Aufgabenblatt 6

Aufgabe 1

a)

1. $(\sqrt{3}+i)^3 = (\sqrt{3}+i)^2 * (\sqrt{3}+i) = (2+6i) * (\sqrt{3}+i)$
   $= (2\sqrt{3}-6) + i(2+6\sqrt{3})$
2. $(2+3i) * \frac{1+2i}{(1-2i)(1+2i)} = \frac{1+2i}{3}(2+3i) = \frac{1}{3}(1+2i)(2+3i)$
   $= \frac{1}{3}((2-6)+i(4+3)) = \frac{1}{3}(-4+7i)$
3. $\frac{1}{-5+2i\sqrt{6}} = \frac{-5-2i\sqrt{6}}{25+24}$
4. $\frac{1}{(1+i)^2} = \frac{1}{2i} = \frac{-2i}{4} = -0.5i$

b)

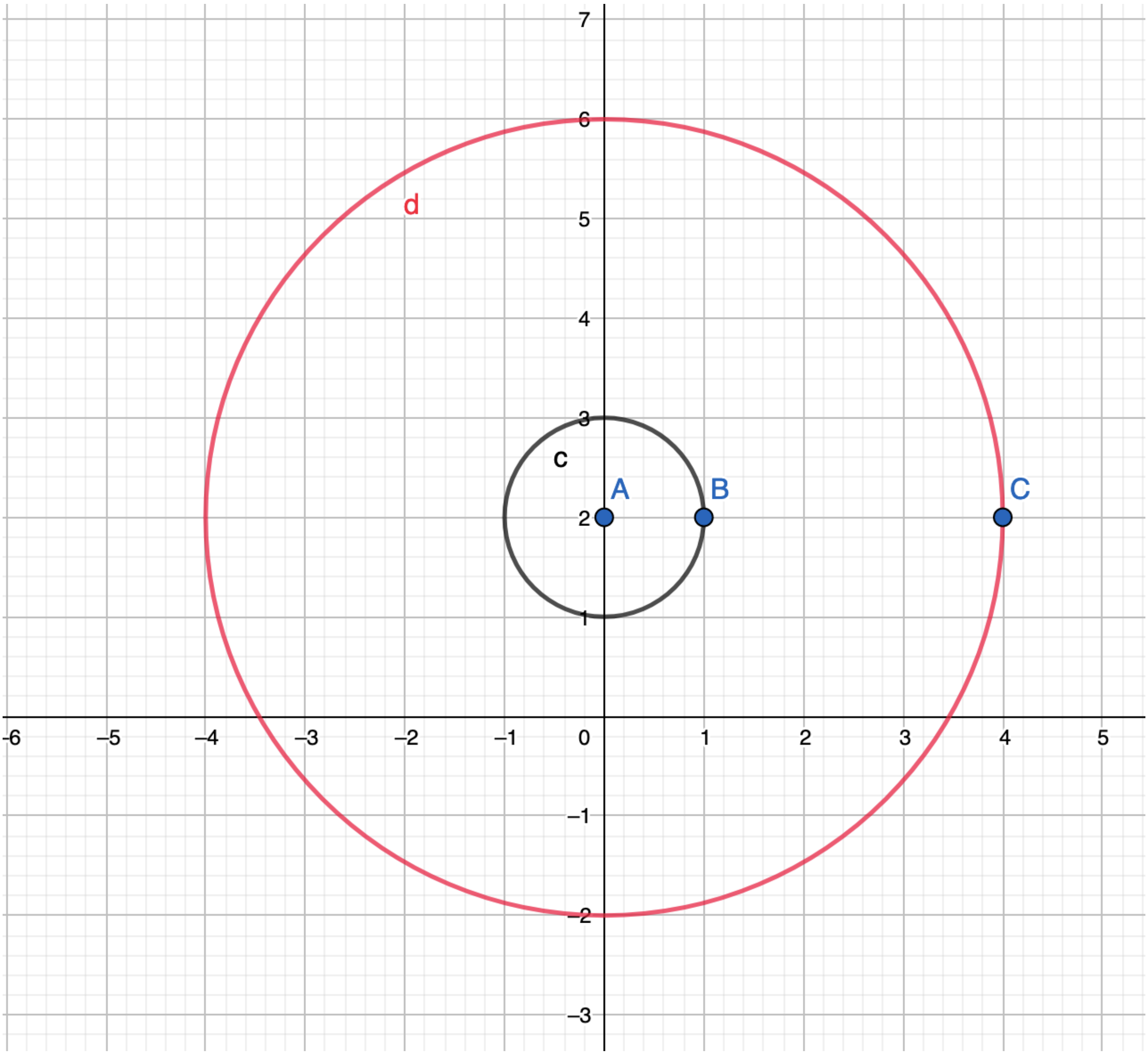
1. $i^3 = -i = 0 - 1i$ Daraus folgt $|z| = 1$ & $arg(z) = -\frac{\pi}{2}$
2. $|z| = \sqrt{6^2} = 6$
   $arg(z) = arctan(\frac{0}{6}) = 0$
3. $|z| = \sqrt{(2\sqrt{3})^2 + 2^2} = \sqrt{14}$
   $arg(z) = arctan(\frac{2}{2\sqrt{3}}) = \frac{\pi}{6}$
4. $|z| = \sqrt{(-\frac{3}{5})^2 + \frac{4}{5}^2} = \sqrt{\frac{25}{25}} = 1$
   $arg(z) = arctan(\frac{\frac{4}{5}}{\frac{3}{5}}) + \pi = arctan(\frac{4}{3}) + \pi$

