Time Series modelling the term structure of interest rate

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Time Series Econometrics Project for December 2019

Time Series Econometrics Project

1 Summary

2 Figure summary

3 ABSTRACT

The purpose of this project is to verify whether interest rates are related to each other by looking at short-term (monthly) and long-term (yearly) interest rates.

4 INTRODUCTION

Firstly, we must check if a series has a Unit Root. For doing this, we must run the Unit Root Testing (for this project I've always run the Augmented Dickey-Fuller test) on all the series. The Null Hypothesis is that a series has a unit root and we reject the Null Hypothesis if the p-values (returned by the unit root test) is less than 0.05. From the test we obtain the following values.

	t-Statistic		*Prob.*
M1	-2.959225		0.0395
	1%	-3.441493	
	5%	-2.866348	
	10%	-2.569390	
M2	-2.125743		0.2347
	1%	-3.441533	
	5%	-2.866365	
	10%	-2.569399	
M3	-2.262989		0.1846
	1%	-3.441533	
	5%	-2.866365	
	10%	-2.569399	
M6	-2.430617		0.1337
	1%	-3.441493	
	5%	-2.866348	
	10%	-2.569390	
Y1	-2.199465		0.2068
	1%	-3.441513	
	5%	-2.866356	
	10%	-2.569395	
Y2	-1.987172		0.2926
	1%	-3.441513	
	5%	-2.866356	
	10%	-2.569395	
Y3	-1.825867		0.3679
	1%	-3.441513	
	5%	-2.866356	
	10%	-2.569395	
Y4	-1.442928		0.5619
	1%	-3.441493	
	5%	-2.866348	
	10%	-2.569390	
Y5	-1.410224		0.5782
	1%	-3.441493	
	5%	-2.866348	
	10%	-2.569390	

Table 1: Unit Root Test of all the interest rate

So, we can say that only M1 has not a unit root because has a Prob. less than 0.05[1] (and we reject the Null Hypothesis), so it is *I*(0). All other series have a unit root.

We can plot all the series in a graph like this:

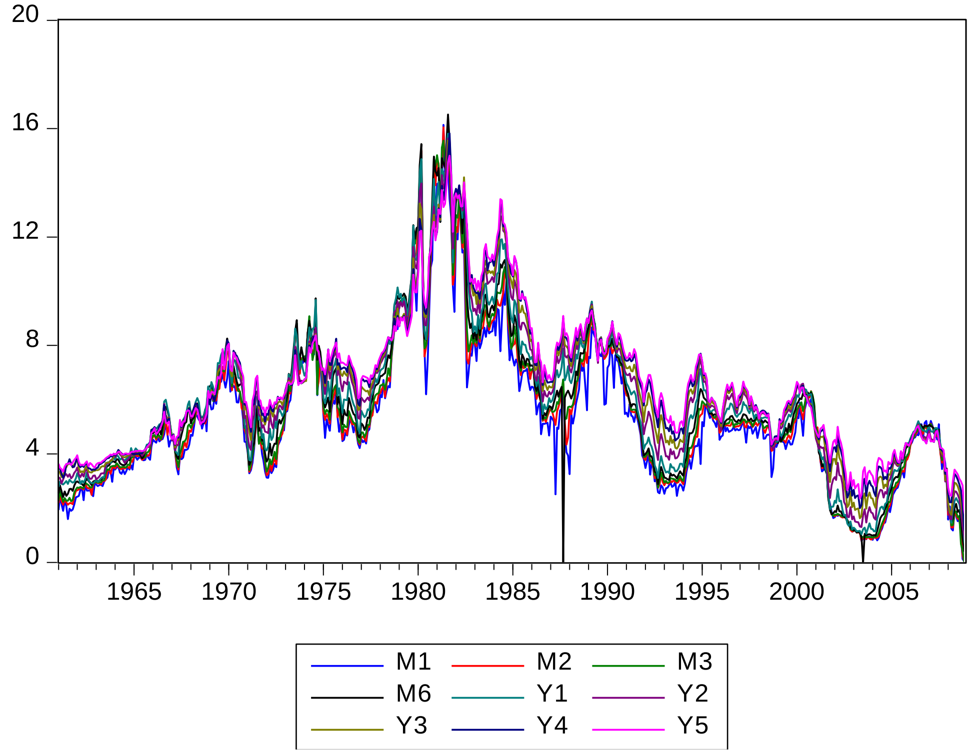

Figure 1: Plot of all the interest rates

Graphically we can see that there is no trend in these series.

