2025년 1학기 13주차(2)

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Internal Flow

Internal flows through pipes, elbows, tees, valves, etc, as in this oil refinery, are found in nearly every industry.

Objectives

- Have a deeper understanding of laminar and turbulent flow in pipes and the analysis of fully developed flow.
- Calculate the major and minor losses associated with pipe flow in piping networks and determine the pumping power requirements.
- Understand various velocity and flow rate measurement techniques and learn their advantages and disadvantages.

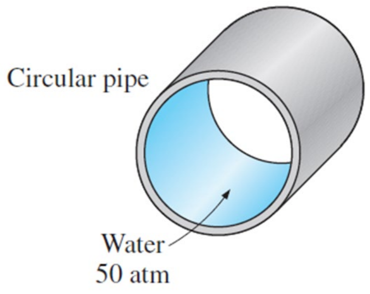
Introduction

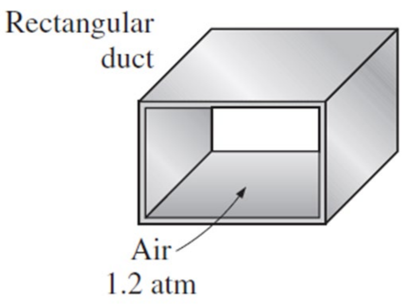
Liquid or gas flow through *pipes* or *ducts* is commonly used in heating and cooling applications and fluid distribution networks.

The fluid in such applications is usually forced to flow by a fan or pump through a flow section.

We pay particular attention to *friction*, which is directly related to the *pressure drop* and *head loss* during flow through pipes and ducts.

The pressure drop is then used to determine the *pumping power requirement*.





Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any significant distortion, but noncircular pipes cannot.

Introduction

Theoretical solutions are obtained only for a few simple cases such as fully developed laminar flow in a circular pipe.

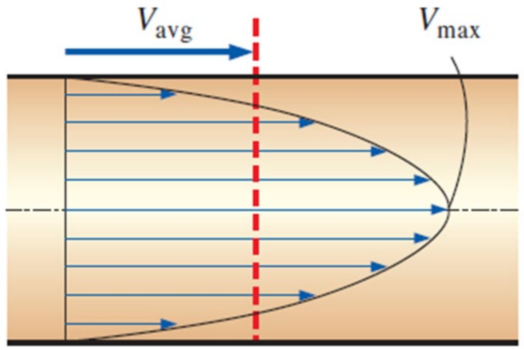
Therefore, we must rely on experimental results and empirical relations for most fluid flow problems rather than closed-form analytical solutions.

$$\dot{m} = \rho V_{\text{avg}} A_c = \int_{A_c} \rho u(r)\, dA_c$$

The value of the average speed V_{avg} at some streamwise cross-section is determined from the requirement that the *conservation of mass* principle be satisfied.

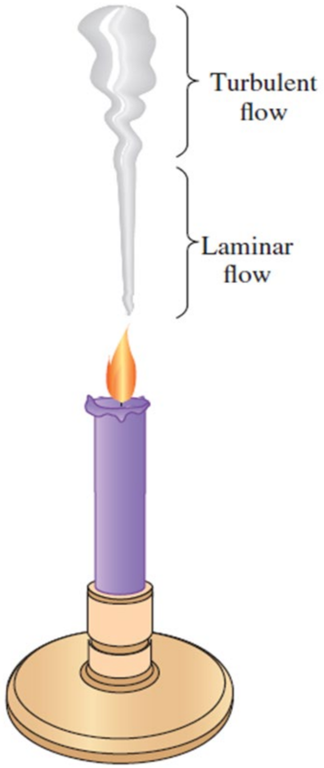
$$V_{\text{avg}} = \frac{\int_{A_c} \rho u(r)\, dA_c}{\rho A_c} = \frac{\int_0^R \rho u(r) 2\pi r\, dr}{\rho \pi R^2} = \frac{2}{R^2}\int_0^R u(r) r\, dr$$

The average speed for incompressible flow in a circular pipe of radius *R*.



Average speed V_{avg} is defined as the average speed through a cross section. For fully developed laminar

Laminar and Turbulent Flows



Laminar and turbulent flow regimes of candle smoke.

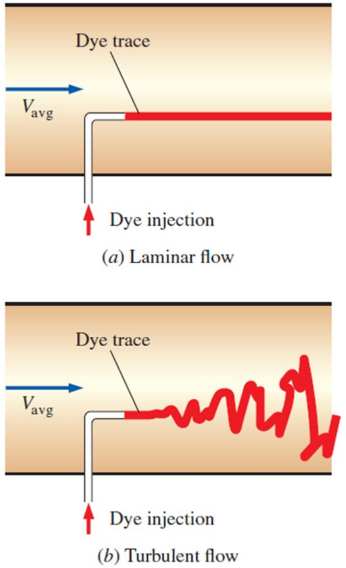
Laminar: Smooth streamlines and highly ordered motion.

Turbulent: Velocity fluctuations and highly disordered motion.

Transition: The flow fluctuates between laminar and turbulent flows.

Most flows encountered in practice are turbulent.

Laminar flow is encountered when highly viscous fluids such as oils flow in small pipes or narrow passages.



The behavior of colored fluid injected into the flow in laminar and turbulent flows in a pipe.

