

1      **Bumblebees minimize control challenges by combining active and passive**  
2      **modes in unsteady winds**

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10     **Abstract**

11     The natural wind environment that volant insects encounter is unsteady and highly  
12     complex, posing significant flight-control and stability challenges. Unsteady airflows can  
13     range from structured chains of discrete vortices shed in the wake of an object to fully  
14     developed chaotic turbulence. It is critical to understand the flight-control strategies  
15     insect employ to safely navigate in natural environments. We combined experiments on  
16     free flying bumblebees with high fidelity numerical simulations and lower-order  
17     modeling to identify the salient mechanics that mediate insect flight in unsteady winds.  
18     We trained bumblebees to fly upwind towards an artificial flower in a wind tunnel  
19     under steady wind and in a von Kármán street (23Hz) formed in the wake of a cylinder.  
20     The bees displayed significantly higher movement in the unsteady vortex street  
21     compared to steady winds. Correlation analysis revealed that at lower frequencies  
22     (<10Hz) in both steady and unsteady winds the bees mediated lateral movement with  
23     body roll - typical casting motion. At frequencies in unsteady winds there was a negative  
24     correlation between body roll and lateral accelerations. Numerical simulations of a  
25     bumblebee in similar conditions permitted the separation of the passive and active  
26     components of the flight trajectories. Comparison between the free-flying and  
27     'numerical' bees revealed a novel mechanism that enables bees to passively ride out  
28     high-frequency perturbations while performing active maneuvers and corrections at  
29     lower frequencies. The capacity of maintaining stability by combining passive and active  
30     modes at different timescales provides a viable means for volant animals and machines  
31     to tackle the control challenges posed by complex airflows.

32     **Significance**

33     Laboratory studies of insect flight typically employ still air or steady flow; natural flows  
34     are unsteady and turbulent. This raises fundamental questions about how insects  
35     control flight in realistic environments. We used a combination of experiments on freely  
36     flying bees, high-fidelity numerical simulations and reduced order modeling to  
37     disentangle the active and passive components of the flight dynamics of bumblebees. In  
38     unsteady winds bumblebees perform active maneuvers at lower frequencies while  
39     interacting passively with wind unsteadiness at higher frequencies. Our findings reveal a  
40     novel flight-control mechanism implemented by bumblebees to cope with complex  
41     airflows.

43 winds, one that could be widespread among volant organisms and suitable for artificial  
44 flying systems operating in outdoor environments.

45 **Introduction**

46 Volant insects are able to maintain stable flight in natural environments, despite  
47 frequently encountering unpredictable, complex airflows. While laboratory studies of  
48 insect flight typically employ steady flow, natural flows are often unsteady, containing  
49 de-stabilizing vortices and gusts over a wide range of spatial and temporal scales. These  
50 unsteady flows are generated by the interaction between wind and obstacles (1, 2),  
51 which results in chains of discrete vortices in the near wake of trees, branches and  
52 flowers, and fully developed, turbulent flow farther away.

53 The fact that unsteady airflow is ubiquitous in natural environments raises the  
54 fundamental question of whether insects need to actively respond to the full spectrum of  
55 unsteady flows that they experience in the natural world, or whether some of these  
56 disturbances can be handled with minimal energetic cost via passive mechanisms.  
57 Several recent studies have quantified the flight trajectories and kinematics of insects  
58 subjected to various flow perturbations, such as discrete gusts (3), von Kármán vortex  
59 streets (4, 5), or steady vortices (6). However, kinematics alone do not allow one to  
60 easily distinguish between passive body motions induced by a fluid disturbance and  
61 active, voluntary motions generated by the insect to maneuver or correct for a  
62 perturbation, particularly when the flow environment is unpredictable or unknown.

63 In this study, we focus on disentangling the active vs. passive components of a  
64 bumblebee's motion while flying through the unsteady, but structured, von Kármán  
65 vortex street generated behind a circular cylinder in flow. Bumblebees are ideal model  
66 organisms for studying insect flight in complex airflow, since they are relentless foragers  
67 that continue to fly in a wide range of weather conditions (7, 8). Prior work has shown  
68 that many flying animals, including bumblebees, are most sensitive to aerial  
69 disturbances along the lateral axis and rotations along the body's longitudinal axis (i.e.  
70 roll). Rolling motion and lateral translation are also critical components of voluntary  
71 maneuvering, and some insects have been shown to employ roll as the preferred means  
72 to perform rapid flight maneuvers (9, 10). We therefore chose to generate a von Kármán  
73 vortex street behind a vertical cylinder, which provides the greatest lateral and rolling  
74 perturbations (5), and chose a cylinder size ( $d = 25$  mm) corresponding to a typical  
75 bumblebee's wing span, which produces a chain of vortices shed at a rate of 23 Hz (5).  
76 We compare the rolling and lateral flight dynamics of bumblebees flying in the vortex  
77 street to their flight in steady flow. The relative importance of active control  
78 mechanisms vs. passive dynamics could most readily be tested by varying the level of  
79 active control that an insect can employ as it flies through unsteady flow, and  
80 quantifying the resulting change in its body dynamics and flight trajectory. However,  
81 such manipulations are unfeasible in living specimens, but numerical simulations of  
82 flying insects offer a suitable alternative. Simulations that combine flapping wing  
83 aerodynamics and free-flight dynamics provide a powerful tool that allows for  
84 simultaneous manipulation of all kinematic parameters of the insect model,  
85 aerodynamic forces and torques, and the ambient flow field. These types of simulations  
86 have provided significant insight into both the fine scale flow structures that form over  
87 flapping wings and the overall dynamics of insect flight, see (11, 12) for review, but such  
88 simulations have only been performed in still air hitherto. Here, we develop a model of a  
89 flapping bumblebee flying through a von Kármán vortex street that is nominally

90 identical to the one utilized in experiments on real bees. The model accounts for flow  
91 properties at all length scales in the oncoming flow and resolves the resulting lateral and  
92 roll flight dynamics of the insect due to its interaction with the wind. Such high fidelity  
93 simulations require extensive computational resources, and over 8192 cores and >280  
94 million grid points were used in this study, the highest number that has been used to  
95 resolve flapping flight to date.

96 We compare the flight trajectories and body motions of live bumblebees flying in the  
97 wake of a cylinder to those of the model bee in similar flow conditions with no active  
98 control, to parsimoniously estimate the role of active control and identify the control  
99 strategies being implemented by flying bumblebees.

100

## 101 **Results**

### 102 **Bumblebee Flight Dynamics**

103 During free flight in both steady and unsteady flows, the flight path of the bees typically  
104 consisted of large-amplitude motions at low frequencies (i.e., “casting”) that combined  
105 lateral translation and rolling (Fig. 1a; (5)). In unsteady flow, higher frequency  
106 translational and rolling motions were superimposed on top of these low-frequency  
107 casting motions (Fig. 1c, Supplementary Video 1), and earlier investigations revealed  
108 that these high-frequency motions occur primarily at the 23-Hz which is the vortex  
109 shedding frequency of the cylinder (5). In contrast, the lateral casting movements  
110 typically occur at frequencies below 10 Hz, and this was used as the cut-off frequency  
111 for examining the low- vs. high-frequency components of motion. The overall results  
112 were found to be nominally insensitive to the choice of cut-off frequency, so long as it  
113 was smaller than the frequency of the von Kármán street.

114 The magnitude of lateral accelerations associated with low-frequency, casting motions  
115 was similar in both steady and unsteady flow ( $p = 0.12$ , Fig 2a). High-frequency motions  
116 ( $> 10$  Hz) were not observed in steady flow. However in unsteady flow, the lateral  
117 accelerations associated with high-frequency motions were significantly higher than  
118 those associated with casting motions ( $p < 10^{-7}$ , Fig. 2a). The mean absolute roll angles of  
119 bees were also similar in both airflow conditions during casting ( $p = 0.48$ , Fig. 2a). The  
120 absolute roll angles measured at higher frequencies in unsteady flow were significantly  
121 smaller than those at lower frequencies ( $p < 10^{-6}$ , Fig. 2b).

