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Time delay cosmography

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Abstract Gravitational time delays, observed in strong lens systems where the variable background source is multiply-imaged by a massive galaxy in the foreground, provide direct measurements of cosmological distance that are very complementary to other cosmographic probes. The success of the technique depends on the availability and size of a suitable sample of lensed quasars or supernovae, precise measurements of the time delays, accurate modeling of the lens mass distributions, and our ability to characterize the density environment along the line of sight to the source. We review the progress made during the last 15 years, during which the first competitive cosmological inferences with time delays were made, and look ahead to the potential of significantly larger lens samples in the near future.

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1 Introduction [TT]

The measurement of cosmic distances is central to our understanding of cosmography, i.e. the description of the geometry and kinematics of the universe. The discovery of the period luminosity relation for cepheids led to the realization that the universe is much bigger than the Milky Way and that it is currently expanding. Relative distance measurements based on supernova Ia

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light curves were the turning point in the discovery of the acceleration of the universe (Riess et al, 1998; Perlmutter et al, 1999).

In the two decades since the discovery of the acceleration of the universe, distance measurements have improved steadily. For example, the Hubble constant has now been measured to 3% precision (Riess et al, 2011; Freedman et al, 2012) while the distance to the last scattering surface of the cosmic microwave backgrond is now known to better than 1% precision Bennett et al (2013); **?**. This precision is more than sufficient for all purposes related to our understanding of phenomena occurring within the universe, like galaxy evolution.

In spite of all this progress, the most fundamental question still remains unanswered. What is causing the acceleration? Is this *dark energy* something akin to Einstein's cosmological constant or is it a dynamical component? Answering this question from an empirical standpoint will require further improvements in the precision of distance measurements (Weinberg et al, 2013). [EXPLAIN DEGENERACY IN CMB DATA AND NEED FOR LOWER REDSHIFT PROBES]

Many dedicated experiments are currently under way or being planned with ths goal in mind.

Precision, however, is not sufficient by itself. In addition to controlling the known statistical uncertainties, modern day experiments need to control stystematic errors in order to fullfill their potential, including the infamous unknow unknowns. The most direct way to demonstrate *accuracy* is to compare independent measurements with comparable *precision*. [MENTION LITTLE TENSION BETWEEN WMAP9 AND PLANCK: MAKE PLOT SHOWING H0 LOCALLY VS H0 FROM CMB?]

Ideally, the comparison between independent measurements should be carried out blindly, so as to minimize experimenter bias. Two blind mutually blind measurements agreeing that the equation of state parameter w is not -1 would be a very convincing demonstration that the dark energy is not the cosmological constant. Conversely, the significant disagreement of two independent measurements, could open the door to the discovery of new physics.

In this review we focus on gravitational time delay as a tool for cosmography. Gravitational time delays are a natural phenomenon in general relativity and provide a direct and elegant way to measure absolute distances out to cosmological redshift. When the line of sight to a distant source of light is suitably well aligned with an intervening massive system, multiple images appear to the observer. The arrival time of the images depends on the interplay of the geometric and gravitational delays specific to the configuration. If the emission from the source is variable in time, the difference in arrival time is measurable, and can be converted into the so-called time delay distance $D_{\Delta\mathrm{t}}$, a combination of angular diameter distances to the deflector and source. $D_{\Delta\mathrm{t}}$ is inversely proportional to the Hubble Constant H_0 and it is more weakly dependent on other cosmological parameters. The sensitivity to H_0 and independence to the local distance ladder method make time delays a very valuable cosmological tool for precise and accurate cosmology. [EXPLAIN MORE IN

DETAIL INDEPENDENCE OF DISTANCE LADDER AND TD] As several authors have pointed out (Linder, 2011; ?; Weinberg et al, 2013), achieving sub-percent precision and accuracy on the measurement of the Hubble constant is a powerful addition to stage III and IV dark energy experiments.

This review is organized as follows. In Section 1 we summarize the history of time delay cosmography up until the turn of the millennium, in order to give a sense of the early challenges and how they were overcome. In Section 3, we review the theoretical foundations of the method, in terms of the gravitational optics version of Fermat's principle. In Section 4 we describe in some detail the elements of a modern time delay distance measurement, emphasizing recent advances and remaining challenges. In Section 5 we elucidate the connection between time delay distance measurements and cosmological parameters, discussing complementarity with other cosmological probes. Section 6 critically examines the future of the method, discussing prospects for increasing the precision, testing for accuracy, and synergy with other future probes of dark energy. A brief summary is given in Section 7. Owing to space limitations, we could only present a selection of all the beautiful work that has been published on this topic in the past decades. We refer the readers to recent (Bartelmann, 2010; Ellis, 2010; Treu, 2010; Treu et al, 2012; Jackson, 2013, 2015; Treu and Ellis, 2015) and not-so-recent (Blandford and Narayan, 1992; Courbin et al, 2002; Kochanek and Schechter, 2004; Falco, 2005; Schneider et al, 2006) excellent reviews and textbooks (Schneider et al, 1992) for additional information and historical context.

2 A brief history of time delay cosmography [TT]

Refsdal (?, Ref64)irst suggested in 1964 that lens time delays could be used to measure absolute distances out to cosmological distances, and therefore the Hubble Constant to leading order. Unfortunately, no strong lensing systems were known at that time, and therefore his intuition remained purely theoretical for over a decade.

