2025년 1학기 11주차(2)

스마트팜 응용역학

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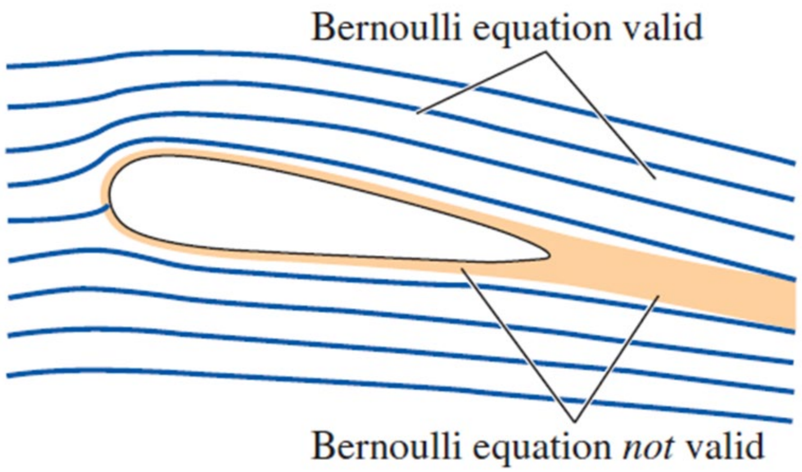
The Bernoulli Equation

Bernoulli equation: An approximate relation between pressure, velocity magnitude, and elevation, and is valid in regions of steady, incompressible flow where net frictional forces are negligible.

Despite its simplicity, it has proven to be a very powerful tool in fluid mechanics.

The Bernoulli approximation is typically useful in flow regions outside of boundary layers and wakes, where the fluid motion is governed by the combined effects of pressure and gravity forces.



The *Bernoulli equation* is an *approximate* equation that is valid only in *inviscid regions of flow* where net viscous forces are negligibly small compared to inertial, gravitational, or pressure forces. Such regions occur outside of *boundary layers* and *wakes*.

The Bernoulli Equation

Acceleration of a Fluid Particle.

In two-dimensional flow, the acceleration can be decomposed into two components:

streamwise acceleration a_s along the streamline and

normal acceleration a_n in the direction normal to the streamline, which is

given as $a_{\mathrm{n}} = V^2/R.$

Streamwise acceleration is due to a change in speed along a streamline, and normal acceleration is due to a change in direction.

For particles that move along a *straight path,* $a_n = 0$ since the radius of curvature is infinity and thus there is no change in direction. The Bernoulli equation results from a force balance along a streamline.

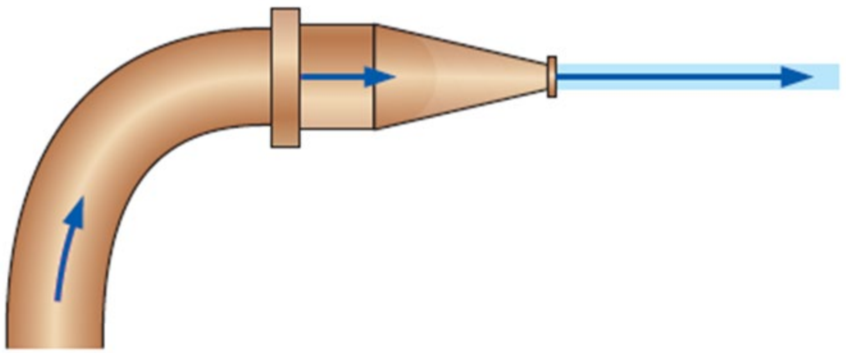
The Bernoulli Equation

$$dV = \frac{\partial V}{\partial s} ds + \frac{\partial V}{\partial t} dt \qquad \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} + \frac{\partial V}{\partial t}$$

$$\partial V / \partial t = 0 \qquad V = V(s)$$

$$a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} = \frac{\partial V}{\partial s} V = V \frac{dV}{ds}$$

$$V = ds/dt$$



Acceleration in steady flow is due to the change of velocity with position.

During steady flow, a fluid may not accelerate in time at a fixed point, but it may accelerate in space.

The Bernoulli Equation

Derivation of the Bernoulli Equation.

$$\sum F_s = m\,a_s \qquad P\,dA-(P+dP)\,dA-dW\,\sin\theta = dm\,V\frac{dV}{ds}$$

$$dm=\rho d\mathrm{V}=\rho\,dA\,ds \qquad dW=dm\,g=\rho g\,dA\,ds$$

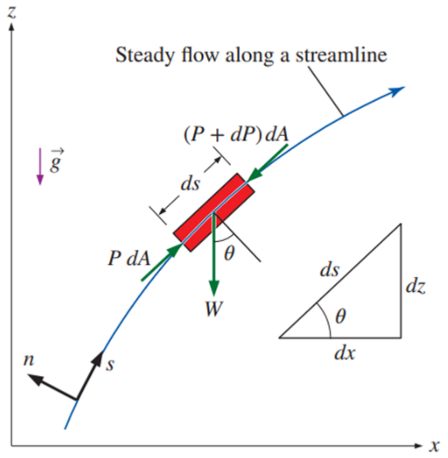
$$\sin\theta = dz/ds \qquad -dP\,dA-\rho g\,dA\,ds\frac{dz}{ds}=\rho\,dA\,ds\,V\frac{dV}{ds}$$

$$-dP-\rho g\,dz=\rho V\,dV \qquad V\,dV=\tfrac{1}{2}d(V^2)$$

$$\frac{dP}{\rho}+\frac{1}{2}d\left(V^2\right)+g\,dz=0$$

Steady flow

$$\int\frac{dP}{\rho}+\frac{V^2}{2}+gz=\text{constant (along a streamline)}$$



The forces acting on a fluid particle along a streamline.

The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when compressibility and frictional effects are negligible.

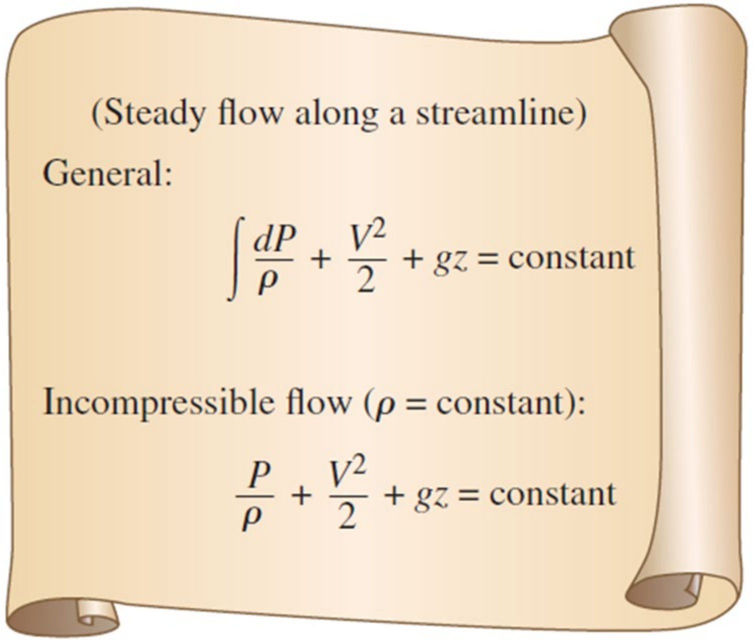
Steady, incompressible flow "Bernoulli equation"

$$\frac{P}{\rho}+\frac{V^2}{2}+gz=\text{constant (along a streamline)}$$

The Bernoulli equation between any two points on the same streamline.

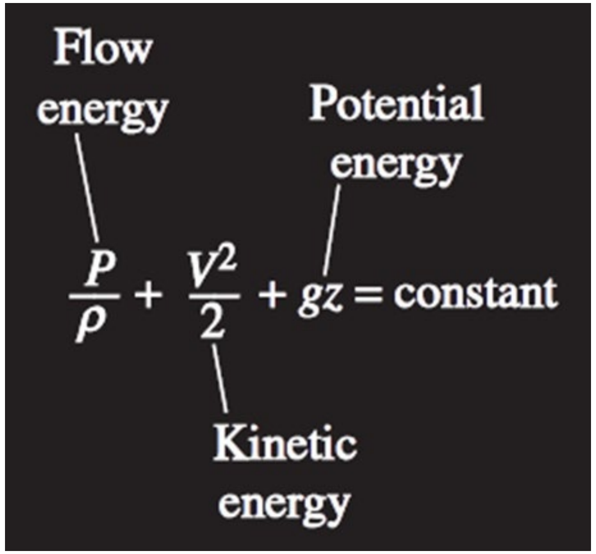
$$\frac{P_1}{\rho}+\frac{V_1^2}{2}+g\,z_1=\frac{P_2}{\rho}+\frac{P_2^2}{2}+g\,z_2$$

The Bernoulli Equation



The incompressible Bernoulli equation is derived assuming incompressible flow, and thus it should not be used for flows with significant compressibility effects.

The Bernoulli Equation



The Bernoulli equation states that the sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow.

The **Bernoulli equation** can be viewed as the "***conservation of mechanical energy principle***."

This is equivalent to the general conservation of energy principle for systems that do not involve any conversion of mechanical energy and thermal energy to each other, and thus the mechanical energy and thermal energy are conserved separately.

The Bernoulli equation states that during steady, incompressible flow with negligible friction, the various forms of mechanical energy are converted to each other, but their sum remains constant.

There is no dissipation of mechanical energy during such flows since there is no friction that converts mechanical energy to sensible thermal (internal) energy.

The Bernoulli equation is commonly used in practice since a variety of practical fluid flow problems can be analyzed to reasonable accuracy with it.

The Bernoulli Equation

Force Balance across Streamlines.

Force balance in the direction *n* normal to the streamline yields the following relation applicable *across* the streamlines for steady, incompressible flow:

$$\frac{P}{\rho} + \int \frac{V^2}{R} dn + gz = \text{constant} \qquad (\text{across streamlines})$$

For flow along a straight line, $R \to \infty$

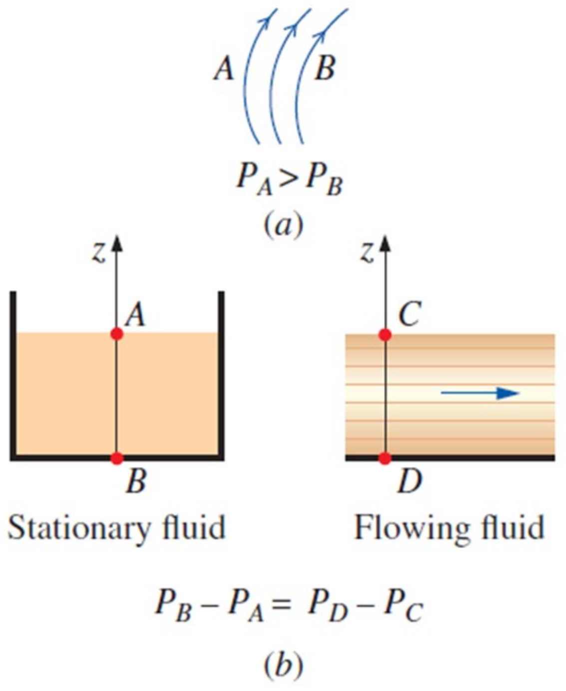
and this equation reduces to

$P/\rho + gz = \text{constant}$ or

$P = -\rho gz + \text{constant}$ which is an expression

for the variation of hydrostatic pressure with vertical distance for a stationary fluid body.