c) $|z| = \sqrt{1^2 + \sqrt{3}^2}$
$\arg(z) = \arctan(\sqrt{3}) = \frac{\pi}{3}$
$z^{50} = |z|^{50}(arg(z))^{50} = 2^{50} * e^{i\frac{\pi}{3}50}$
sieht nicht besonders einfacher aus

Aufgabe 2

a) $1 < |x+(y-2)i| < 2$
$1 < \sqrt{x^2+(y-2)^2} < 2 \;\big|()^2$

$1 < x^2 + (y-2)^2 < 4$



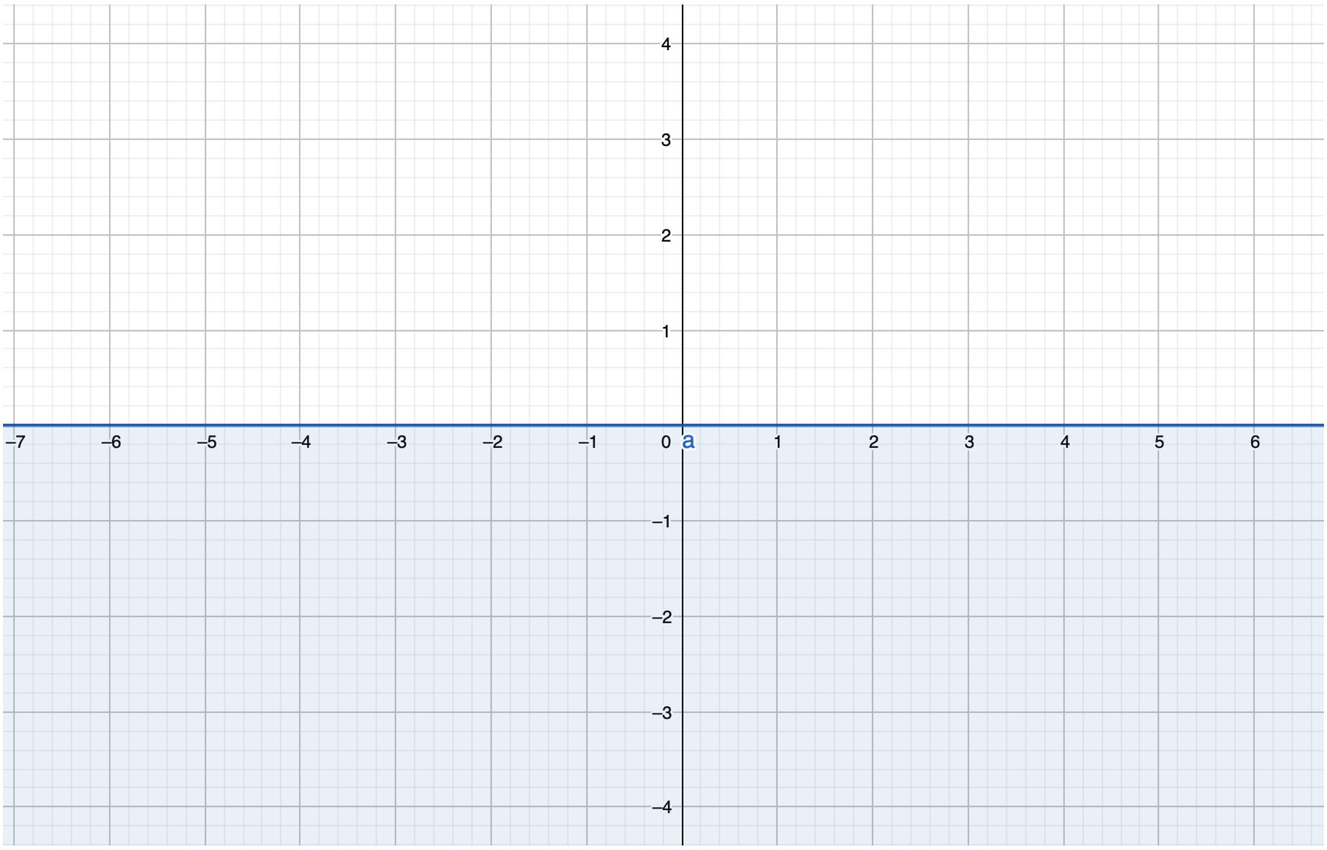
Fläche zwischen dem roten Kreis und dem schwarzen Kreis.