For the development of this project I have chosen to use only rates M2 and Y3 because all the series behave in the same way, therefore it is indifferent which to choose. Their graph is as follows:

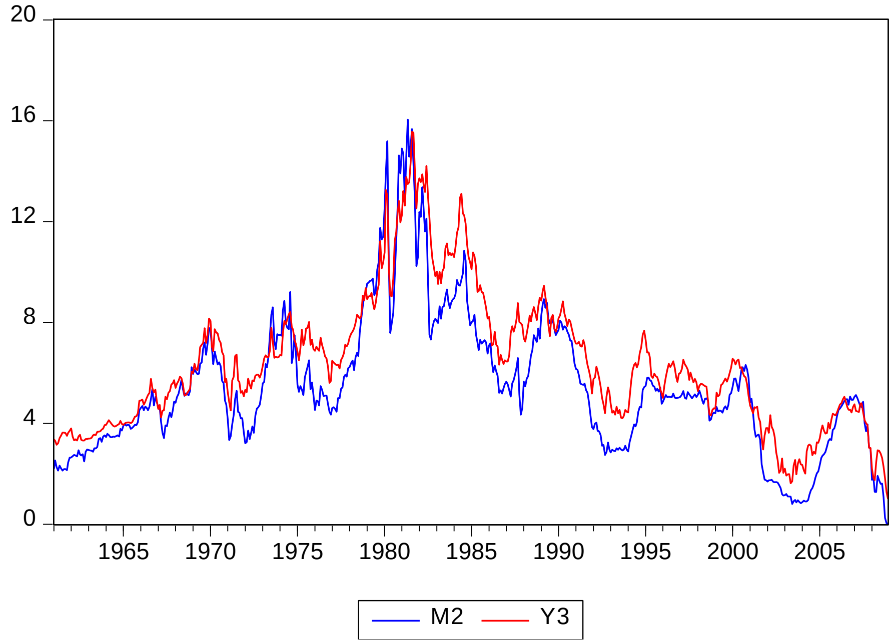

Figure 2: Plot of series M2 and Y3

[1] I've chosen this value as threshold value of 5% level of confidence.

5 CHECK IF INTEREST RATES ARE *I*(1)

To check if the two series are I(1) we must run the Unit Root Test.

5.1 UNIT ROOT TESTING

The chosen test type is the Augmented Dicken-Fuller test and I've check for unit root in level, including only intercept and with the Schwartz Info Criteria.

By running the unit root test on M2 and Y3 we obtain the following result.

Null Hypothesis: M2 has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.125743	0.2347
Test critical values:	1% level	-3.441533	
	5% level	-2.866365	
	10% level	-2.569399	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(M2)
Method: Least Squares
Date: 12/06/19 Time: 20:03
Sample (adjusted): 1961M04 2008M12
Included observations: 573 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
M2(-1)	-0.017114	0.008051	-2.125743	0.0340
D(M2(-1))	0.170229	0.041582	4.093832	0.0000
D(M2(-2))	-0.116236	0.041702	-2.787311	0.0055
C	0.089804	0.049104	1.828877	0.0679

R-squared	0.044108	Mean dependent var	-0.003939
Adjusted R-squared	0.039069	S.D. dependent var	0.533851
S.E. of regression	0.523319	Akaike info criterion	1.549706
Sum squared resid	155.8280	Schwarz criterion	1.580078
Log likelihood	-439.9906	Hannan-Quinn criter.	1.561553
F-statistic	8.751929	Durbin-Watson stat	1.986254
Prob(F-statistic)	0.000011		

Figure 4: Unit Root test for M2

Null Hypothesis: Y3 has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.825867	0.3679
Test critical values:	1% level	-3.441513	
	5% level	-2.866356	
	10% level	-2.569395	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(Y3)
Method: Least Squares
Date: 12/06/19 Time: 20:04
Sample (adjusted): 1961M03 2008M12
Included observations: 574 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Y3(-1)	-0.011922	0.006529	-1.825867	0.0684
D(Y3(-1))	0.137645	0.041560	3.311985	0.0010
C	0.072317	0.044976	1.607900	0.1084

R-squared	0.022767	Mean dependent var	-0.004030
Adjusted R-squared	0.019344	S.D. dependent var	0.416418
S.E. of regression	0.412371	Akaike info criterion	1.071426
Sum squared resid	97.09848	Schwarz criterion	1.094175
Log likelihood	-304.4994	Hannan-Quinn criter.	1.080300
F-statistic	6.651397	Durbin-Watson stat	1.976272
Prob(F-statistic)	0.001395		

Figure 3: Unit Root Test for Y3

Since the p-value (Prob.) is greater than 0.05 for both the interest rates, we refuse to reject the Null Hypothesis. So, M2 and Y3 has not a Unit Root and from this we can say that they are I(1).