Pressure decreases towards the center of curvature when streamlines are curved (*a*), but the variation of pressure with elevation in steady, incompressible flow along a straight line (*b*) is the same as that in stationary fluid.



The Bernoulli Equation

Unsteady, Compressible Flow.

The Bernoulli equation for *unsteady, compressible flow:*

$$Unsteady, compressible\ flow: \quad \int \frac{dP}{\rho} + \int \frac{\partial V}{\partial t} ds + \frac{V^2}{2} + gz = f(t)$$

The Bernoulli Equation

Static, Dynamic, and Stagnation Pressures.

The kinetic and potential energies of the fluid can be converted to flow energy (and vice versa) during flow, causing the pressure to change. Multiplying the Bernoulli equation by the density gives.

$$P+\rho\frac{V^2}{2}+\rho gz=\text{constant (along a streamline)}$$

***P* is the static pressure:** It does not incorporate any dynamic effects; it represents the actual thermodynamic pressure of the fluid. This is the same as the pressure used in thermodynamics and equations of state.

$\rho V^2/2$ **is the kinetic energy per unit volume or the dynamic pressure:** It

represents the pressure rise when the fluid in motion is brought to a stop isentropically.

The Bernoulli Equation

Static, Dynamic, and Stagnation Pressures.

The kinetic and potential energies of the fluid can be converted to flow energy (and vice versa) during flow, causing the pressure to change. Multiplying the Bernoulli equation by the density gives.

$$P+\rho\frac{V^2}{2}+\rho gz=\text{constant (along a streamline)}$$

ρgz **is the potential energy per unit volume or the hydrostatic pressure:** It is not pressure in a real sense since its value depends on the reference level selected; it accounts for the elevation effects, that is, fluid weight on pressure. (Be careful of the sign—unlike hydrostatic pressure ρgh which *increases* with fluid depth *h*, the hydrostatic pressure term ρgz *decreases* with fluid depth.)

The Bernoulli Equation

Static, Dynamic, and Stagnation Pressures.

The kinetic and potential energies of the fluid can be converted to flow energy (and vice versa) during flow, causing the pressure to change. Multiplying the Bernoulli equation by the density gives.

$$P + \rho \frac{V^2}{2} + \rho g z = \text{constant (along a streamline)}$$

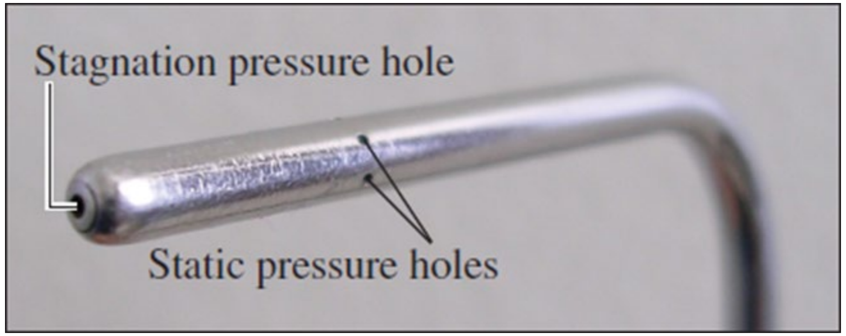
Total pressure: The sum of the static, dynamic, and hydrostatic pressures. Therefore, the Bernoulli equation states that *the total pressure along a streamline is constant*.

The Bernoulli Equation

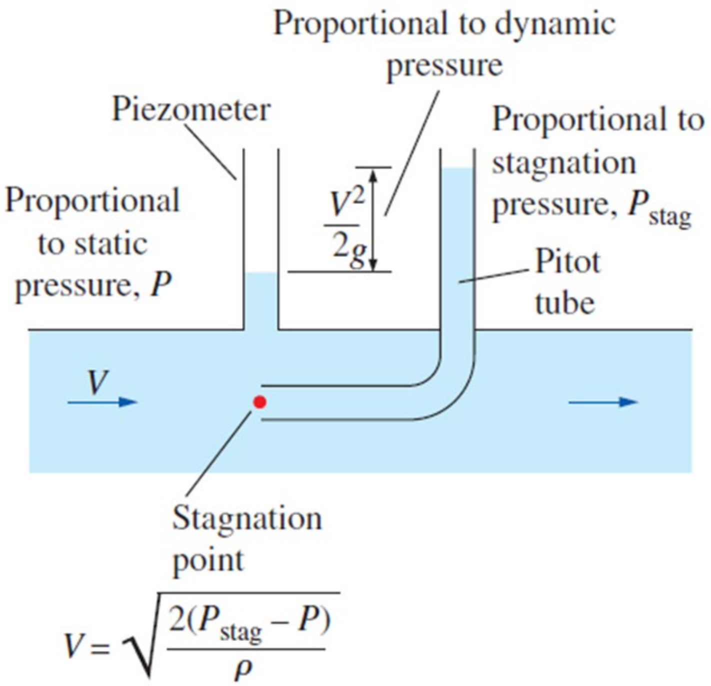
Stagnation pressure: The sum of the static and dynamic pressures. It represents the pressure at a point where the fluid is brought to a complete stop isentropically.

$$P_{\text{stag}} = p + \rho \frac{V^2}{2} \quad (\text{kPa})$$

$$V = \sqrt{\frac{2\left(P_{\text{stag}} - P\right)}{\rho}}$$

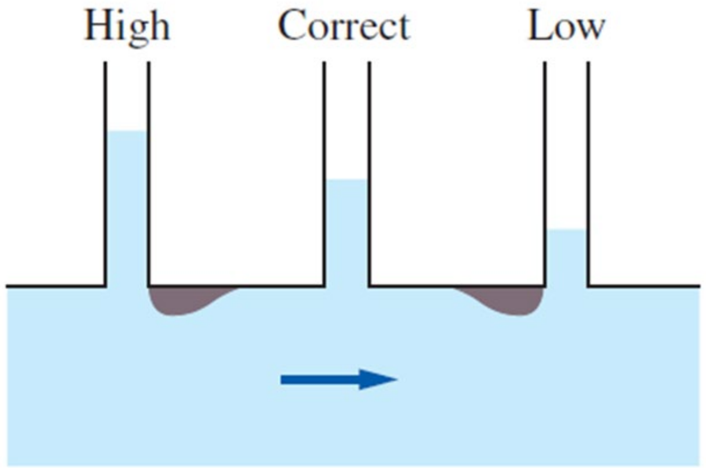


Close-up of a **Pitot-static probe**, showing the stagnation pressure hole and two of the five static circumferential pressure holes.

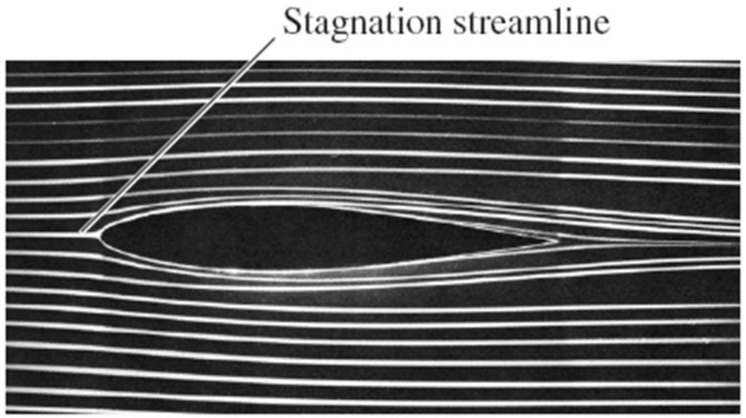


The static, dynamic, and stagnation pressures measured using **piezometer tubes**.

The Bernoulli Equation



Careless drilling of the static pressure tap may result in an erroneous reading of the static pressure head.



Courtesy of ONERA. Photo by Werlé.

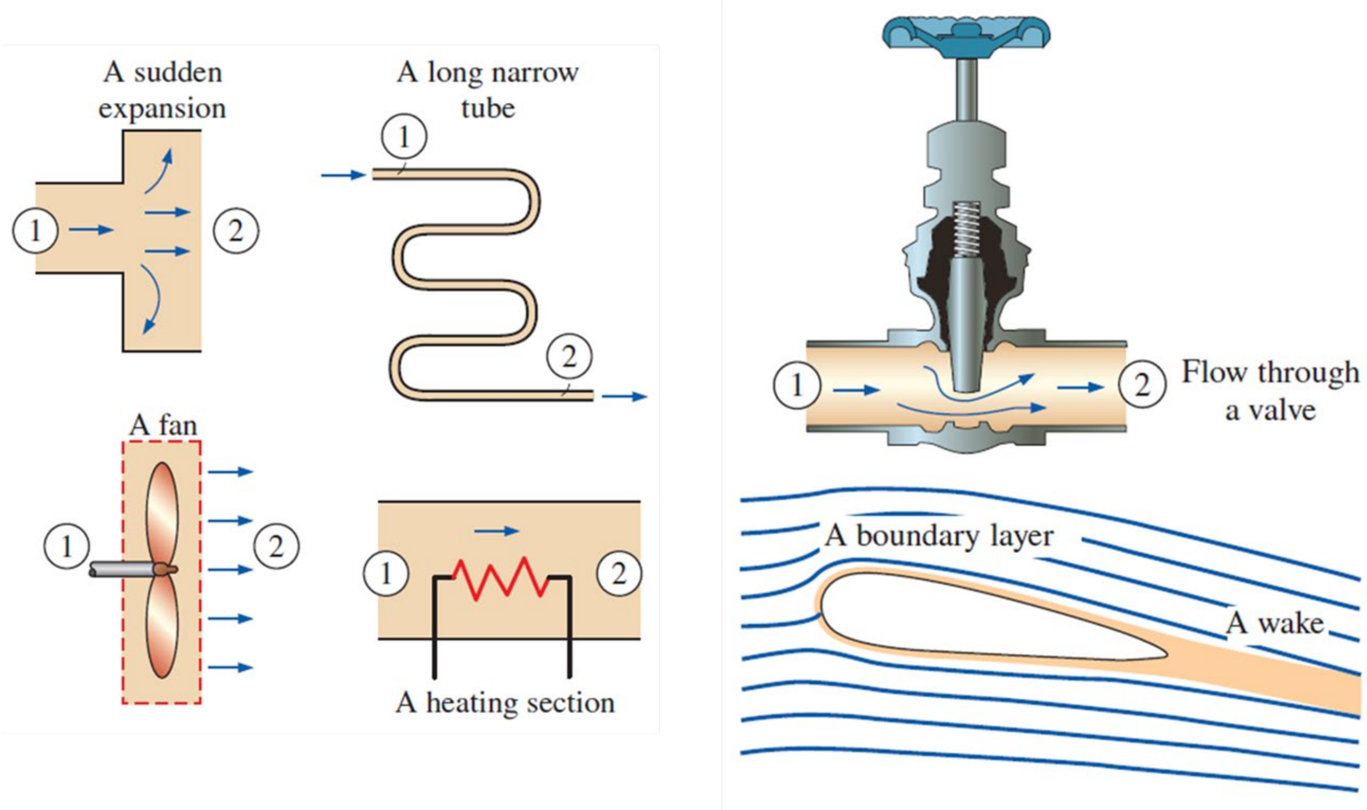
Streaklines produced by colored fluid introduced upstream of an airfoil; since the flow is steady, the streaklines are the same as streamlines and pathlines. The **stagnation streamline** is marked.