The prospects of using time delays for cosmography suddenly brightened in the late seventies, with the discovery of the first strongly lensed quasars (Walsh et al, 1979). Even though they were not the strongly lensed supernovae that Refsdal had in mind, quasars fluxes are sufficiently variable (Vanderriest et al, 1982) that people were able to start to put Refsdal's idea in practice (Vanderriest et al, 1989). For completeness, we should mention that the first multiply imaged supernova has been discovered in 2014, fifty years afer Refsdal's initial suggestion (Kelly et al, 2015a), lensed by a foreground cluster of galaxies. The time delays are being measured at the time of writing (Rodney et al, 2015; Kelly et al, 2015b). However, it is unclear at the moment whether the cluster potential can be constrained with sufficient precision to yield interesting cosmological information (Treu et al, 2016). Therefore, in this review, we will restrict our case to the much more common and better understood case of a variable quasar being lensed by a foreground elliptical galaxy.

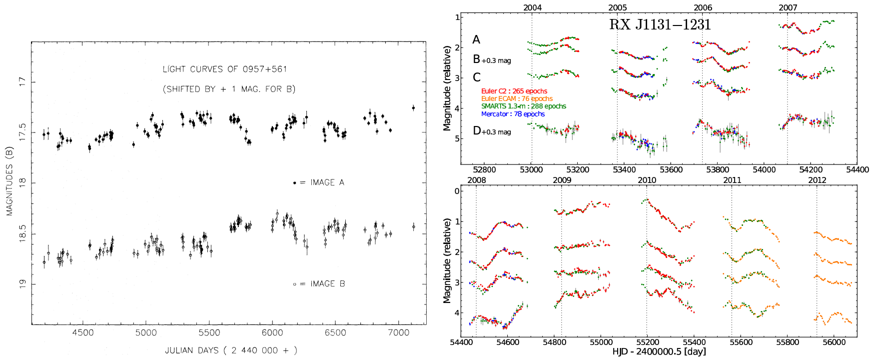

Fig. 1 Comparison between one of the early light curves (left panel, from Vanderriest et al, 1989), and a modern light curve from COSMOGRAIL (right panel, from Tewes et al, 2013b)). Note the improved photometric precision, cadence, and duration of the light curves, allowing for unambiguous determination of the time-delay to within 1-2% precision.

Discovery and monitoring of lensed quasars continued in the eighties and nineties, powered by heroic efforts. By the end of the millennium the number of known strongly lensed systems was in double digits (Courbin et al, 2002) , and the first truly robust time delays were measured (Kundic et al, 1997; Schechter et al, 1997).

The discovery of multiply imaged quasars finally took off at the beginning of the current Millennium with the improvement of panoramic search technology in dedicated or existing surveys (Browne et al, 2003; Oguri et al, 2006; Agnello et al, 2015).

The period of time delay cosmography was marred by controversies over systematic errors. The measurement of time delays was particularly controversial during the nineties as the quality of the early data allowed for multiple values (Press et al, 1992), owing to the combined effects of gaps in the data, and microlensing noise in the optical light curves. This problem was definitely solved at the turn of the millennium, with the beginning of modern monitoring campaigns, characterized by high cadence, high precision, and long duration, both at optical and radio wavelengths (Fassnacht et al, 1999, 2002; Burud et al, 2002; Eigenbrod et al, 2005), as illustrated in Figure 1. We discuss in more detail modern monitoring campaigns in Section **??**.

Finally, when robust time delays started to become available, the focus of the controversy shifted to the modeling of the gravitational potential of the lens. Typically, in the mid nineneties, the only constraints available to modelers were the quasar image positions and to lesser extent flux ratios (limited by microlensing, variability and differential extinction). Thus, the best one could do was to assume some simple form for the lens potential like a singular isothermal sphere, thus breaking the mass sheet degeneracy, and to neglect the effects of structure along the line of sight. Given these necessary but oversimplistic assumptions, random errors grossly underestimated the total uncertainty, leading to measurements apparently inconsistent with those

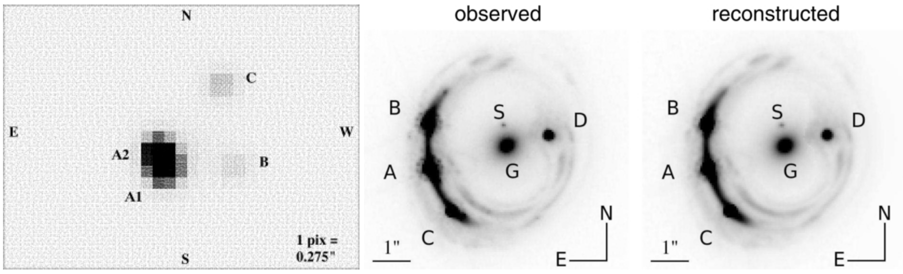

Fig. 2 Comparison between imaging data available in the nineties (eft panel, from Schechter et al, 1997) and in the most recent studies (middle and right panels, from Suyu et al, 2014)). With modern data the structure of the quasar host galaxy can be modeled in great detail, providing thousands of constraints on the deflection angle, and thus on the derivatives of the gravitational potential.

obtained by other groups or other techniques (Kochanek and Schechter, 2004). Since then, two methods have been pursued in order to break degeneracies in more flexible modeling of the lensing data and obtain realistic estimates on the uncertainties. One consists in using large samples of systems with relatively weak priors (Oguri, 2007). The other method consists in obtaining high quality data for each lens system, such as detailed imaging of the quasar host galaxy (Keeton et al, 2000; Wucknitz et al, 2004; Suyu et al, 2006), or non-lensing data like the deflector stellar velocity dispersion (Treu and Koopmans, 2002) and the properties of galaxies along the line of sight (Keeton and Zabludoff, 2004; Suyu et al, 2010). We discuss these approaches in Section **??**. The astounding improvement in data quality over the past two decades is illustrated in Figure 2.