b) $\sqrt{x^2+(y-1)^2} > \sqrt{x^2+(y+1)^2} \; |()^2 - x^2$

$y^2 - 2y + 1 > y^2 + 2y + 1$

$-y > y$, dass heisst wenn y < 0 ist

$\mathbb{L} = \{a, b \in \mathbb{R} \mid b < 0\}$



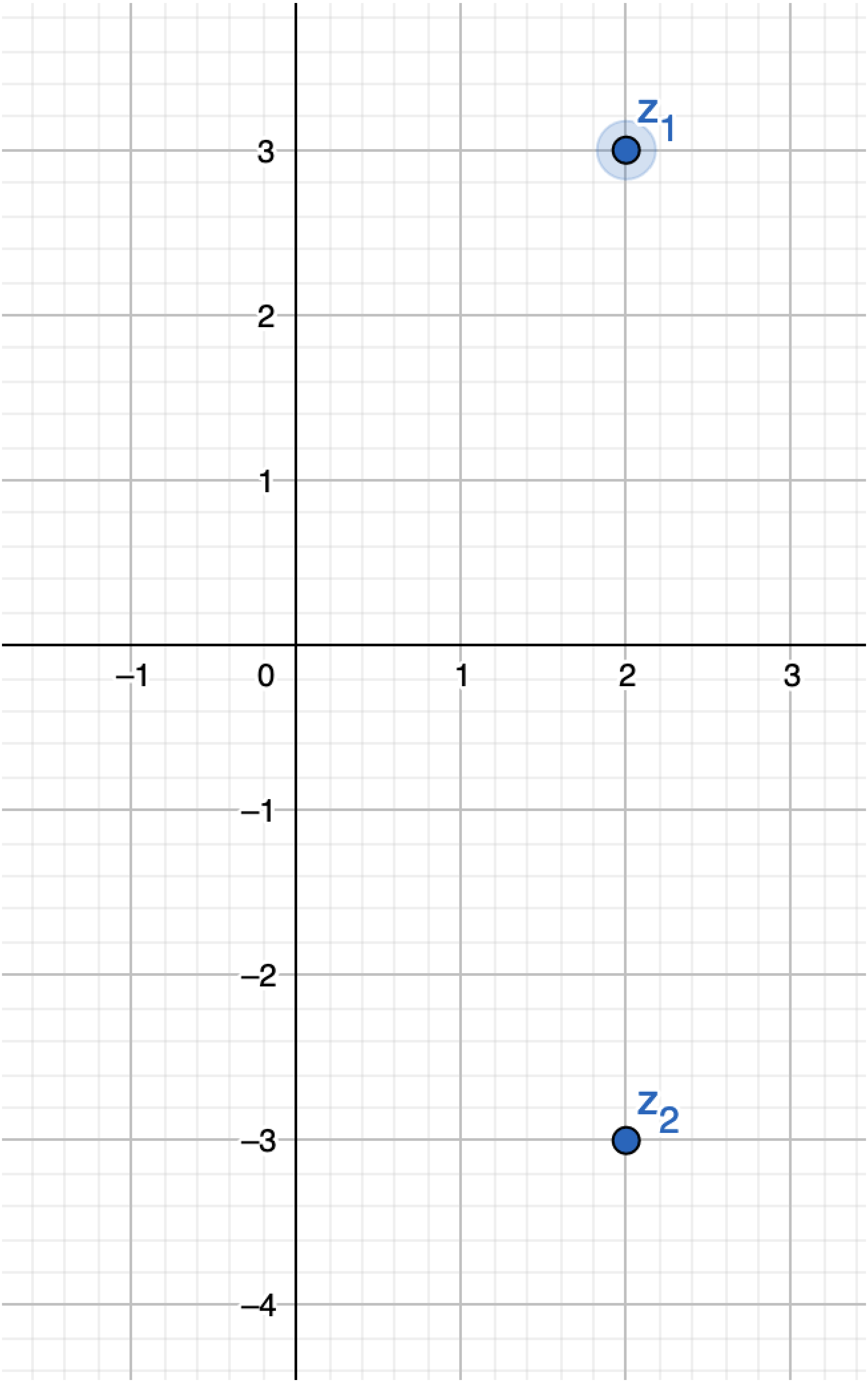
c) $2z^2 - 8z + 26 = 0$

$$2z^2 - 8z + 26 = 0$$

$$z_{1,2} = \frac{8 \pm \sqrt{64 - 4*2*26}}{2*2}$$

$$z_{1,2} = \frac{8 \pm \sqrt{144}*\sqrt{-1}}{2*2}$$