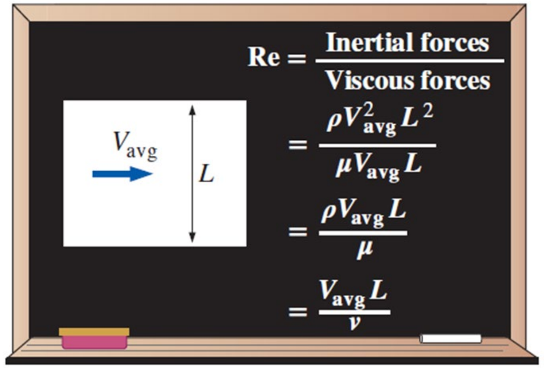
Laminar and Turbulent Flows

Reynolds Number

The transition from laminar to turbulent flow depends on the *geometry, surface roughness, flow speed, surface temperature,* and *type of fluid.*

The flow regime depends mainly on the ratio of *inertial forces* to *viscous forces* (**Reynolds number**).

$$\mathrm{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{\mathrm{avg}} D}{\nu} = \frac{\rho V_{\mathrm{avg}} D}{\mu}$$



The Reynolds number can be viewed as the ratio of inertial forces to viscous forces acting on a fluid element.

At large Reynolds numbers, the inertial forces, which are proportional to the fluid density and the square of the fluid speed, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid (**turbulent**).

At small or moderate Reynolds numbers, the viscous forces are large enough to suppress these fluctuations and to keep the fluid “in line” (**laminar**).

Critical Reynolds number, $\mathrm{Re}_{\mathrm{cr}}$:

The Reynolds number at which the flow becomes turbulent.

The value of the critical Reynolds number is different for different geometries and flow conditions.

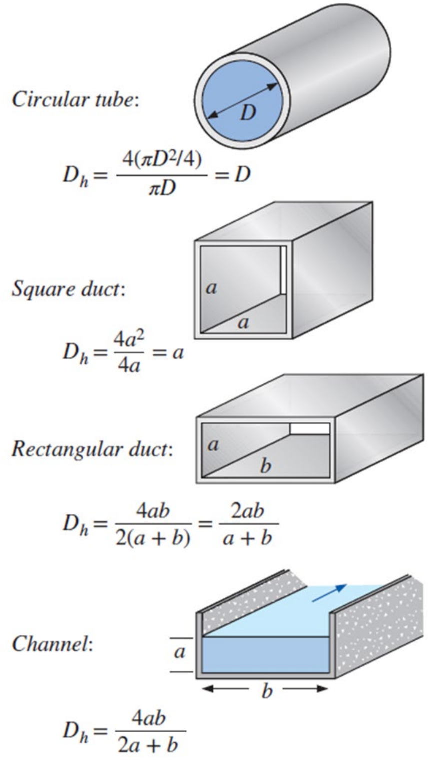
Laminar and Turbulent Flows

For flow through noncircular pipes, the Reynolds number is based on the **hydraulic diameter.**

Hydraulic diameter: $$D_h = \frac{4A_c}{p}$$

Circular pipes: $$D_h = \frac{4A_c}{p} = \frac{4(\pi D^2/4)}{\pi D} = D$$

Average speed : $$V_{avg} = \frac{\dot{m}}{\rho A_c}$$

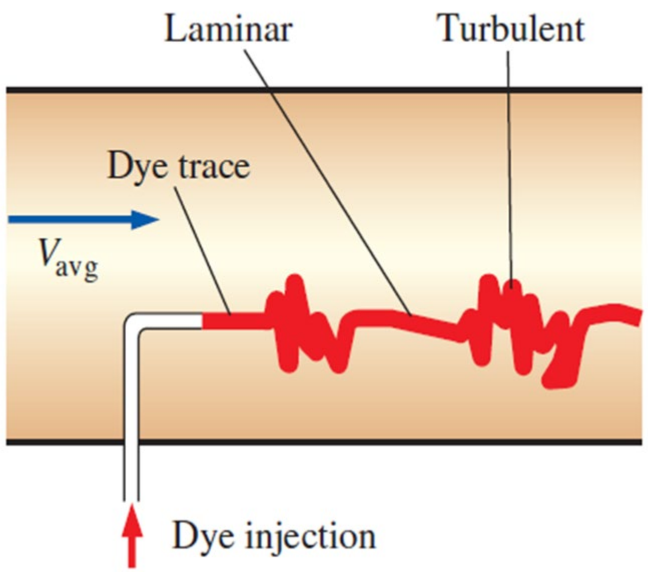


The hydraulic diameter $D_h = 4A_c/p$ is defined such that it reduces to ordinary diameter for circular tubes.

Laminar and Turbulent Flows

For flow in a circular pipe:

$$\begin{aligned} \mathrm{Re} \lesssim 2{,}300 &\quad \text{laminar flow} \\ 2300 \lesssim \mathrm{Re} \lesssim 4{,}000 &\quad \text{transitional flow} \\ \mathrm{Re} \gtrsim 4{,}000 &\quad \text{turbulent flow} \end{aligned}$$



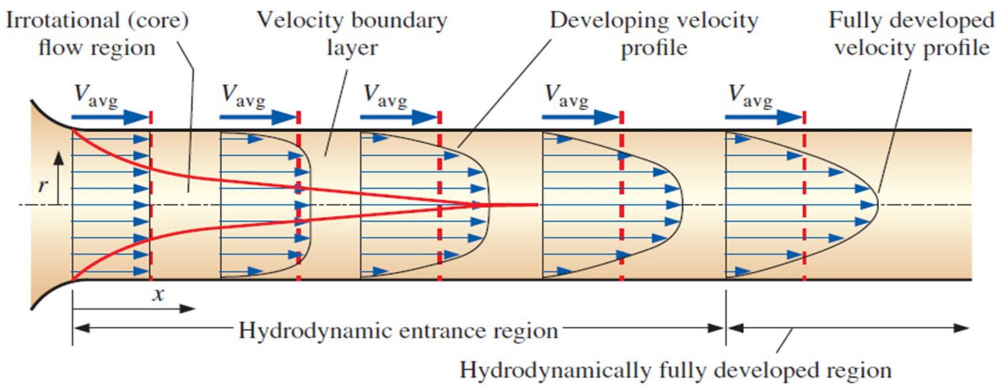
In the transitional flow region of $2300 \le \mathrm{Re} \le 4000$, the flow switches between laminar and turbulent seemingly randomly.

