Aufgabenblatt 7

Aufgabe 1

a) $f(x) = \frac{x}{1+x^2}$

$f(x)' = \frac{1-x^2}{(1+x^2)^2}$

b)

$f(x) = \frac{sin(x)}{x}$

$f'(x) = \frac{cos(x)x - sin(x)}{x^2}$

c)

$f(x) = e^{-2x}\cos(3x)$

$f'(x) = -2e^{-2x}cos(3x) + e^{-2x}sin(3x)3$

d)

$f(x) = \ln(x + \sqrt{1+x^2})$

$u = ln(x)$

$u' = \frac{1}{x}$

$v = x + \sqrt{1+x^2}$

$v' = 1 + \frac{1}{2\sqrt{1+x^2}} + 2x$

$f'(x) = \frac{\frac{x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}} = \frac{x}{1+x^2+x*\sqrt{1+x^2}} = \frac{1}{\sqrt{1+x^2}}$

e) $f(x) = \arctan(\frac{1}{x^2})$

$f'(x) = \arctan'(\frac{1}{x^2}) * -\frac{2}{x^3} = \frac{1}{1+\frac{1}{x^4}} \times -\frac{2}{x^3} = -\frac{2x}{(1+\frac{1}{x^4})x^4} = -\frac{2x}{x^4+1}$

Aufgabe 2

a) $f(x) = \frac{1}{8}x^3 - \frac{3}{2}x + 2$

$f'(x) = \frac{3}{8}x^2 - 1.5$

Tangente bei $x_0 = 0$:

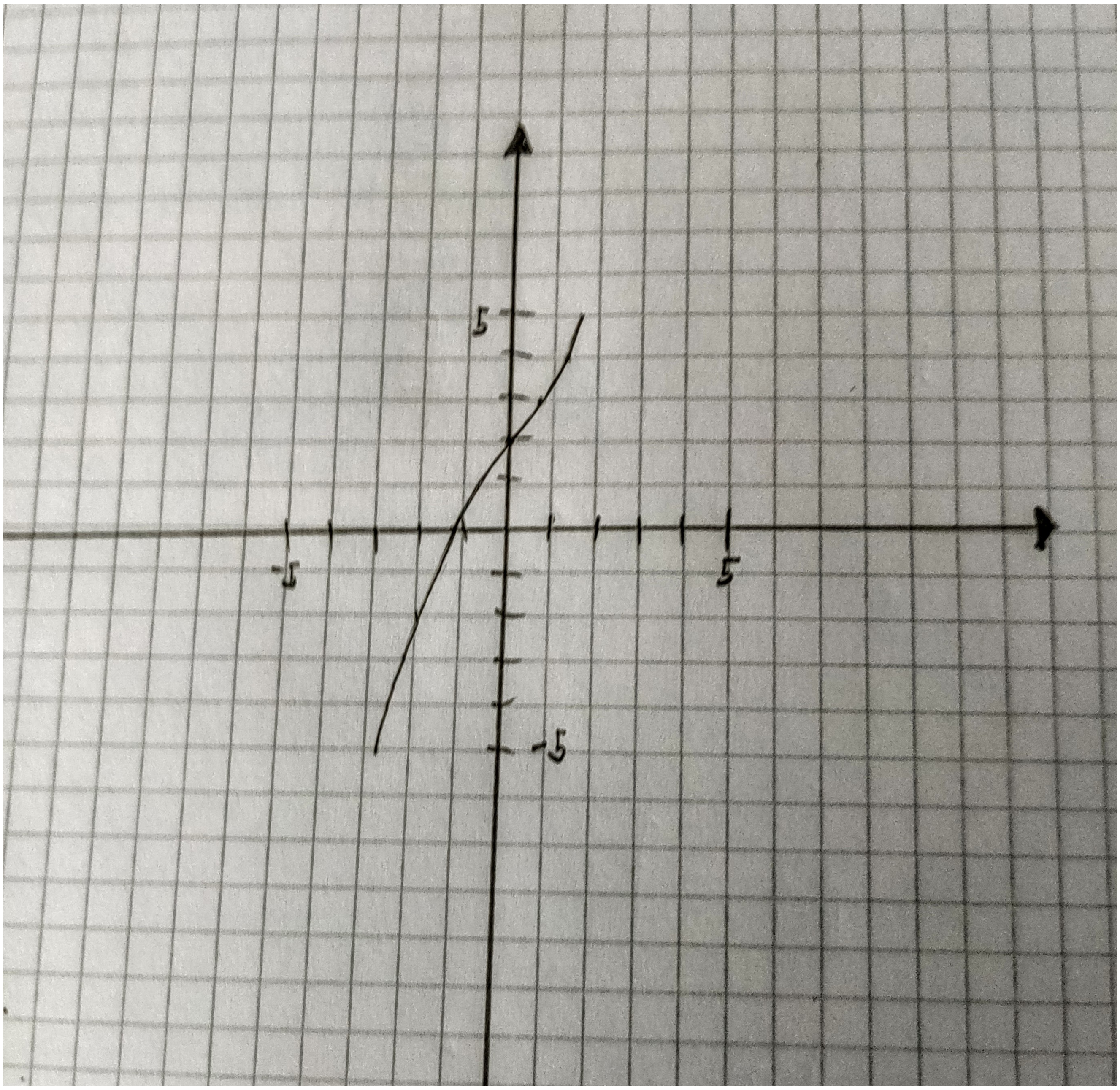
$y = 2 - 1.5x$

Tangente bei $x_1 = 2$:

$y = f(2) + f'(2)(x-2) = 0$

$\frac{1}{8}x^3 - \frac{3}{2}x + 2 = 0$
$x_1 = -4$

Tangente bei $x_2 = -2$:
$\frac{1}{8}x^3 - \frac{3}{2}x - 2 = 0$
$x_1 = 4$



b) $f(x) = cosh(x) := \frac{1}{2}(e^x + e^{-x}) = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$
$f'(x) = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$
Tangente bei $x_0 = 0$:
$y = f(0) + f'(0)x = 1$
Tangente bei $x_1 = ln(2)$:
$y = f(ln(x)) + f'(ln(2))(x - ln(2))$
$= 1.25 + 0.75(x - ln(2))$

Aufgabe 3

a) Nicht differenzierbar

b) Differenzierbar, da:

$$\lim_{x\to 0} \frac{\sin(x)}{x} = \lim_{x\to 0} cos(x) = 1$$

$$f'(x) = \begin{cases} \frac{cos(x)x + sin(x)}{x^2} \text{ für} x \neq 0 \\ 0 \text{ für } x = 0 \end{cases}$$

$$f'(0) = 0$$

c) Differenzierbar, da:

$$\lim_{x\searrow 0} x\ln(x) = \lim_{x\searrow 0} \frac{ln(x)}{\frac{1}{x}} = \lim_{x\searrow 0} \frac{x^{-1}}{-x^{-2}} = \lim_{x\searrow 0} -x = 0$$

$$f'(x) = \begin{cases} \ln(x) + 1 \text{ für} x \neq 0 \\ 0 \text{ für } x = 0 \end{cases}$$

$$f'(x) = 0$$

d)Differenzierbar, da:

$$lim_{x\to 0} \frac{1}{x^2} = \infty$$

$$lim_{x\to\infty} exp(-x) = 0$$

$$f'(0) = 0$$