The Bernoulli Equation

Limitations on the Use of the Bernoulli Equation:

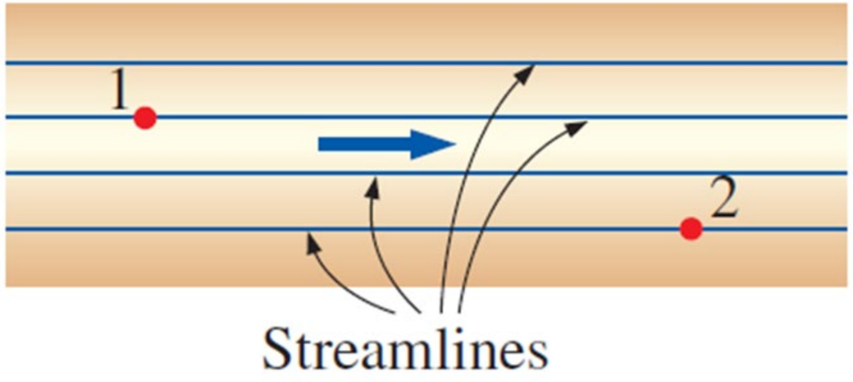
1. **Steady flow:** The Bernoulli equation is applicable to *steady flow*.
2. **Frictionless flow:** Every flow involves some friction, no matter how small, and *frictional effects* may or may not be negligible.
3. **No shaft work:** The Bernoulli equation is not applicable in a flow section that involves a pump, turbine, fan, or any other machine or impeller since such devices destroy the streamlines and carry out energy interactions with the fluid particles. When these devices exist, the energy equation should be used instead.
4. **Incompressible flow:** Density is taken constant in the derivation of the Bernoulli equation. The flow is incompressible for liquids and also by gases at Mach numbers less than about 0.3.
5. **No heat transfer:** The density of a gas is inversely proportional to temperature, and thus the Bernoulli equation should not be used for flow sections that involve significant temperature change such as heating or cooling sections.
6. **Flow along a streamline:** Strictly speaking, the Bernoulli equation is applicable along a streamline. However, when a region of the flow is *irrotational* and there is negligibly small *vorticity* in the flow field, the Bernoulli equation becomes applicable across streamlines as well.

The Bernoulli Equation



Frictional effects, heat transfer, and components that disturb the streamlined structure of flow make the Bernoulli equation invalid. It should *not* be used in any of the flows shown here.

The Bernoulli Equation



$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

When the flow is irrotational, the Bernoulli equation becomes applicable between any two points along the flow (not just on the same streamline).

The Bernoulli Equation

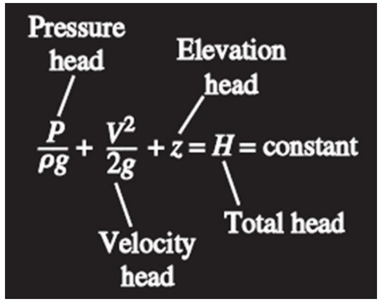
Hydraulic Grade Line (HGL) and Energy Grade Line (EGL).

It is often convenient to represent the level of mechanical energy graphically using ***heights*** to facilitate visualization of the various terms of the Bernoulli equation. Dividing each term of the Bernoulli equation by *g* gives.

$$\frac{P}{\rho g}+\frac{V^2}{2g}+z=H=\text{constant}\quad(\text{along a streamline})$$

$P/\rho g$ **is the pressure head;** it represents the height of a fluid column that produces the static pressure *P*.

$V^2/2g$ **is the velocity head;** it represents the elevation needed for a fluid to reach speed *V* during frictionless free fall.



z **is the elevation head;** it represents the potential energy of the fluid.

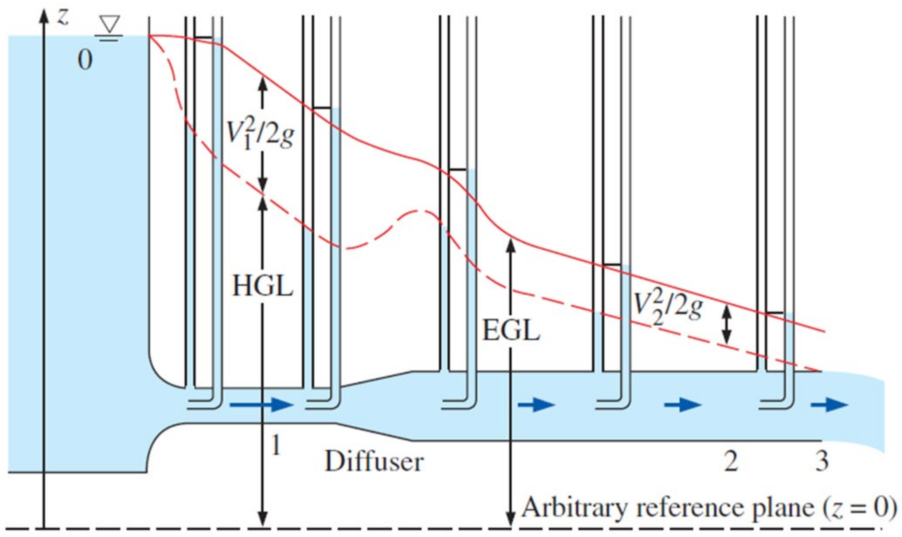
An alternative form of the Bernoulli equation is expressed in terms of heads as: ***The sum of the pressure, velocity, and elevation heads is constant along a streamline.***

The Bernoulli Equation

Hydraulic grade line (HGL), $P/\rho g + z$ The line that represents the sum of the static pressure and the elevation heads.

Energy grade line (EGL), $P/\rho g + V^2/2g + z$ The line that represents the total head of the fluid.

Dynamic head, $V^2/2g$ The difference between the heights of EGL and HGL.



The *hydraulic grade* line (HGL) and the *energy grade line* (EGL) for free discharge from a reservoir through a horizontal pipe with a diffuser.

The Bernoulli Equation

Notes on HGL and EGL.

For *stationary bodies* such as reservoirs or lakes, the EGL and HGL coincide with the free surface of the liquid.

The EGL is always a distance $V^2/2g$ above the HGL. These two curves approach each other as the speed decreases, and they diverge as the speed increases.

In an *idealized Bernoulli-type flow*, EGL is horizontal and its height remains constant.

For *open-channel flow*, the HGL coincides with the free surface of the liquid, and

and the EGL is a distance $V^2/2g$ above the free surface.

The Bernoulli Equation

Notes on HGL and EGL.

At a *pipe exit*, the pressure head is zero (atmospheric pressure) and thus the HGL coincides with the pipe outlet.

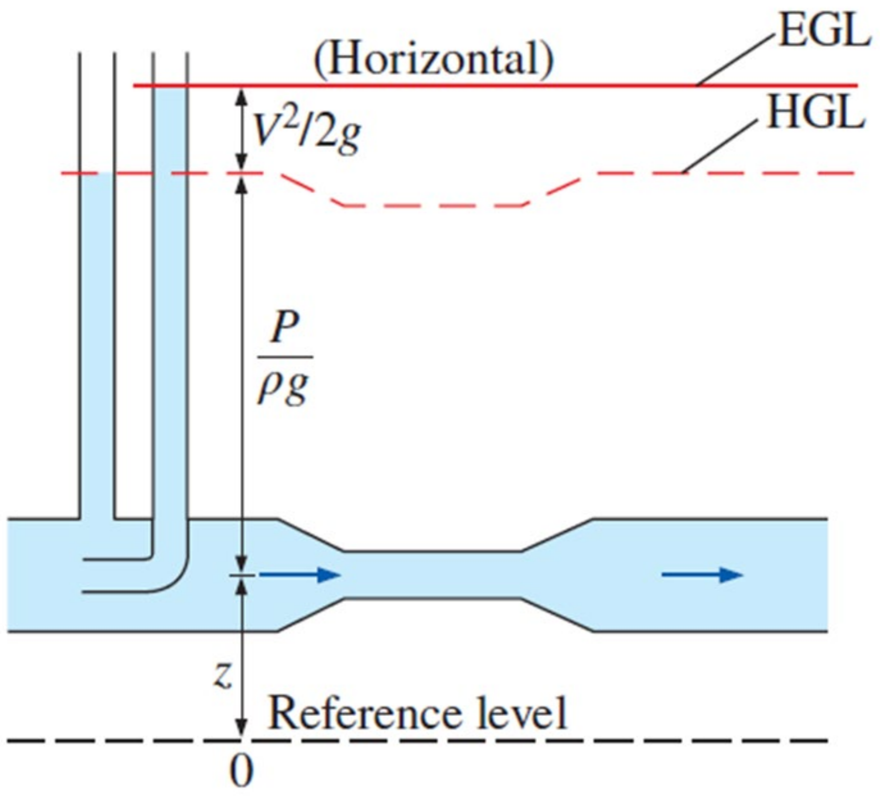
The *mechanical energy loss* due to frictional effects (conversion to thermal energy) causes the EGL and HGL to slope downward in the direction of flow. The slope is a measure of the head loss in the pipe. A component, such as a valve, that generates significant frictional effects causes a sudden drop in both EGL and HGL at that location.

The Bernoulli Equation

A *steep jump/drop* occurs in EGL and HGL whenever mechanical energy is added or removed to or from the fluid (pump, turbine).