The Entrance Region

Velocity boundary layer: The region of the flow in which the effects of the viscous shearing forces caused by fluid viscosity are felt.

Boundary layer region: The viscous effects and the velocity changes are significant.

Irrotational (core) flow region: The frictional effects are negligible and the velocity remains essentially constant in the radial direction.



The development of the velocity boundary layer in a pipe. The developed average velocity profile is parabolic in laminar flow, but somewhat flatter or fuller in turbulent flow.

The Entrance Region

Hydrodynamic entrance region: The region from the pipe inlet to the point at which the boundary layer merges at the centerline.

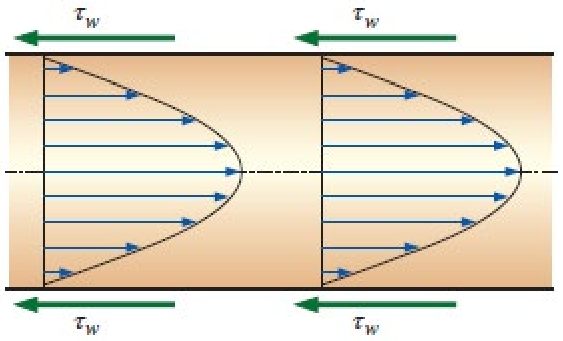
Hydrodynamic entry length L_h**:** The length of this region.

Hydrodynamically developing flow: Flow in the entrance region. This is the region where the velocity profile develops.

Hydrodynamically fully developed region: The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged.

Fully developed: When both the velocity profile the normalized temperature profile remain unchanged.

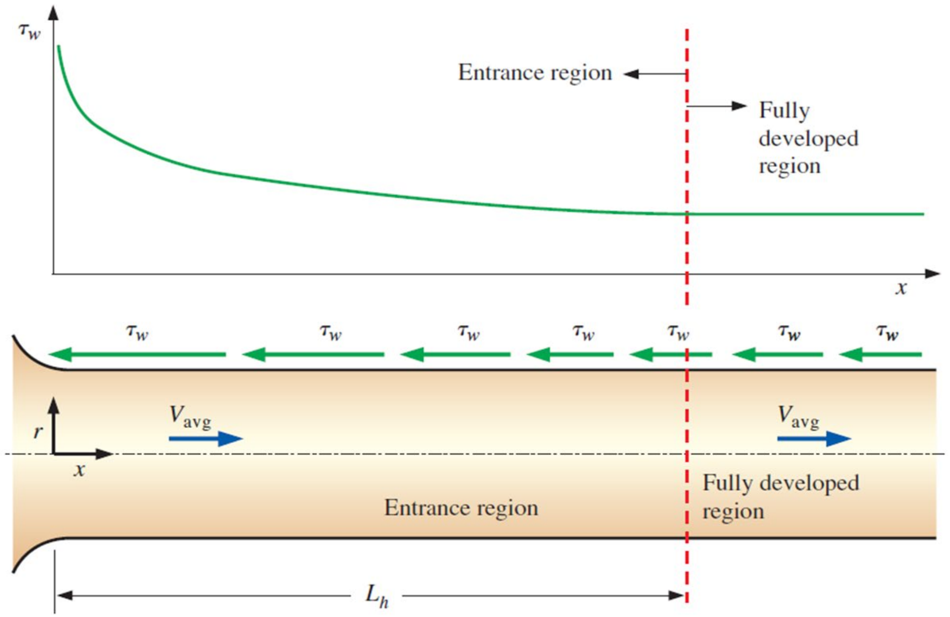
Hydrodynamically fully developed: $$\frac{\partial u(r,x)}{\partial x} = 0 \quad \rightarrow \quad u = u(r)$$



In the fully developed flow region of a pipe, the velocity profile does not change downstream, and thus the wall shear stress remains constant as well.

The Entrance Region

The pressure drop is *higher* in the entrance regions of a pipe, and the effect of the entrance region is always to *increase* the average friction factor for the entire pipe.



The variation of wall shear stress in the flow direction for flow in a pipe from the entrance region into the fully developed region.

The Entrance Region

Entry Lengths.

The hydrodynamic entry length is usually taken to be the distance from the pipe entrance to where the wall shear stress (and thus the friction factor) reaches within about 2 percent of the fully developed value.

$$\frac{L_{h,\text{잼aminar}}}{D} \cong 0.05\text{Re}$$

hydrodynamic entry length for laminar flow.

$$\frac{L_{h,\text{잼urbulent}}}{D} = 1.359\text{Re}^{1/4}$$

hydrodynamic entry length for turbulent flow.

$$\frac{L_{h,\text{잼urbulent}}}{D} \approx 10$$

hydrodynamic entry length for turbulent flow, an approximation

The pipes used in practice are usually several times the length of the entrance region, and thus the flow through the pipes is often assumed to be fully developed for the entire length of the pipe.

This simplistic approach gives *reasonable* results for long pipes but sometimes poor results for short ones since it underpredicts the wall shear stress and thus the friction factor.