$$z_{1,2} = \frac{8 \pm 12i}{2 * 2} = 2 \pm 3i$$



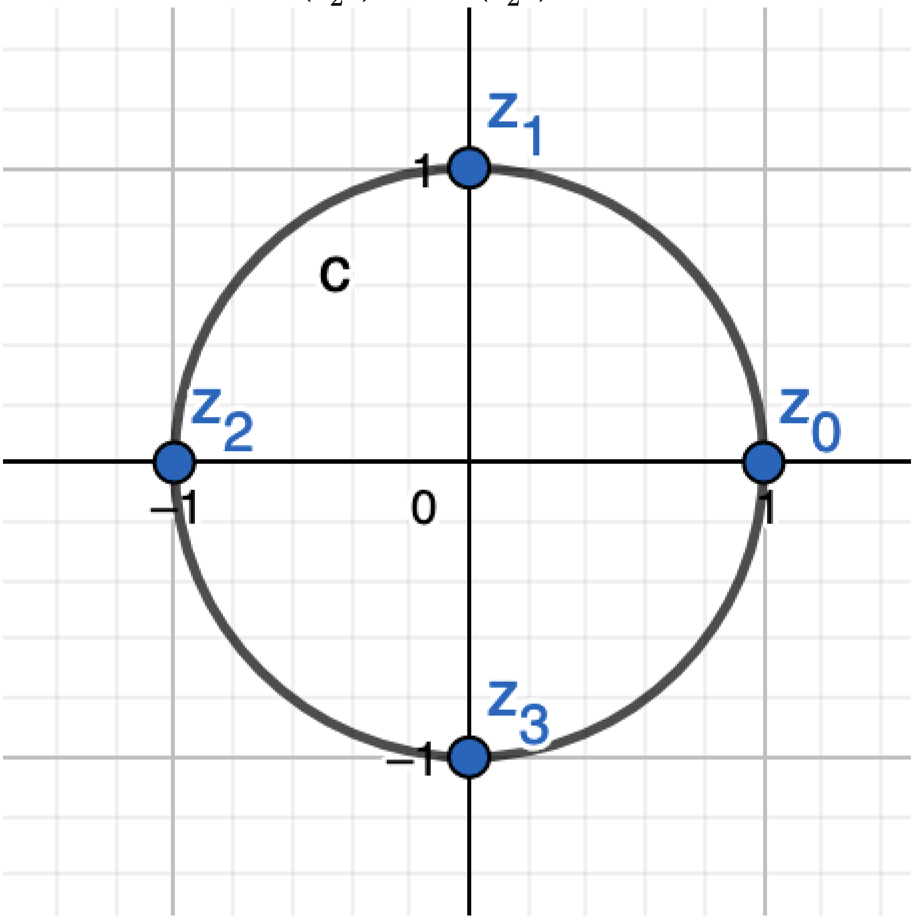
z1
z2
3
2
1
0
−1
−2
−3
−4
−1
1
2
3

d) $z_0 = e^{i0*\frac{2\pi}{4}} = 1$

$z_1 = e^{i1*\frac{2\pi}{4}} = \cos(\frac{\pi}{2}) + i\sin(\frac{\pi}{2}) = i$

$z_2 = e^{i2*\frac{2\pi}{4}} = \cos(\pi) + \sin(\pi)i = -1$

$z_3 = e^{32*\frac{2\pi}{4}} = \cos(\frac{3\pi}{2}) + \sin(\frac{3\pi}{2})i = -i$



e)

