

## Lab Assignment 1

## Objectives

1. Become familiar with Racket, a SCHEME interpreter;
2. Be able to set the SCHEME mode and know the default mode we will use for the course;
3. Be able to enter expressions in Racket and understand their evaluation;
4. Practice with function abstractions

## Lab Assignment Activities

### 1. Currency Conversion

- (a) The exchange rate between U.S. dollars and euros is  $1\$ = 0.82\text{€}$ . Write a SCHEME procedure, named `usd-to-euro`, to convert dollars into euros. How many euros will you get when you exchange \$250?
- (b) The exchange rate between euros and Japanese yen is  $1\text{€} = 126.01\text{¥}$ . Write a SCHEME procedure, named `euro-to-yen`, to convert euros into yen. How many yen will you get when you exchange €250?
- (c) Write a procedure, named `usd-to-yen`, using your solutions from parts (a) and (b) to convert U.S. dollars to yen. How many yen would you receive in exchange for \$250?

2. (a) Define a variable representing the mathematical constant  $e$ . You must name your constant `e` and use the approximation 2.71828 as the value for your variable.
- (b) Define a function `tanh` which, given a positive number  $x$ , returns the hyperbolic tangent of  $x$  defined as

$$\tanh(x) = \frac{e^{2x} - 1}{e^{2x} + 1}. \quad (1)$$

You may use the built in SCHEME function (`expt b e`) function to complete this problem. (`expt b e`) computes  $b^e$ .

### 3. Matrices

A *matrix* is a rectangular grid of numbers organized into rows and columns. Matrices are an important tool in algebra and are often used to solve systems of linear equations. Below are examples of a couple of  $2 \times 2$  matrices (matrices with 2 rows and 2 columns) that we will call  $M$  and  $N$ .

$$M = \begin{pmatrix} 2 & -4 \\ -6 & 12 \end{pmatrix} \quad N = \begin{pmatrix} -3 & 1 \\ 2 & 7 \end{pmatrix}$$

- (a) A special value associated with any  $2 \times 2$  matrix is the *determinant*. Given a generic  $2 \times 2$  matrix, the determinant can be computed using the following formula:

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

Using the formula, we can compute the determinant of matrix  $M$  above as  $(2)(12) - (-4)(-6) = 0$ . Write a Scheme procedure, named `(det2x2 a b c d)` to compute the determinant of a generic  $2 \times 2$  matrix. Assume that the matrix elements  $a$ ,  $b$ ,  $c$  and  $d$  are given as four formal parameters. Compute the determinant of  $N$ .

- (b) A matrix is called *invertible* if its determinant is non-zero. Write a procedure, named `(invertible? a b c d)`, that checks whether or not a generic  $2 \times 2$  matrix is invertible. Verify that  $N$  is invertible and  $M$  is not invertible.
- (c) A powerful property of matrices is that certain kinds of matrices may be meaningfully *multiplied* together to get another matrix. (It turns out that matrix multiplication is intimately related to composition of linear functions, but you won't need this interpretation to complete the exercise.) In particular, it is possible to multiply  $2 \times 2$  matrices. Assume we have two matrices:

$$A = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \quad B = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$$

The product of these matrices is defined to be

$$A \cdot B = \begin{pmatrix} a_1 a_2 + b_1 c_2 & a_1 b_2 + b_1 d_2 \\ c_1 a_2 + d_1 c_2 & c_1 b_2 + d_1 d_2 \end{pmatrix}.$$

Given two  $2 \times 2$  matrices, we wish to determine whether or not their product  $A \cdot B$  will be invertible. There are two ways to do this

- i. Compute the product, as described above; then compute its determinant. Define a function named `(prod-inv-direct? a1 b1 c1 d1 a2 b2 c2 d2)` which determines if the product of two matrices is invertible by this method.
- ii. It is a remarkable fact that for two matrices  $A$  and  $B$ ,  $\det(A \cdot B) = \det(A) \times \det(B)$ . Thus, we can compute the determinant of  $A \cdot B$  indirectly (without computing the product of the two matrices) from the determinants of  $A$  and  $B$ . Define a function named `(prod-inv-indirect? a1 b1 c1 d1 a2 b2 c2 d2)` which determines if the product of two matrices is invertible by this method.

Once you have finished:

1. Save your work (the definitions) to a file named `lab1.rkt`
2. Submit your lab solutions for grading via Mimir.
3. If you haven't already, read the Honor Code Pledge on the CSE1729 Moodle site completely.
4. If you haven't already, submit your signed Honor Code Pledge in lab today.

Please note: assignments will not be graded for credit until your Honor Code Agreement is filed-see it under Course Content in Moodle.