Laminar Flow in Pipes

We consider steady, laminar, incompressible flow of a fluid with constant properties in the fully developed region of a straight circular pipe.

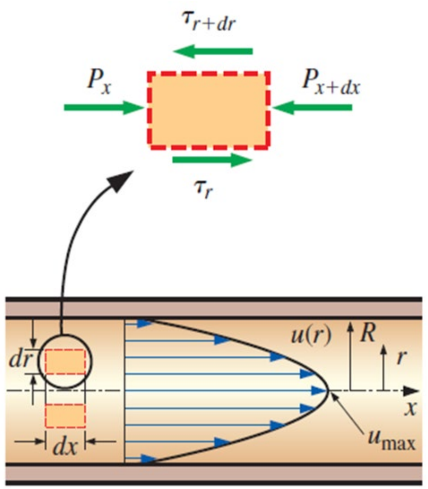
In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and the velocity profile *u*(*r*) remains unchanged in the flow direction. There is no motion in the radial direction, and thus the velocity component in the direction normal to the pipe axis is everywhere zero. There is no acceleration since the flow is steady and fully developed.

$$(2\pi r\, dr\, P)_x - (2\pi r\, dr\, P)_{x+dx} + (2\pi r\, dx\, \tau)_r - (2\pi r\, dx\, \tau)_{r+dr} = 0$$

$$r\frac{P_{x+dx}-P_x}{dx} + \frac{(r\tau)_{r+dr}-(r\tau)_r}{dr} = 0$$

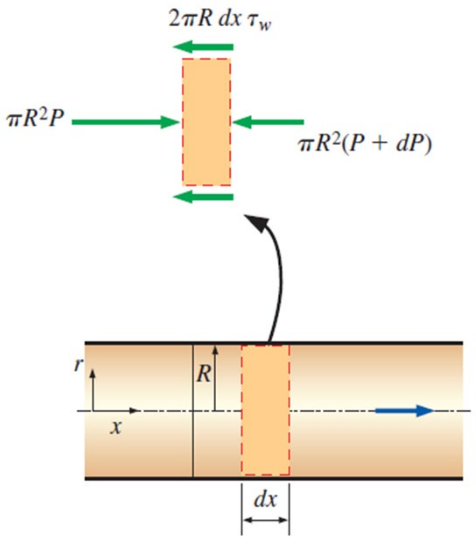
$$r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0 \qquad \tau = -\mu\left(du/dr\right)$$

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx}$$



Free-body diagram of a ring-shaped differential fluid element of radius *r*, thickness *dr*, and length dx oriented coaxially with a horizontal pipe in fully developed laminar flow.

Laminar Flow in Pipes



Force balance:

$$\pi R^2 P - \pi R^2(P + dP) - 2\pi R\,dx\,\tau_w = 0$$

Simplifying:

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

Free-body diagram of a fluid disk element of radius *R* and length *dx* in fully developed laminar flow in a horizontal pipe.

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

$$u(r) = \frac{r^2}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2$$

$$\partial u / \partial r = 0 \text{ at } r = 0$$

$$u = 0 \text{ at } r = R$$

Boundary conditions

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1-\frac{r^2}{R^2}\right)$$

Average speed

$$V_{avg} = \frac{2}{R^2}\int_0^R u(r) r\,dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(-\frac{r^2}{R^2}\right) r\,dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

$$u(r) = 2V_{avg}\left(1-\frac{r^2}{R^2}\right)$$

Velocity profile

$$u_{max} = 2V_{avg}$$

Maximim speed at centerline

Laminar Flow in Pipes

Pressure Drop and Head Loss.

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L} \qquad \textit{Laminar flow:} \qquad \Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

A pressure drop due to viscous effects represents an irreversible pressure loss, and it is called **pressure loss** ΔP_L.

$$\textit{Pressure loss:} \qquad \Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2}$$

$$f = \frac{8\tau_w}{\rho V_{avg}^2}$$ Darcy friction factor

$$\textit{Circular pipe, laminar:} \qquad f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{\mathrm{Re}}$$

