MA424 Modelling in Operations Research
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Summative Project

Foodys is a food delivery company that uses a fleet of scooters for delivering take away food from restaurants to customers' homes. For safety, each scooter has to go through a checkup after it has worked for c consecutive days. Further, scooters are subject to breakdowns: the time a scooter is in service before breaking down follows an exponential distribution with rate μ per day. The company hires two mechanics, who are both qualified for checkups and repairs. However, mechanic 1 is specifically assigned for checkups and mechanic 2 for repairs. Scooters that need a checkup go to mechanic 1 and queue waiting for a checkup. Scooters that break down go to mechanic 2 and queue waiting for repair.

Suppose Foodys owns N scooters out of which n are working during Foodys working hours and the rest $N - n$ are either at mechanic 1 for a checkup, mechanic 2 for repair, or in Foodys parking lot waiting to be used. When a scooter goes for a checkup or breaks down, it is replaced with one of the scooters in the parking lot. The first n working scooters are not subject to a checkup until they have had their first breakdown, but scooters in the parking lot are assigned a checkup time the minute they start working.

1 The first time Foodys is under-resourced

Foodys is concerned about having more than $N - n$ scooters at the mechanics and thus having to work with less than n scooters. Suppose they start with n scooters working and $N - n$ at the parking lot ready to be used when needed. Assume that the time mechanic 1 takes to do a checkup follows an exponential distribution with rate μ_1 per day, and mechanic 2 repair times also follow an exponential distribution with rate μ_2 per day. Let T be the first time that Foodys works with less than n scooters (i.e. they are under-resourced). We want to estimate $E[T]$.

(i) Using $N = 20$, $n = 10$, $c = 2$, $\mu = 0.25$, $\mu_1 = 1.5$, and $\mu_2 = 2$, build a discrete event simulation model to estimate $E[T]$. When you build your simulation model define your variables, events, event lists, output variables. Write down the pseudocode of each event case as we did in lecture. Implement your model by writing an R script to estimate $E[T]$ using $K = 500$ iterations.

(ii) Find an estimate of $E[T]$ where we are 95% confident that it is within 0.3 days of its true value. Give the sample standard deviation of your estimator. How many iterations did you have to run to get this value?

(iii) Give an interval centered around the estimate given in (ii) above where we are 90% confident that the true value of $E[T]$ is within this interval. Show analytically that indeed this is a 90% interval.

(iv) Foodys wants to increase T by making the checkup/repair process more efficient. Without hiring extra mechanics, what advice can you give them? Explain why your advice would work better without performing any simulations to verify this.

2 Reduce the variance of the estimator

Foodys was happy with the estimator that you gave in part 1(i)-(ii) for $E[T]$ but they would like to improve its variance. Write a paragraph proposing a variance reduction estimator based on control variates (no more than one) and explain why it would reduce the variance. Justify your choice of control variate. (Do not perform any simulations).

3 Correct the distribution of the mechanics

Foodys informs you that the distributions for the mechanics are not as given in part 1. For mechanic 1, the density function is $f(x) = \frac{1}{2}(1+x)e^{-x}$, $0 < x < \infty$. For mechanic 2, the distribution function (CDF) is $F(x) = 1 - e^{-\alpha x^{\beta}}$, $0 < x < \infty$, where $\alpha = 3$ and $\beta = 2$.

(i) For each mechanic, use a different method to simulate the service times. Justify why you used that method. Write two R functions, one for each mechanic, that simulate the service times.

(ii) Generate 10000 iid values from each function and plot their histogram with the densities of the distributions to show that they have indeed been simulated correctly (multiply the densities with a constant so that they align with the histogram and explain how you got that constant).

4 CEO's visit to r cities

Foodys CEO needs to visit cities $1, 2, ..., r$ with city 0 being the city she is currently located at. Suppose a non-negative reward $v(i, j)$ is associated with the CEO going from city i to city j. So if the CEO visits the cities in permutation $x_1, ..., x_r$ then the reward of this choice $x = (x_1, ..., x_r)$ is:

$$V(x) = \sum_{i=1}^{r} v(x_{i-1}, x_i),$$

where $x_0 = 0$. Note that there is no reward for coming back to city 0. To generate $v(i, j)$, set seed to 1 and generate $v(i, j)$ using Uniform$(0, 1)$ random variables starting from $i = 0$ and $j = 1, ..., 10$, then $v(1, j)$ for $j = 2, ..., 10$, $v(2, j)$ for $j = 1, 3, 4, ..., 10$ and so on.

(i) Use MCMC and $r = 10$ to simulate high reward itineraries for the CEO. Define the stationary distribution that we want our Markov Chain to converge in such a way that high valued solutions are given extremely high probability and include a tunable parameter. Find 3 values of this parameter where the generated Markov Chain behaves differently in the long run for each of these 3 values and demonstrate this using plots and averages.

(ii) If you could change this parameter during one simulation of the stationary distribution, how would you change it in order to improve the solutions. Demonstrate that you can get better solutions than the fixed 3 values that you used in (i) by running it in R. Present all your results.