Ultimately, the controversies over systematic errors were essential to spur the community to overcome the difficulties and find ways to address them. This is a natural and probably inevitable part of the scientific process. However, the bitterness of some of those controversies during the ninenties and early naughts still resonates today. Unfortunately, some of the scientists that followed the field with excitement at that time, are still under the impression that strong lensing time delays are inherently inaccurate and imprecise. As we have briefly described here, and we will discuss in detail in the next sections, in the last twenty years the field has moved forward considerably implementing many solutions to the lessons learned the hard way.

3 Theoretical background [PJM]

In this section we provide a brief summary of the theory of gravitational lens time delays. We have distilled much of this from the excellent exposition of Schneider and Kochanek (Schneider et al, 2006), as well as the various key papers we cite.

Fermat's Principle of Least Time holds for the propagation of light rays through curved spacetime. The light travel time through an isolated, thin gravitational lens is given by

$$\tau(\boldsymbol{\theta}) = \frac{D_{\Delta t}}{c} \cdot \Phi(\boldsymbol{\theta}, \boldsymbol{\beta}), \tag{1}$$

$$\text{where } \Phi(\boldsymbol{\theta}) = \frac{1}{2}(\boldsymbol{\theta} - \boldsymbol{\beta})^2 - \psi(\boldsymbol{\theta}). \tag{2}$$

Here, $\boldsymbol{\theta}$ denotes the light source's apparent position on the sky, and $\boldsymbol{\beta}$ is the position of the unlensed source. The difference between the observable position $\boldsymbol{\theta}$ and the unobservable position $\boldsymbol{\beta}$ is the scaled deflection angle $\boldsymbol{\alpha}(\boldsymbol{\theta})$, which is typically ~ 1 arcsecond in a galaxy-scale strong gravitational lens system. $\psi(\boldsymbol{\theta})$ is the scaled gravitational potential of the lensing object, projected onto the lens plane. Both $\boldsymbol{\alpha}(\boldsymbol{\theta})$ and $\psi(\boldsymbol{\theta})$ can be predicted given a model for the mass distribution of the lens.

Images form at the extrema of the light travel time, where $\boldsymbol{\nabla}\tau(\boldsymbol{\theta}) = \boldsymbol{\nabla}\Phi(\boldsymbol{\theta}) = 0$ (**?**). For this reason, $\Phi(\boldsymbol{\theta})$ is known as the "Fermat potential." This quantity can also be thought of as the spatially-varying refractive index of the lens. The arrival time itself is not observable, but differences in arrival time between multiple images are. In the above approximation, the "time delay" $\Delta\tau_{\rm AB}$ between image A and image B can be predicted via

$$\Delta\tau_{\rm AB} = \frac{D_{\Delta t}}{c}\Delta\Phi_{\rm AB} \tag{3}$$

where $\Delta\Phi_{\rm AB}$ is the Fermat potential difference between the two image positions. Figure **??** illustrates the origin of the time delay between the images in a gravitational lens system. The small magnitude of the fractional time delay (typically $\Delta\tau \sim 10$ days out of $D_{\Delta t}/c \sim 10^{12}$ days light travel time) is commensurate with the square of the deflection angle (typically $|\boldsymbol{\alpha}| \sim 1$ arcsecond, or $\sim 5 \times 10^{-6}$ radians).

We see from Equation 3 that given a mass model that predicts $\Delta\Phi_{\rm AB}$, we can infer the "time delay distance" $D_{\Delta t}$ from a measured time delay $\Delta\tau_{\rm AB}^{\rm obs}$. This distance is actually a combination of angular diameter distances:[1]

$$D_{\Delta t} = (1 + z_{\rm d})\frac{D_{\rm d}D_{\rm s}}{D_{\rm ds}} \tag{4}$$

These angular diameter distances can be predicted given the redshifts of the lens and source, $z_{\rm d}$ and $z_{\rm s}$, and an assumed world model with cosmological parameters $\boldsymbol{\Omega}$. The time delay distance is primarily sensitive to the Hubble constant, since $D_{\Delta t} \propto H_0^{-1}$.

Knowledge of the lens mass distribution is of vital importance to the success of this cosmological inference: Equation 3 shows that the time delay distance is as sensitive to uncertainty in the predicted Fermat potential as it is the measured time delay itself. More concentrated mass distributions with steeper density profiles produce longer time delays leading to shorter inferred time delay distances (and larger inferred values of H_0).

[1] As Schneider et al (2006) point out, $D_{\Delta t}$ can be written more simply in terms of comoving angular diameter distances, but most of the literature uses the formula in Equation 4.

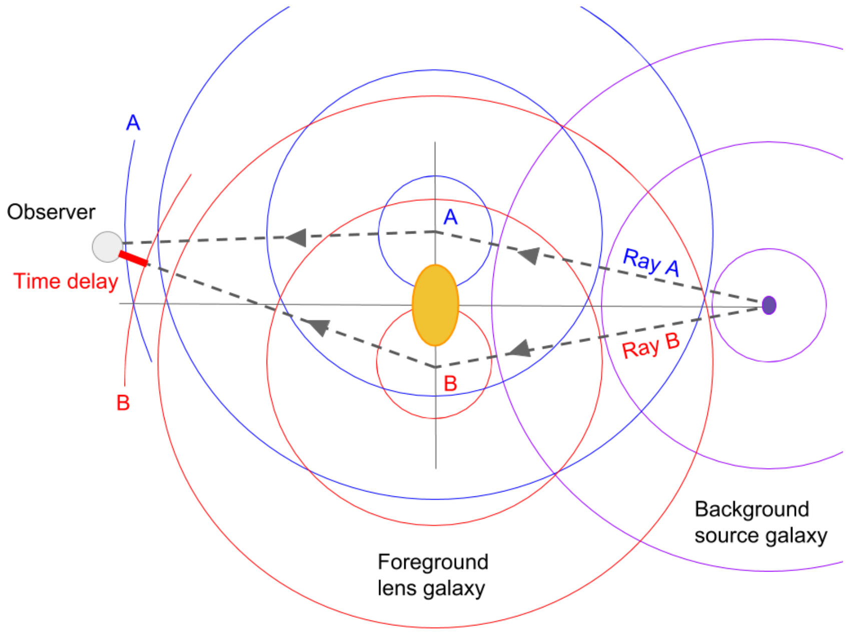

Fig. 3 Schematic diagram, adapted from (?), illustrating the origin of the gravitational time delay.

