t}=\frac{qK^j_t\lambda}{\Pi_{t+1}}-\mathbb{E}_t(W^{E,j}_{t+1})=0 \qquad (30)$$

The first derivative of expected future wealth with respect to capital is [14]

$$\frac{\partial \mathbb{E}_t(W^{E,j}_{t+1})}{\partial K^j_t} = \left((1-\Gamma(\overline{\omega}^j_{t+1}))R^E_{t+1}q - \Gamma'(\overline{\omega}^j_{t+1})\frac{\partial \overline{\omega}^j_{t+1}}{\partial K^j_t} R^E_{t+1} q K^j_t \right) \frac{1}{\Pi_{t+1}} \tag{31}$$

Let me first focus on the second term of (31). From (26), $\Gamma'(\overline{\omega}^j_{t+1})$ is equal to

$$\Gamma'(\overline{\omega}^j_{t+1}) = G'(\overline{\omega}^j_{t+1}) - f(\overline{\omega}^j_{t+1})\overline{\omega}^j_{t+1} + (1 - F(\overline{\omega}^j_{t+1})), \tag{32}$$

where, from (26), $G'(\overline{\omega}^j_{t+1})$ is equal to

$$G'(\overline{\omega}^j_{t+1}) = \frac{\partial \int_0^{\overline{\omega}^j_{t+1}} \omega^j_{t+1} f(\omega^j_{t+1}) \mathrm{d}\omega^j_{t+1}}{\partial \overline{\omega}^j_{t+1}} = \overline{\omega}^j_{t+1} f(\overline{\omega}^j_{t+1}). \tag{33}$$

Combining (33) and (32), $\Gamma'(\overline{\omega}^j_{t+1})$ is equal to

$$\Gamma'(\overline{\omega}^j_{t+1}) = 1 - F(\overline{\omega}^j_{t+1}) \geq 0 \tag{34}$$

From the definition of the default cutoff (6) and the resource constraint of the entrepreneur (8), the term $\frac{\partial \overline{\omega}^j_{t+1}}{\partial K^j_t}$ is equal to

$$\frac{\partial \overline{\omega}^j_{t+1}}{\partial K^j_t} = \frac{R^F_t}{R^E_t q} \frac{qK^j_t - b^j_t}{K^2_t} = \frac{R^F_t n^{E,j}_t}{R^E_t q K^2_t} \geq 0 \tag{35}$$

Substituting (34) and (35) in (31) it is possible to write

$$\frac{\partial \mathbb{E}_t(W^{E,j}_{t+1})}{\partial K^j_t} = \left((1-\Gamma(\overline{\omega}^j_{t+1}))R^E_t q - (1 - F^E(\overline{\omega}^j_{t+1}))\frac{R^F_t n^{E,j}_t}{K_t} \right) \frac{1}{\Pi_{t+1}}$$

[14] Also R^E_{t+1} is a function of capital, however I assume that entrepreneurs take the return on capital as given.

Using the definition of $\Gamma(\overline{\omega}^j_{t+1})$ derived in (26) and the definition of the default cutoff (6)

$$\begin{aligned}
\frac{\partial \mathbb{E}(W^{E,j}_{t+1})}{\partial K^j_t} &= \left(R^E_{t+1} q - (1 - F(\overline{\omega}^j_{t+1})) R^E_{t+1} q \overline{\omega}^j_{t+1} - R^E_{t+1} q G(\overline{\omega}^j_{t+1}) - (1 - F^E(\overline{\omega}^j_{t+1})) \frac{R^F_t n^{E,j}_t}{K^j_t} \right) \frac{1}{\Pi_{t+1}} \\
&= \left(R^E_{t+1} q - (1 - F^E(\overline{\omega}^j_{t+1})) R^F_t \frac{b^j_t + n^{E,j}_t}{K^j_t} - R^E_{t+1} q G(\overline{\omega}^j_{t+1}) \right) \frac{1}{\Pi_{t+1}} \\
&= \left(R^E_{t+1} q (1 - G(\overline{\omega}^j_{t+1})) - R^F_t q (1 - F^E(\overline{\omega}^j_{t+1})) \right) \frac{1}{\Pi_{t+1}} \qquad (36)
\end{aligned}$$

