Aufgabenblatt 10

Aufgabe 1

a)

$\int_0^2 x\sqrt{2x^2+1}dx$

$u = 2x^2 + 1$

$\frac{du}{dx} = 4x \Rightarrow du = 4xdx$

$\int_1^9 u^{0.5}\frac{1}{4}dx = \frac{1}{4}(\frac{u^{1.5}}{1.5}|_1^9) = \frac{1}{4}(18 - \frac{2}{3}) = 8.6\overline{7}$

b)

$\int_0^1 \frac{e^x}{e^x+1}dx$

$u = e^x$

$\frac{du}{dx} = e^x \Rightarrow du = e^x dx$

$\int_1^e \frac{du}{u+1} = \ln(u+1)|_1^e = \ln(e+1) - \ln(2) = \ln(\frac{e+1}{2})$

c)

$\int_1^2 \frac{1}{\sqrt{4-(x-1)^2}}dx$

$u = (x-1)$

$\frac{du}{dx} = 1 \Rightarrow du = dx$

$\int_0^1 \frac{1}{\sqrt{4-u^2}}du = \frac{1}{2}\int_0^1 \frac{1}{\sqrt{1-(\frac{u}{2})^2}}du = \frac{1}{2}\text{arcsin}(\frac{u}{2})|_0^1$

$= \frac{1}{2}(\text{arcsin}(\frac{1}{2}) - \text{arcsin}(0)) = \frac{\pi}{12}$

d)

$\int_0^{\frac{\pi}{2}} \cos(x)\sin^5(x)dx$

$u = sin(x)$

$\frac{du}{dx} = cos(x)$

$\int_0^1 u^5 du = \frac{1}{6}u^6|_0^1 = \frac{1}{6}$

Aufgabe 2

a)

$f(x) = \frac{x^2+x+18}{x(x^2+2x-3)} = \frac{x^2+x+18}{x(x+3)(x-1)} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-1}$

Passendes A, B, C durch einsetzen:
$A(x-1)(x+3) + Bx(x-1) + Cx(x+3) = x^2 + x + 18$

$x = 1$ einsetzen: $4C = 20 \Rightarrow C = 5$
$x = -3$ einsetzen: $12B = 24 \Rightarrow B = 2$
$x = 0$ einsetzen: $-3A = 18 \Rightarrow A = -6$

$f(x) = \frac{x^2+x+18}{x(x^2+2x-3)} = \frac{x^2+x+18}{x(x+3)(x-1)} = \frac{-6}{x} + \frac{2}{x+3} + \frac{5}{x-1}$

b)
$f(x) = \frac{-2x^3+x^2+2x+3}{(x^2+1)(x^2+2x+2)} = \frac{A}{x^2+1} + \frac{B(2x)}{x^2+1} + \frac{C}{x^2+2x+2} + \frac{D(2x+2)}{x^2+2x+2}$
Partialbruchansatz vorgegeben

$A(x^2+2x+2) + C(x^2+1) + D(2x+2)(x^2+1) = -2x^3 + x^2 + 2x + 3$
$= A((x+1)^2+1) + C(x^2+1) + D(2x+2)(x^2+1)$ ohne B, da ich weiss dass $B = 0$

$x = -1$ einsetzen: $A + 2C = 4 \Rightarrow A = 4 - 2C$
$x = 0$ einsetzen: $2A + C + 2D = 3$
$A = 4 - 2C$ einsetzen: $2(4-2C) + C + 2D = 3 \Rightarrow D = \frac{3C-5}{2}$
$D = \frac{3C-5}{2}$ und $A = 4 - 2C$ einsetzen:
$(4-2C)((x+1)^2+1) + C(x^2+1) + (\frac{3C-5}{2})(2x+2)(x^2+1) = -2x^3 + x^2 + 2x$
$\Rightarrow C = \frac{3x^3+2x^2-x-3}{x(3x^2+2x-1)}$
$A = 4 - 2(\frac{3x^3+2x^2-x-3}{x(3x^2+2x-1)})$
$D = 3(\frac{3x^3+2x^2-x-3}{x(3x^2+2x-1)}) - 5$ sieht falsch aus

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Aufgabe 3
a)
$u = 5\pi x$
$\frac{du}{dx} = 5\pi \Rightarrow du = 5\pi dx$
$5\pi \int_0^{2.5\pi} \sin(u) du = 5\pi(cos(u)|_0^{2.5\pi}) = 5\pi(cos(2.5\pi) - 1) = -5\pi$

b)
$\int_e^a \frac{1}{x\ln(x)} dx$

$u = x\ln(x)$

$du = \ln(x)dx$

c)

$\int_0^b 1 * \arcsin(x)dx = -\int_0^b \frac{x}{\sqrt{1-x^2}}dx + x\arcsin(x)|_0^b$

$u = 1 - x^2$

$\frac{du}{dx} = -2x$

$-2\int_0^{1-b^2} \frac{1}{\sqrt{u}}du + x\arcsin(x)|_0^b = 2\sqrt{1-x^2} + x\arcsin(x)|_0^b$

d)

$f(x) = \frac{1}{x^2+2x+2} = \frac{1}{1+(x+1)^2}$

$\int_0^\infty \frac{dx}{x^2+2x+2} = arctan(x+1)|_0^\infty$