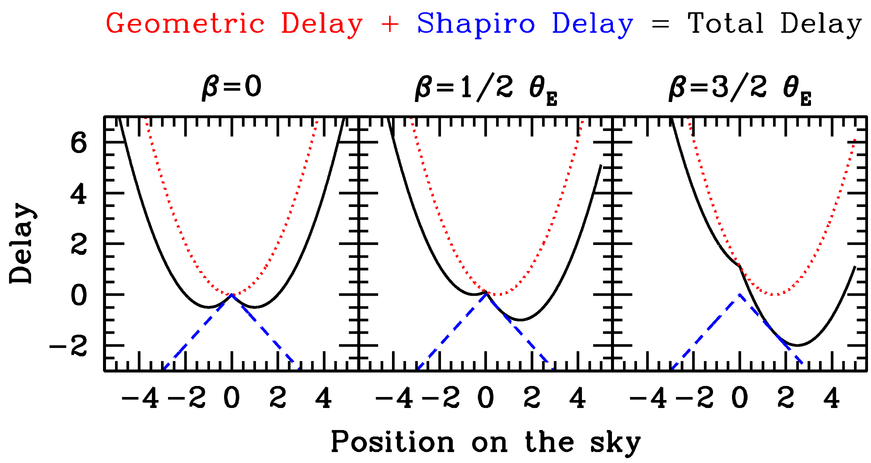

Fig. 4 Geometric and general relativistic (Shapiro) contributions to the lens time delay, from (?). Images form at minima and saddle points of the delay surface, shown here in cross-section. Different source positions result in different geometrical delays as well as shifted image positions.

Moreover, there is significant risk of systematic error when modeling lens mass distributions. While image positions remain invariant under the "mass sheet transformation" (?) (?, and its generalization, the source size transformation), the time delays predicted by the model can change significantly. The mass sheet transformation and its effect on the time delay is as follows:

$$\kappa(\boldsymbol{\theta}) \rightarrow \kappa(\boldsymbol{\theta})' = (1 - \lambda) + \lambda\kappa(\boldsymbol{\theta}) \tag{5}$$

$$\Delta\tau \rightarrow \Delta\tau' = \lambda\Delta\tau. \tag{6}$$

This means that if we allow our model the freedom to generate both the $\kappa(\boldsymbol{\theta})$ and $\kappa(\boldsymbol{\theta})'$ mass distributions, our image position data will not favor one over the other: they will be equally likely given the data. This model degeneracy can only be broken by additional information. One source of information would be independent measurements of the mass distribution: stellar kinematics is the obvious choice. Another way to break the degeneracy is to include prior knowledge of the lens mass distribution, from measurements of other, similar galaxies to the lens. This type of information is typically encoded as a simply-parametrized model, such as an elliptically-symmetric mass distribution with power law density profile (as opposed to a free form density map). Assuming a particular density profile partially breaks the mass sheet degeneracy: it remains to be seen how much systematic error in the time delay that assumption introduces.

The form of the mass sheet transformation given by Equation 5 is a rescaling plus an offset. One way to achieve such a transformation is therefore to change the overall mass of the lens (by a factor of λ), and at the same time add a "mass sheet," a constant convergence $(1 - \lambda)$. Both these variations are possible in nature: lens galaxies come in a range of masses, and the combined gravitational lensing effect of all the other galaxies, groups and filaments along the line of sight to the source can, in the weak lensing limit, be approximated by a constant "external convergence" (associate with an "external shear", capable of further distorting the lensed images). However, as Schneider and Sluse (2013) point out, these physical effects only complicate the modeling problem, and should not be confused with the mathematical degeneracy between lens model parameters that is associated with the mass sheet transformation, and which would be present regardless of any external weak lensing effects. Having said that, any additional external physical mass component must also be taken into account when modeling the lens.

Any independent information about the physical mass of the deflector galaxy, such as the kinematics of its stars, can play an important role in breaking the degeneracy in the mass model, which must now be able to self-consistently predict the lensing effect (image distortions and time delays) and the internal dynamics of the lens galaxy, and take into account the weak lensing effects of structures along the line of sight. Schneider and Sluse (2013) provide demonstrations of the scale of this problem: very good data (both imaging and spectroscopic), as well as careful treatment of the models used, will be needed to obtain accurate results. In Section 4.2 we review the recent choices and approximations that have been made when constructing such models.

4 Modern time delay distance measurement [PJM]

Since 2010, it has been recognized that accurate cosmography with individual lens systems involves the following key analysis steps.

Time Delay Estimation The light curve extracted from monitoring observations is used as input to an inference of the time delay between the multiple images.

Lens Galaxy Mass Modeling High resolution imaging and spectroscopic data are used to constrain a model for the lens galaxy mass distribution, which can be used to predict Fermat potential differences. Both the Einstein ring image and the stellar velocity dispersion are important.

Environment and Line of Sight Modeling Additional observational information about the field of view around the lens system is used to account for the weak lensing effects due to massive structures in the lens plane and along the line of sight.

Cosmological parameter inference can then proceed – although in practice the separation between this final step and the ones above is not clean. Practitioners aspire to a joint inference of lens, source, environment and cosmological parameters from all the data simultaneously, but have to date broken the problem down into the above steps. In the next three sections we describe current state of the art, limitations, and principal sources of systematic error of these three key measurement parts of the problem.