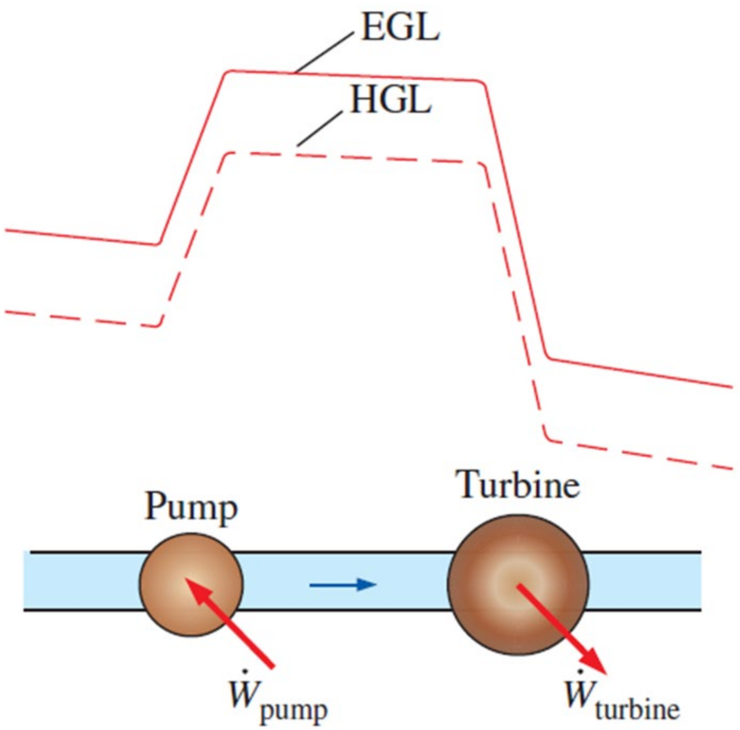
The (gage) pressure of a fluid is zero at locations where the HGL *intersects* the fluid. The pressure in a flow section that lies above the HGL is negative, and the pressure in a section that lies below the HGL is positive.

The Bernoulli Equation



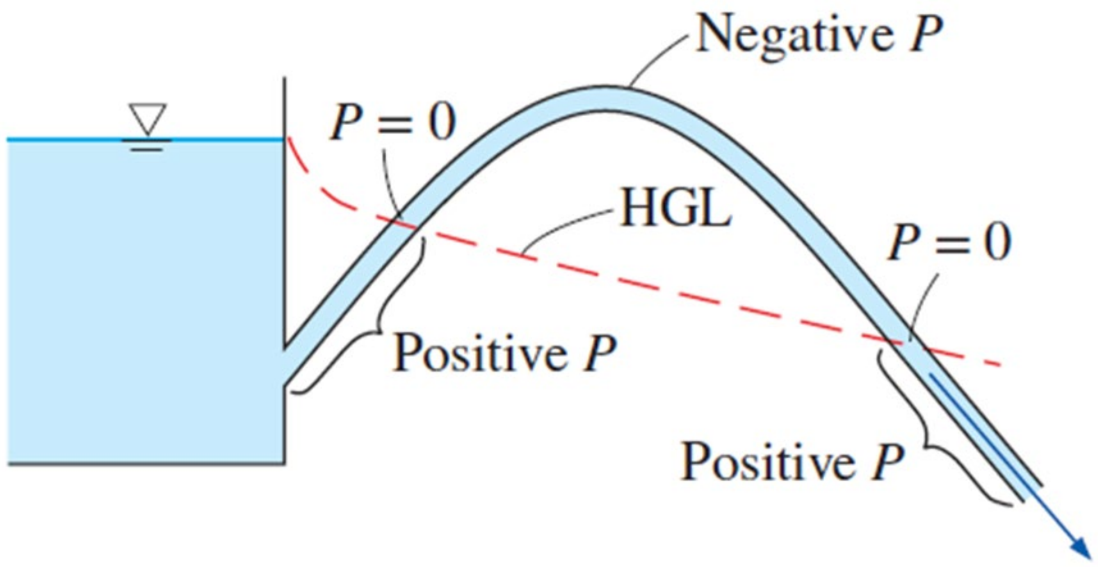
In an idealized Bernoulli-type flow, EGL is horizontal and its height remains constant. But this is not the case for HGL when the flow speed varies along the flow.

The Bernoulli Equation



A steep jump occurs in EGL and HGL whenever mechanical energy is added to the fluid by a pump, and a steep drop occurs whenever mechanical energy is removed from the fluid by a turbine.

The Bernoulli Equation

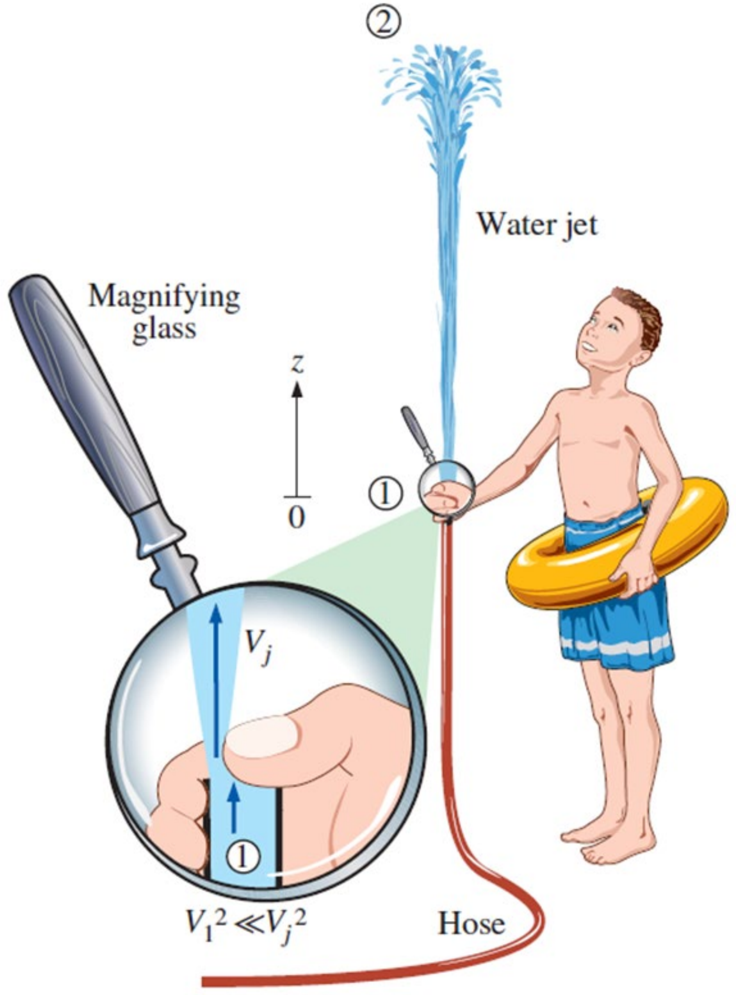


The gage pressure of a fluid is zero at locations where the HGL *intersects* the fluid, and the pressure is negative (vacuum) in a flow section that lies above the HGL.

The Bernoulli Equation

$$\frac{P_1}{\rho g} + \cancelto{\text{ignore}}{\frac{V_1^2}{2g}} + \cancelto{0}{z_1} = \frac{P_2}{\rho g} + \cancelto{0}{\frac{V_2^2}{2g}} + z_2 \rightarrow \frac{P_1}{\rho g} = \frac{P_{\text{atm}}}{\rho g} + z_2$$

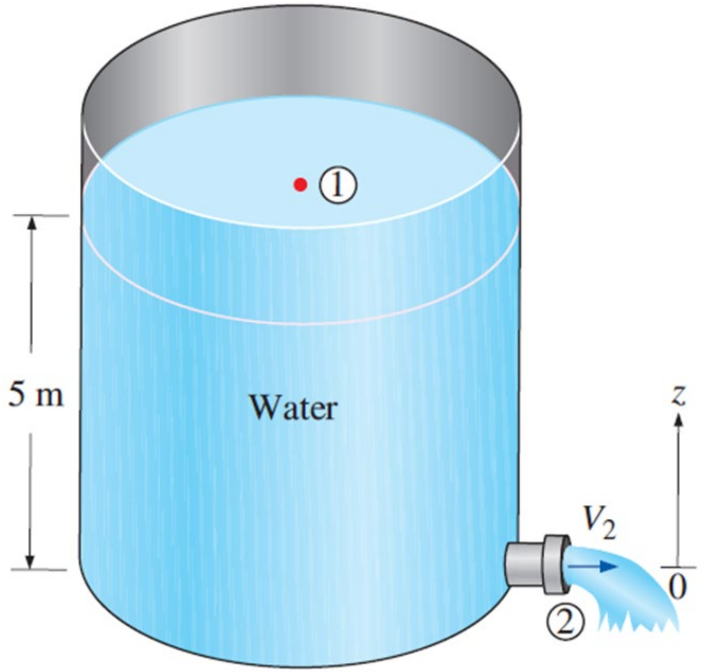
$$z_2 = \frac{P - P_{\text{atm}}}{\rho g} = \frac{P_{1,\text{gage}}}{\rho g}$$



Example: Spraying Water into the Air

The Bernoulli Equation

Example: Water Discharge from a Large Tank



$$\cancel{\frac{P_1}{\rho g}} + \underbrace{\cancel{\frac{V_1^2}{2g}}}_{\text{ignore}} + z_1 = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + \overbrace{\cancel{z_2}}^{0} \rightarrow \quad z_1 = \frac{V_2^2}{2g} \qquad V_2 = \sqrt{2gz_1}$$

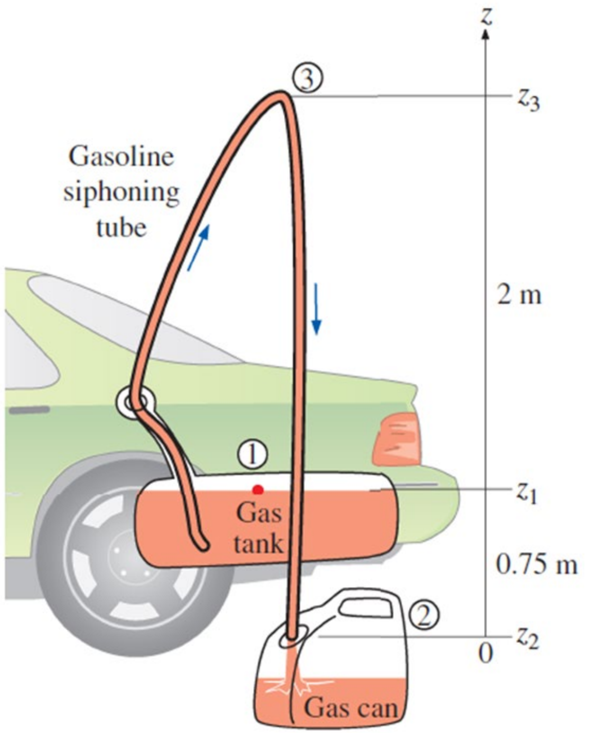
The relation $V = \sqrt{2gz}$ is called the **Torricelli equation.**

The Bernoulli Equation

$$\cancel{\frac{P_1}{\rho g}} + \frac{\cancel{V_1^2}^{\approx 0}}{2g} + z_1 = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + \cancel{z_2}^{0} \rightarrow \quad z_1 = \frac{V_2^2}{2g}$$

$$V_2 = \sqrt{2gz_1}$$

Example: Siphoning Out Gasoline from a Fuel Tank.