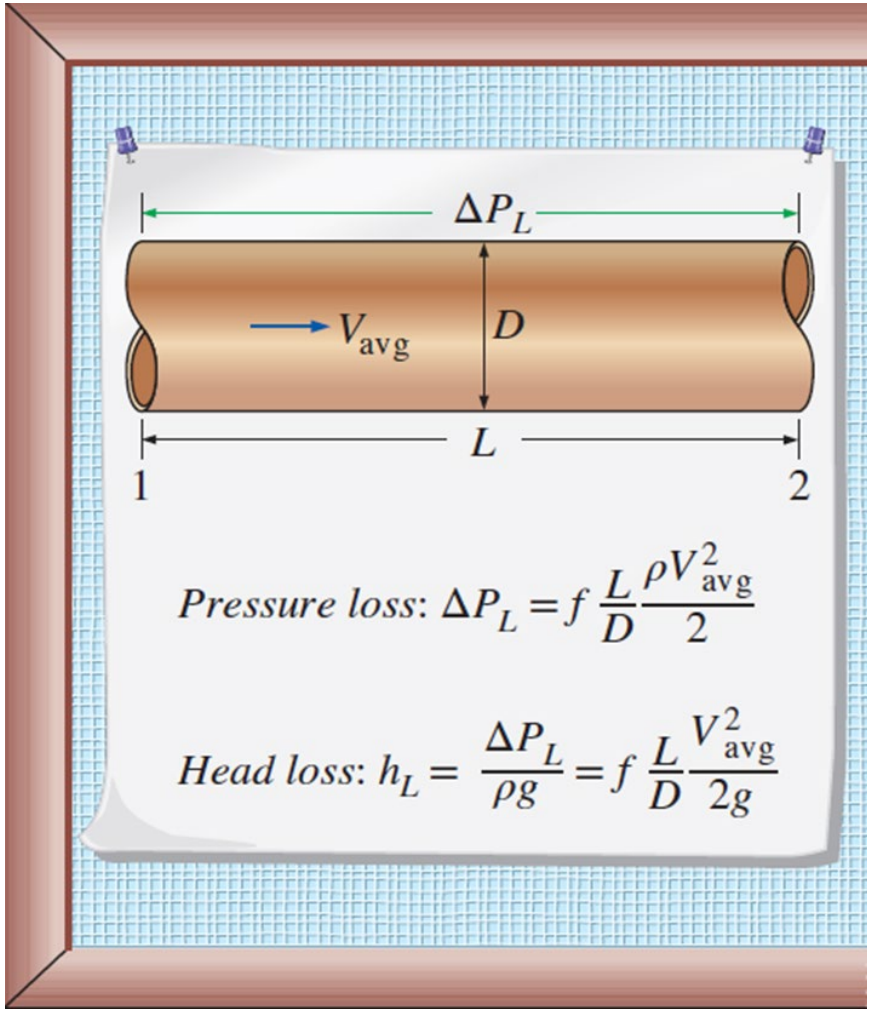
$\rho V_{avg}^2/2$ dynamic pressure

$$\textit{Head loss:} \qquad h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V_{avg}^2}{2g}$$

In laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the pipe surface.

The head loss represents the additional height that the fluid needs to be raised by a pump in order to overcome the frictional losses in the pipe.

Laminar Flow in Pipes



The relation for pressure loss (and head loss) is one of the most general relations in fluid mechanics, and it is valid for laminar or turbulent flows, circular or noncircular pipes, and pipes with smooth or rough surfaces.

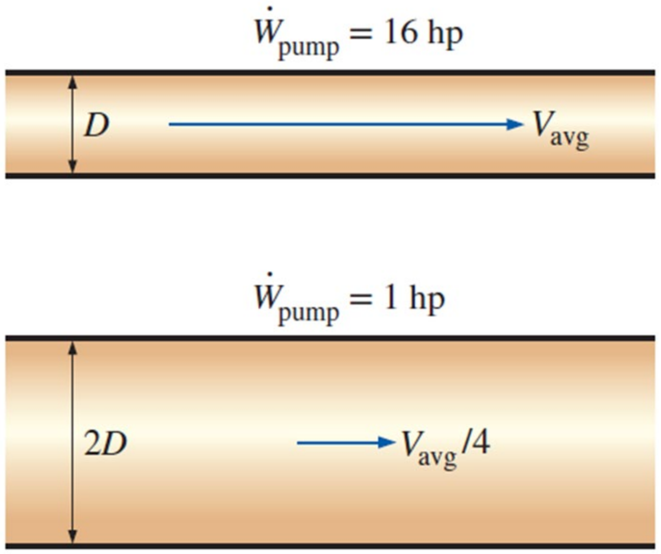
Laminar Flow in Pipes

$$\dot{W}_{\text{pump},\,L} = \dot{V}\,\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L$$

Horizontal pipe:
$$V_{avg} = \frac{(P_1 - P_2)R^2}{8\mu L} = \frac{(P_1 - P_2)D^2}{32\mu L} = \frac{\Delta P D^2}{32\mu L}$$

$$\dot{V} = V_{\text{avg}} A_c = \frac{(P_1 - P_2)R^2}{8\mu L}\pi R^2 = \frac{(P_1 - P_2)\pi D^4}{128\mu L} = \frac{\Delta P \pi D^4}{128\mu L}$$

Poiseuille's law



The pumping power requirement for a laminar flow piping system can be reduced by a factor of 16 by doubling the pipe diameter.

For a specified flow rate, the pressure drop and thus the required pumping power is proportional to the length of the pipe and the viscosity of the fluid, but it is inversely proportional to the fourth power of the diameter of the pipe.

Laminar Flow in Pipes

The pressure drop ΔP equals the pressure loss ΔP_L in the case of a horizontal pipe, but this is not the case for inclined pipes or pipes with variable cross-sectional area.

This can be demonstrated by writing the energy equation for steady, incompressible one-dimensional flow in terms of heads as,

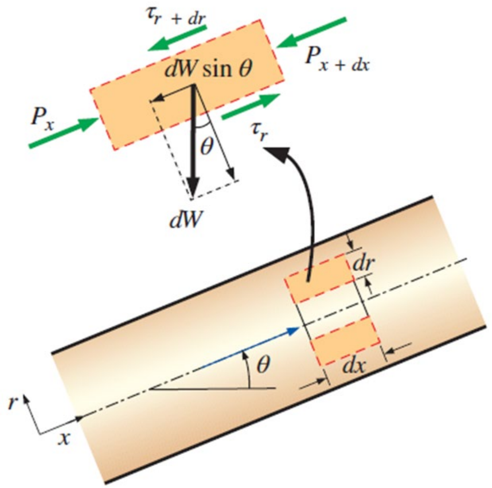
$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump},\, u} = \frac{P_2}{\rho g} + \alpha \frac{V_2^2}{2g} + z_2 + h_{\text{turbine},\, e} + h_L$$

$$P_1 - P_2 = \rho\left(\alpha_2 V_2^2 - \alpha_1 V_1^2\right)/2 + \rho g\left[\left(z_2 - z_1\right) + h_{\text{turbine},\, e} - h_{\text{pump},\, u} + h_L\right]$$

Therefore, the pressure drop $\Delta P = P_1 - P_2$ and pressure loss $\Delta P_L = pgh_L$ for a given flow section are equivalent if (1) the flow section is horizontal so that there are no hydrostatic or gravity effects $\left(z_1 = z_2\right)$, (2) the flow section does not involve any work devices such as a pump or a turbine since they change the fluid pressure $(h_{\text{pump},\, u} = h_{\text{turbine},\, e} = 0)$, (3) the cross-sectional area of the flow section is constant and thus the average flow velocity is constant $\left(V_1 = V_2\right)$ and (4) the velocity profiles at sections 1 and 2 are the same shape $\left(\alpha_1 = \alpha_2\right)$.