4.1 Measuring time delays [PJM]

The measurement of gravitational time delays involves two steps: taking monitoring observations of the system over a period of several years, and then inferring the time delays between the multiple images from these data.

4.1.1 Monitoring Observations and Results

AGN show intrinsic time variability on many scales, with the variability amplitude increasing with timescale. Long monitoring campaigns can build up high statistical significance as more and more light curve features can be brought into play. However, such long campaigns are difficult to carry out in practice, because a large number of guaranteed observing nights are required (even if the total exposure time is modest). Scheduling such a program has proved difficult, due to the competing demands of the rest of the astronomy community. The highest precision time delays have come from monitoring campaigns carried out with dedicated facilities, that is, observatories that were either able to commit to the long term monitoring proposal submitted, or that were actually operated in part by the monitoring collaboration.

Monitoring of the CLASS lens B1608+656 in the radio with the Very Large Array enabled the breakthrough time delay measurements of (Fassnacht et al, 2002). In its first season, this program yielded measurements of all three time delays in this quadruple image system with precision of 6–10% (Fassnacht et al, 1999); with the variability of the source increasing over the subsequent two seasons, (Fassnacht et al, 2002) were able to reduce this uncertainty to 2–5%. Such high precision was the result of a dedicated campaign during Fassnacht's residence at NRAO, and which consisted of 8-month seasons, with a mean observation spacing of around 3 days. The light curves were calibrated to 0.6% accuracy.

While time delays had previously been measured in ten other lens systems, this was the first time that all the delays in a quad had been obtained; moreover, it brought the time delay uncertainty below the systematic uncertainty due to the lens model, prompting new efforts in this direction beyond what (Koopmans and Fassnacht, 1999) were able to do.

While B1608+656 is not the only radio lens with measured time delays, a combination of factors led the observational focus to shift towards monitoring in the optical. With the sample of known, bright lensed quasars increasing in size, networks of 1-2m class optical telescopes began to be investigated. The variability in these systems is somewhat more reliable, and while microlensing and image resolution present observational challenges, the access to data was found to be less restrictive. The COSMOGRAIL project took on the task of measuring lens time delays with few-percent precision in this way: Eigenbrod et al (2005) showed that microlensing was likely not to be an insurmountable task, and **?** provided the proof of concept with a 4% precision time delay measurement in SDSS J1650+4251.

One of the keys to the success of this program has been the simultaneous deconvolution of the individual frames in the imaging dataset, using a mixture model to describe the point-like quasar images and extended lens and AGN host galaxies (Magain et al, 1998). Another is the dedicated nature of the network of telescopes employed, and the careful calibration of the photometry across this distributed system. Seasons of 8–12 months duration over campaigns of up to 9 years have been achieved, with typical mean observation gaps of around 3–4 days.

The COSMOGRAIL team have now published high precision time delays in WFI J2033−4723 (Vuissoz et al, 2008, 3.8%), HE 0435−1223 (Courbin et al, 2011, 5.6%), SDSS J1206+4332 (Eulaers et al, 2013, 2.7%) and RX J1131−1231 (Tewes et al, 2013b, 1.5%), and SDSS J1001+5027 (Rathna Kumar et al, 2013, 2.8%), with more due to follow. Typically multiple years of monitoring is needed to obtain an accurate time delay, as the variability fluctuates and the reliability of the measurement converges (see the discussion in e.g. Tewes et al, 2013b).

A consistent picture seems to emerge from both the VLA and COSMOGRAIL len monitoring projects, that high precision gravitational time delay measurement requires campaigns consisting of multiple, long seasons, with around 3-day cadence. The baseline observing strategy for the Large Synoptic Survey telescope is somewhat different to this, with seasons expected to be around 4–5 months in length, and gaps between observation nights only reaching 4–5 days when images in all filters are taken into account. The "Time Delay Challenge" project was designed to test the measurability of lens time delays with such light curves (?), in a blind test offered to the astronomical community. From the ten algorithms entered by seven teams, it was concluded that time delay estimates of the precision and accuracy needed for time delay cosmography would indeed be possible, in around 400 LSST lensed quasar systems (Liao et al, 2015). This result came with two caveats: 1) the single filter light curve data presented in the challenge is representative of the multi-filter data we actually expect, and 2) that "outliers" (catastrophic time delay mis-estimates) will be able to be caught during the measurement process. A second challenge to test these assumptions is in preparation.

4.1.2 Lightcurve Analysis Methods

How were the time delays surveyed in the previous section derived from the light curve data? Interest in this particular inference problem has been high since the controversies of the late 1990's. Fassnacht et al (1999) used the "dispersion method" of Pelt et al (1996), a technique that involves shifting one observed light curve relative to another (both in time and in amplitude) and minimizing the dispersion between adjacent points in the resulting composite curve. Uncertainties were estimated by Monte Carlo resampling of the data, assuming the minimum dispersion time delay and magnification ratio to be true. In order to take into account the slowly varying incoherent microlensing signals present in their optical light curve data, the COSMOGRAIL team have investigated three analysis techniques that all involve interpolation of the light curves in some way (Tewes et al, 2013a): free-knot splines, Gaussian processes and simple linear interpolation have all been tested, within a common "python curve-shifting" (PyCS) framework.[2] These agree with each other given light curves of sufficient length, providing an argument for multiple-season monitoring campaigns.