122 During low-frequency casting, there was a strong, positive correlation between the  
123 instantaneous roll angle of bees and their lateral acceleration ( $C = 0.89 \pm 0.16$  in steady  
124 flow,  $C = 0.81 \pm 0.16$  in unsteady flow), and the strength of this correlation was similar in  
125 steady and unsteady flow ( $p = 0.20$ ). However, for higher-frequency motions in unsteady  
126 flow, there was a strong, negative correlation between roll angle and lateral acceleration  
127 ( $C = -0.81 \pm 0.13$ ), which was significantly different from the correlation for low-  
128 frequency motion in unsteady flow ( $p < 10^{-17}$ , Fig. 2c).

129 Due to the high cross correlations, to estimate the relationship between roll angle and  
130 lateral acceleration, we performed a linear regression between these variables, for low-

frequency casting motions in both flow conditions and for high-frequency motions in unsteady flow (Fig. 1b, 1d, 1e). We found the slope of the regression across all flight trials and compared mean values between flow conditions. For low-frequency casting motions, a statistically similar, positive regression slope was present in both wind conditions ( $p = 0.866$ , Fig. 2d). However, at higher frequencies in unsteady flow, the slope of this relationship was negative, and significantly different from the slope for low-frequency motions in unsteady flow ( $p < 10^{-14}$ , Fig. 2d).

### Assessing the Presence of Active Flight Control

The high-fidelity numerical simulations were conducted to evaluate the passive flight dynamics of a flapping bumblebee with no active control, flying in a von Kármán vortex street with the same characteristics as the experimental unsteady flow condition. Flight was simulated over 0.5 sec (76 wing beats), during which time the bumblebee interacted with a number of vortices in the von Kármán street (Fig. 3, Supp. Video 2, Supp. Fig. 1). The main outcome of the numerical simulation is the time evolution of the aerodynamic forces and torques acting on the insect, as well as the resulting lateral and rolling motions of the ‘numerical’ bee. Because the simulations did not include voluntary low-frequency casting motions, the flight dynamics of the model bee in the simulations were high-pass filtered and analyzed in the same way as the high-frequency motions of real bees flying in unsteady flow.

The high-pass filtered lateral and rotational positions and accelerations from the simulation showed good agreement with experimental measurements (Fig. 4 a - d). The time-varying lateral and rolling motions were nominally within the standard deviations of the high-frequency components of motion measured on freely flying bees in unsteady wind, and mean values of absolute roll angle and lateral acceleration were not significantly different from measurements on real bees (roll angle:  $p = 0.65$ ; lateral acceleration:  $p = 0.37$ ).

157

## Discussion

Aerial locomotion at small scales is very changeling and insects utilize a variety of unsteady aerodynamic mechanisms to generate the necessary forces to not only stay aloft but also perform coordinated maneuvers. In the natural environment volant organisms are also posed with wind unsteadiness. What are the added challenges imposed by wind unsteadiness and what mechanisms do insects employ to mitigate them?

### Low-frequency motions

In both steady and unsteady winds, bumblebees performed casting movements consisting of large, lateral displacements at low frequencies (Fig. 1a & Supp. Video 1). Such casting motions have also been observed in other insects, including wasps and honeybees (13, 14). While the significance of casting is not completely understood, some insects such as wasps perform casting flights to aid visual processing (13). In the experiments conducted here, the high, positive cross-correlation between roll angle and

172 lateral acceleration at the low frequencies associated with casting behavior (Fig. 2c)  
173 suggests a causative relationship between these motions.

174 The “helicopter model” of flight control, in which the net aerodynamic force vector  
175 remains fixed relative to the body and body attitude is adjusted to alter the direction of  
176 motion, has been demonstrated in several species of insects and birds (15, 16). Based on  
177 this model, acceleration or deceleration along the longitudinal axis requires a change in  
178 body pitch, and lateral maneuvers require a body rotation around the roll axis, to  
179 reorient the aerodynamic force vector. Our data on low-frequency casting motions in  
180 bumblebees supports the helicopter model of flight control. Considering level  
181 translation only along the lateral axis, the relationship between body roll ( $\psi$ , in rad)  
182 (refer to Supp. Fig. 1 for coordinate system) and lateral acceleration based on the  
183 helicopter model is ( $g \tan \psi$ ) (refer S2.1 for derivation), where  $g$  is the acceleration due  
184 to gravity. The roll angles observed during casting were sufficiently small, and upon  
185 applying the small angle approximation, the relationship between the roll angle and  
186 lateral acceleration becomes linear with the proportionality coefficient nominally equal  
187 to the acceleration due to gravity (see S2.1 for derivation). In our experiments, since the  
188 bees maintained altitude while casting in both steady and unsteady flow, the slope of the  
189 regression line between body roll and lateral acceleration was close to the expected  
190 value of  $9.8 \text{ m/s}^2/\text{rad}$  (steady airflow =  $8.6 \pm 1.2 \text{ m/s}^2$ , unsteady airflow =  $8.8 \pm 1.5 \text{ m/s}^2$ ;  
191 Fig. 2d).

192 The high correlation and positive relationship between low-frequency rolling motions  
193 and lateral acceleration in both steady and unsteady winds (Fig. 2d) suggests that the  
194 bees were performing the same type of controlled maneuvers at low frequencies in both  
195 flow situations. Additionally, the presence of casting motions even in the absence of  
196 external flow perturbations (i.e., in steady flow) indicates that these motions were most  
197 likely voluntary (see Fig. 5a and Supp. Video 3 for schematic representation and  
198 animation of casting motion, respectively). Based on the regression between lateral  
199 acceleration and roll, bees are able to generate a lateral acceleration of around  $1 \text{ m/s}^2$   
200 (typical accelerations measured during casting) by rolling approximately 6 degrees  
201 towards the direction of intended motion.

## 202 **High-frequency motions**

203 In the unsteady flow condition, we observed high-frequency rolling motions and lateral  
204 accelerations superimposed on top of the slow casting motions, with a clear separation  
205 between these two types of oscillations in terms of their temporal scale (Fig. 1c and  
206 Supp. Video 1). These high-frequency oscillations in unsteady flow have previously been  
207 shown to occur primarily at the vortex-shedding frequency associated with the wake of  
208 the cylinder (5). The most obvious difference between the low-frequency casting  
209 motions and the higher-frequency oscillations is in the phase relation between rolling  
210 and lateral motion. During casting in both flow conditions, we found a strong, positive  
211 correlation between roll and lateral acceleration (i.e., the bee rolls to the right and  
212 accelerates to the right), whereas at higher frequencies, there is an equally strong but  
213 negative correlation between these variables (Fig. 2c). The slope of the regression line  
214 between lateral acceleration and roll angle is negative and much larger in magnitude for  
215 the high-frequency motions ( $-80 \pm 17 \text{ m/s}^2/\text{rad}$ ; Fig. 2d); thus, mean lateral accelerations

216 are significantly larger and mean roll angles significantly smaller for high- vs. low-  
217 frequency motions (Fig. 2). These findings suggest that the high-frequency oscillations  
218 are fundamentally different from the voluntary, low-frequency casting motions.

219 The high-frequency oscillations corresponding to the vortex shedding rate could be  
220 purely passive motions induced by the external airflow, or they could represent a  
221 combination of passive, externally-induced motions and active, corrective maneuvers.  
222 To distinguish between these hypotheses, we performed high-fidelity numerical  
223 simulations based on the assumption that if the high-frequency motions were purely  
224 passive, then a ‘numerical’ bumblebee with no flight control would exhibit similar  
225 motion dynamics as measured in the experiments. The close similarity in temporal  
226 patterns of lateral and rolling motions exhibited by real and simulated bumblebees over  
227 several cycles of flow disturbance (approximately 20 wing beats; Fig. 4 a-d) lends  
228 support to the notion that the high-frequency motions are indeed predominantly due to  
229 passive interactions with vortices in the von Kármán street. This suggests that bees do  
230 not actively respond to these flow perturbations on a wingbeat-by-wingbeat basis.