6 COINTEGRATION TEST

To avoid inconsistent estimates, the series must be I(1).

In order to check if the interest rates are cointegrated, we must run the Single-Equation Cointegration Test on the two series.

Date: 12/09/19 Time: 13:44
Series: Y3 M2
Sample: 1961M01 2008M12
Included observations: 576
Null hypothesis: Series are not cointegrated
Cointegrating equation deterministics: C
Automatic lags specification based on Schwarz criterion (maxlag=18)

Dependent	tau-statistic	Prob.*	z-statistic	Prob.*
Y3	-5.473176	0.0000	-57.15238	0.0000
M2	-5.670049	0.0000	-60.94962	0.0000

*MacKinnon (1996) p-values.

Intermediate Results:

	Y3	M2
Rho - 1	-0.099395	-0.105999
Rho S.E.	0.018160	0.018695
Residual variance	0.145695	0.166911
Long-run residual variance	0.145695	0.166911
Number of lags	0	0
Number of observations	575	575
Number of stochastic trends**	2	2

**Number of stochastic trends in asymptotic distribution

Figure 5: Cointegration Test for M2 and Y3

I've chosen as Dependent variable Y3. Since Prob. is less than 0.05, we reject the Null Hypothesis that say the Series are not cointegrated. So, the interest rates are cointegrated.

7 CHECK IF SPREADS IS *I*(0)

To check if the spreads is *I*(0) we have to define a new series called "SPREAD" obtained by subtracting M2 to Y3, as follow:

$$spread = Y3 - M2$$

This new series has the following graph:

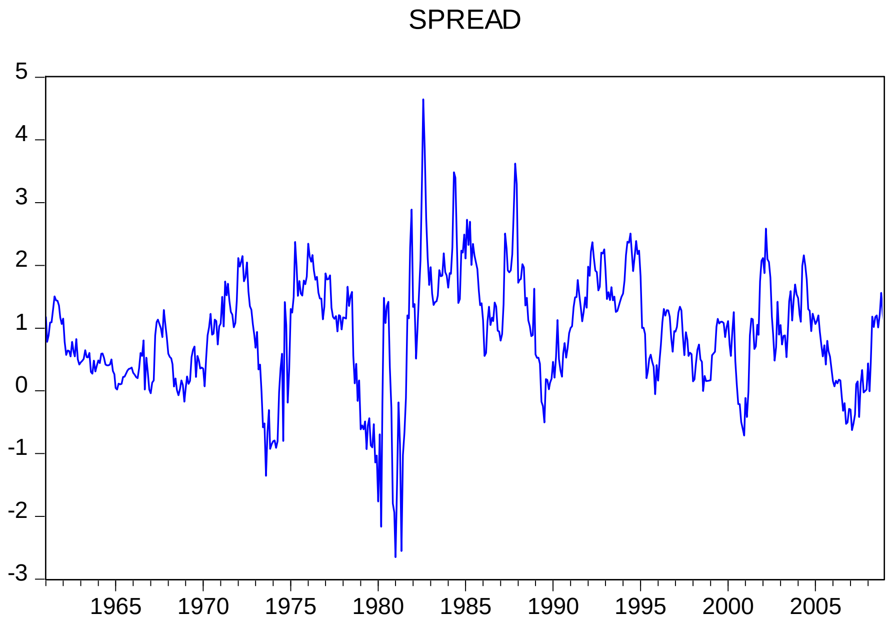

Figure 6: Plot of series "SPREAD"

Now we can run the Unit Root Test in order to check if it has a Unit Root.

Null Hypothesis: SPREAD has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.675533	0.0000
Test critical values:	1% level	-3.441493	
	5% level	-2.866348	
	10% level	-2.569390	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(SPREAD)
Method: Least Squares
Date: 12/09/19 Time: 15:24
Sample (adjusted): 1961M02 2008M12
Included observations: 575 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SPREAD(-1)	-0.106377	0.018743	-5.675533	0.0000
C	0.096501	0.024161	3.994054	0.0001

R-squared	0.053224	Mean dependent var	-0.000333
Adjusted R-squared	0.051572	S.D. dependent var	0.421227
S.E. of regression	0.410221	Akaike info criterion	1.059233
Sum squared resid	96.42540	Schwarz criterion	1.074379
Log likelihood	-302.5295	Hannan-Quinn criter.	1.065140
F-statistic	32.21168	Durbin-Watson stat	1.989445
Prob(F-statistic)	0.000000		

Figure 7: Unit Root Test for SPREAD

Since Prob. is less then 0.05, we reject the Null Hypothesis. Therefore, the spread has not a unit root, so it is *I*(0).