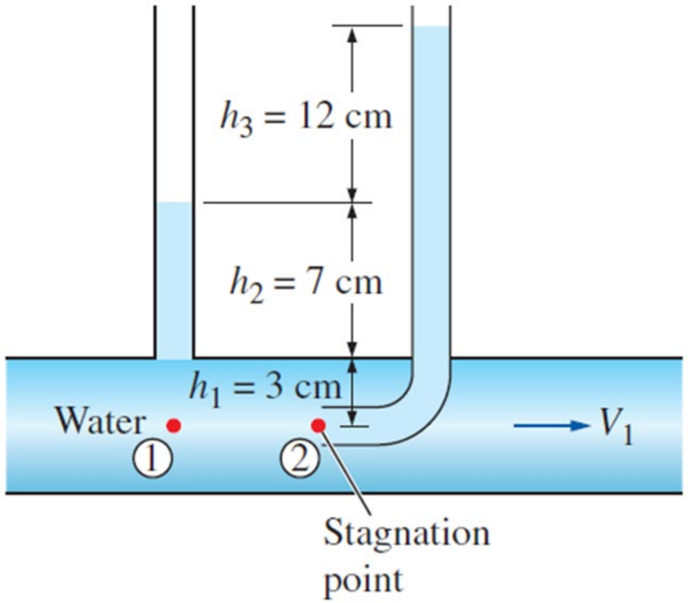


$$\frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + \cancel{z_2}^{0} = \frac{P_3}{\rho g} + \cancel{\frac{V_3^2}{2g}} + z_3 \quad \rightarrow \quad \frac{P_{atm}}{\rho g} = \frac{P_3}{\rho g} + z_3$$

$$P_3 = P_{atm} - \rho g z_3$$

The Bernoulli Equation

Example: Velocity Measurement by a Pitot Tube.



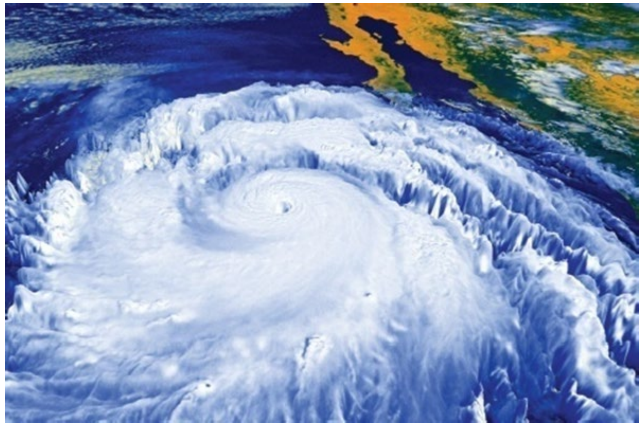
$$P_1 = \rho g \left(h_1 + h_2 \right)$$

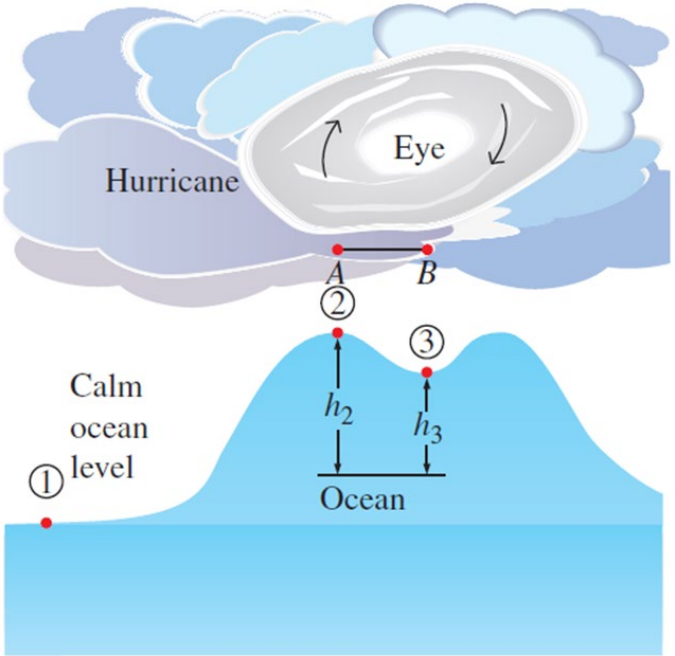
$$P_2 = \rho g \left(h_1 + h_2 + h_3 \right)$$

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{p_2}{\rho g} + \frac{\cancel{V_2^2}^{\,0}}{2g} + \cancel{z_2} \rightarrow \frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g}$$

The Bernoulli Equation

Example: The Rise of the Ocean Due to a Hurricane.

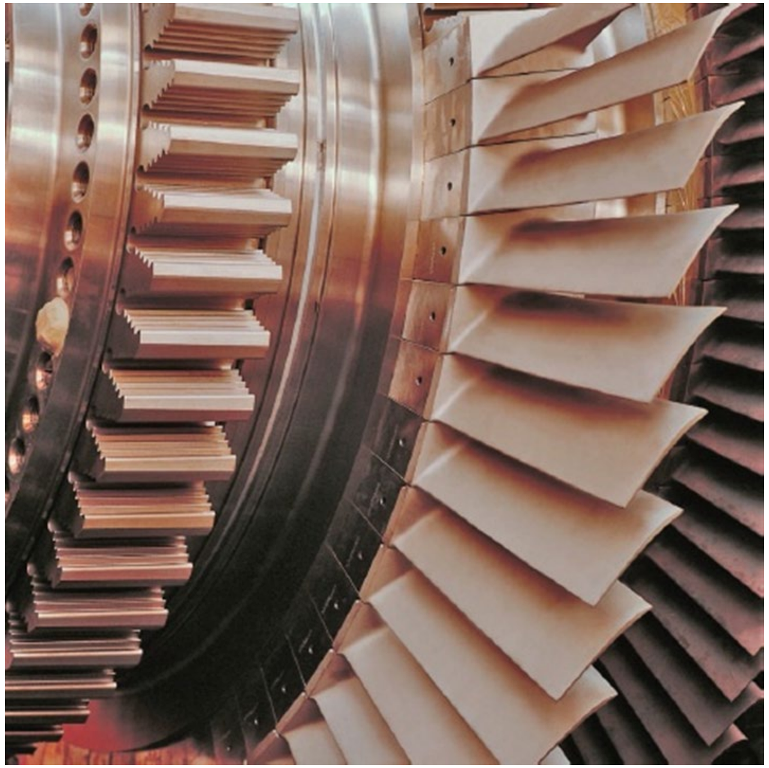




The eye of hurricane Linda (1997 in the Pacific Ocean near Baja California) is clearly visible in this satellite photo.

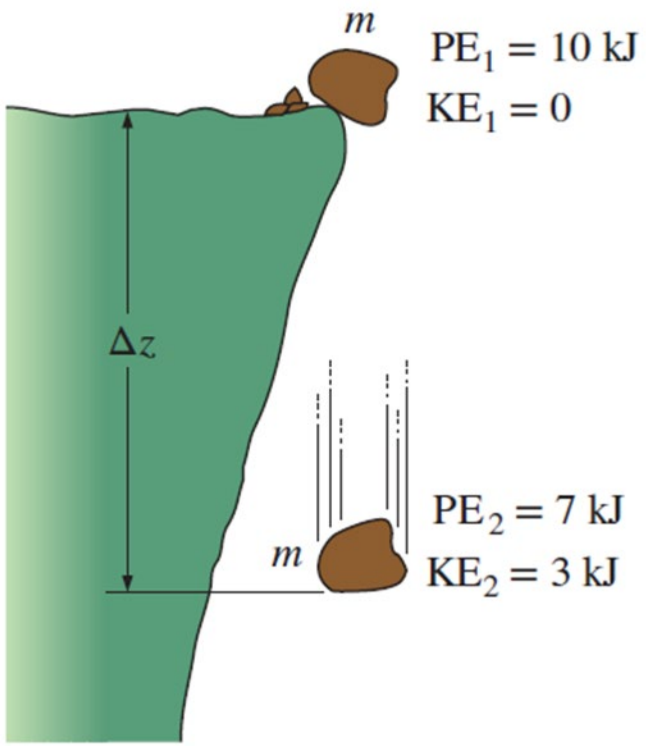
$$\frac{p_A}{\rho g}+\frac{V_A^{\ 2}}{2g}+\cancel{z_A}=\frac{p_B}{\rho g}+\cancelto{0}{\frac{V_B^{\ 2}}{2g}}+\cancel{z_B}\rightarrow\frac{P_B-P_A}{\rho g}=\frac{V_A^{\ 2}}{2g}$$

The Bernoulli Equation



Compressible flow of a gas through turbine blades is often modeled as isentropic, and the compressible form of the Bernoulli equation is a reasonable approximation.

General Energy Equation



$$E_{in} - E_{out} = \Delta E$$

$$\dot{Q}_{net\ in} + \dot{W}_{net\ in} = \frac{dE_{sys}}{dt}$$

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{d}{dt}\int_{\text{sys}} \rho e\, dV$$

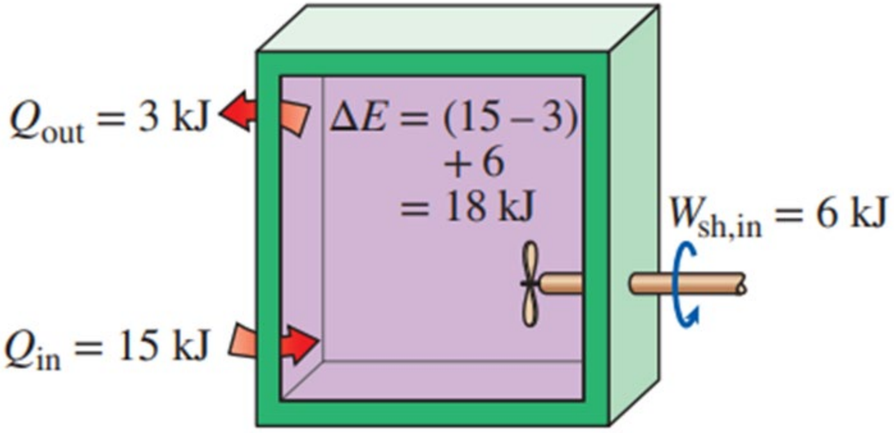
$$\dot{Q}_{\text{net in}} = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}$$

$$\dot{W}_{\text{net in}} = \dot{W}_{\text{in}} - \dot{W}_{\text{out}}$$

$$e = u + ke + pe = u + \frac{V^2}{2} + gz$$

The first law of thermodynamics (the conservation of energy principle):
Energy cannot be created or destroyed during a process; it can only change forms.

General Energy Equation



The energy change of a system during a process is equal to the *net* work and heat transfer between the system and its surroundings.