231 To further illustrate the mechanics of the passive, high-frequency motions that bees  
232 exhibit in unsteady flow, we propose an idealized “sailboat” model. The governing  
233 equations for the high-frequency motions exhibited by the insect can be derived by  
234 considering a ‘numerical’ bumblebee subjected to a periodic lateral wind disturbance.  
235 This is a reasonable comparison since the von Kármán street induces similar lateral  
236 disturbances on a bee inflight. The relationship between the lateral acceleration and roll  
237 angle experienced would be as follows (see S2.2 for derivation):

$$G = \frac{F_y}{m\psi} = \frac{-4\pi^2 f_v^2 I_{xx}}{\bar{L}m} \quad (\text{Eq. 1})$$

238 Here;  $G$  is the relation between lateral acceleration and roll angle ( $\psi$ , in rad),  $m$  is the  
239 mass of the bee,  $F_y$  is the lateral acceleration,  $I_{xx}$  is the roll moment of inertia,  $\bar{L}$  is the  
240 moment arm, or the distance between the center of area and center of mass of the bee,  
241 and  $f_v$  is the frequency of the von Kármán street. The model asserts that the force  
242 induced by a lateral wind on the wings (which are located above the center of mass for  
243 much of the stroke cycle), and the misalignment between the center of pressure and  
244 center of mass of the body, leads to a lateral displacement and a corresponding roll, akin  
245 to a sailboat rolling when subjected to a side wind (see Fig. 5b and Supplementary Video  
246 4). This model preserves the phase relationship between roll angle and lateral  
247 acceleration while buffeting in the experiments. By substituting parameters for the  
248 “mean” bumblebee from the experiments into Eq. 1 the proportionality between the  
249 lateral acceleration and the roll angle due to the wind disturbance is around -79  
250 m/s<sup>2</sup>/rad (see S2.2 for elaboration), which is comparable to that measured in the free  
251 flying bees (-74 ± 13 m/s<sup>2</sup>/rad), see Fig. 2d. In Eq. 1 the distance between the center of  
252 aerodynamic pressure and center of mass critically influences the relationship, thus the  
253 dynamics of the high frequency motion may differ vastly between insects.

254 **Bimodal Flight Control**

255 While foraging in the outdoor environment, flying organisms are likely to be subjected  
256 to wind induced disturbances over a wide range of frequencies and scales, however the

magnitude of fluctuations over different frequencies and scales is still unclear due to lack of outdoor measurements. Nevertheless, performing active corrections for disturbances at all time scales is not only infeasible due to sensorimotor delays but also suboptimal. In this respect, it is vital to optimize the flight control system such that behaviorally relevant tasks can continue to be performed with minimal trade off. In the context of insect flight through unsteady winds, we distinguish between two distinct modes, i.e., the active and passive. Passive interactions with high frequency disturbances may be a suitable strategy since they impart relatively small net displacements that are likely to average over shorter time scales – as in this study where the high frequency displacements were smaller and tended to average over a few centiseconds, see Fig 1c & 5. Do the bees perform any corrective maneuvers when flying in the von Kármán street? The unfiltered time histories of the numerical simulations reveal that though the high frequency motions were relatively small in magnitude, the unsteady wind also introduces instabilities at lower frequencies, see SF2, and if uncorrected, these instabilities tend to increase over time. Therefore to maintain stable flight, some level of flight control is necessary but at frequencies much lower than the frequency of the von Kármán street. The von Kármán street induced disturbances at 23 Hz or 43 ms, which is within the perceptive limits and realms of sensorimotor response times of insects (3). Fruitflies subjected to mechanically induced rapid roll perturbations responded within 5 ms with active changes in wing kinematics (17). The passive response to higher frequencies disturbances noted in our experiments could be attributed to the relatively smaller magnitude of perturbations presented as well as the vast difference in organism properties. Additionally, insect responses to aerial perturbations have been shown to be context-specific whereby escape responses are mediated by specialized neural controllers that can be triggered within much smaller time scales (10).

Combination of active and passive modes in unsteady fluid environments is not uncommon in the animal kingdom. Classical experiments on fish swimming in von Kármán streets reveal a characteristic locomotion pattern known as Kármán gaiting where fish passively comply with the vortex street resulting in a combined bending and swaying of the body (18). This passive mechanism significantly reduces sensorimotor demands however additional active stabilization is still necessary but at much longer timescales, see (19) for review. The flight trajectories of the bees here do not resemble Kármán gaiting, but the mechanism may be considered analogous. The sensorimotor basis for the implementation of such mechanisms in insects is still an open question while we have a better understanding of aquatic locomotion.

Measurements made on Kármán gaiting fish reveal relatively limited additional cost of locomotion in unsteady fluids (19) but the energetic cost of flight in steady vs unsteady wind is still unclear. However unlike in aquatic locomotion, the variations in orientation angles while maneuvering and high frequency motions inflight would result in a reduction in altitude due to the reorientation of the aerodynamic force vector. Thus apart from implementing corrections to maintain body posture in unsteady wind, an overall increase in force production may be necessary to maintain altitude. Additionally performing active corrections may impose further metabolic cost but measurements on bumblebees in rarified medium and in load-lifting tests reveal that they can increase force production with relatively limited energetic expense (20, 21). Taken together we

302 observe a novel flight control mechanism implemented by bumblebees to cope with  
303 complex and unpredictable winds, a mechanism that could be widespread among volant  
304 organisms and one that could be suitable for artificial miniature aerial vehicles flying in  
305 outdoor environments.

306

## 307 MATERIALS AND METHODS

### 308 *Study specimens*

309 Bumblebees (*Bombus impatiens*) from a commercial breeder (BioBest) were maintained  
310 in the lab and given continuous access to a foraging chamber, where they could feed  
311 freely from an artificial flower containing linalool-scented nectar. Nominally similar  
312 sized bumblebees (body length = 14 mm ± 0.5 mm, mass = 165 mg ± 10%) were selected  
313 for flight experiments. The bees were placed in a transparent chamber (0.4 x 0.4 x 0.4  
314 m) without access to food, for approximately two hours prior to the experiment

### 315 *Flight Tests*

316 All experiments were conducted in a 6 m-long, suction-type, open-return wind tunnel  
317 with a 0.9 x 0.5 x 0.5 m working section. Once sufficiently starved, each bee was placed  
318 in the wind tunnel (with no airflow) where it could feed from an artificial flower  
319 resembling the one in the foraging chamber. Once feeding commenced, the bee was  
320 allowed to feed for approximately 10 seconds, then separated from the nectar source  
321 and released at the downstream end of the wind tunnel. Upon release, if the bee did not  
322 fly towards the artificial flower, it was manually re-introduced to the nectar source and  
323 subsequently separated. This procedure was repeated until direct flights to the nectar  
324 source were observed. Once consistent behavior was established, wind was introduced  
325 and bees were filmed as they flew upstream. The wind-speed was set to ~2.55 m/s,  
326 which represents an intermediate cruising velocity for bumblebees (8, 22).

327 Each bee was flown in two airflow conditions, steady and unsteady. In the wind tunnel  
328 with unimpeded (steady) flow, a uniform velocity profile was present across the  
329 interrogation volume (< 2 % variation in mean flow speed) and turbulence intensity  
330 (standard deviation/mean wind speed) was less than 1.2 %. With a cylinder (d = 25  
331 mm) positioned vertically at the inlet of the test section, a von Kármán street developed  
332 in the wake and vortex shedding occurred at 23 Hz, in agreement with the predicted  
333 vortex shedding Strouhal number of 0.19 (23, 24). The vertical cylinder induced lateral  
334 velocity fluctuations at the shedding rate over a range of ± 1.2 m/s. Refer to (5) for  
335 further elaboration of flow conditions.

336 We filmed bees and quantified airflow within a specific interrogation volume (a cube  
337 with side lengths of 100 mm, located 100 mm downstream from the cylinder). The  
338 downstream distance was chosen to avoid the recirculating region in the near wake of  
339 the cylinder and to allow the formation of a periodic von Kármán vortex street. A total  
340 of 13 bees were subjected to this assay, and paired trials in steady vs. unsteady flow  
341 were obtained for each individual.

342

343 ***Kinematic reconstruction and analysis***

344 Prior to experiments, a triangular marker was affixed to the dorsal surface of the thorax  
345 of each bee, to aid in quantifying its position and orientation during flight. The markers  
346 consisted of three black points representing the vertices of an isosceles triangle  
347 (measuring 2.7 x 2.3 mm) set upon a white background. During flight trials, bees were  
348 filmed as they flew through the interrogation volume using two Photron SA3 high-speed  
349 cameras sampling at 1000 Hz, placed above the wind tunnel at approximately 30° from  
350 the vertical. The recorded flight sequences were digitized using an open-source  
351 MATLAB-based routine, DLTdv5 (25), utilizing the automated tracking feature to  
352 localize the three black points on the triangular marker throughout each sequence.  
353 Digitization error in localizing the centroids of marker points is expected to be of the  
354 order of 1-2 pixels, which is much smaller than the mean number of pixels separating  
355 the markers (~30). The digitized position data were initially passed through a 4<sup>th</sup>-order,  
356 Butterworth, low-pass filter to remove any higher frequency errors due to the  
357 digitization process, with a cutoff frequency of 30 Hz, which is lower than the Nyquist  
358 frequency (500 Hz). For analysis of displacements at higher frequencies, the  
359 displacements were filtered using a high pass filter with 10Hz cut-off.