Substituting in the first order condition of the Lagrangian with respect to capital (29)

$$0 = R^E_{t+1}(1 - G(\overline{\omega}^j_{t+1})) - R^F_t(1 - F^E(\overline{\omega}^j_{t+1})) + \Lambda_t(\lambda - R^E_{t+1}(1 - G(\overline{\omega}^j_{t+1})) + R^F_t(1 - F^E(\overline{\omega}^j_{t+1}))) \qquad (37)$$

Implying that

$$\begin{aligned}
\Lambda_t &= \frac{R^E_{t+1}(1 - G(\overline{\omega}^j_{t+1})) - R^F_t(1 - F^E(\overline{\omega}^j_{t+1}))}{R^E_{t+1}(1 - G(\overline{\omega}^j_{t+1})) + R^F_t(1 - F^E(\overline{\omega}^j_{t+1})) - \lambda} \\
&= 1 + \frac{\lambda}{R^E_{t+1}(1 - G(\overline{\omega}^j_{t+1})) - R^F_t(1 - F^E(\overline{\omega}^j_{t+1})) - \lambda} \qquad (38)
\end{aligned}$$

The incentive constraint is binding if $\Lambda_t > 0$. This condition is satisfied if $R^E_{t+1}(1 - G(\overline{\omega}^j_{t+1})) - R^F_t(1 - F^E(\overline{\omega}^j_{t+1})) > 0$. Note that if the incentive constraint is not binding, $R^F_t > R^E_{t+1}$ because $G(\overline{\omega}^j_{t+1})) \leq F^E(\overline{\omega}^j_{t+1})$.

The incentive constraint is binding if $R^F_t \leq R^E(n^E_t)$. To see this, assume that it is not binding, so that $\Lambda_t = 0$. From (37), the loan demand is defined by

$$R^E_{t+1}(1 - G(\overline{\omega}^j_{t+1})) - R^F_t(1 - F^E(\overline{\omega}^j_{t+1})) = 0$$

The second order condition is

$$\begin{aligned}
SOC &= - R^E_{t+1} \overline{\omega}^j_{t+1} f(\overline{\omega}^j_{t+1}) + R^F_t f(\overline{\omega}^j_{t+1}) \\
&= - R^F_t \frac{b^j_t}{q K^j_t} f(\overline{\omega}^j_{t+1}) + R^F_t f(\overline{\omega}^j_{t+1}) \geq 0,
\end{aligned}$$

where the second equality is obtained substituting the definition of the default cutoff (6).

Since the second order condition is positive, the result obtained from the first order condition is a minimum. This implies that the optimal choice is either $b_t^j = \infty$ or $b_t^j = 0$.

If the incentive constraint is satisfied, the entrepreneurial expected future wealth is

$$
\begin{aligned}
\mathbb{E}(W_{t+1}^{E,j}) =& R_{t+1}^E qK_t^j - (1 - F^E(\overline{\omega}_{t+1}^j))\overline{\omega}_{t+1}^j qK_t^j R_{t+1}^E - G^E(\overline{\omega}_{t+1}^j) R_{t+1}^E qK_t^j \\
=& (1 - G^E(\overline{\omega}_{t+1}^j)) R_{t+1}^E qK^j - (1 - F^E(\overline{\omega}_{t+1}^j)) R_t^F b_t^j \\
=& (1 - G^E(\overline{\omega}_{t+1}^j)) R_{t+1}^E qK_t^j - (1 - G^E(\overline{\omega}_{t+1}^j)) R_{t+1}^E b_t^j \\
=& (1 - G^E(\overline{\omega}_{t+1}^j)) R_{t+1}^E n_t^{E,j}
\end{aligned}
$$

However, if the entrepreneur does not ask any loan

$$\mathbb{E}(W^{E,j}) = R_{t+1}^E n_t^{E,j},$$

and the incentive constraint is always satisfied. Therefore, if the incentive constraint is not binding, it is always optimal for the entrepreneur not to obtain loans.