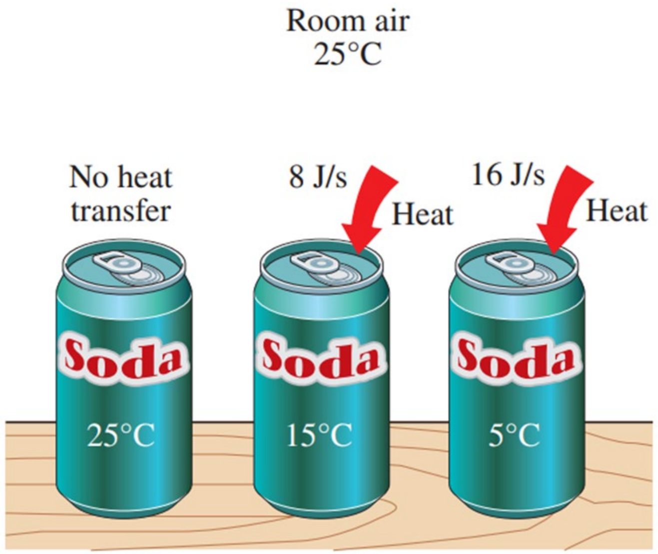
General Energy Equation

Energy Transfer by Heat, *Q*

Thermal energy: The sensible and latent forms of internal energy.

Heat Transfer: The transfer of energy from one system to another as a result of a temperature difference.

The direction of heat transfer is always from the higher-temperature body to the lower-temperature one.



Adiabatic process: A process during which there is no heat transfer.

Heat transfer rate: The time rate of heat transfer.

Temperature difference is the driving force for heat transfer. The larger the temperature difference, the higher is the rate of heat transfer. Condensation of water vapor from the room is shown for the coldest can.

General Energy Equation

Energy Transfer by Work, *W*.

Work: The energy transfer associated with a force acting through a distance.

A rising piston, a rotating shaft, and an electric wire crossing the system boundaries are all associated with work interactions.

Power: The time rate of doing work.

Car engines and hydraulic, steam, and gas turbines produce work; compressors, pumps, fans, and mixers consume work.

$$W_{\text{total}} = W_{\text{shaft}} + W_{\text{pressure}} + W_{\text{viscous}} + W_{\text{other}}$$

W_{shaft} The work transmitted by a rotating shaft.

$W_{pressure}$ The work done by the pressure forces on the control surface.

$W_{viscous}$ The work done by the normal and shear components of viscous forces on the control surface.

W_{other} The work done by other forces such as electric, magnetic, and surface tension.

General Energy Equation

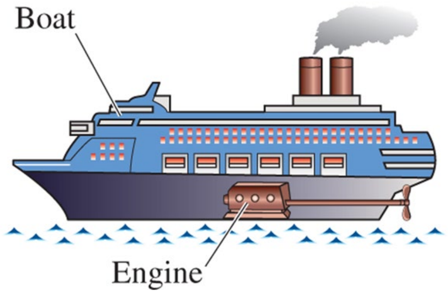
Shaft Work A force *F* acting through a moment arm *r* generates a torque T $T = Fr \rightarrow F = \frac{T}{r}$

This force acts through a distance *s* $s = (2\pi r)n$

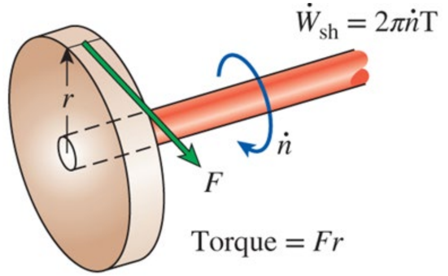
Shaft work $W_{sh} = Fs = \left(\frac{T}{r}\right)(2\pi rn) = 2\pi nT \quad (\mathrm{kJ})$

The power transmitted through the shaft is the shaft work done per unit time:

$$\dot{W}_{shaft} = \omega T_{shift} = 2\pi \dot{n} T_{shaft} \qquad \dot{W}_{sh} = 2\pi \dot{n} T \quad (\mathrm{kW})$$



Energy transmission through rotating shafts is commonly encountered in practice.



Shaft work is proportional to the torque applied and the number of revolutions of the shaft.

General Energy Equation

Work Done by Pressure Forces

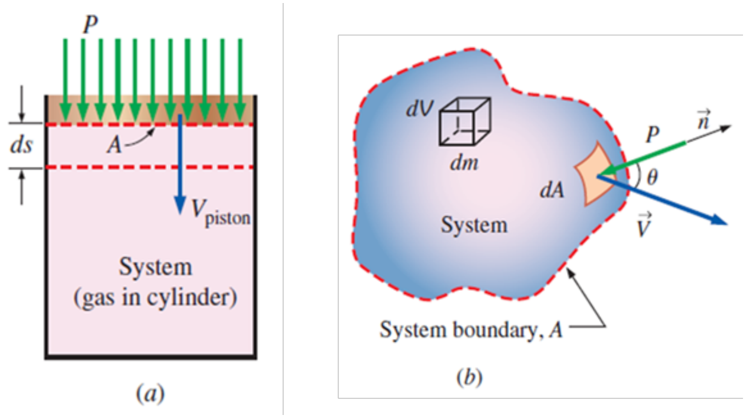
$$\delta W_{\text{boundary}} = PAds$$

$$\delta \dot{W}_{\text{pressure}} = \delta \dot{W}_{\text{boundary}} = PAV_{\text{piston}} \qquad V_{\text{piston}} = ds/dt$$

$$\delta \dot{W}_{pressure} = -PdAV_n = -PdA\left(\vec{V}\cdot\vec{n}\right)$$

$$\dot{W}_{pressure,net\,in} = -\int_A P(\vec{V}\cdot\vec{n})dA = -\int_A \frac{P}{\rho}\rho\left(\vec{V}\cdot\vec{n}\right)dA$$

$$\dot{W}_{net.in} = \dot{W}_{shaft,net\ in} + \dot{W}_{pressure,net\,in} = \dot{W}_{shaft,\ net\ in} - \int_A P(\vec{V}\cdot\vec{n})dA$$



The pressure force acting on (*a*) the moving boundary of a system in a piston-cylinder device, and (*b*) the differential surface area of a system of arbitrary shape.

General Energy Equation

The conservation of energy equation is obtained by replacing *B* in the Reynolds transport theorem by energy *E* and *b* by *e*.

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} + \dot{W}_{\text{pressure, net in}} = \frac{dE_{\text{sys}}}{dt}$$

$$e = u + ke + pe = u + V^2/2 + gz$$

$$\frac{dE_{sys}}{dt} = \frac{d}{dt}\int_{CV} e\rho dV + \int_{CS} e\rho(\vec{V}_r \cdot \vec{n})A$$

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} + \dot{W}_{\text{pressure, net in}} = \frac{d}{dt}\int_{\text{CV}} e\rho d\text{V} + \int_{\text{CS}} e\rho(\vec{V}_r \cdot \vec{n})dA$$

$$\frac{dB_{\text{sys}}}{dt} = \frac{d}{dt}\int_{\text{CV}} b\rho dV + \int_{\text{CS}} b\rho(\vec{V}_r \cdot \vec{n})\,dA$$

$$B = E \qquad b = e \qquad b = e$$

$$\frac{dE_{\text{sys}}}{dt} = \frac{d}{dt}\int_{\text{CV}} e\rho dV + \int_{\text{CS}} e\rho(\vec{V}_r \cdot \vec{n})\,dA$$

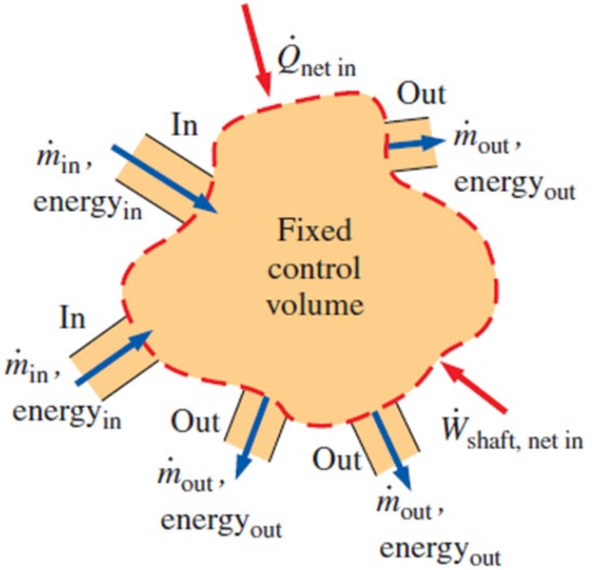
$$\left(\begin{array}{c}\text{The net rate of energy}\\ \text{transfer into a CV by}\\ \text{heat and work transfer}\end{array}\right) = \left(\begin{array}{c}\text{The time rate of}\\ \text{change of the energy}\\ \text{content of the CV}\end{array}\right) + \left(\begin{array}{c}\text{The net flow rate of}\\ \text{energy out of the control}\\ \text{surface by mass flow}\end{array}\right)$$

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt}\int_{\text{CV}} e\rho d\text{V} + \int_{\text{CS}}\left(\frac{P}{\rho} + e\right)\rho(\vec{V}_r \cdot \vec{n})dA$$

$$Fixed\ CV:\ \dot{Q}_{net\ in} + \dot{W}_{shaft,\ net\ in} = \frac{d}{dt}\int_{CV} e\rho dV + \int_{CS}\left(\frac{P}{\rho} + e\right)\rho(\vec{V}_r \cdot \vec{n})dA$$

General Energy Equation

In a typical engineering problem, the control volume may contain many inlets and outlets; energy flows in at each inlet, and energy flows out at each outlet. Energy also enters the control volume through net heat transfer and net shaft work.



$$\dot{m} = \int_{Ac} \rho(\vec{V}\cdot\vec{n})dA_c$$

$$\dot{Q}_{net\,in} + \dot{W}_{shaft,\ net\ in} = \frac{d}{dt}\int_{CV} e\rho dV + \sum_{out}\dot{m}\left(\frac{P}{\rho}+e\right) - \sum_{in}\dot{m}\left(\frac{P}{\rho}+e\right) \qquad e = u + \mathrm{ke} + \mathrm{pe}$$

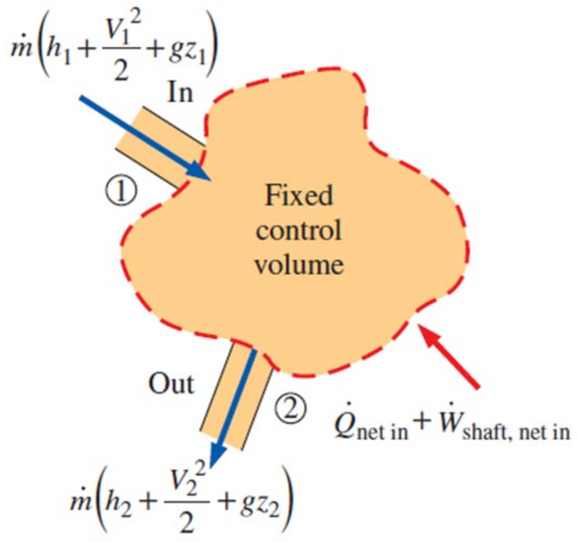
$$\dot{Q}_{net\,in} + \dot{W}_{shaft,\ net\ in} = \frac{d}{dt}\int_{CV} e\rho dV + \sum_{out}\dot{m}\left(\frac{P}{\rho}+u+\frac{V^2}{2}+gz\right) - \sum_{in}\dot{m}\left(\frac{P}{\rho}+u+\frac{V^2}{2}+gz\right)$$

$$\dot{Q}_{net\,in} + \dot{W}_{shaft,\ net\ in} = \frac{d}{dt}\int_{CV} e\rho dV + \sum_{out}\dot{m}\left(h+\frac{V^2}{2}+gz\right) - \sum_{in}\dot{m}\left(h+\frac{V^2}{2}+gz\right)$$

$$h = u + Pv = \mathrm{u} + \mathrm{P}/\rho$$

Energy Analysis Of Steady Flows

A control volume with only one inlet and one outlet and energy interactions.



