Symbol Tables

Direct Access Tables

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Definition

- A Symbol Table is a data structure with records including a key and allowing operations such as
  - **Insertion** of a new record
  - **Search** of a record with a given key
  - Delete, select, order, union
- Sometimes symbol tables are denoted with the term **dictionary**
  - Many applications need fast searches
  - Dictionaries are very important in computer engineering

Applications

- Symbol tables have several applications

Applications	Target, i.e., searching	Key	Return Value
Dictionary	Definition	Word	Definition
Book index	Relevant pages	Word	Page list
DNS Lookup	IP address given its URL	URL	IP address
Reverse DNS Lookup	URL given its IP address	IP address	URL
File system	File on disk	File name	Disk location
Web search	Web page	Keyword	Page list

Implementations

- Symbol tables have several implementations
- Linear structures
  - Direct Access Tables
  - Arrays
    - Unordered
    - Ordered
  - Lists
    - Unordered
    - Ordered
  - Hash Tables
- Tree structures
  - Binary Search Trees (BSTs)
  - Balanced Trees
    - 2-3-4
    - RB-tree
    - B-tree

Already studied - To be done - Not analyzed in this course

Complexity

- Different data structures have different performances

Worst case complexity

Data Structure	Insert	Search
Direct Access Table	1	1
Unordered Array	1	n
Ordered Array Linear Search	n	n
Ordered Array Binary Search	n	$log_2 n$
Unordered List	1	n
Ordered List	n	n
BST	n	n
RB-tree	$log_2 n$	$log_2 n$
Hashing	1	n

Complexity

Average case complexity

Data Structure	Insert	Search	
		Hit	Miss
Direct Access Table	1	1	1
Unordered Array	1	$n/2$	n
Ordered Array Linear Search	$n/2$	$n/2$	$n/2$
Orderer Array Binary Search	$n/2$	$log_2 n$	$log_2 n$
Unordered List	1	$n/2$	n
Ordered List	$n/2$	$n/2$	$n/2$
BST	$log_2 n$	$log_2 n$	$log_2 n$
RB-tree	$log_2 n$	$log_2 n$	$log_2 n$
Hashing	1	1	1

Direct Access Tables

- All search algorithms analyzed so far in the course are based on comparisons
  - For example searching for a key into an array, a list or a BST implies comparing this key with the element or node keys visiting the data structure with a specific logic
- **Direct Access Tables** and **Hash Tables** use a different paradigm
  - They compute the position of the key within the data structure by applying a function to the key