360 Instantaneous velocity and accelerations of the bees were calculated through numeric  
361 differentiation of the digitized position, see SF1 for coordinate system. The influence of  
362 flow conditions on the body orientation and rotation rates of bees was assessed by  
363 evaluating variation in roll, pitch and yaw angles of the triangular markers, using a rigid  
364 body assumption. The method detailed in (5) was used to calculate the instantaneous  
365 orientation and rotation rates of the bees, and to evaluate the errors associated with  
366 data capture and subsequent analysis. Subsequently, during analysis the lateral and  
367 rotational velocities and accelerations were separated into low (<10 Hz) and high  
368 frequency components of motion by applying a 4<sup>th</sup> order Butterworth 10-Hz low- or  
369 high-pass filter.

370 ***Numerical Modeling***

371 The numerical method employed in this study is described in greater detail in (26, 27),  
372 and the bumblebee model has been introduced in (28). In this section we provide a  
373 short overview of the model, we refer to Supplementary Figure 1 for further elaboration.  
374 The bumblebee is approximated by three rigid elements: the body and two wings, which  
375 move with respect to each other. The characteristic size of the insect is the wing length,  
376 which is equal to  $R = 13.2$  mm. The wings follow a prescribed periodic flapping motion  
377 with frequency  $f = 152$  Hz. Their positional angle varies as  $\Phi = 24^\circ + 115^\circ / \sin(2\pi ft)$ , the  
378 elevation with respect to the stroke plane is constant and equal to  $\theta = 12.55^\circ$ . The  
379 feathering angle  $\alpha$  is equal to  $70^\circ$  during upstroke and  $-40^\circ$  during downstroke, with  
380 sinusoidal variation at the reversals. The angle between the body longitudinal axis and  
381 the horizontal plane is equal to  $24.5^\circ$ , and the angle between the wing stroke plane and  
382 the horizontal plane is equal to  $28^\circ$ .

383 These wing kinematics ensure trimmed flight at  $u_\infty = 2.5$  m/s, assuming that the body  
384 mass is 175 mg. The position of the insect varies in time according to the solid-body  
385 dynamics model (26). In this study, only two degrees of freedom are taken into account:

386 lateral displacement and roll rotation about the longitudinal axis of the body. The roll  
387 moment of inertia is equal to  $I_{xx}=10.92 \times 10^{-10} \text{ kg m}^2$ . As with the experimental data  
388 gathered from bees in unsteady flow, the calculated translational and rotational  
389 positions and accelerations of the simulated bee were initially filtered using 4<sup>th</sup> order  
390 Butterworth low-pass filter with cut-off frequency 30 Hz. Subsequently to isolate the  
391 higher frequency motions, the data was high-pass filtered using a 4<sup>th</sup> order Butterworth  
392 10 Hz high-pass filter.

393 To generate unsteady flow in the simulation, a vertical cylinder (diameter=26.4 mm, Re  
394 = 4200) is placed in front of the insect, at approximately 80 mm from the insect's center  
395 of mass (Fig. 3, Supplementary Fig. 1). The cylinder has a salient detail on one side in  
396 order to break the symmetry of the flow. The flow is governed by the incompressible  
397 Navier-Stokes equations, and the surrounding air has density  $\rho=1.177\text{kg/m}^3$  and  
398 kinematic viscosity  $\eta = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$ . The flow domain is a rectangular channel of  
399 length 132 mm, having a 105.6 x 52.8 mm cross-section. The no-slip boundary condition  
400 at the surfaces of the insect and cylinder as well as the flow outlet condition, are  
401 imposed using the volume penalization method (see (26, 27) for details). The penalized  
402 equations are solved numerically using a Fourier pseudo-spectral method. The  
403 boundary condition on the side-walls of the channel is periodic. The flow domain is  
404 discretized using a uniform Cartesian grid consisting of 960 x 768 x 384 points (> 280  
405 million grid points). The volume penalization parameter is equal to  $\eta=10^{-3}$ . The  
406 resulting simulated flow environment accurately reproduced the unsteady flow  
407 environment used in experiments, with a vortex shedding rate and transient velocities  
408 matching those measured in the experiments.

409 **Statistical Analysis**

410 Statistical significance of experimental results was analyzed by performing paired t-tests  
411 ( $n = 14$  individuals in all cases) in MATLAB, between the low-frequency components of  
412 flight in steady vs. unsteady flow, and between the low- and high-frequency components  
413 of flight in unsteady flow. The mean lateral and rotational displacement and  
414 accelerations obtained from the simulation were compared to those measured on freely  
415 flying bees in the experiments by performing a one-sample t-test.

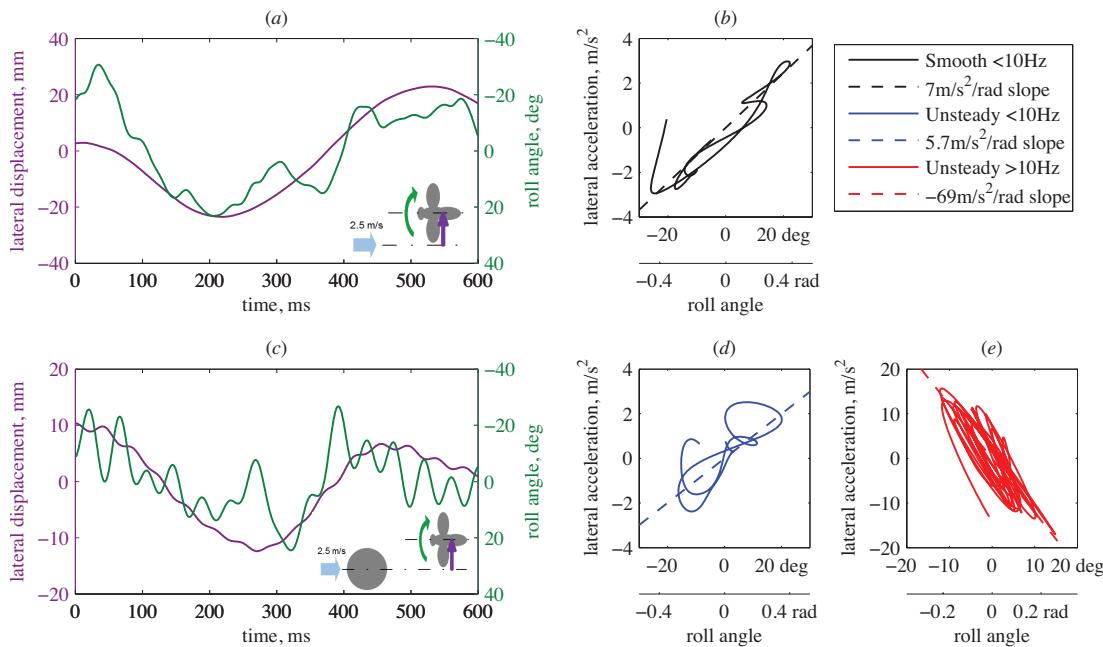
416 **References**

- 417 1. Stull RB (1988) *An Introduction to Boundary Layer Meteorology* ed Stull RB  
418 (Springer Netherlands, Dordrecht) doi:10.1007/978-94-009-3027-8.
- 419 2. Watkins S, Milbank J, Loxton BJ, Melbourne WH (2006) Atmospheric Winds and  
420 Their Implications for Microair Vehicles. *AIAA J* 44(11):2591–2600.
- 421 3. Vance JT, Faruque I, Humbert JS (2013) Kinematic strategies for mitigating gust  
422 perturbations in insects. *Bioinspir Biomim* 8(1):016004.
- 423 4. Ortega-Jimenez VM, Greeter JSM, Mittal R, Hedrick TL (2013) Hawkmoth flight  
424 stability in turbulent vortex streets. *J Exp Biol* 216(Pt 24):4567–79.
- 425 5. Ravi S, Crall JD, Fisher A, Combes SA (2013) Rolling with the flow: bumblebees  
426 flying in unsteady wakes. *J Exp Biol* 216(Pt 22):4299–309.