The net rate of energy transfer to a control volume by heat transfer and work during steady flow is equal to the difference between the rates of outgoing and incoming energy flows by mass flow.

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

$$\dot{Q}_{net\,in} + \dot{W}_{shaft,\ net\ in} = \dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g\left(z_2 - z_1\right)\right)$$

$$q_{net\,in} + w_{shaft,\ net\ in} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g\left(z_2 - z_1\right) \qquad h = u + P/\rho$$

$$w_{shaft,\ net\,in} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + \left(u_2 - u_1 - q_{net\,in}\right)$$

Energy Analysis Of Steady Flows

Ideal flow (no mechanical energy loss): $q_{\text{net in}} = u_2 - u_1$

Real flow (with mechanical energy loss): $e_{\text{mech, loss}} = u_2 - u_1 - q_{\text{net in}}$

$$e_{\text{mech, in}} = e_{\text{mech, out}} + e_{\text{mech, loss}}$$

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} gz_2 + e_{\text{mech, loss}}$$

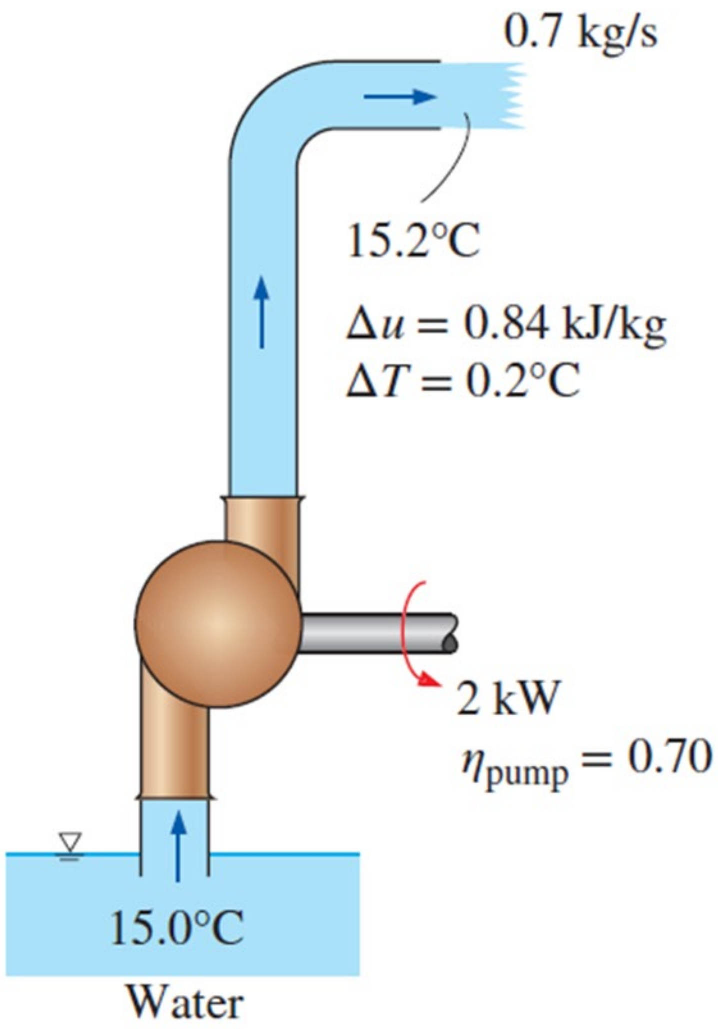
$$w_{shaft,\ net\ in} = w_{pump} - w_{turbine}$$

$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 + w_{\text{pump}} = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + w_{\text{turbine}} + e_{\text{mech, loss}}$$

$$\dot{m}\left(\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech, loss}}$$

$$\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech loss, pump}} + \dot{E}_{\text{mech loss, turbine}} + \dot{E}_{\text{mech loss, piping}}$$

Energy Analysis Of Steady Flows



펌프 유체에 전달된 유용한 일률:

$$\dot{W}_{\text{fluid}} = \eta_{\text{pump}} \cdot \dot{W}_{\text{shaft}} = 0.7 \times 2 = 1.4\,\text{kW}$$

내부 에너지 상승에 사용된 일률:

$$\dot{Q}_{\text{internal}} = \dot{m} \cdot \Delta u = 0.7 \cdot 0.84 = 0.588\,\text{kW}$$

Mechanical Loss = 2 - 1.4 – 0.588 = 0.012 kW (Friction, sound etc.)
Loss sum = 0.588 + 0.012 = 0.6 kW

The lost mechanical energy in a fluid flow system results in an increase in the internal energy of the fluid and thus in a rise of fluid temperature.

Energy Analysis Of Steady Flows

A typical power plant has numerous pipes, elbows, valves, pumps, and turbines, all of which have irreversible losses.

Energy Analysis Of Steady Flows

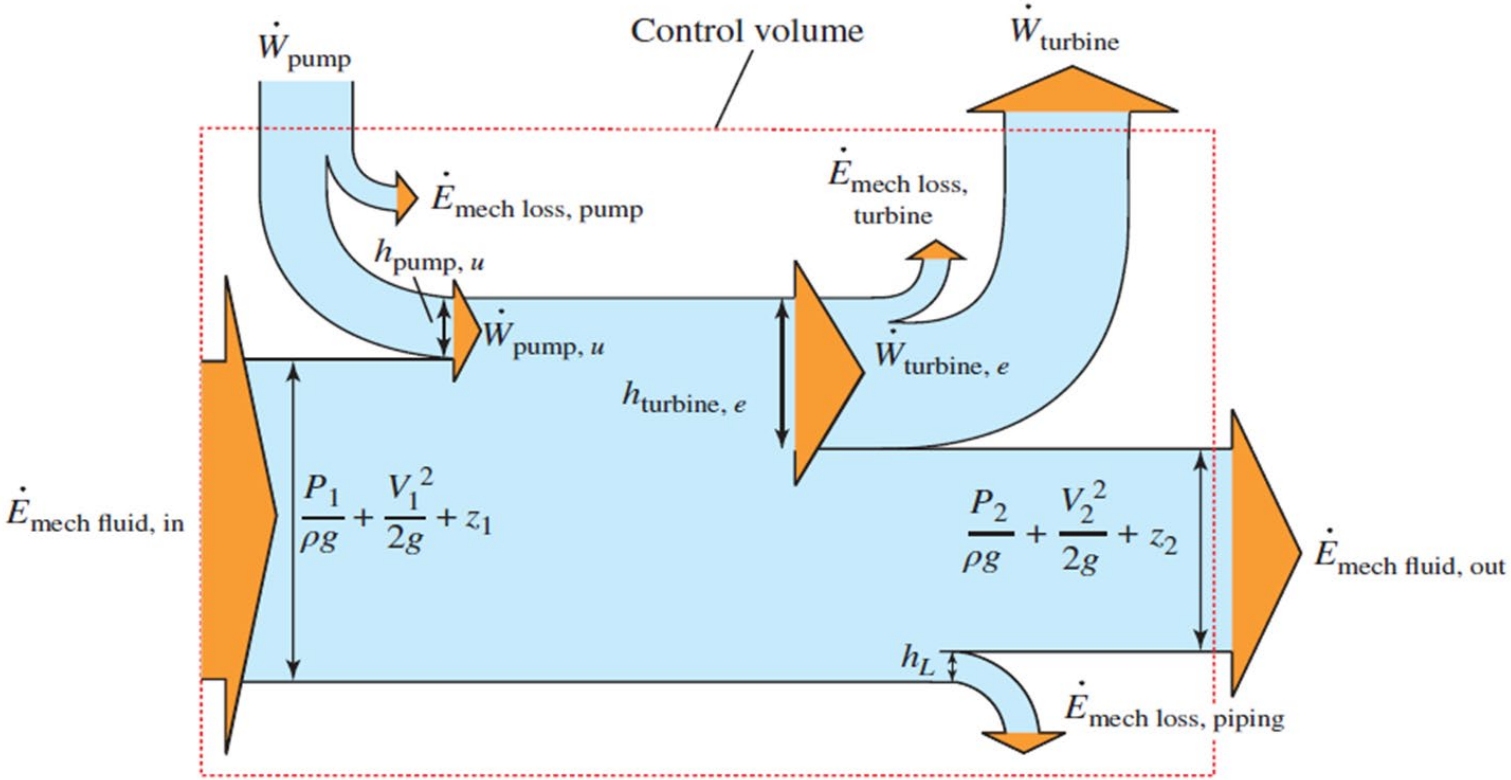
Energy equation in terms of ***heads***

$$\frac{P_1}{\rho_1 g}+\frac{V_1^{\,2}}{2g}+z_1+h_{pump,u}=\frac{P_2}{\rho_2 g}+\frac{V_2^{\,2}}{2g}+z_2+h_{turbine,e}+h_L$$

where

- $h_{pump,\,u}=\frac{w_{pump,\,u}}{g}=\frac{\dot{W}_{pump,\,u}}{\dot{m}g}=\frac{\eta_{pump}\dot{W}_{pump}}{\dot{m}g}$ is the *useful head delivered to the fluid by the pump*. Because of irreversible losses in the pump, $h_{pump,\,u}$ is *less* than $\dot{W}_{pump}/\dot{m}g$ by the factor η_{pump}.
- $h_{turbine,\,e}=\frac{w_{turbine,\,e}}{g}=\frac{\dot{W}_{turbine,\,e}}{\dot{m}g}=\frac{\dot{W}_{turbine}}{\eta_{turbine}\dot{m}g}$ is the *extracted head removed from the fluid by the turbine*. Because of irreversible losses in the turbine, $h_{turbine,\,e}$ is *greater* than $w_{turbine,\,e}/\dot{m}g$ by the factor η_{turbine}.
- $h_L=\frac{e_{mech\ loss,\ piping}}{g}=\frac{\dot{E}_{mech\ loss,\ piping}}{\dot{m}g}$ is the *irreversible head loss* between 1 and 2 due to all components of the piping system other than the pump or turbine.