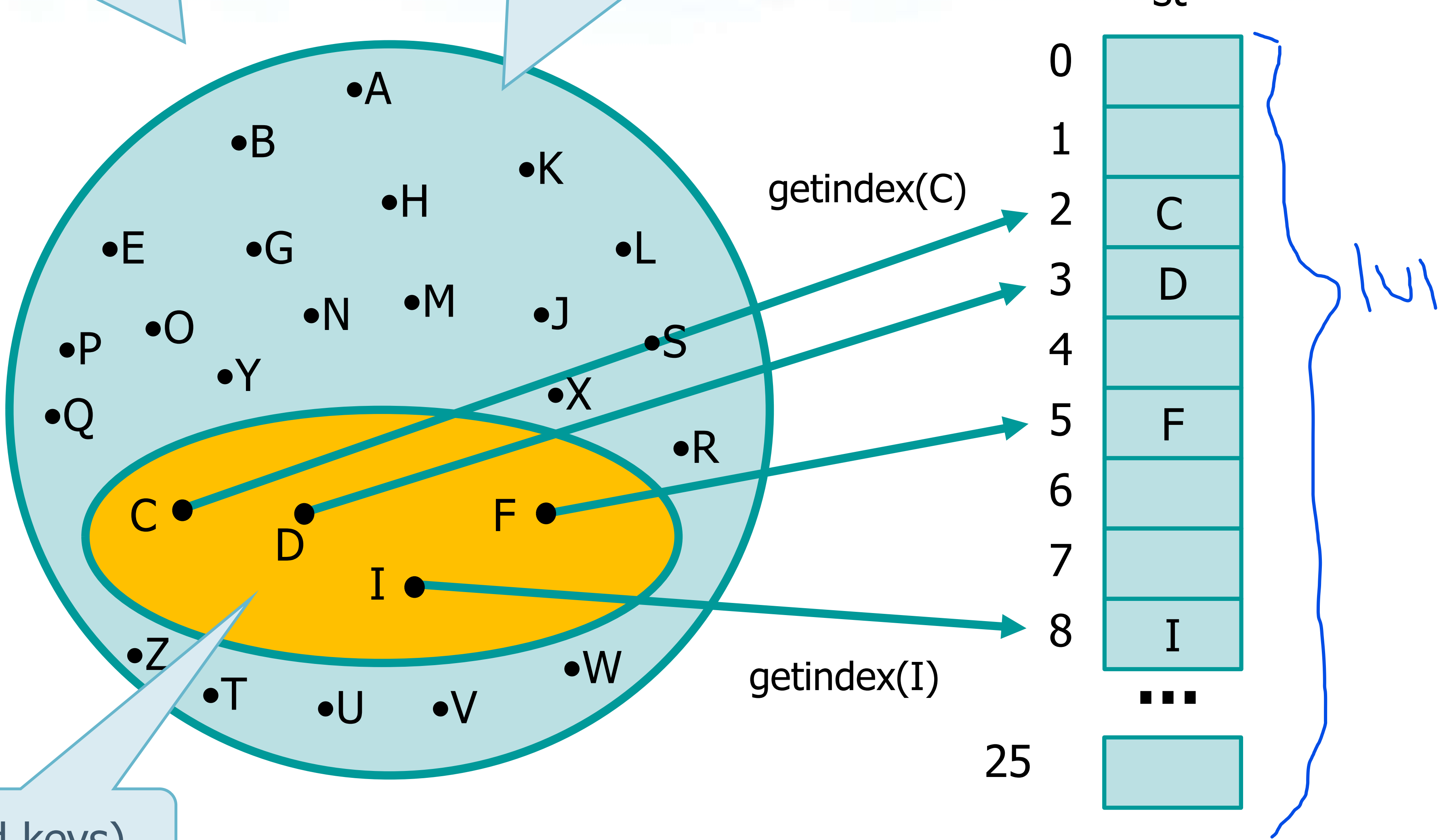
Direct Access Tables

- Problem definition
  - Suppose we need to store a key $\boldsymbol{k}$ belonging to a universe $\boldsymbol{U}$ of key in a table, with
    - $k \in U$
    - No two elements have the same key
    - U has cardinality $|U|$
- Core ideas
  - We can use an array to store the keys (and the related data fields)
    - The array (**st**) has size equal to $|U|$
  - We need to map each key ($k \in U$) into a specific element of the array

Direct Access Tables

There is a 1:1 mapping between $k \in U$ and elements in st

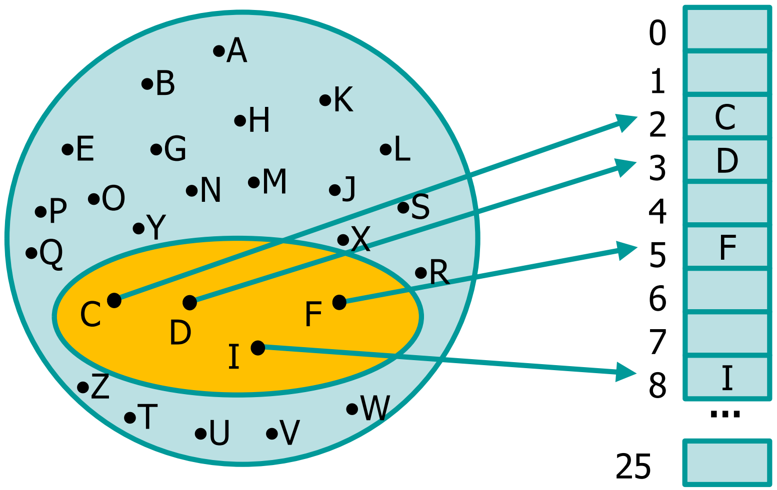
U (Universe of keys)