This implies that the incentive constraint is binding if at $b_t = 0$ the marginal return on borrowing is higher than the marginal cost

$$R_t^F \leq R^E(n_t^E).$$

Assuming that $R_t^F \leq R^E(n_t^E)$, the demand for loan demand is implicitly defined by the incentive participation constrain

$$(1 - \Gamma(\overline{\omega}_{t+1}^j)) R_{t+1}^E = \lambda$$

C.1.2 Properties of the Loan Demand

This section shows the properties of the loan demand characterized in Section C.1.1.

Proof of Proposition 1

Proposition 1 states that loan demand falls with the borrowing rate. Let the function $\mathcal{I}$

be

$$\mathcal{I}_t = (1 - \Gamma(\overline{\omega}^j_{t+1}))R^E_{t+1} - \lambda$$

The derivative of the loan demand with respect to the borrowing rate is

$$\frac{\mathrm{d}b_t}{\mathrm{d}R^F_t} = -\frac{\frac{\partial \mathcal{I}_t}{\partial R^F_t}}{\frac{\partial \mathcal{I}_t}{\partial b_t}} \tag{39}$$

The numerator of (39) is

$$\frac{\partial \mathcal{I}_t}{\partial R^F_t} = -R^E_{t+1}\Gamma'(\overline{\omega}_{t+1})\frac{\partial \overline{\omega}_{t+1}}{\partial R^F_t} \tag{40}$$

From the definition of the default cutoff (6), the term $\frac{\partial \overline{\omega}_{t+1}}{\partial R^F_t}$ is

$$\frac{\partial \overline{\omega}_{t+1}}{\partial R^F_t} = \frac{b_t}{R^E_{t+1} q K_t} \tag{41}$$

Substituting in (40) $\Gamma'(\overline{\omega}_{t+1})$ from (34) and $\frac{\partial \overline{\omega}_{t+1}}{\partial R^F_t}$ using (41)

$$\begin{aligned}\frac{\partial \mathcal{I}_t}{\partial R^F_t} &= -R^E_{t+1}(1 - F(\overline{\omega}_{t+1}))\frac{b_t}{R^E_{t+1} q K_t} \\ &= -(1 - F(\overline{\omega}_{t+1}))\frac{b_t}{qK_t} \leq 0 \end{aligned} \tag{42}$$

The denominator of (39) is

$$\frac{\partial \mathcal{I}_t}{\partial b_t} = -R^E_{t+1}\Gamma'(\overline{\omega}_{t+1})\frac{\partial \overline{\omega}_{t+1}}{\partial b_t} + (1 - \Gamma(\overline{\omega}_{t+1}))R^{E\prime}_{t+1} \tag{43}$$

The term $\frac{\partial \overline{\omega}_{t+1}}{\partial b_t}$ is

$$\frac{\partial \overline{\omega}_{t+1}}{\partial b_t} = \frac{R^F_t}{q}\frac{R^E_{t+1} q K_t - (R^{E\prime}_{t+1}K_t + R^E_{t+1})b_t}{(R^E_{t+1}K_t)^2} = \frac{R^F_t}{q}\frac{R^E_{t+1} n^E_t - R^{E\prime}_{t+1}K_t}{(R^E_{t+1}K_t)^2} \geq 0 \tag{44}$$

Substituting in (43) $\Gamma'(\overline{\omega}_{t+1})$ and $\frac{\partial \overline{\omega}_{t+1}}{\partial b_t}$ using (34) and (44)

$$\frac{\partial \mathcal{I}_t}{\partial b_t} = -R^E_{t+1}(1 - F(\overline{\omega}_{t+1}))\frac{R^F_t}{q}\frac{R^E_{t+1} n^E_t - R^{E\prime}_{t+1}K_t}{(R^E_{t+1}K_t)^2} + (1 - \Gamma(\overline{\omega}_{t+1}))R^{E\prime}_{t+1} \leq 0 \tag{45}$$

Substituting (42) and (45) in (39)

$$\frac{\mathrm{d}b_t}{\mathrm{d}R_t^F} = -\frac{(1-F(\overline{\omega}_{t+1}))\frac{b_t}{qK_t}}{R_{t+1}^E(1-F(\overline{\omega}_{t+1}))\frac{R_t^F}{q}\frac{R_{t+1}^E n_t^E - R_{t+1}^{E\prime}K_t}{(R_{t+1}^E K_t)^2} - (1-\Gamma(\overline{\omega}_{t+1}))R_{t+1}^{E\prime}} \leq 0$$

Since $\frac{\mathrm{d}b_t}{\mathrm{d}R_t^F} \leq 0$, loan demand is a decreasing function of the borrowing rate.