- 427 6. Ortega-Jimenez VM, Mittal R, Hedrick TL (2014) Hawkmoth flight performance in  
428 tornado-like whirlwind vortices. *Bioinspir Biomim* 9(2):025003.
- 429 7. Crall J, Combes S (2013) Blown in the wind: Bumblebee temporal foraging  
430 patterns in naturally varying wind conditions. *Integrative and Comparative  
431 Biology* (OXFORD UNIV. PRESS INC), p E270.
- 432 8. Riley JR, et al. (1999) Compensation for wind drift by bumble-bees. *Nature*  
433 400(6740):126–126.
- 434 9. Dudley R (2002) Mechanisms and implications of animal flight maneuverability.  
435 *Integr Comp Biol* 42(1):135–40.
- 436 10. Muijres FT, Elzinga MJ, Melis JM, Dickinson MH (2014) Flies evade looming  
437 targets by executing rapid visually directed banked turns. *Science*  
438 344(6180):172–7.
- 439 11. Shyy W, Aono H, Kang C-K, Liu H (2013) *An Introduction to Flapping Wing  
440 Aerodynamics* (Cambridge University Press). Volume 37 .
- 441 12. Sun M (2014) Insect flight dynamics: Stability and control. *Rev Mod Phys*  
442 86(2):615–646.
- 443 13. Boeddeker N, Dittmar L, Stürzl W, Egelhaaf M (2010) The fine structure of  
444 honeybee head and body yaw movements in a homing task. *Proc Biol Sci*  
445 277(1689):1899–906.
- 446 14. Braun E, Dittmar L, Boeddeker N, Egelhaaf M (2012) Prototypical components of  
447 honeybee homing flight behavior depend on the visual appearance of objects  
448 surrounding the goal. *Front Behav Neurosci* 6:1.
- 449 15. Thomas ALR, Taylor GK (2001) Animal Flight Dynamics I. Stability in Gliding  
450 Flight. *J Theor Biol* 212:399–424.
- 451 16. Ros IG, Bassman LC, Badger MA, Pierson AN, Biewener AA (2011) Pigeons steer  
452 like helicopters and generate down- and upstroke lift during low speed turns.  
453 *Proc Natl Acad Sci U S A* 108(50):19990–5.
- 454 17. Beatus T, Guckenheimer JM, Cohen I (2015) Controlling roll perturbations in fruit  
455 flies. *J R Soc Interface* 12(105):20150075–20150075.
- 456 18. Liao JC, Beal DN, Lauder G V, Triantafyllou MS (2003) Fish exploiting vortices  
457 decrease muscle activity. *Science* 302(5650):1566–9.
- 458 19. Liao JC (2007) A review of fish swimming mechanics and behaviour in altered  
459 flows. *Philos Trans R Soc Lond B Biol Sci* 362(1487):1973–93.
- 460 20. Dillon ME, Dudley R (2014) Surpassing Mt. Everest: extreme flight performance  
461 of alpine bumble-bees. *Biol Lett* 10(2):20130922.
- 462 21. Hedenstrom A, Ellington CP, Wolf TJ (2001) Wing wear, aerodynamics and flight  
463 energetics in bumblebees (*Bombus terrestris*): an experimental study. *Funct Ecol*

- 464 15(4):417–422.
- 465 22. Ellington CP (1991) Limitations on animal flight performance. *J Exp Biol* 160:71–  
466 91.
- 467 23. Roshko A (1961) Experiments on the flow past a circular cylinder at very high  
468 Reynolds number. *J Fluid Mech* 10(03):345.
- 469 24. Vickery BJ (1966) Fluctuating lift and drag on a long cylinder of square cross-  
470 section in a smooth and in a turbulent stream. *J Fluid Mech* 25(03):481.
- 471 25. Hedrick TL (2008) Software techniques for two- and three-dimensional  
472 kinematic measurements of biological and biomimetic systems. *Bioinspir Biomim*  
473 3(3):034001.
- 474 26. Engels T, Kolomenskiy D, Schneider K, Sesterhenn J (2015) FluSI: A novel parallel  
475 simulation tool for flapping insect flight using a Fourier method with volume  
476 penalization. *arXiv* 1506.06513.
- 477 27. Kolomenskiy D, Schneider K (2009) A Fourier spectral method for the Navier-  
478 Stokes equations with volume penalization for moving solid obstacles. *J Comput*  
479 *Phys* 228(16):5687–5709.
- 480 28. Engels T, Kolomenskiy D, Schneider K, Lehmann F-O, Sesterhenn J (2016)  
481 Bumblebee Flight in Heavy Turbulence. *Phys Rev Lett* 116(2):028103.

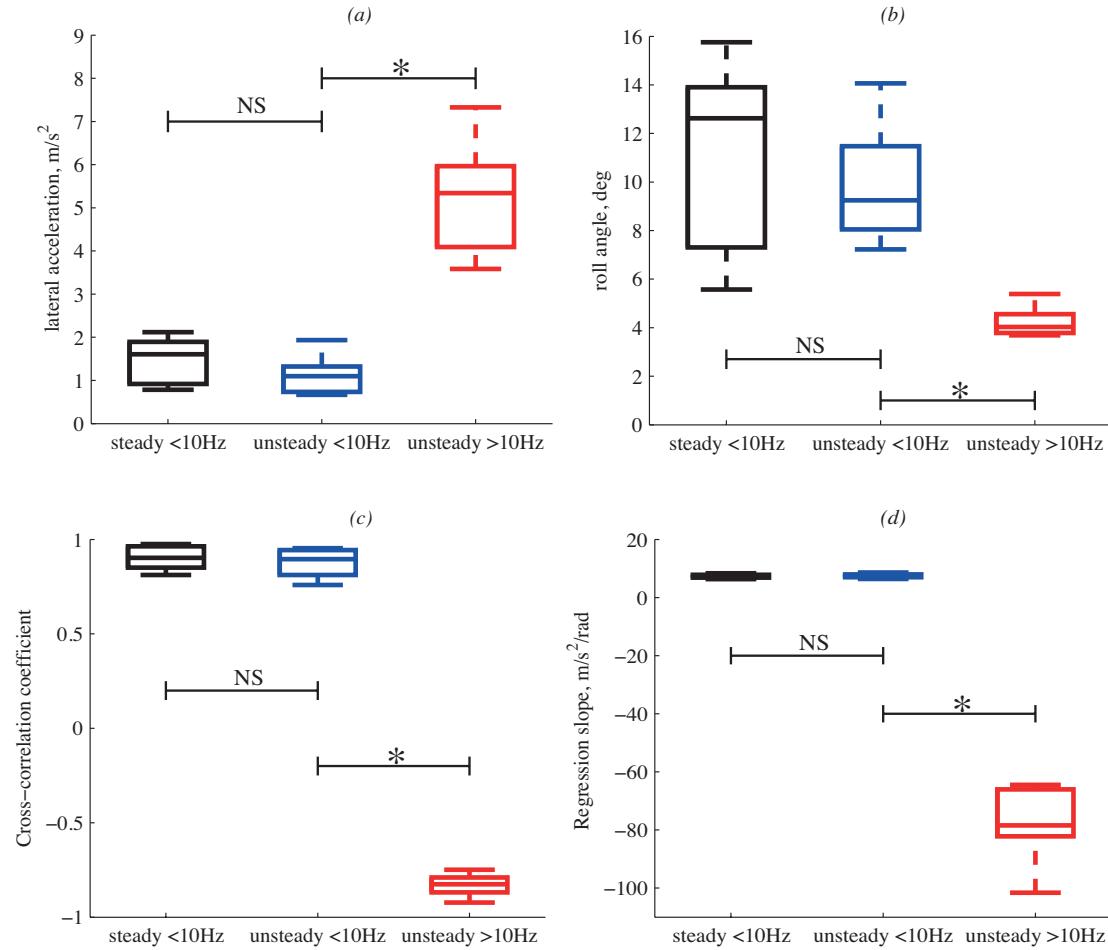
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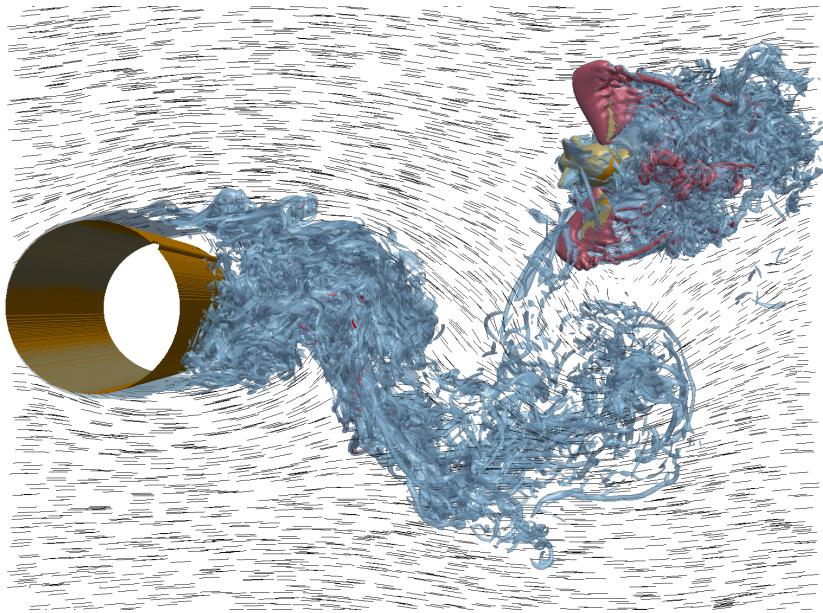
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*Figure 1: (a) Sample time series of lateral displacement and roll angle of a freely flying bumblebee in steady flow. (b) Instantaneous lateral acceleration vs. roll angle for lower-frequency (<10Hz) motions of the flight shown in (a). (c) Sample time series of lateral displacement and roll angle for the same bee flying in the same flow field but with higher-frequency oscillations. (d) Instantaneous lateral acceleration vs. roll angle for the higher-frequency flight. (e) Instantaneous lateral acceleration vs. roll angle for the higher-frequency flight.*