K (used keys)

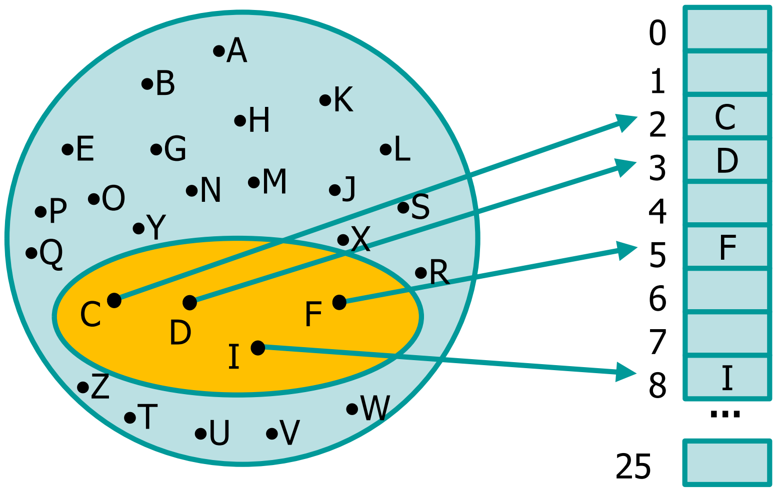
Direct Access Tables

- We have two problems
  - As the array **st** has size equal to $|U|$, the cardinality of U must be small to be able to allocate the array **st**
  - We always use $|U|$ elements even when we want to store a small subset of $|U|$



Direct Access Tables

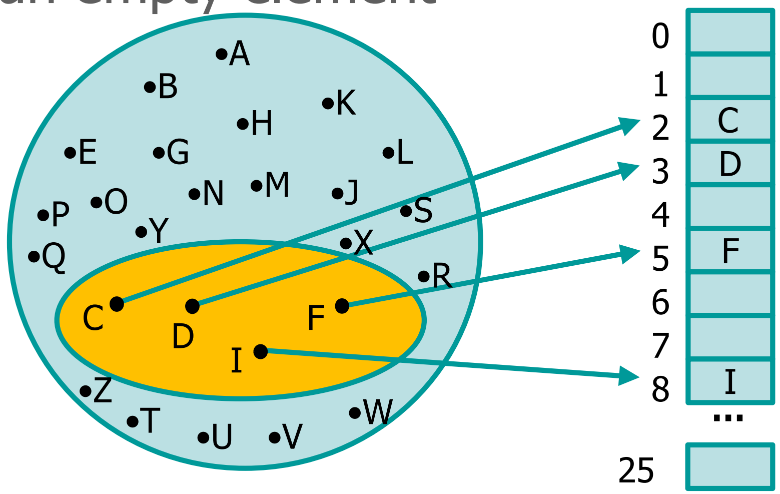
- We have two problems
  - We need to understand how to map keys into elements
    - This may be simple in specific cases, but the keys are not necessarily integer values
    - The mapping between keys and array indices may be complex



Direct Access Tables

- To create the mapping key-index we have to design a function (**getindex**) that given a key k
  - Returns an integer from 0 to $|U| - 1$, acting as an array index
    - If the key k is in the table
      - **st[getindex(k)]** stores it
    - If the key k is not in the table
      - **st[getindex(k)]** stores an empty element

