Homework #5

Due date: 18:00, November 14th, Monday, 2016

Problem statement

Let F_n be the n^{th} Fibonacci number. Then,

$$\text{LEMMA: } \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^n = \begin{bmatrix} F_{n-1} & F_n \\ F_n & F_{n+1} \end{bmatrix} \quad n \geq 1$$

(Proof omitted)

According to this lemma, the n^{th} Fibonacci number can be computed in $O(\log n)$ time by the fast exponential algorithm:

Let M be a $2 * 2$ square matrix, then:

$M^0 = I$ where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity matrix

$M^n = M * M^{n-1}, n > 0$ is odd

$= (M^2)^{n/2}, n > 0$ is even

Requirements

1. Represent the $2 * 2$ square matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ by

   ```
   struct matrix { int a,b,c,d; };
   ```

2. You shall write three functions, say

   ```
   struct matrix mul(matrix m1,matrix m2);
   ```
   // compute $m1 * m2$

   ```
   struct matrix pow(matrix m,int n);
   ```
   // compute M^n

   ```
   int F(int n);
   ```
   // compute F_n

3. The input will be in the range of 0~46
4. Plagiarism is not allowed!

Submission

Be sure to upload your source code to E3 by the due date and name your file as "**xxxxxxx_hw5.c**", where **xxxxxxx** is your student ID.

Sample run

Enter an integer >=0: 10
F(10) = 55

Enter an integer >=0: 20
F(20) = 6765

Enter an integer >=0: 30
F(30) = 832040

Enter an integer >=0: 40
F(40) = 102334155

Enter an integer >=0: 46
F(46) = 1836311903

Enter an integer >=0: ^Z