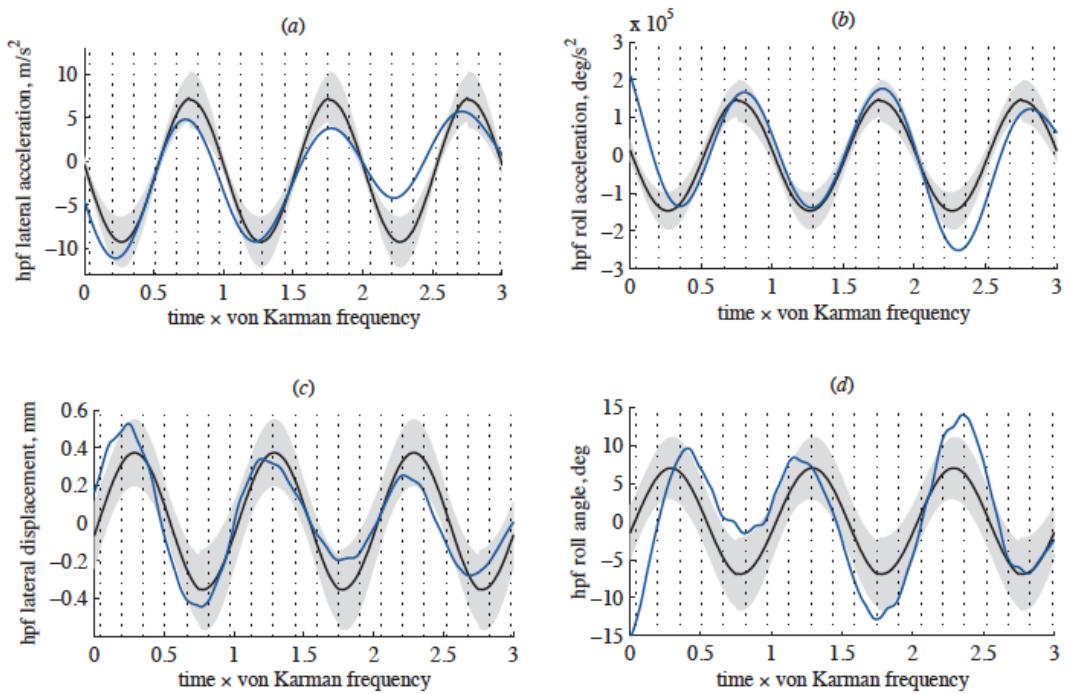
*unsteady wake of a vertical cylinder. (d) Instantaneous lateral acceleration vs. roll angle for lower- (d) and higher-frequency (e) motions of the flight shown in (c). In (b), (d) and (e) the linear regression between lateral acceleration and roll angle is shown by a dashed line.*



*Figure 2: Mean absolute lateral acceleration (a) and (b) roll angle of lower-frequency ( $<10\text{ Hz}$ ) casting motions in steady and unsteady flow, and higher-frequency ( $>10\text{ Hz}$ ) motions in unsteady flow. (c) Correlation coefficients (zero time-lag, normalized cross correlation) for the relationship between instantaneous lateral acceleration and roll angle for lower-frequency ( $<10\text{ Hz}$ ) casting motions in steady and unsteady flow, and higher-frequency ( $>10\text{ Hz}$ ) motions in unsteady flow. (d) The slope of the regression line between lateral acceleration and roll angle, in the same flow conditions as in (c). Asterisks indicate a significant difference at the  $p < 0.05$  level,  $n = 14$  for all conditions.*



*Figure 3: Birds eye view of the flow field obtained with numerical simulation. Isosurfaces of dimensionless vorticity magnitude are shown as, light blue  $|\omega| = 5$ , red  $|\omega| = 40$ . Arrows show the velocity in the horizontal plane that passes through the insect's center of gravity. The insect is colored in orange. Flow is shown at  $t = 106.5$  ms after the beginning of the simulation.*



*Figure 4: Time evolution of high pass filtered ( $hpft > 10Hz$ ) lateral accelerations (a) and displacements (b), and of roll accelerations (c) and angles (d) from the numerical simulation and experiments on freely flying bees. Blue line shows results of the numerical simulation (for motions  $> 10$  Hz), and black line/gray shading*

show the mean and standard deviation of measurements from experiments in unsteady flow (motions > 10 Hz). Dashed vertical lines represent wing beat periods.

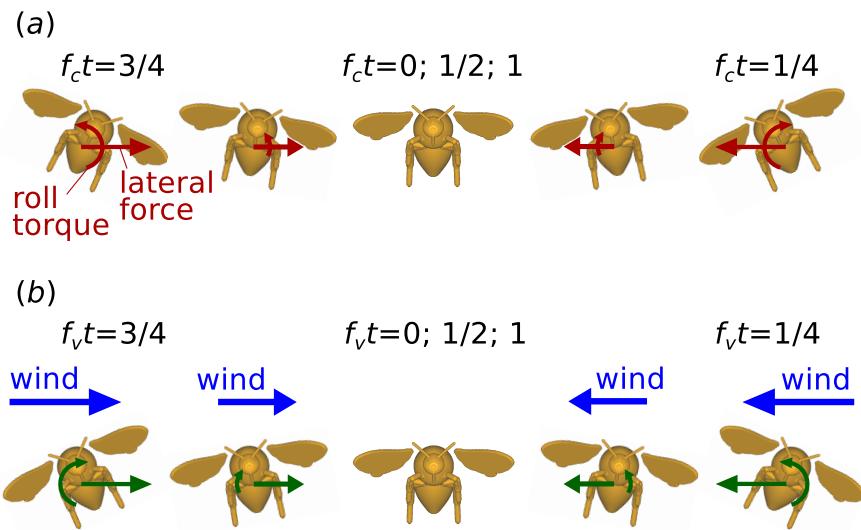


Figure 5: Schematic representation of the relationship between lateral and roll, accelerations and displacements, during (a) low frequency casting maneuvers (helicopter model) and (b) while high frequency buffeting in unsteady winds (sailboat model). The red and green arrows in (a) and (b) respectively represent the self-initiated and wind-induced forces and torques respectively. Here  $f_c$  (< 10 Hz) and  $f_v$  ( $\approx$  23 Hz) are casting and von Kármán frequencies respectively and  $t$  is normalized time.