The time delay challenge prompted seven analysis teams to develop and test algorithms for time delay estimation. These are outlined in the TDC1 analysis paper of Liao et al (2015), but we give a very brief summary here as well, along with updated references. The PyCS team tried a three-step approach (visual inspection and interactive curve shifting, followed by spline fitting, followed by an additional spline model regression analysis of the residuals), and submitted an entry after each step (Bonvin et al, 2016). Two other teams applied similar curve-shifting approaches: both Aghamousa and Shafieloo (2015) and Rathna Kumar et al (2015) devised smoothing and cross-correlation schemes that they find to be both fast and reliable. Jackson applied the dispersion method of Pelt et al (1996), but carefully supervised via visual inspection to check for catastrophic failures. The three remaining teams used Gaussian Processes (GPs) to model the light curves. Tak et al (2016) used

[2] The COSMOGRAIL curve shifting analysis code is available from `http://cosmograil.org`

a custom Gibbs sampler to infer the hyper-parameters describing the GP for the AGN variability and polynomials for the microlensing signals, although they ignored microlensing during the challenge itself. Romero-Wolf & Moustakas implemented a very similar model, also ignored microlensing, and used a freely-available ensemble sampler for the inference. Hojjati and Linder (2014) used GPs for both the AGN and microlensing variability, and optimized the hyper-parameters rather than sampling them.

Two factors were important in the minimisation of catastrophic time delay mis-estimation: explicitly including microlensing in the model, and visual inspection of the results. An additional promising avenue for future challenges ought to be ensemble analysis, to exploit 1) the intrinsic correlations between, for example, AGN variability, color and brightness, and 2) the fact that the cosmological parameters are common to all lens systems.

4.2 Modeling the lens mass distribution [TT]

Once the time delays have been measured, the second main ingredient entering the determination of time delay distances is the mass model of the main deflector. In the early days of time delay cosmography one could only rely on the relative positions of the multiple images as constraints (since in general the flux ratios are affected by micro and millilensing, variability, and differential dust extinction, and are therefore highly uncertain). Even for a quadruply imaged quasars, the six positional constraints are insufficient to determine Fermat potential differences to the desired level of precision and accuracy.

There are two classes of solution to the problem of underconstrained lens models. One is to analyze large samples of lenses with physically motivated priors and exploit the fact that cosmological parameters are the same for all lenses to remove model degeneracies. A number of attempts along these lines have been made in the past (**?**), and it is easy to imagine that this solution will be popular in the future, when large samples of lenses with measured time delays will be available.

The alternative solution is to increase dramatically the number of emprical constraints per lens system by means of dedicated high resolution imaging and spectroscopic observations (Suyu et al, 2010, 2013, 2014). We describe this approach in detail below.

For simplicity, in this section we describe only the case of a single deflector in a single plane, leaving line of sight and environmental effects for a later section.

[NEEDS TO BE DONE HOLISTICALLY EVEN THOUGH SO FAR IT HAS BEEN DONE SEPARATELY IN PRACTICE]

4.2.1 High Resolution Imaging Observations

Lensed quasars reside in a host galaxy. For typical redshifts of lens and source, the host galaxy apparent size is of order arcseconds. Images with sufficient depth and resolution to isolate the bright point source and detect the lower surface brightness host galaxy, often reveal extended lensed features connecting the point-like images themselves (e.g. Figure **??**). In the best conditions these images cover hundreds if not thousand resolution elements. The distortion of the detailed features of the lensed images are a direct measurement of the variation of the deflection angle between the images. In principle, for data with infinite signal-to-noise ratio and resolution one could imagine integrating the gradient of the deflection angle along a path betwen a pair of images to obtain the difference in Fermat potential. In practice, in the presence of noisy data and limited resolution, forward modeling approaches have been the most successful so far, as discussed below. From an observational point of view, it has been demonstrated that images with $0.1'' - 0.2''$ FWHM resolution provide good results, provided that the point spread function can be appropriately modeled or reconstructed as part of the lens model itself. The Hubble Space Telescope in the optical/near infrared (Suyu et al, 2010, 2013, 2014; **?**) and the Very Large Baseline Interferometer in the radio (Wucknitz et al, 2004) have been the main sources of images for this application. Recent progress in adaptive optics imaging at Keck (Chen et al, 2016), the beautiful data being obtained for lensed source by ALMA (**?**), and the many facilities currently being constructed or planned (**?**), indicate that the prospects to scale up the number of systems with available high resolution images are bright.

4.2.2 Lens Modeling Techniques

Conceptually a detailed model of a lensed quasar and its host galaxy needs to describe three different physical components: i) the surface brightness of the source; ii) the surface brightness of the deflector; iii) the gravitational potential of the deflector. It is useful to conceptualize the problem in this way, in order to understand where the information needed to break the degeneracy in the intrepretation of the data comes from. Lensing is achromatic and preserves surface brightness so any feature that belongs to the source (including in line of sight velocity) should appear in all the multiple images (appropriately distorted). Likewise, the deflector is typically a massive early-type galaxy with smooth surface brightness distribution and approximately uniform colors (except for dust, see, e.g. Suyu et al, 2010).

Each of the three components is typically described in terms of one or two choices: i) simply parametrized functions such as a Sersic profile for the surface brightness of the lens or the source, and a singular isothermal ellipsoid for the gravitational potential of the deflector (Keeton); ii) as combination of basis sets like surface brightness pixels, potential pixels, or gauss-hermite functions (Coles, Birrer, Nightingale, Collett). These so-called "pixellated" models require regularization to avoid overfitting noise in the data. Hybrid approaches have been proposed where some of the components are simply parametrized and others are pixellated (Dye & Warren; Treu & Koopmans), or where pixels are used as corrections to simply parametrized models (Suyu). The variety of approaches in the literature reflect the inevitable tensions between the need to impose as many physically motivated assumptions as possible while retaining sufficient flexibility to obtain a realistic estimate of the uncertainties. If the model is too constrained by the assumption it will lead to underestimated errors, if it is more flexible than necessary it will lead to a loss of information.