Energy Analysis Of Steady Flows



Mechanical energy flow chart for a fluid flow system that involves a pump and a turbine. Vertical dimensions show each energy term expressed as an equivalent column height of fluid, that is, head.

Energy Analysis Of Steady Flows

$$\frac{P_1}{\rho_1 g}+\frac{V_1^2}{2g}+z_1+h_{pump,u}=\frac{P_2}{\rho_2 g}+\frac{V_2^2}{2g}+z_2+h_{turbine,e}+h_L \qquad (5?74)$$

Special Case: Incompressible Flow with No Mechanical Work Devices and Negligible Friction.

When piping losses are negligible, there is negligible dissipation of mechanical energy into thermal energy, and thus $h_L = e_{mech\ loss,\ piping}/g \cong 0.$

Also, $h_{pump,u} = h_{turbine,e} = 0$ when there are no mechanical work devices such as fans, pumps, or turbines. Then Eq. 5-74 reduces to

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad \text{or} \quad \frac{P}{\rho g}+\frac{V^2}{2g}+z_1=\text{constant}$$

This is the **Bernoulli equation** derived earlier using Newton’s second law of motion.

Thus, the Bernoulli equation can be thought of as a degenerate form of the energy equation.

Energy Analysis Of Steady Flows

Kinetic Energy Correction Factor, α

The kinetic energy of a fluid stream obtained from $V^2/2$ is not the same as the actual kinetic energy of the fluid stream since the square of a sum is not equal to the sum of the squares of its components.

This error can be corrected by replacing the kinetic energy terms $V^2/2$ in the energy equation by $\alpha V_{\text{avg}}^2/2$, where α is the **kinetic energy correction factor.**

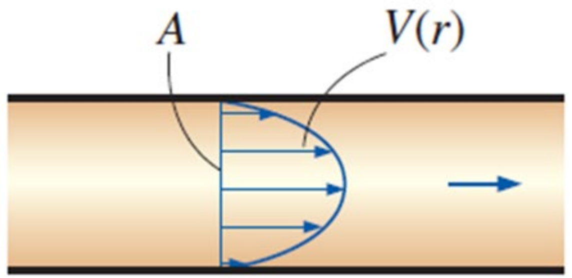
The correction factor is 2.0 for fully developed laminar pipe flow, and it ranges between 1.04 and 1.11 for fully developed turbulent flow in a round pipe.

When the kinetic energy correction factors are included, the energy equations for ***steady incompressible flow*** become.

$$\dot{m}\left(\frac{P_1}{\rho}+\alpha_1\frac{V_1^2}{2}+gz_1\right)+\dot{W}_{\text{pump}}=\dot{m}\left(\frac{P_2}{\rho}+\alpha_2\frac{V_2^2}{2}+gz_2\right)+\dot{W}_{\text{turbine}}+\dot{E}_{\text{mech, loss}}$$

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{pump,u}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{turbine,e}+h_L$$

Energy Analysis Of Steady Flows



$$\dot{m} = \rho V_{\text{avg}} A, \qquad \rho = \text{constant}$$

$$\dot{\text{KE}}_{\text{act}} = \int \text{ke}\,\delta\dot{m} = \int_A \frac{1}{2}[V(r)]^2 [\rho V(r)\, dA]$$

$$= \frac{1}{2}\rho \int_A [V(r)]^3\, dA$$

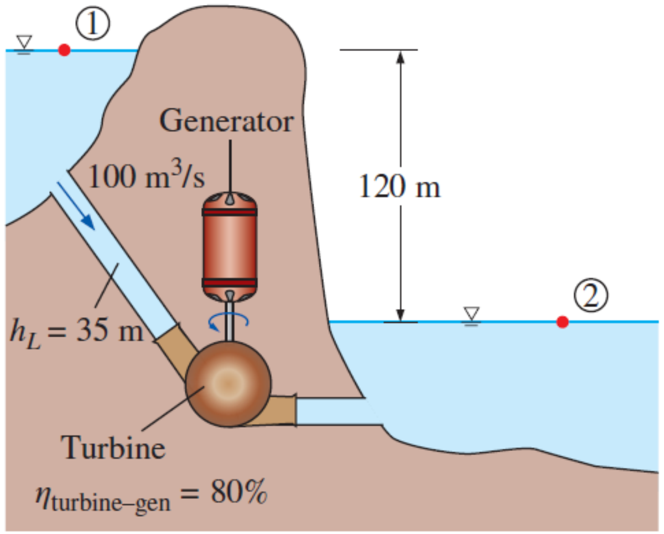
$$\dot{\text{KE}}_{\text{avg}} = \frac{1}{2}\dot{m}V_{\text{avg}}^2 = \frac{1}{2}\rho A V_{\text{avg}}^3$$

$$\alpha = \frac{\dot{\text{KE}}_{\text{act}}}{\dot{\text{KE}}_{\text{avg}}} = \frac{1}{A}\int_A \left(\frac{V(r)}{V_{\text{avg}}}\right)^3 dA$$

The determination of the *kinetic energy correction factor* using the actual velocity distribution $V(r)$ and the average speed V_{avg} at a cross section.

Energy Analysis Of Steady Flows

Example: Hydroelectric Power Generation from a Dam.



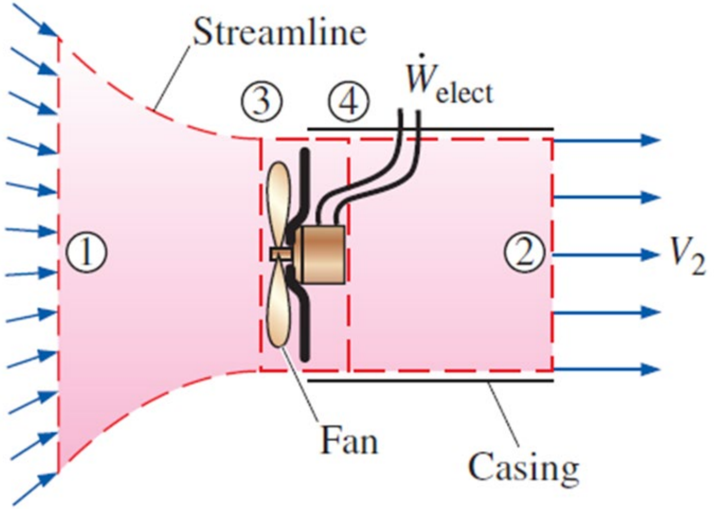
$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + z_1 + \cancel{h_{pump,u}}^{\,0} = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \cancel{\frac{V_2^2}{2g}} + \cancel{z_2}^{\,0} + h_{\text{turbine, e}} + h_L$$

$$h_{turbine,\,e} = z_1 - h_L$$

$$\dot{W}_{turbine,\,e} = \dot{m} g h_{turbine,\,e} \qquad \dot{W}_{\text{electric}} = \eta_{\text{turbine-gen}} \dot{W}_{\text{turbine, e}}$$

Energy Analysis Of Steady Flows

Example: Fan Selection for Air Cooling of a Computer.



Energy equation between 3 and 4

$$\dot{m}\frac{P_3}{\rho} + \dot{W}_{\text{fan}} = \dot{m}\frac{P_4}{\rho} + \dot{E}_{\text{mech loss, fan}}$$

$$\dot{W}_{\text{fan, u}} = \dot{m}\frac{p_4 - P_3}{\rho}$$

Energy equation between 1 and 2

$$\dot{m}\left(\cancel{\frac{P_1}{\rho}} + \alpha_1 \frac{\cancelto{0}{V_1^2}}{2} + \cancel{gz_1}\right) + \dot{W}_{\text{fan}} = \dot{m}\left(\cancel{\frac{P_2}{\rho}} + \alpha_2 \frac{V_2^{\,2}}{2} + \cancel{gz_2}\right) + \cancelto{0}{\dot{W}_{\text{turbine}}} + \dot{E}_{\text{mech loss, lfan}}$$

$$\dot{W}_{\text{fan, u}} = \dot{m}\alpha_2 \frac{V_2^{\,2}}{2} \qquad \dot{W}_{\text{elect}} = \frac{\dot{W}_{\text{fan, u}}}{\eta_{\text{fan-motor}}} \qquad \dot{W}_{\text{fan}} - \dot{E}_{\text{mech loss, fan}} = \dot{W}_{\text{fan, u}}$$

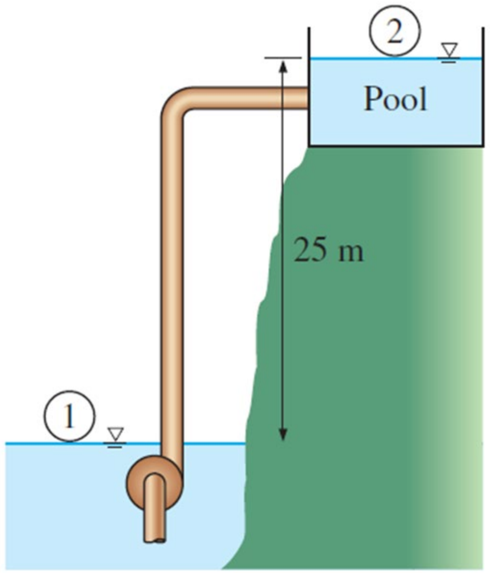
Energy Analysis Of Steady Flows

Example: Pumping Water from a Lake to a Reservoir.

$$\dot{W}_{\text{pump u}} = \eta_{\text{pump}} \dot{W}_{\text{shaft}}$$

Energy equation between 1 and 2

$$\dot{m}\left(\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine, e}} + \dot{E}_{\text{mech loos, piping}}$$



$$\dot{W}_{\text{pump, u}} = \dot{m}gz_2 + \dot{E}_{\text{mech loos, piping}}$$

$$\dot{E}_{\text{mech loos, piping}} = \dot{m}gh_L$$

For the pump

$$\Delta P = P_{\text{out}} - P_{\text{in}} = \frac{\dot{W}_{\text{pump, u}}}{\dot{V}}$$

Summary

- The Bernoulli Equation:
  - Acceleration of a Fluid Particle.
  - Derivation of the Bernoulli Equation.
  - Force Balance across Streamlines.
  - Unsteady, compressible flow.
  - Static, Dynamic, and Stagnation Pressures.
  - Limitations on the Use of the Bernoulli Equation.
  - Hydraulic Grade Line (HGL) and Energy Grade Line (EGL).
  - Applications of the Bernouli Equation.

Summary

- General Energy Equation:
  - Energy Transfer by Heat, Q.
  - Energy Transfer by Work, W.
  - Shaft Work.
  - Work Done by Pressure Forces.

- Energy Analysis of Steady Flows:
  - Special Case: Incompressible Flow with No Mechanical Work Devices and Negligible Friction.