# Supplementary material

## S1 Numerical simulation

The flow configuration used for the numerical simulation of uncontrolled flight is visualized in figure SF1, and a sample flow visualization is shown in figure 4. This section provides an extended discussion of the results of the numerical simulation.

Figure SF2 displays the time evolution of the accelerations and the displacements of the body from the beginning of the simulation. The insect is initially at rest, in-line with the cylinder, and its roll angle is zero. The asymmetry in the wake of the cylinder increases in time. The vortex street begins to have a significant influence on the insect after  $t = 50$  ms.

The profiles of linear and angular accelerations are shown in figure SF2(*top row*) with blue dashed lines. They both consist of slow quasi-periodic oscillations superimposed with a high-frequency noisy component. The frequency of quasi-periodic oscillations is about 24 Hz, which is the vortex shedding frequency of the cylinder. Note that it is slightly larger in the numerical simulation than in the experiment, because the diameter of the cylinder is also larger by 5%. This acceleration is a direct consequence

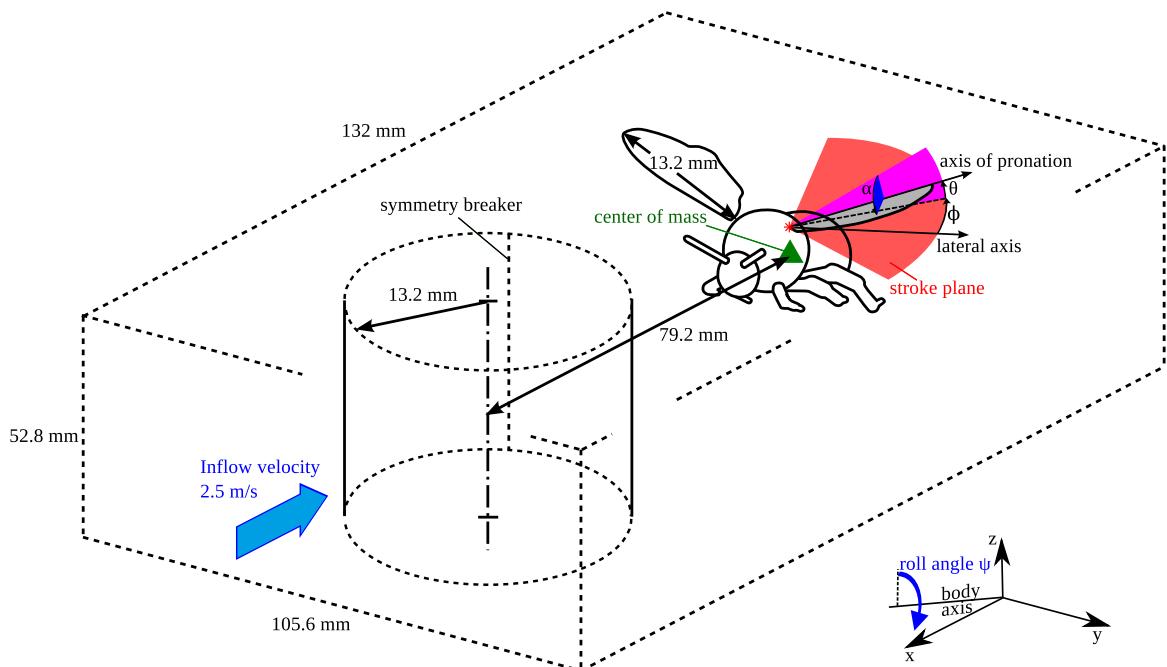


Figure SF1: Schematic drawing of the bumblebee model. See Materials and Methods section for the description of the numerical setup.

of the periodic sidewash by the vortex street. The high-frequency oscillations occur on the time scale even shorter than the wingbeat frequency. They are due to the aerodynamic interaction between the wings and the small-scale turbulence in the wake. In the wind tunnel experiment, the time resolution of the inverse dynamics analysis is limited by the frame rate of the video sequences. Therefore, in order to establish a comparison between the numerical simulation and the experiment, we applied the 4th order Butterworth low-pass filter at 30 Hz to the time series of the accelerations. The results are shown with the light blue lines. We focus on the time interval of three shedding periods (125 ms) after the beginning of the quasi-periodic state at  $t = 95$  ms.

The time evolution of the lateral displacement and of the roll angle is shown in figure SF2(*bottom row*). Note that oscillations are only visible at the von Kármán shedding frequency and not at the flapping frequency, because time integration has a low-pass filtering effect. It is also important to keep in mind that we are considering a fluid-structure interaction problem which does not necessarily have a time-periodic solution. An impulsive start from the quiescent flow initial condition and turbulence in the wake of the cylinder produce a slight imbalance between the consequent cycles of right and left sidewash during the beginning of the motion. These effects result in a slow large-amplitude drift of the insect model. However we are only interested in a time interval shorter than the slow casting motion, because the bees can definitely control their kinematics at that long time scale. Therefore we apply the 4th order Butterworth high-pass filter at 10 Hz to the time series of lateral and angular displacement in order to focus on the quasi-periodic oscillations at the von Kármán shedding frequency. The result is shown with solid lines in figure SF2(*bottom row*). Again, we focus on a time subsequence of 125 ms to compare with the experiment.

## S2 Reduced-order modeling of roll-sideslip oscillations

In this section we derive the equations that describe the dynamics of the low-frequency motion (casting) and high-frequency motion. The time series of accelerations and of the roll angle, obtained in the experiments, show scale separation between slow casting motion and rapid buffeting at the von Kármán shedding frequency. We hypothesize that these are two conceptually different modes:

- *Active (helicopter) mode*, which is a voluntary low-frequency casting motion;
- *Passive mode*, which is an involuntary passive high-frequency advection motion.

We propose idealized analytical models of these two modes.

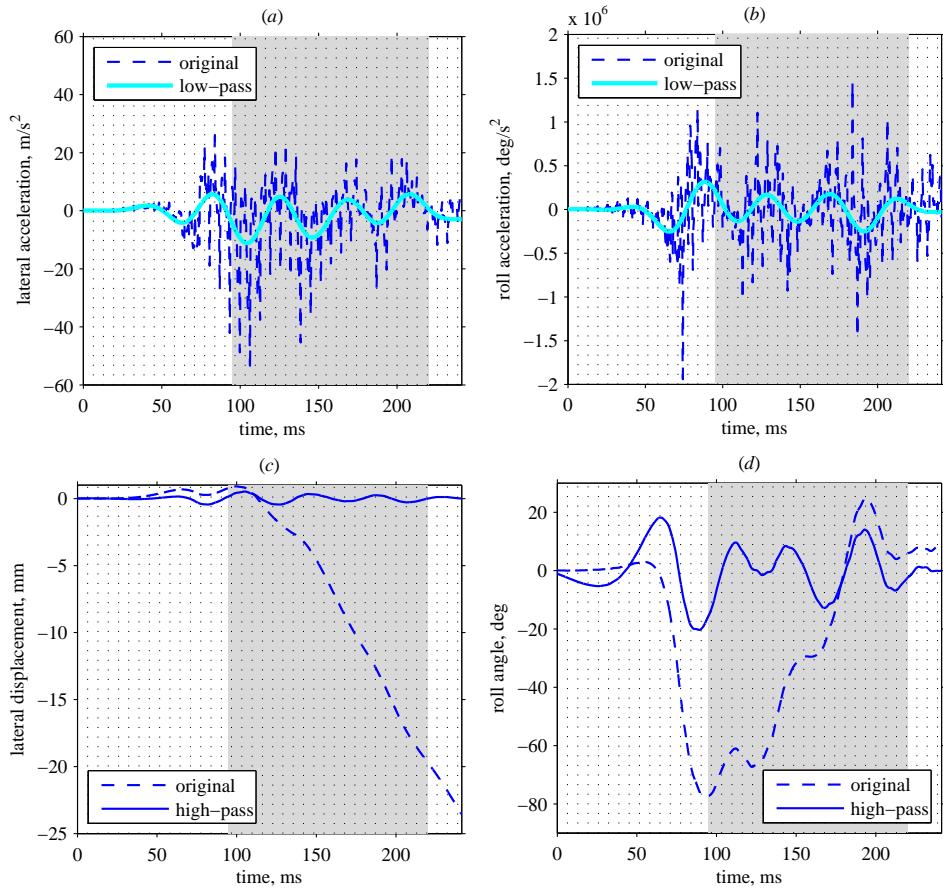


Figure SF2: Time evolution of the accelerations and displacements in the numerical simulation. Top row: lateral acceleration (a) and roll acceleration (b). Bottom row: lateral displacement (c) and roll angle (d). The gray shaded area indicates the time window used for comparison with the experiment. Vertical dotted lines indicate the wingbeat periods.

