Homework #3

Problem statement

Given an unsigned integer $n \geq 2$, factor it into primes and use the factorization to determine the number, sum, and product of divisors of n.

Let $n = p_1^{e_1} p_2^{e_1} \dots p_k^{e_k}$, where $p_1 < p_2 < \cdots p_k$ are primes and $e_i > 0$, be the prime factorization of n. Then,

the number of divisors of n $= \prod_{i=1}^{k}(1 + e_i)$ … (1)

and

the sum of divisors of n $= \prod_{i=1}^{k}\sum_{j=0}^{e_i} p_i^j$ … (2)

and

the product of divisors of n $= n^{\left(\frac{(number\ of\ divisors\ of\ n)}{2}\right)}$

if $(number\ of\ divisors\ of\ n)$ is an even number … (3)

and

the product of divisors of n $= n^{\left(\frac{((number\ of\ divisors\ of\ n) - 1)}{2}\right)} * \sqrt{n}$

if $(number\ of\ divisors\ of\ n)$ is an odd number … (4)

For example, $20 = 2^2 * 5$ has 6 divisors, namely, 1, 2, 4, 5, 10 and 20, that sum up to 42, and product is equal to 8000, agreeing with formula $(1): (1 + 2)(1 + 1) = 6$ and formula $(2): (2^0 + 2^1 + 2^2)(5^0 + 5^1) = 42$ and formula $(3): 20^{\frac{6}{2}} = 8000$.

Requirements

1. The easiest factorization method is the trial division algorithm that consists of the following loop:

   While (*not finish yet*) {

       Find the next prime p

       Find the largest integer e such that p^e divides n

       Reduce n to $\dfrac{n}{p^e}$

   }

   For example, let $n = 20 = 2^2 * 5$, the values of p, e, and n at the end of each iteration are shown below:

1st iteration	$p = 2$	$e = 2$	$n = 5$
2st iteration	$p = 3$	$e = 0$	$n = 5$
3st iteration	$p = 5$	$e = 1$	$n = 1$

2. The product of divisors of n is also an unsigned integer.
3. Be careful of integer overflow especially the product of divisors of n.
4. Write a C program that is capable of handling input.
5. Properly comment your program.
6. Plagiarism is not allowed!
7. See the sample run below for the required output format.

Submission

Be sure to upload your source code to E3 by the due date and name your file as "**xxxxxxx_hw3.c**", where **xxxxxxx** is your student ID.

Sample run

Enter an unsigned integer >= 2: 20
Prime factorization of 20 = 2^2x5^1
Number of divisors = 6
Sum of divisors = 42
Product of divisors = 8000

Enter an unsigned integer >= 2: 81
Prime factorization of 81 = 3^4
Number of divisors = 5
Sum of divisors = 121
Product of divisors = 59049

Enter an unsigned integer >= 2: 1421
Prime factorization of 1421 = 7^2x29^1
Number of divisors = 6
Sum of divisors = 1710
Product of divisors = 2869341461

Enter an unsigned integer >= 2: 84
Prime factorization of 84 = 2^2x3^1x7^1
Number of divisors = 12
Sum of divisors = 224
Product of divisors overflow!

Enter an unsigned integer >= 2: 3287037600
Prime factorization of 3287037600 = 2^5x3^2x5^2x7^3x11^3
Number of divisors = 864
Sum of divisors = 1982896512
Product of divisors overflow!

Enter an unsigned integer >= 2: 4198216889
Prime factorization of 4198216889 = 60917^1x68917^1
Number of divisors = 4
Sum of divisors = 4198346724
Product of divisors overflow!

Enter an unsigned integer >= 2: 4294967279
Prime factorization of 4294967279 = 4294967279^1
Number of divisors = 2
Sum of divisors = 4294967280
Product of divisors = 4294967279

Enter an unsigned integer >= 2: ^Z