Laminar Flow in Pipes

Effect of Gravity on Velocity and Flow Rate in Laminar Flow.



Free-body diagram of a ring-shaped differential fluid element of radius *r*, thickness *dr*, and length *dx* oriented coaxially with an inclined pipe in fully developed laminar flow.

$$dW_x = dW \ \sin\theta = \rho g dV_{\text{element}} \ \sin\ \theta = \rho g(2\pi r\ dr\ dx)\ \sin\ \theta$$

$$(2\pi r\ dr\ P)_x - (2\pi r\ dr\ P)_{x+dx} + (2\pi r\ dx\ \tau)_r$$
$$-(2\pi r\ dx\ \tau)_{r+dr} - \rho g(2\pi r\ dr\ dx)\ \sin\theta = 0$$

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} + \rho g\ \sin\theta$$

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx} + \rho g\ \sin\theta\right)\left(1 - \frac{r^2}{R^2}\right)$$

$$V_{avg} = \frac{(\Delta P - \rho g L\ \sin\theta)D^2}{32\mu L}$$

$$\dot{V} = \frac{(\Delta P - \rho g L\ \sin\theta)\pi D^4}{128\mu L}$$

Laminar Flow in Pipes

Laminar Flow in Circular Pipes

(Fully developed flow with no pump or turbine in the flow section, and $\Delta P = P_1 - P_2$)

Horizontal pipe: $\dot{V} = \dfrac{\Delta P\, \pi D^4}{128\mu L}$

Inclined pipe: $\dot{V} = \dfrac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$

Uphill flow: $\theta > 0$ and $\sin\theta > 0$

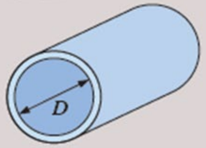
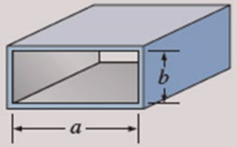
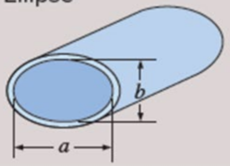
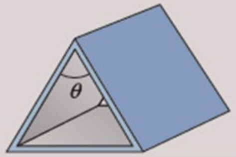
Downhill flow: $\theta < 0$ and $\sin\theta < 0$

The relations developed for fully developed laminar flow through horizontal pipes can also be used for inclined pipes by replacing ΔP with $\Delta P - \rho g L \sin\theta$.

Laminar Flow in Pipes

The friction factor *f* relations are given in Table 8-1 for *fully developed laminar flow* in pipes of various cross sections. The Reynolds number for flow in these pipes is based on the hydraulic diameter $D_h = 4A_c/p$, where A_c is the cross-sectional area of the pipe and *p* is its wetted perimeter.

Friction factor for fully developed *laminar flow* in pipes of various cross sections ($D_h = 4A_c/p$ and $\mathrm{Re} = V_{avg} D_h/\nu$)

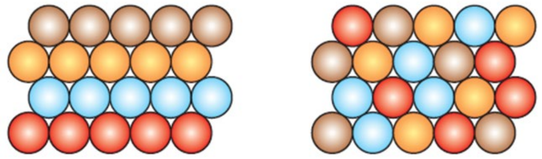
Tube Geometry	a/b or θ°	Friction Factor f
Circle 	—	64.00/Re
Rectangle 	a/b	
	1	56.92/Re
	2	62.20/Re
	3	68.36/Re
	4	72.92/Re
	6	78.80/Re
	8	82.32/Re
	∞	96.00/Re
Ellipse 	a/b	
	1	64.00/Re
	2	67.28/Re
	4	72.96/Re
	8	76.60/Re
	16	78.16/Re
Isosceles triangle 	θ	
	10°	50.80/Re
	30°	52.28/Re
	60°	53.32/Re
	90°	52.60/Re
	120°	50.96/Re

Turbulent Flow in Pipes

Most flows encountered in engineering practice are turbulent, and thus it is important to understand how turbulence affects wall shear stress.

Turbulent flow is a complex mechanism dominated by fluctuations, and it is still not fully understood.

We must rely on experiments and the empirical or semi-empirical correlations developed for various situations.



(*a*) Before turbulence   (*b*) After turbulence

The intense mixing in turbulent flow brings fluid particles at different momentums into close contact and thus enhances momentum transfer.

Turbulent flow is characterized by disorderly and rapid fluctuations of swirling regions of fluid, called **eddies**, throughout the flow.

These fluctuations provide an additional mechanism for momentum and energy transfer.

In turbulent flow, the swirling eddies transport mass, momentum, and energy to other regions of flow much more rapidly than molecular diffusion, greatly enhancing mass, momentum, and heat transfer.

As a result, turbulent flow is associated with much higher values of friction, heat transfer, and mass transfer coefficients.