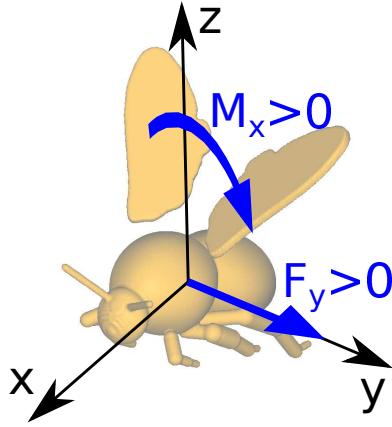


Figure SF3: Coordinate system attached to the insect body, used in the reduced-order models. Blue arrows show the positive directions of roll torque and side force.

## S2.1 Active (helicopter) mode

The insect drives this motion by asymmetrically changing the kinematics of its right and left wings. Thus, the insect controls the roll moment  $M_x$ . Let us assume it varies in time as a sine wave with given frequency  $f_c$ ,

$$M_x(t) = \hat{M} \sin(2\pi f_c t). \quad (1)$$

The body coordinate system is shown in figure SF3. To obtain the time evolution of the roll angle  $\psi(t)$ , we integrate the equation of roll motion,

$$I_{xx} \ddot{\psi} = M_x(t), \quad (2)$$

where  $I_{xx}$  is the moment of inertia. Using the time periodicity condition  $\psi(t + 1/f_c) = \psi(t)$ , we obtain

$$\psi = \hat{\psi} \sin(2\pi f_c t), \quad \text{where} \quad \hat{\psi} = -\frac{\hat{M}}{4\pi^2 f_c^2 I_{xx}}. \quad (3)$$

When the body roll angle  $\psi$  is non-zero, the side force  $F_y$  becomes non-zero. Let us assume that  $\psi$  varies in a range where we can use the small angle approximation (see figures 1 and 2 of the article). Since the vertical aerodynamic force supports the body weight,  $F_z = mg$ , we obtain

$$F_y = F_z \tan \psi \sim mg\psi. \quad (4)$$

Note that the aerodynamic force due the asymmetric wing kinematics of the right and left wings may also contribute to the lateral force. In our simple model we neglect this secondary effect. Using (3), we obtain

$$F_y = \hat{F} \sin(2\pi f_c t), \quad \text{where} \quad \hat{F} \sim -\frac{mg\hat{M}}{4\pi^2 f_c^2 I_{xx}}. \quad (5)$$

Finally, we integrate the equation of motion,

$$m\ddot{y} = F_y(t), \quad (6)$$

to obtain the time evolution of the lateral displacement,

$$y = \hat{y} \sin(2\pi f_c t), \quad \text{where } \hat{y} \sim \frac{g\hat{M}}{16\pi^4 f_c^4 I_{xx}}. \quad (7)$$

The motion described by (7) and (3) is visualized in figure 6(a). Note that while  $\hat{y}$  is positive,  $\hat{\psi}$  is negative. This implies that the leftmost lateral position (maximum positive  $y$ ) corresponds to the maximum roll to the right (maximum negative  $\psi$ ). From (7) and (3) we also obtain

$$\frac{\hat{y}}{\hat{\psi}} = -\frac{g}{4\pi^2 f_c^2}. \quad (8)$$

For example, at  $f_c = 2\text{Hz}$ , we obtain  $\hat{y}/\hat{\psi} = -62 \text{ mm/rad}$ . Hence, the roll amplitude of  $\hat{\psi} = 20^\circ$  would be accompanied by  $\hat{y} = -22\text{mm}$  lateral displacement amplitude. Whereas low-frequency casting motion only requires relatively small roll, high-frequency casting motion with the same lateral displacement amplitude would require unrealistically large roll angle.

Finally, for comparison with the experiment, it is important to notice that the gain coefficient in this model is a constant,

$$G = \frac{F_y}{m\psi} = g \approx 9.81\text{m/s}^2/\text{rad}. \quad (9)$$

## S2.2 Passive mode

In this case, we suppose that the insect is only driven by the aerodynamic drag due to the external flow. A major simplifying assumption is to neglect the time variation of the shape of the insect. In addition, we assume that the roll angle is sufficiently small so that any nonlinear coupling between the lower-frequency and the high-frequency motions is negligible. The insect model is placed in a von Kármán street, which induces periodic side wind with frequency  $f_v$ . Let us assume that, in the oscillating side wind, the elementary aerodynamic force acting on a surface element  $\sigma$  at a point  $\xi$  is equal to

$$\mathbf{q}(\xi, t) = \hat{\mathbf{q}}(\xi) \sin(2\pi f_v t). \quad (10)$$

By integration over the surface of the body, we obtain the force

$$\mathbf{F}(t) = \int_{\Sigma} \hat{\mathbf{q}}(\xi) \sin(2\pi f_v t) d\sigma(\xi) \quad (11)$$

and the torque

$$\mathbf{M}(t) = \int_{\Sigma} ((\xi - \mathbf{x}_{cg}) \times \hat{\mathbf{q}}(\xi)) \sin(2\pi f_v t) d\sigma(\xi), \quad (12)$$

where  $\Sigma$  is the surface of the insect and  $\mathbf{x}_{cg}$  is its center of gravity. We are only interested in the  $y$  component of  $\mathbf{F}(t)$  and  $x$  component of  $\mathbf{M}(t)$ . Let us define the moment arm  $L$ ,

$$L = M_x(t)/F_y(t). \quad (13)$$

The plus sign implies that the torque is negative if the force is positive and the center of pressure is situated above the center of gravity. We approximate  $L$  by a constant  $\bar{L}$ . A practical estimate of  $\bar{L}$  is given later in the text. We obtain the following approximate formulae for the side force and roll torque:

$$F_y = \hat{F} \sin(2\pi f_v t), \quad \text{where } \hat{F} = \int_{\Sigma} \hat{q}_y(\xi) d\sigma(\xi) \quad (14)$$

and

$$M_x = \hat{M} \sin(2\pi f_v t), \quad \text{where } \hat{M} = \hat{F} \bar{L}. \quad (15)$$

From the solid dynamics equations (2) and (6), requiring that the motion is periodic in time, we find

$$y = \hat{y} \sin(2\pi f_v t), \quad \text{where } \hat{y} = -\frac{\hat{F}}{4\pi^2 f_v^2 m} \bar{L} \quad (16)$$

and

$$\psi = \hat{\psi} \sin(2\pi f_v t), \quad \text{where } \hat{\psi} = -\frac{\hat{F} \bar{L}}{4\pi^2 f_v^2 I_{xx}}. \quad (17)$$

This motion is visualized in figure 6(b). Note that now the sign of  $\hat{y}$  is the opposite of  $\hat{\psi}$ . This implies that the leftmost lateral position (maximum positive  $y$ ) corresponds to the maximum roll to the left (maximum negative  $\psi$ ).

It is instructive to write the proportionality between the lateral displacement and the roll angle,

$$\frac{\hat{y}}{\hat{\psi}} = \frac{I_{xx}}{\bar{L}m}. \quad (18)$$

It does not depend of the frequency, in contrast to (8), but it depends on the shape of the body. In the experiments, the average body mass was equal to  $m = 165\text{mg}$ , moment of inertia  $I_{xx} = 8.49 \cdot 10^{-10}\text{kgm}^2$ , and the vortex shedding frequency  $f_v = 23\text{Hz}$ . The moment arm is estimated to be equal to  $\bar{L} = 1.4\text{mm}$ , using the numerical simulation. We thus obtain  $\hat{y}/\hat{\psi} = 3.7\text{mm/rad}$ . The roll angle amplitude of  $20^\circ$  corresponds now to the lateral displacement amplitude of only  $1.3\text{mm}$ .

Finally, let us calculate the proportionality coefficient

$$\Pi = \frac{F_y}{m\psi} = -\frac{4\pi^2 f_v^2 I_{xx}}{\bar{L}m} \approx -76.6\text{m/s}^2/\text{rad}, \quad (19)$$

which is a counterpart of the gain  $G$  defined by (9).