Proof of Proposition 2

Proposition 2 states that the entrepreneurial default rate increases with the borrowing rate.

The entrepreneurial default rate $F(\overline{\omega}_{t+1})$ is an increasing function of $\overline{\omega}_{t+1}$. To see this, I take the derivative of the default rate (5) with respect to the default threshold

$$F'(\overline{\omega}_{t+1}) = f(\overline{\omega}_{t+1}) = \frac{1}{\overline{\omega}_{t+1}\sigma_t}\phi\left(\frac{\log(\overline{\omega}_{t+1}) + 0.5\sigma_t^2}{\sigma_t}\right) \geq 0, \tag{46}$$

where $\phi(\cdot)$ is the p.d.f. of the standard normal distribution.

In order to show that the entrepreneurial default rate increases with the borrowing rate, I need to show that the default cutoff $\overline{\omega}_{t+1}$ increases with the borrowing rate. The equilibrium value of the default cutoff can be obtained from the loan demand function (9)

$$\Gamma(\overline{\omega}_{t+1}) = 1 - \frac{\lambda}{R_{t+1}^E}$$

Taking the inverse of $\Gamma(\overline{\omega}_{t+1})$

$$\overline{\omega}_{t+1} = \Gamma^{-1}\left(1 - \frac{\lambda}{R_{t+1}^E}\right) \tag{47}$$

Taking the derivative of (47)

$$\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}R_t^F} = \frac{1}{1 - F\left(1 - \frac{\lambda}{R_{t+1}^E}\right)}\frac{1}{(R_{t+1}^E)^2}R_{t+1}^{E\prime}\frac{1}{q}\frac{\mathrm{d}b_t}{\mathrm{d}R_t^F} \geq 0 \tag{48}$$

Since the default rate increases with the default threshold that is increasing in the loan rate, the default rate increases with the loan rate.

Proof of Proposition 3

Proposition 3 states that if $\sigma_{t+1} \leq 1$ and R^F_{t+1} is low enough, such that the default rate of the entrepreneurs F is lower than 15%, an increase in uncertainty implies a stronger rise in the default rate of the entrepreneurs when the default cutoff is higher.

From the definition of the definition of the default rate (5), the effect of an increase of the default cutoff on the default rate is

$$F'(\overline{\omega}_{t+1}) = \frac{1}{\overline{\omega}_{t+1}\sigma_{t+1}}\phi\left(\frac{\log(\overline{\omega}_{t+1}) + 0.5\sigma^2_{t+1}}{\sigma_{t+1}}\right) \tag{49}$$

In order to show that an increase in uncertainty has a stronger effect on the default rate when the default cutoff is higher, the following expression has to be positive

$$\begin{aligned}\frac{\mathrm{d}F'_{t+1}}{\mathrm{d}\sigma_{t+1}} = &-\frac{\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}\sigma_{t+1} + \overline{\omega}_{t+1}}{\overline{\omega}^2_{t+1}\sigma^2_{t+1}}\phi\left(\frac{\log(\overline{\omega}_{t+1}) + 0.5\sigma^2_{t+1}}{\sigma_{t+1}}\right)\\ &+ \phi'\left(\frac{\log(\overline{\omega}_{t+1}) + 0.5\sigma^2_{t+1}}{\sigma_{t+1}}\right)\frac{1}{\overline{\omega}_{t+1}\sigma_{t+1}}\frac{\frac{1}{\overline{\omega}_{t+1}}\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}\sigma_{t+1} + 0.5\sigma^2_{t+1} - \log(\overline{\omega}_{t+1})}{\sigma^2_{t+1}}\end{aligned} \tag{50}$$