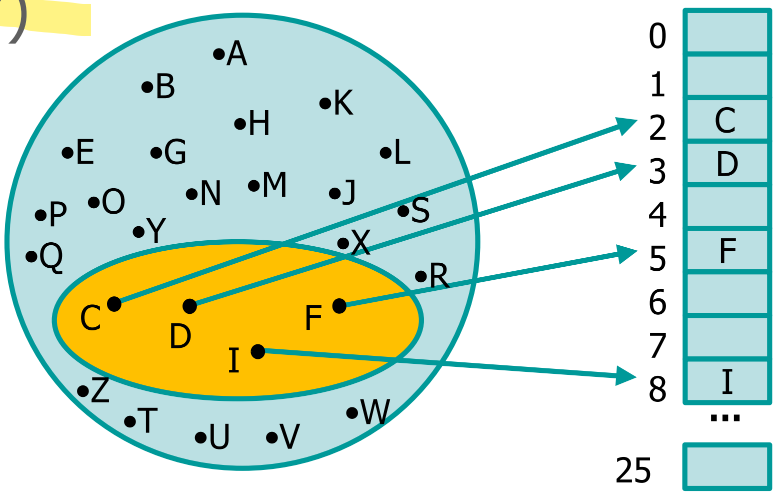
There is a 1:1 mapping between $k \in U$ and elements in st



Direct Access Tables

- This looks simple enough, but **getindex** must be general
  - If keys are integers from 0 to $|U| - 1$
    - getindex (k) = k
  - If keys are small (capital) letters in the English alphabet (i.e., a-z or A-Z)
    - getindex (k) = k – ((int) 'a')
    - getindex (k) = k – ((int) 'A')

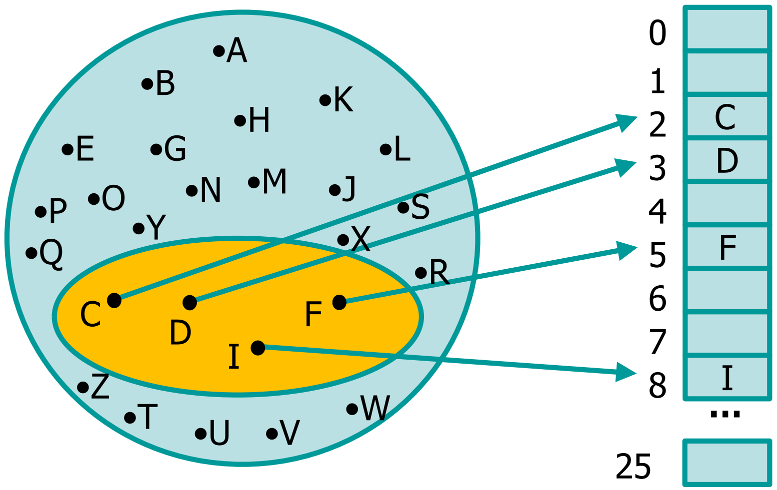
ASCII for 'a' is 97, thus 'a' is mapped onto 0 and 'z' is mapped onto 26. Same consideration for 'A' (ASCII 65).



Direct Access Tables

- If keys are generic values
  - Function **getindex** has to map those keys into integer values in the range $[0, |U| - 1]$
  - This may be very complex

we want the array not to be too big,
but at the same time getindex without collision
(more than one key withing the same element)



Advantages

- Complexity plays in favour of direct access tables
  - Insert, search, and delete complexity
    - $T(n) = \Theta(1)$
  - Init complexity
    - $T(n) = \Theta(|U|)$
  - Memory usage
    - $S(n) = \Theta(|U|)$

Disadvantages

- Limits are due to
  - For large $|U|$ the array **st** cannot be allocated
    - Direct access tables can be used only for small |U|
    - Thus, if |U| is large direct tables cannot be used
  - If $k \ll |U|$ there is a memory loss
- Funcion **getindex** has to be properly designed depending on the key type

Disadvantages

- Direct access tables have restricted practical applications
  - Used to convert keys into integers (and vice-versa) with a cost equal to 1
- When $|U|$ is large or keys are complex, direct access tables must be extended into **Hash Tables**
  - With hash-tables the **1:1** mapping between keys and array indices **is lost**
  - We must map "many" elements in a "small" table