Once the choice of modeling parametrization is set, exploring the likelihood is numerically non-trivial, often requiring weeks to months of CPU time. Fortunately, there are techniques to speed up the calculations by limiting the number of non-linear parameters. For example, for a given lens model, the transformation between source and image plane can be described as a linear operation, or the pixellated corrections to the potential can be found by linearizing the lens equation. [Dye, Nightingale, Birrer, Suyu, etc]

Ideally, modeling choices should be explored systematically as well, since they can potentially introduce systematic errors. This is currently being done in the most advanced studies, at great expense in term of computing time and investigator time. As we discuss in Section 6, speeding up the modeling phase and reducing the investigator time per system will be key to analyzing the large statistical samples expected in the future.

4.2.3 The Role of Stellar kinematics

As introduced in Sectionsec:theory, stellar kinematics provide a qualitatively different input and therefore very valuable in breaking degeneracies in the intepretation of lensing data (e.g. the mass-sheet degeneracy), and in estimating systematic uncertainties. Of course, translating kinematic data into estimates of gravitational potential has its own uncertainties and degeneracies (e.g. the mass anisotropy degeneracy for pressure supported systems), but the combination of the two datasets in the context of a single mass model has been proven to be very effective (Treu and Koopmans, 2004). Even a single measurement of stellar velocity dispersion, interpreted via simple spherical Jeans modeling has been shown to substantially reduce modeling uncertainties (**?**Koopmans et al, 2003; Suyu et al, 2014). It is clear that getting spatially resolved kinematic data will allow for better constraints on the lens model and thus of the resulting cosmological inference.

[PJM: I THINK THIS SECTION IS THE PLACE TO DESCRIBE THE PARTICULAR CHOICES THAT HAVE BEEN MADE FOR JOINT ANALYSIS OF LENSING AND KINEMATICS DATA – THE THEORY SECTION IS BETTER LEFT MORE GENERAL. WE COULD SKETCH THE SUYU ET AL 2013 MODEL AND THEN CRITIQUE IT, AND ALSO NOTE ANY ALTERNATIVE ASSUMPTIONS MADE BY OTHERS (BUT I DON'T THINK THEY WILL BE VERY DIFFERENT).]

4.3 Lens environments and line of sight effects [PJM]

The analysis of B1608+656 by Suyu et al (2010) explicitly took into account the weak lensing effects of external structures. Such a correction had been suggested by Fassnacht et al (2006), who identified 4 galaxy groups along the line of sight in a spectroscopic survey of the B1608+656 field, and estimated that they could, if left unaccounted for, bias any inferred Hubble constant high by around 5%. Exactly how to model this effect has been the topic of a number of papers since 2010: the problem is how to incorporate our knowledge of where the galaxies are along the line of sight without introducing additional bias due to the necessary assumptions about how their (dark) mass is distributed, and how the rest of the mass budget in the field adds up.

Suyu et al (2010) attempted to solve these problems by comparing the B1608+656 field with a large number of fields with similar galaxy number overdensity drawn from the Millennium Simulation, modeling the line of sight effects with a single external convergence parameter and accepting a somewhat broad prior distribution for it, in return for not having to make strong assumptions about the structure of the galaxy groups in the field. The external convergence in the simulated fields was calculated by ray-tracing by Hilbert et al (2009), and the comparison in galaxy overdensity was enabled by the analysis of galaxy number counts in archival HST images by Fassnacht et al (2011), who found that the B1608+656 field was overdense by a factor of two. The resulting prior PDF for the $\kappa_{\rm ext}$ parameter had median 0.10 with the 68% credible interval spanning 0.05 to 0.18.

Since this initial analysis, a number of improvements have been suggested and investigated. All have in common the desire to bring more information to bear on the problem, in order to increase the precision (while continuing to avoid introducing bias). Greene et al (2013) showed that weighting the galaxy counts by distance, photometric redshift and stellar mass can significantly reduce the uncertainty in $\kappa_{\rm ext}$, by up to 50%. Colbert et al (2013) claim an additional 30% improvement by including knowledge of the stellar mass to halo mass relation in galaxies, and modeling each galaxy halo's contribution to $\kappa_{\rm ext}$ individually in a 3-D reconstruction of the mass in the field which is then calibrated to simulations in something like the high resolution limit of the number counts approach.

While research into these methods continues, one problem in particular remains outstanding. The methods that involve calibration to numerical simulations are dependent on the cosmological parameters assumed in that simulation, while all methods involve modeling line of sight structures at various distances as part of an evolving universe, whose dynamics depend on cosmological parameters. We face two options: either treat these cosmological parameters self-consistently as hyperparameters in a joint analysis of the time delays and the lens environments, or demonstrate that they can be decoupled via various simplifying assumptions that introduce sub-dominant systematic error.

5 From time delay distances to cosmological parameters [PJM]

Early approaches to inferring cosmological parameters from time delay lens observations focused on measuring the Hubble constant in a Friedman-Robertson-Walker model with asserted (fixed) density parameters.[3] With better data came the recognition that time delay lenses were really probes of cosmological distance (Koopmans et al, 2003; Suyu et al, 2010), and the emphasis shifted to inferring the set of cosmological parameters that are needed to predict the kinematics of the expansion of the Universe out to the redshift of the source.

The ~~amount of cosmological information in an individual lens is still relatively small, and the~~ parameter most strongly constrained is the Hubble constant, but as sample sizes increase we expect ensembles of lenses to support the inference of several cosmological parameters (or combinations thereof).