The term $\phi'(x)$ can be written as

$$\phi'(x) = -\frac{1}{\sqrt{2\pi}}xe^{-0.5x^2} \tag{51}$$

The term $\phi(x)$ can be written as

$$\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-0.5x^2} \tag{52}$$

Substituting (51) and (52) in (50)

$$\begin{aligned}\frac{\mathrm{d}F'_{t+1}}{\mathrm{d}\sigma_{t+1}} =& \frac{1}{\sqrt{2\pi}}e^{-0.5x^2}\frac{1}{\overline{\omega}\sigma^2}\Bigg(-\frac{\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}\sigma_{t+1} + \overline{\omega}_{t+1}}{\overline{\omega}_{t+1}}\\ &+\frac{\log(\overline{\omega}_{t+1}) + 0.5\sigma^2_{t+1}}{\sigma_{t+1}}\frac{\log(\overline{\omega}_{t+1}) - 0.5\sigma^2_{t+1} - \frac{1}{\overline{\omega}_{t+1}}\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}\sigma_{t+1}}{\sigma_{t+1}}\Bigg)\end{aligned} \tag{53}$$

Assuming that $\sigma_{t+1} \leq 1$ and that R^F_{t+1} is low enough such that $\frac{\log(\overline{\omega}_{t+1})+0.5\sigma^2_{t+1}}{\sigma_{t+1}} \leq -1$, (53) is positive. The last assumption implies that the default rate of entrepreneurs $F(\overline{\omega}) < 0.15$.

In order to see that this assumption is sufficient, first note that

$$-\frac{\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}\sigma_{t+1}}{\overline{\omega}_{t+1}}+\frac{1}{\overline{\omega}_{t+1}}\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}},$$

is positive if $\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}\geq 0$. The effect of uncertainty on the default cutoff is

$$\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}=\frac{R_t^F}{q}\frac{\mathrm{d}b_t}{\mathrm{d}\sigma_{t+1}}\frac{R_{t+1}^E qK_t-R^E b-R_{t+1}^{E\prime}K_t b_t}{(R_{t+1}^E K_t)^2}=\frac{R_t^F}{q}\frac{\mathrm{d}b_t}{\mathrm{d}\sigma_{t+1}}\frac{R_{t+1}^E n^E-R_{t+1}^{E\prime}K_t b_t}{(R_{t+1}^E K_t)^2}.$$

The effect of uncertainty on the default cutoff is positive if $\frac{\mathrm{d}b_t}{\mathrm{d}\sigma_{t+1}}\geq 0$. The effect of uncertainty on loan demand is

$$\frac{\mathrm{d}b_t}{\mathrm{d}\sigma_{t+1}}=-\frac{\frac{\partial\mathcal{I}_t}{\partial\sigma_{t+1}}}{\frac{\partial\mathcal{I}_t}{\partial b_t}} \tag{54}$$

The numerator of (54) is

$$\frac{\partial\mathcal{I}_t}{\partial\sigma_{t+1}}=-\frac{\partial\Gamma(\overline{\omega}_{t+1})}{\partial\sigma_{t+1}}R_{t+1}^E, \tag{55}$$

where, using the definition of $\Gamma(\overline{\omega}_{t+1})$ (26), $\frac{\partial\Gamma(\overline{\omega}_{t+1})}{\partial\sigma_{t+1}}$ is

$$\begin{aligned}\frac{\partial\Gamma(\overline{\omega}_{t+1})}{\partial\sigma_{t+1}}=&-\frac{\partial F(\overline{\omega}_{t+1})}{\partial\sigma_{t+1}}\overline{\omega}_{t+1}+\frac{\partial G(\overline{\omega}_{t+1})}{\partial\sigma_{t+1}}\\=&-F'(\overline{\omega}_{t+1})\overline{\omega}_{t+1}^2\frac{0.5\sigma_{t+1}^2-\log(\overline{\omega}_{t+1})}{\sigma_{t+1}}+F'(\overline{\omega}_{t+1})\overline{\omega}_{t+1}^2\frac{-0.5\sigma_{t+1}^2-\log(\overline{\omega}_{t+1})}{\sigma_{t+1}}\\=&-F'(\overline{\omega}_{t+1})\overline{\omega}_{t+1}^2\sigma_{t+1}\leq 0\end{aligned}$$

