misc

SCHWARZ $[0,1] \xrightarrow{f} \mathbb{R}$ continuous, $f(x+h)+f(x-h)-2f(x)=o(h^2)$ for all $x \in (0,1)$ implies $f(x)=ax+b$ is affine.

proof wlog $f(0)=f(1)=0$. let $g_\varepsilon(t)=f(t)-\varepsilon t(1-t)$. then $\frac{g(t+h)+g(t-h)-2g(t)}{h^2} \to 2\varepsilon$ for $t \in (0,1)$ meaning g_ε has maximum at 0 or 1. hence $g_\varepsilon \leq 0$ for all ε and $f \leq 0$. similarly $f \geq 0$. ■

claim $a_n = o(n)$ (weakly) increasing sequence of positive integers $\implies \dfrac{n}{a_n}$ contains all positive integers.

proof as ka_n has the same properties, it suffices to show $a_n = n$ has a solution. at $n=1$ we have $\geq$, and at some point $<$. we cannot however have a flip from $>$ to $<$ as $a_n > n \implies a_{n+1} \geq n+1$. ■