In Figure 5 we reproduce the current best constraints on cosmological parameters, from the two best-measured systems, B1608+656 and RXJ1131 (Suyu et al, 2014). When this figure was made, the available precision from just these two lenses was about the same as that from SDSS DR7 BAO (Percival et al, 2010) and the "Constitution" set of Type Ia supernovae (Hicken et al, 2009). When all three of the curvature density Ω_k, Dark Energy density Ω_{DE} and equation of state w_0 parameters are allowed to vary, along with H_0, we see that the time delay lenses provide similar constraints to BAO and complementary constraints to the SNe: the time delays and the BAO signal depend on angular diameter distances and H_0, while the supernovae probe luminosity distances.

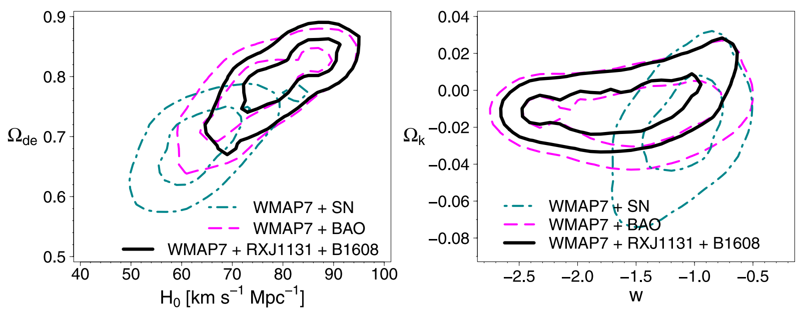

Fig. 5 Current cosmological parameter constraints from time delay lenses. The marginalized posterior PDFs given the combined B1608+656 and RXJ1131 datasets and the assumption of an open CDM cosmology with unknown dark energy equation of state is shown in two sets of two parameter dimensions, and compared to those given contemporary BAO and Type Ia supernova data. Figure reproduced from Suyu et al (2014).

One important feature of the cosmological parameter inference carried out in the RXJ1131 analysis of Suyu et al (2013) is that it was blinded. Following

[3] The original investigation by Refsdal (1964) involved the "assumption that the linear distance–red-shift relation is valid."

the simple methodology suggested in the blind Type Ia supernova analysis of Conley et al (2006), all cosmological parameter PDFs were plotted with centroids offset to the origin until the team agreed (after notably lengthy discussions about systematic errors) to “open the box.” Such attempts to avoid “unconscious experimenter bias” introduced by stopping systematics analysis when the “right answer” is obtained have long been advocated in particle physics (Klein and Roodman, 2005), and seem likely to become the standard in cosmology as well (e.g. Heymans et al, 2006; The Dark Energy Survey Collaboration et al, 2015).

While the sample of very well-measured lenses was being painstakingly expanded from zero to two, the exploration of statistical approaches to dealing with large samples of lenses began. Compressing the image configuration and time delay in double image systems into a single summary statistic, Oguri (2007) derived a scaleable method for measuring the Hubble constant (but not the other cosmological parameters) from samples of lenses, finding $H_0 = 68 \pm 6\,(\mathrm{stat.}) \pm 8\,(\mathrm{syst.})\,\mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$ from a sample of 16 lenses with measured time delays. The systematic uncertainties associated with this result may be hard to reduce given the approximations made: while the summary statistic is model-independent, the interpretation is not. An alternative approach is to work with more flexible lens models, fit the data for each one, and combine the whole sample in a joint inference. This is the approach taken by Saha et al (2006), who found $72^{8}_{11}\,\mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$ from 10 lenses (again assuming fixed curvature and dark energy parameters). With the non-parametric (pixelated) mass models used, the degeneracy between density profile slope and predicted time delay is broken by the choice of pixel value prior PDF. This assumption was tested by Read et al (2007), who used a hydrodynamic simulation of an elliptical galaxy to generate mock image position and time delay data, and confirm the accuracy of the previous study’s Hubble constant uncertainties. With improved time delay estimates in a sample of ten lenses, Rathna Kumar et al (2015) reduced the uncertainty to $\pm 6\,\mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$. While focused only on the Hubble constant and carried out unblind, and with the lens environment and line of sight mass structures remain unaccounted for and further tests on realistic simulated galaxies warranted, these ensemble studies point the way towards a future of considerably larger sample sizes.

6 Outlook [TT]

Motivation.

What’s the point? Arent’ other probes already doing it? Our place in the cosmology ecosystem. Refer back to current constraints from Suyu et al (2013) in Section **??**, which show other probes. Discuss place relative to other distance indicators like Cepheids, BAO, Sne. Linder SL + SNe plots.

Then complementarity with growth of structure probes like weak lensing, clusters etc etc. How important is H0? Weinberg et al 2013, Kim et al 2014. Figure 48 from Weinberg?

Importance of multiple INDEPENDENT measurements for discovery of new physics, and to ensure overall accuracy of cosmological parameters. Define accuracy. Define precision. Segue.

6.1 Precision [PJM

The primary limit to the precision Raw precision from Coe & Moustakas 2009. Discuss: Sample size. Stage 3, stage 4 surveys. Monitoring solutions.

Extrapolations to N lenses assuming X% precision per time delay distance, forecasts.

FIGURE: Forecasts for 10,50,100,1000 lenses for various cosmological models (w, wa+w0, curvature etc etc). CosmoSIS forecasts (ackn. Dave & Elise, ask them).

Note on approximation of forecasting using just Ddt, and not DA as well: above plot is conservative. Cite Jee et al paper 1 for pointing this out, show figure from Jee et al paper 2?

6.2 Accuracy [PJM]

Discussion of systematic uncertainties:

1) Time delay measurement. Light curve quality.

2) Lens mass modeling. Percent-level systematics due to model assumptions (ie MSD). IFU observations, resolved stellar kinematics. Ensembles.

3) Environment and line of sight

Other things: time delay perturbations (someone's noise is somebody else's signal..) The importance of blinding.

7 Summary [TT]

Short summary.

Bulleted list of pithy conclusions.

Wise prophesy.

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