Aufgabenblatt 4

Aufgabe 1

a) $a_n = \frac{1}{\sqrt{n^3+1}}$
$\frac{1}{\sqrt{n_0^3+1}} < \epsilon$
$\frac{1}{\epsilon^2} < n_0^3 + 1$
$\sqrt[3]{\frac{1}{\epsilon^2} - 1} < n_0$
Epsilon $\epsilon = 10^{-3}$ einsetzen:
$\sqrt[3]{\frac{1}{10^{-3^2}} - 1} = 100 < n_0$
$n_0 = 101$

Epsilon $\epsilon = 10^{-6}$ einsetzen:
$\sqrt[3]{\frac{1}{10^{-6^2}} - 1} = 10000 < n_0$
$n_0 = 10001$
Für jedes ϵ gibt es ein n_0, so dass $\sqrt[3]{\frac{1}{\epsilon^2} - 1} < n_0$ und $|a_n| < \epsilon$ für alle $n \geq n_0$.

b)
$1.\overline{359} = 1 + \sum_{k=1}^{\infty} \frac{359}{10^{3k}} = 1 + 359 * \frac{\frac{1}{10^3}}{1-\frac{1}{10^3}} = 1 + \frac{359}{999}$
$1.35 + 10^{-2} * \sum_{k=1}^{\infty} \frac{9}{10^k} = 1.35 + 10^{-2} * 9 * \frac{\frac{1}{10^2}}{1-\frac{1}{10^2}} = \frac{34}{25}$

Aufgabe 2
a)
$a_n = \frac{3n*n^2+3}{4n^2-6n+1}$
$a_n = \frac{2+n-\frac{3}{n}}{4n-6+\frac{1}{n}}$
$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\frac{2}{n}+1-\frac{3}{n^2}}{4-\frac{6}{n}+\frac{1}{n^2}} = \frac{0+1+0}{4-0+0} = \frac{1}{4}$
b) $\lim_{n\to\infty} \frac{(n^3+1)^{\frac{1}{2}}-n}{2n(n)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{(n^3+1)^{\frac{1}{2}}-n}{2n(n)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{\frac{\sqrt{n^3+1}}{n^{1.5}} - n^{-0.5}}{2} = \frac{\frac{\infty}{\infty}-0}{2} = \frac{1}{2}$
c)
$c_n = \frac{4^{n+1}(1.5+(-1)^n*\frac{1}{2^{n-2}})}{4^{n+1}(\frac{8*3^n}{4^{n+1}+1})}$
$lim_{n\to\infty} c_n = lim_{n\to\infty} \frac{1.5+(-1)^n*\frac{1}{2^{n-2}}}{\frac{8*3^n}{4^{n+1}}+1} = \frac{1.5+0}{0+1} = 1.5$
d)
$s_n = \sum_{k=1}^{n} \frac{2^{k+1}}{(-5)^k} = 2 * \sum_{k=1}^{n} \frac{2^k}{(-5)^k}$
$lim_{n\to\infty} s_n = 2 * \lim_{n\to\infty} \sum_{k=1}^{n} \frac{2^k}{(-5)^k} = 2 * \frac{\frac{2}{-5}}{1-\frac{2}{-5}} = 2 * \frac{\frac{2}{5}}{\frac{7}{5}} = \frac{4}{7}$

Aufgabe 3

Induktionsverankerung:

$\frac{2}{1(2)(3)} = \frac{1}{2} - \frac{1}{(2)(3)} = \frac{1}{3}$ ✓

Induktionsschritt:

Ang.: $\sum_{k=1}^{n} \frac{2}{k(k+1)(k+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)}$

Es gilt zu beweisen $\sum_{k=1}^{n+1} \frac{2}{k(k+1)(k+2)} = \frac{1}{2} - \frac{1}{(n+2)(n+3)} = \frac{1}{2} - \frac{1}{n^2+5n+6}$

$\sum_{k=1}^{n} \frac{2}{k(k+1)(k+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)} \,\Big|\, + \frac{2}{(n+1)(n+2)(n+3)} = \frac{2}{(n^3+6n^2+11n+6)}$

$\sum_{k=1}^{n+1} \frac{2}{k(k+1)(k+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)} * \frac{(n+3)}{(n+3)} + \frac{2}{(n+1)(n+2)(n+3)} = \frac{1}{2} - \frac{(n+1)}{(n+1)(n+2)(n+3)} = \frac{1}{2} - \frac{1}{(n+2)(n+3)}$

qed

Grenzwertsberechnung:

$\sum_{k=1}^{\infty} \frac{1}{k(k+1)(k+2)} = \lim_{n\to\infty}(\frac{1}{2} - \frac{1}{(n+2)(n+3)}) = \frac{1}{2} + \lim_{n\to\infty} -2 * \lim_{n\to\infty} \frac{1}{(n+2)} \lim_{n\to\infty} \frac{1}{(n+3)})$

$\sum_{k=1}^{\infty} \frac{1}{k(k+1)(k+2)} = \lim_{n\to\infty}(\frac{1}{2} - \frac{1}{(n+2)(n+3)}) = \frac{1}{2} + -2 * 0 * 0 = \frac{1}{2}$