Substituting in (55)

$$\frac{\partial\mathcal{I}_t}{\partial\sigma_{t+1}}=F'(\overline{\omega}_{t+1})\overline{\omega}_{t+1}^2\sigma_{t+1}R_{t+1}^E \tag{56}$$

Substituting (56) and (45) in (54)

$$\frac{\mathrm{d}b_t}{\mathrm{d}\sigma_{t+1}}=\frac{F'(\overline{\omega}_{t+1})\overline{\omega}_{t+1}^2\sigma_{t+1}R_{t+1}^E}{R_{t+1}^E(1-F(\overline{\omega}_{t+1}))\frac{R_t^F}{q}\frac{R_{t+1}^E n_t^E-R_{t+1}^{E\prime}K_t}{(R_{t+1}^E K_t)^2}-(1-\Gamma(\overline{\omega}_{t+1}))R_{t+1}^{E\prime}}\geq 0$$

Therefore, $\left(-\frac{\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}\sigma_{t+1}}{\overline{\omega}_{t+1}}+\frac{1}{\overline{\omega}_{t+1}}\frac{\mathrm{d}\overline{\omega}_{t+1}}{\mathrm{d}\sigma_{t+1}}\right)\geq 0$. Finally, (50) is positive because , as implied by the assumption $\frac{\log(\overline{\omega}_{t+1})+0.5\sigma^2_{t+1}}{\sigma_{t+1}} \leq -1$

$$-1 - \frac{\log(\overline{\omega}_{t+1}) - 0.5\sigma^2_{t+1}}{\sigma_{t+1}} \geq 0$$

D Reverse bounded Pareto distribution

Suppose that γ follows a Pareto distribution with scale parameter $a > 0$ and support $\gamma \in [\gamma_s, \infty)$. Its p.d.f. and its c.d.f. are

$$f_\gamma(\gamma) = \frac{a\gamma_s^a}{\gamma^{a+1}}$$

$$F_\gamma(\gamma) = 1 - \left(\frac{\gamma_s}{\gamma}\right)^a$$

A bounded Pareto distribution is a distribution obtained from restricting the domain of the Pareto distribution. Let S and H be the lower bound and the upper bounds of the bounded Pareto distribution. The resulting p.d.f. and c.d.f. are

$$f_{\gamma B}(\gamma) = \frac{f_\gamma(\gamma)}{F_\gamma(H) - F_\gamma(S)} = \frac{\frac{a\gamma_s^a}{\gamma^{a+1}}}{1 - \left(\frac{\gamma_s}{H}\right)^a - \left[1 - \left(\frac{\gamma_s}{S}\right)^a\right]} = \frac{aS^a\gamma^{-a-1}}{1 - \left(\frac{S}{H}\right)^a}$$

$$F_{\gamma B}(\gamma) = \frac{F_\gamma(\gamma) - F_\gamma(S)}{F_\gamma(H) - F_\gamma(S)} = \frac{1 - \left(\frac{\gamma_s}{\gamma}\right)^a - \left[1 - \left(\frac{\gamma_s}{S}\right)^a\right]}{1 - \left(\frac{\gamma_s}{H}\right)^a - \left[1 - \left(\frac{\gamma_s}{S}\right)^a\right]} = \frac{1 - S^a\gamma^{-a}}{1 - \left(\frac{S}{H}\right)^a}$$

This distribution is characterized by a positive skewness and a long right tail. A market share distribution that features many small bankers and a few large bankers can be obtained flipping the distribution around the y-axis and shifting it to the right by $S + H$. This lead to a reverse bounded Pareto distribution whose domain is (S, H). The p.d.f. and the c.d.f. of this distribution are

$$f_{\gamma BR}(\gamma) \equiv f_{\gamma B}(-\gamma + H + S) = \frac{aS^a(-\gamma + H + S)^{-a-1}}{1 - \left(\frac{S}{H}\right)^a}$$

$$F_{\gamma BR}(\gamma) = \int_S^\gamma \frac{aS^a(-\gamma + H + S)^{-a-1}}{1 - \left(\frac{S}{H}\right)} d\gamma = \frac{S^a(-\gamma + H + S)^{-a} - S^a H^{-a}}{1 - \left(\frac{S}{H}\right)^a}$$