8 CORRELATION BETWEEN INTEREST RATES

8.1 VAR

Firstly, we must estimate a standard VAR with Endogenous Variable “M2 Y3”, Exogenous Variable “c” and Lag Intervals for Endogenous “1 2”.

Vector Autoregression Estimates
Date: 12/09/19 Time: 13:39
Sample (adjusted): 1961M03 2008M12
Included observations: 574 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	M2	Y3
M2(-1)	0.951758	0.022187
	(0.05323)	(0.04257)
	[17.8808]	[0.52116]
M2(-2)	-0.022637	0.018329
	(0.05342)	(0.04273)
	[-0.42371]	[0.42894]
Y3(-1)	0.353127	1.100199
	(0.06809)	(0.05446)
	[5.18647]	[20.2033]
Y3(-2)	-0.297961	-0.151888
	(0.06750)	(0.05399)
	[-4.41441]	[-2.81350]
C	0.032250	0.104398
	(0.05931)	(0.04744)
	[0.54374]	[2.20068]
R-squared	0.965524	0.976162
Adj. R-squared	0.965282	0.975995
Sum sq. resids	150.6279	96.35725
S.E. equation	0.514513	0.411515
F-statistic	3983.818	5825.156
Log likelihood	-430.5173	-302.3001
Akaike AIC	1.517482	1.070732
Schwarz SC	1.555397	1.108647
Mean dependent	5.447459	6.357638
S.D. dependent	2.761324	2.656021

Determinant resid covariance (dof adj.)	0.026510
Determinant resid covariance	0.026050
Log likelihood	-582.0401
Akaike information criterion	2.062858
Schwarz criterion	2.138687
Number of coefficients	10

Figure 8: VAR Estimation for M2 and Y3

From this we can view, from the Lag Length Criteria, the correct number of lags.

VAR Lag Order Selection Criteria
Endogenous variables: M2 Y3
Exogenous variables: C
Date: 12/06/19 Time: 20:10
Sample: 1961M01 2008M12
Included observations: 568

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-2115.601	NA	5.933331	7.456340	7.471629	7.462306
1	-599.9211	3015.348	0.028949	2.133525	2.179392	2.151424
2	-581.0818	37.34687	0.027475	2.081274	2.157720*	2.111105*
3	-575.6442	10.74111	0.027336	2.076212	2.183236	2.117976
4	-567.6406	15.75371	0.026954*	2.062115*	2.199717	2.115811
5	-566.0492	3.121095	0.027183	2.070596	2.238776	2.136225
6	-562.0043	7.904558	0.027179	2.070438	2.269197	2.148000
7	-555.7991	12.08281*	0.026969	2.062673	2.292010	2.152167
8	-552.8822	5.659038	0.027073	2.066487	2.326402	2.167914

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Figure 9: VAR Lag Lenght Criteria

I've chosen the Schwarz information criteria because the Akaike may select larger value that may give an inconsistent estimation. So, the number of lags is 2. This information will be used for tuning the VAR Estimation.

8.2 Vector Error Correction Estimates

Now we can Estimate a VAR with VAR type equals to "Vector Error Correction", with Endogenous Variable "M2 Y3", with Cointegration Option: "Intercept (no trend) in CE – no intercept in VAR" and with Lag Intervals equals to "1 2" (the number chosen previously).

