GROUP PROJECT SUBMISSION 1

Group Members

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Introduction

The concept of credit risk management, which includes credit valuation adjustment, was developed due to the increased number of country and corporate defaults and financial fallouts. In recent times, there have been cases of sovereign entity defaults, such as Argentina (2001) and Russia (1998). At the same time, a high number of large companies collapsed before, during, and after the financial crisis of 2007/08, including WorldCom, Lehman Brothers, and Enron.
Initially, research in credit risk focused on the identification of such a risk. Specifically, the focus was on counterparty credit risk, which refers to the risk that a counterparty may default on its financial obligations.
Prior to the 2008 financial crisis, market participants treated large derivative counterparties as too big to fail and, therefore, never considered their counterparty credit risk. The risk was often ignored due to the high credit rating of counterparties and the small size of derivative exposures. The assumption was that the counterparties could not default on their financial obligations like other parties.
However, during the 2008 financial crisis, the market experienced dozens of corporate collapses, including large derivative counterparties. As a result, market participants started incorporating credit valuation adjustment when calculating the value of over-the-counter derivative instruments. In other words, CVA is the market value of counterparty credit risk. This price depends on counterparty credit spreads as well as on the market risk factors that drive derivatives' values and, therefore, exposure.
Credit valuation adjustment is a change to the market value of derivative instruments to account for **counterparty credit risk**. It represents the discount to the standard derivative value that a buyer would offer after taking into account the possibility of a counterparty's default. CVA is the most widely known of the **valuation adjustments**, collectively known as XVA.

Credit Valuation Adjustment (CVA), is calculated as the difference between the value of a portfolio which we assume is risk-free, and a portfolio where we account for default risk. As such, CVA can be thought of as the market value of the counterparty credit risk. As you might expect, the higher your exposure, the higher your CVA, since you stand to lose more.

For a mathematical definition of counterparty risk, we define two variables: δ and τ. δ will be our recovery rate – this is the fraction of our portfolio that we would receive if the counterparty defaults. In other words, if our portfolio is worth (t) at time t and the counterparty defaults at time t, we would only receive $\delta V(t)$. So, we would lose $(1-\delta)(t)$. Our second variable, τ, is a stopping time. This will be the time that the counterparty defaults. If the counterparty never defaults, $\tau = \infty$. We only lose out if the counterparty defaults before the time we close-out our position with them. Our CVA is thus as follows:

GROUP PROJECT SUBMISSION 1

CVA = $\mathbb{E}\mathbb{Q}[e{-}r\tau(1 - \delta)V(\tau)\mathbb{I}\{\tau{\le}T\}]$

The goal of Submission 1 is to price a European up-and-out call option held with a risky counterparty. This is a type of call option whose payoff is reduced to 0 if the share price becomes too high over its lifetime. Note that this limits the final payoff of the option, and as a consequence it becomes cheaper than a vanilla call option.

Observe that the payoff of the option is dependent on the value of the share price between the inception of the option and maturity. This means that the option payoff is dependent on the history of the share price, and not just on its terminal value. As a result, you will need to simulate entire share price paths to estimate the price of this option.

You may make the assumptions of the Black-Scholes-Merton model (i.e. assume that both the stock and counterparty firm values follow Geometric Brownian Motion [GBM] with constant drift and volatilities, and default only occurs at maturity).

We are going to be modeling CVA for a number of different levels of correlation between the stock on which the option is written and the value of the counterparty and we will be implementing the Monte Carlo algorithms for CVA.

The biggest issue we face with CVA is the difficulty in calculating it, as there are a number of factors which affect it, such as credit spreads and market factors. As a result, we introduce an extension to the Black-Scholes model, known as the Merton model.

Methodology

We have different approaches or methods for calculating Credit Valuation Adjustment, (CVA), value of a derivative.

1. Simple approach
The simple method calculates the mark to market value of the instrument. The calculation is then repeated to adjust the discount rates by the counterparty's credit spread. Calculate the difference between the two resulting values to obtain the credit valuation adjustment.

2. Swaption-type valuation
The swaption-type is a more complex credit valuation adjustment methodology that requires advanced knowledge of derivative valuations and access to specific market data. It uses the counterparty credit spread to estimate the replacement value of the asset.

3. Simulation modeling
This involves the simulation of market risk factors and risk factor scenarios. The derivatives are then revalued using multiple simulation scenarios. The expected exposure profile of each counterparty is determined by aggregating the resulting matrix. Each counterparty's expected exposure profile is adjusted to derive the collateralized expected exposure profile.

GROUP PROJECT SUBMISSION 1

We are going to use the simulation modeling approach, a code that implements a Monte Carlo simulation for calculating CVA for our first project. The program used for solving the problem of our first group project work is python and before you can work in the python environment, necessary or requisite libraries need to be made available using import tool. Since we are working within the Merton model, we are going to be simulating our assets using a standard normal distribution – this is done in the same way as in the Black-Scholes model. With the import tool, the libraries to be brought include numpy array, norm from scipy.stats library, random and matplotlib.pyplot for plotting the requisite plots. After importing the libraries, information including Market information, like the risk_free rate,r, Share specific information, like volatility, sigma and shareprice, Call option specific information like strike price, K, and time of maturity,T, and Firm specific information like debt, recovery rate etc., should be entered for future use. Functions for later valuations like terminal value and all payoff need to be defined for future use using the def function. Random seed is set zero using np.random.seed() function. We set our seed so that we can easily compare our answers, we create an array of the correlations we are going to be testing.

A code is introduced for calculating the probability of default, which is given by:

$\Phi(-d2)$

We then calculate our closed-form solution for the call option on the share, as per our usual Black-Scholes formula as the analytical solution for villa European call option.

Using the following parameters:

- Option maturity,T, is one year
- The option is struck at-the-money
- The up-and-out barrier for the option is $150
- The current share price, K, is $100
- The risk-free continuously compounded interest rate, r, is 8%
- The volatility for the underlying share, sigma, is 30%
- The volatility for the counterparty's firm value is 25%
- The counterparty's debt, D, due in one year, is $175
- The correlation between the counterparty and the stock is constant at 0.2
- The recovery rate with the counterparty is 25%.

GROUP PROJECT SUBMISSION 1

Having imported all necessary libraries, the random seed is set to zero. Correlated array is determined that will be used to determine the estimates. The for loop enable us to run our Monte Carlo estimation.

We then determine the probability of default. After determining the probability we calculate our closed-form solution for the call option on the share, as per our usual Black-Scholes formula for the analytical solution for villa European call option. We are then able to compute our CVA assuming 0 correlation. We use the formula given by:

$(1 - \delta)X0\Phi(-d2)$,

where $X0$ is the closed-form solution to the price of the call.

Finally, we plot our Monte Carlo CVA estimates for different correlations, alongside three standard deviation error bounds and the 0 correlation CVA.

Conclusion

Credit valuation adjustment (CVA) is the market value of counterparty credit risk and it is the difference between the risk-free portfolio value and the true portfolio value that takes into account the possibility of a counterparty's default. Normal Black schole's cannot be used to calculate CVA and therefore and extended version called Merton is used. Monte Carlo estimates, was used to simulate the price of the option incorporating counterparty risk, given by the default-free price less the CVA.

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