If you have any questions please come and ask! There is no such a thing in science as not to ask when in doubt.

Gravitational waves

Let's start by reading Section 8.1.1 of Maggiore's book. Take now two masses m_1 and m_2, connected by a spring of constant k along the x-axis. The oscillations are damped by a friction with $F = -a\, dx/dt$. At equilibrium the two masses are at a distance L. The system is hit by a gravitational wave propagating in the z direction, with $h_+ = h \cos(\omega t)$ along x and $h_\times = 0$.

- Find the equation of motion for $x(t)$. It should resemble a damped harmonic oscillator, with an undamped angular frequency ω_0 dependent on k, m_1 and m_2, and a source term given by GWs.
- Solve this equation in the form $x(t) = C \cos(\omega t + \phi)$, and find the resonant frequency;
- Averaging over one oscillation compute the total energy of the system and the radiated energy;
- What qualitatively happens in $h_\times \neq 0$ but $h_+ = 0$?
- Take $L = 10$ m, $h = 10^{-21}$, $\omega_0 = 2\pi$kHz, $m_1 = m_2 = 10^3$kg, and a quality factor $Q \sim 10^6$, see Maggiore. Do you think we can detect gravitational waves with such a system?

Cosmology

All light in the Universe gets deflected, especially those photons that travel the longest like the CMB ones. In this exercise we want to understand the basic of CMB lensing.

- Verify all the equations in Dodelson&Schmidt, Section 13.3, on CMB lensing. You might need additional expressions from other parts of the same chapter.
- Evaluate the lensed CMB temperature power spectrum using Eq. 13.21. In principle you could compute the lensing power spectrum starting from the attached power spectrum at $z = 0$ and the following cosmological background: $\Omega_m = 0.3$, $\Omega_\Lambda = 1 - \Omega_m - \Omega_r$, and the radiation corresponds to a black body of $T_0 = 2.7$ K. The value of the Hubble constant is $H_0 = 68$ km/s/Mpc. You will find the CMB lensing power spectrum in the attached files as well. The latter could differ from your calculation at low ℓ due to some assumptions made in the Section 13.3, but the two should be broadly consistent.
- Repeat the first point for CMB polarization. This time what gets shifted is the polarization tensor I_{ij}. Assuming there are no initial B modes, compute the resulting E and B power spectrum. Plot your results.