Vector Error Correction Estimates
Date: 12/09/19 Time: 09:13
Sample (adjusted): 1961M04 2008M12
Included observations: 573 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
M2(-1)	1.000000	
Y3(-1)	-0.986179	
	(0.06019)	
	[-16.3852]	
C	0.813581	
	(0.41460)	
	[1.96231]	
Error Correction:	D(M2)	D(Y3)
CointEq1	-0.063623	0.041414
	(0.02480)	(0.01994)
	[-2.56523]	[2.07708]
D(M2(-1))	0.056500	0.003299
	(0.05426)	(0.04362)
	[1.04119]	[0.07562]
D(M2(-2))	-0.031466	0.012370
	(0.05315)	(0.04273)
	[-0.59200]	[0.28950]
D(Y3(-1))	0.284645	0.140301
	(0.06700)	(0.05387)
	[4.24812]	[2.60467]
D(Y3(-2))	-0.150444	-0.109573
	(0.06807)	(0.05472)
	[-2.21015]	[-2.00239]
R-squared	0.089229	0.034071
Adj. R-squared	0.082816	0.027269
Sum sq. resids	148.4724	95.95052
S.E. equation	0.511268	0.411007
F-statistic	13.91192	5.008761
Log likelihood	-426.1374	-301.0611
Akaike AIC	1.504843	1.068276
Schwarz SC	1.542808	1.106242
Mean dependent	-0.003939	-0.003750
S.D. dependent	0.533851	0.416728
Determinant resid covariance (dof adj.)		0.026216
Determinant resid covariance		0.025761
Log likelihood		-577.8286
Akaike information criterion		2.062229
Schwarz criterion		2.160940
Number of coefficients		13

Figure 10: Vector Error Correction Estimate for M2 and Y3

8.3 Impulse Response Function

From the Vector Error Correction Estimate output we can run the Impulse Response Function with option: Impulses “m2 y3”, responses “m2 y3”, with Decomposition Method “Cholesky – no dof adjustment” and with Cholesky Ordering “m2 y3”. From this it is possible to see if the rates are in some way correlated. (And from the theory we know that there exists a correlation between short- and long-term rates)

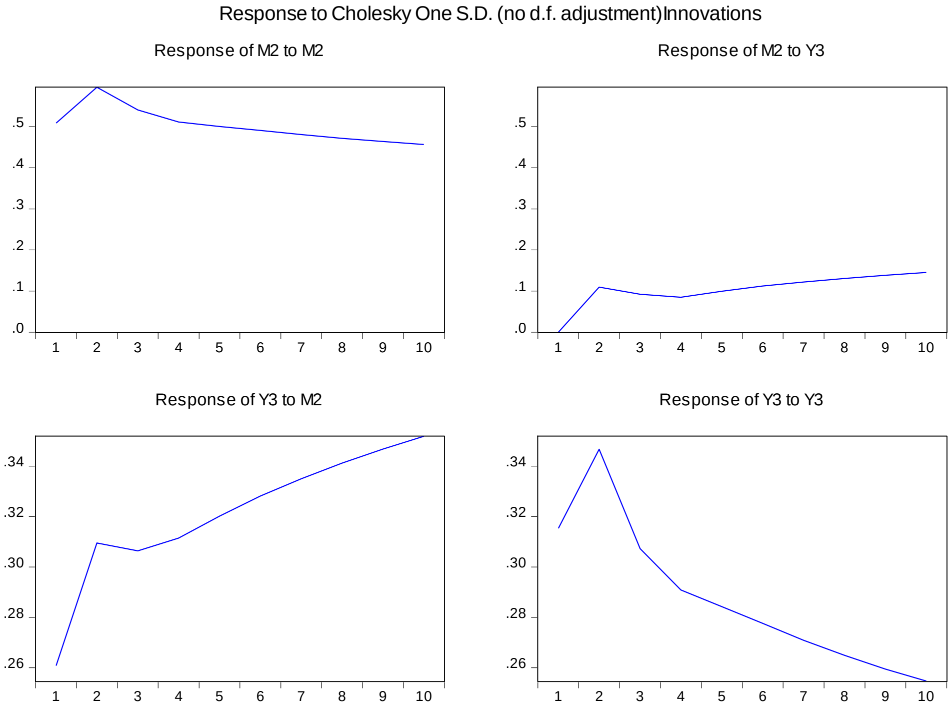

Figure 11: Impulse Response Function for VECM

8.4 Conclusion

From the Impulse Response Function, we can view that Y3 response to M2. So, the long-term responds to the short term. In addition, we can see that short-term react to long-term in short run with temporary shocks. We can also check this by running the Granger Causality Test.

Pairwise Granger Causality Tests
Date: 12/04/19   Time: 14:19
Sample: 1961M01 2008M12
Lags: 2

Null Hypothesis:	Obs	F-Statistic	Prob.
Y3 does not Granger Cause M2	574	14.1010	1.E-06
M2 does not Granger Cause Y3		2.18853	0.1130

Figure 12: Granger Causality Test for M2 and Y3

The Null Hypothesis is “Y3 does not Granger Cause M2” and with a Prob. less than 0.05 we reject the Null Hypothesis. So, we can say that Y3 Granger cause M2.