$z^4 = -4$

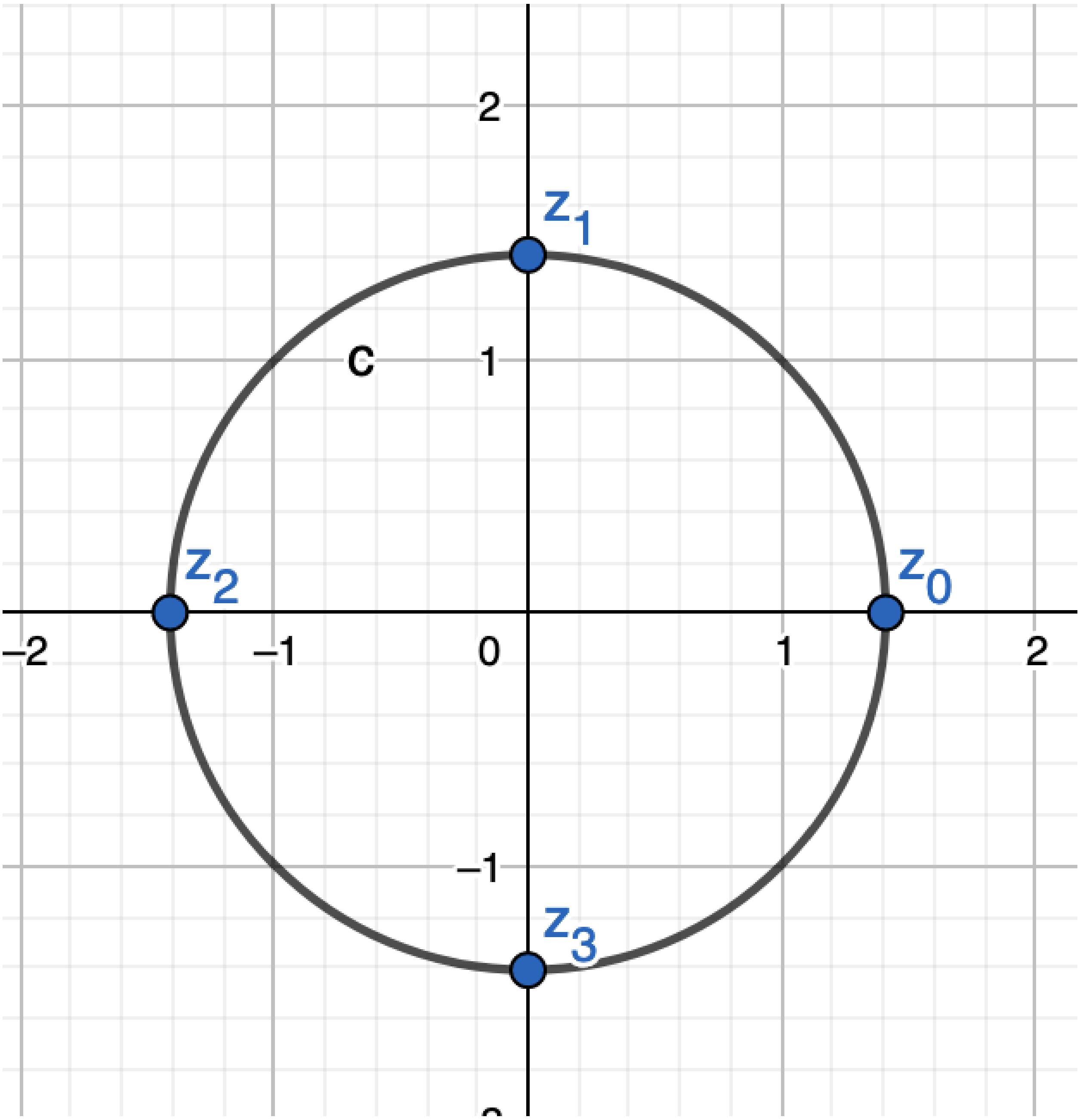
$r = \frac{a}{e^{i\pi}} = 4$

$z_0 = \sqrt[4]{4} * e^{i\frac{0+0*2\pi}{4}} = \sqrt{2}$

$z_1 = \sqrt[4]{4} * e^{i\frac{0+1*2\pi}{4}} = \sqrt{2}e^{i\frac{0+2\pi}{4}} = \sqrt{2}e^{i\frac{\pi}{2}} = \sqrt{2}i$

$z_2 = \sqrt{2}e^{i\pi} = -\sqrt{2}$

$z_3 = \sqrt[4]{4} * e^{i\frac{6\pi}{4}} = -\sqrt{2}i$



Aufgabe 3

a)

Über $\mathbb{R}$: $p(x) = x^4 - 2x^2 - 15 = (x^2 - 5)(x^2 - 3)$
$= (x - \sqrt{5})(x + \sqrt{5})(x^2 + 3)$

b)

Über $\mathbb{R}$: $p(x) = x^3 - x^2 - 8x + 12 = (x - 2)(x^2 + x - 6) = (x - 2)^2(x + 3)$

Ich bin mir bei a) und b) nicht sicher wie es über $\mathbb{C}$ auflösbar sein soll, wenn es so bereits in lineare Faktoren, die nicht weiter zerlegbar sind, faktoriert werden kann...

c)

Über $\mathbb{C}$: $p(x) = x^4 + 4 = (x - \sqrt{2})(x - \sqrt{2}e^{i\frac{\pi}{2}})(x + \sqrt{2})(x + \sqrt{2}e^{i\frac{\pi}{2}})$

Über $\mathbb{R}$:

$(x - \sqrt{2})(x + \sqrt{2})(x - \sqrt{2}i)(x + \sqrt{2}i) = (x - \sqrt{2})(x + \sqrt{2})(x^2 + x\sqrt{2}i - x\sqrt{2}i$ -

$= (x - \sqrt{2})(x + \sqrt{2})(x^2 - 2)$