Turbulent Flow in Pipes

(a)

(c)

(b)

Water exiting a tube: *(a) laminar flow* at low flow rate, *(b) turbulent flow at* high flow rate, and *(c) same as (b)* but with a short shutter exposure to capture individual eddies.

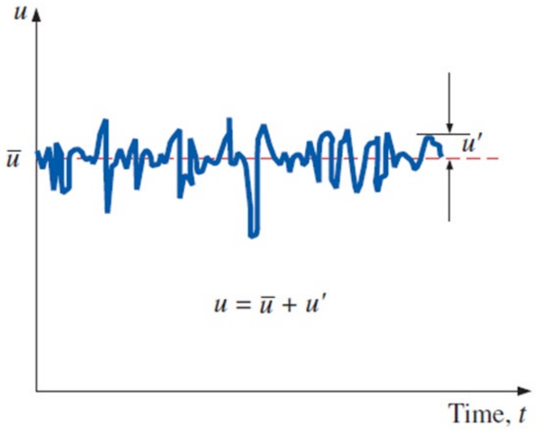
Turbulent Flow in Pipes

$u = \bar{u} + u'$ $\quad$ *average value* $\bar{u}$

fluctuating component u'

$\upsilon = \bar{\upsilon} + \upsilon', P = \bar{P} + P'$

$T = \bar{T} + T'$

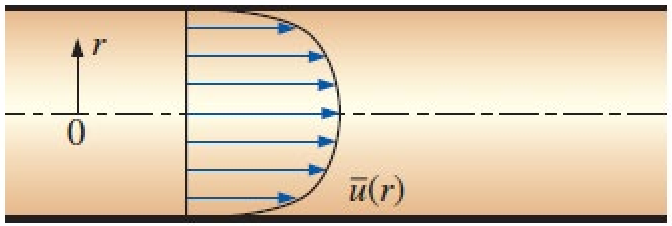


Fluctuations of the velocity component *u* with time at a specified location in turbulent flow.

$$\tau_{\text{total}} = \tau_{\text{lam}} + \tau_{\text{turb}}$$

The *laminar component*: accounts for the friction between layers in the flow direction.

The *turbulent component:* accounts for the friction between the fluctuating fluid particles and the fluid body (related to the fluctuation components of velocity).



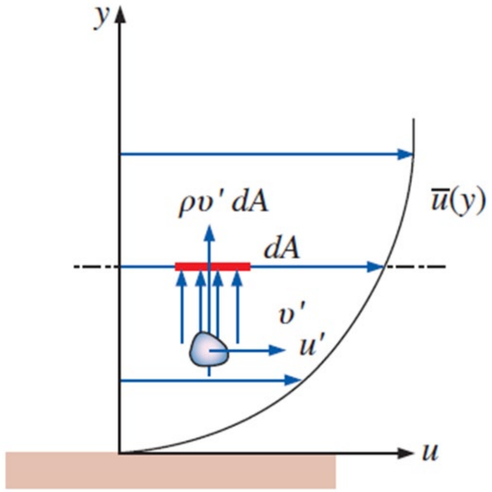
The average or mean velocity profile for turbulent flow in a pipe.

Turbulent Flow in Pipes

Turbulent Shear Stress.

$\tau_{\text{turb}} = -\rho\overline{u'v'}$ turbulent shear stress

Terms such as $-\rho\overline{u'v'}$ or $-\rho\overline{u'^2}$ are called Reynolds stresses or turbulent stresses.



Fluid particle moving upward through a differential area *dA* as a result of the velocity fluctuation v'.

$$\tau_{\text{turb}} = -\rho\overline{u'v'} = \mu_t \frac{\partial \overline{u}}{\partial y}$$ Turbulent shear stress

μ_t **eddy viscosity** or **turbulent viscosity:** accounts for momentum transport by turbulent eddies.

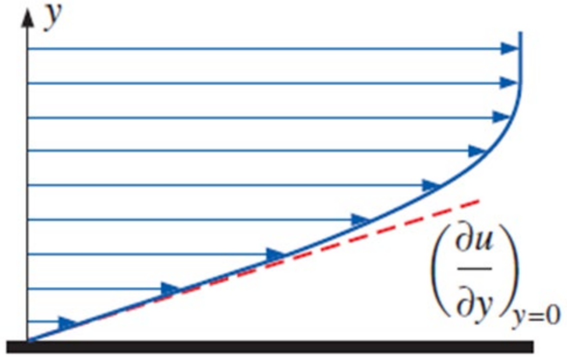
$$\tau_{\text{total}} = (\mu + \mu_t)\frac{\partial u}{\partial y} = \rho(\nu + \nu_t)\frac{\partial u}{\partial y}$$ Total shear stress

$\nu_t = \mu_t/\rho$ **kinematic eddy viscosity** or **kinematic turbulent viscosity** (also called the *eddy diffusivity of momentum*).

Turbulent Flow in Pipes

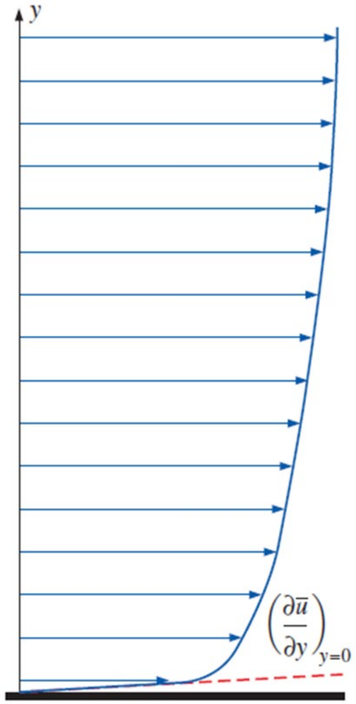
$$\tau_{\text{turb}} = \mu_t \frac{\partial \bar{u}}{\partial y} = \rho l_m^2 \left(\frac{\partial \bar{u}}{\partial y} \right)^2$$

mixing length l_m**:** related to the average size of the eddies that are primarily responsible for mixing.



Laminar flow

The velocity gradients at the wall, and thus the wall shear stress, are much larger for turbulent flow than they are for laminar flow, even though the turbulent boundary layer is thicker than the laminar one for the same value of free-stream speed.



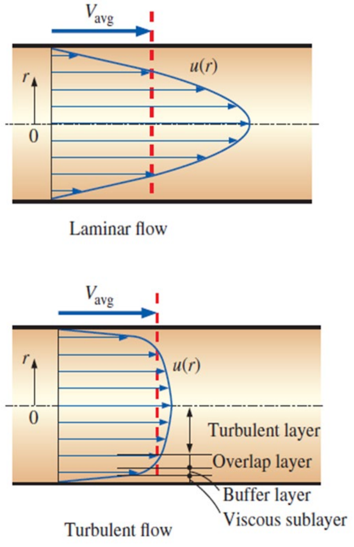
Turbulent flow

Molecular diffusivity of momentum v (as well as μ) is a fluid property, and and its value is listed in fluid handbooks.

Eddy diffusivity V_t (as well as μ_t), however, is *not* a fluid property, and its value depends on flow conditions. Eddy diffusivity μ_t decreases toward the wall, becoming zero at the wall. Its value ranges from zero at the wall to several thousand times the value of the molecular diffusivity in the core region.

Turbulent Flow in Pipes

Turbulent Velocity Profile.



The velocity profile in fully developed pipe flow is parabolic in laminar flow, but much fuller in turbulent flow. Note that *u* of r in the turbulent case is the *time-averaged* velocity component in the axial direction (the overbar on *u* has been dropped for simplicity).

The very thin layer next to the wall where viscous effects are dominant is the **viscous** (or **laminar** or **linear** or **wall**) sublayer.

The velocity profile in this layer is very nearly *linear*, and the flow is streamlined.

Next to the viscous sublayer is the **buffer layer,** in which turbulent effects are becoming significant, but the flow is still dominated by viscous effects.

Above the buffer layer is the **overlap** (or **transition**) **layer,** also called the **inertial sublayer,** in which the turbulent effects are much more significant, but still not dominant.

Above that is the **outer** (or **turbulent**) **layer** in the remaining part of the flow in which turbulent effects dominate over molecular diffusion (viscous) effects.

Turbulent Flow in Pipes

$$\tau_w = \mu \frac{u}{y} = \rho \nu \frac{u}{y} \quad \text{or} \quad \frac{\tau_w}{\rho} = \frac{\nu u}{y}$$

$$u^* = \sqrt{\tau_w / \rho}$$ friction velocity

Viscous sublayer: $$\frac{u}{u^*} = \frac{y u^*}{\nu}$$ law of the wall

Thickness of viscous sublayer: $$y = \delta_{sublayer} = \frac{5\nu}{u^*} = \frac{25\nu}{u_\delta}$$

The thickness of the viscous sublayer is proportional to the kinematic viscosity and inversely proportional to the average flow speed.

ν / u^* Viscous length; it is used to nondimensionalize the distance *y* from the surface.

Nondimensionalized variables: $$y^+ = \frac{y u^*}{\nu} \quad \text{and} \quad u^+ = \frac{u}{u^*}$$

Normalized law of the wall: $$u^+ = y^+$$

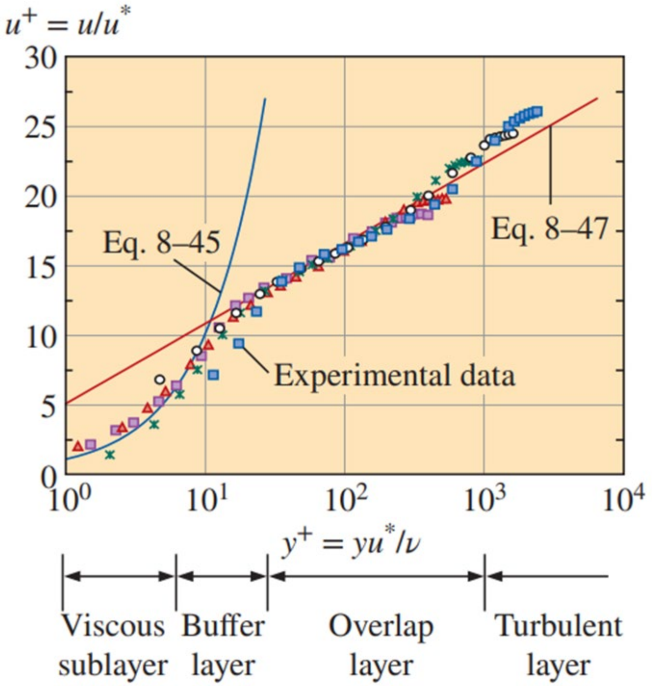
Turbulent Flow in Pipes

The logarithmic law:

$$\frac{u}{u^*} = \frac{1}{\kappa}\ln\frac{yu^*}{\nu} + B$$

Overlap layer :

$$\frac{u}{u^*} = 2.5\ln\frac{yu^*}{\nu} + 5.0 \quad or \quad u^+ = 2.5\ln y^+ + 5.0$$



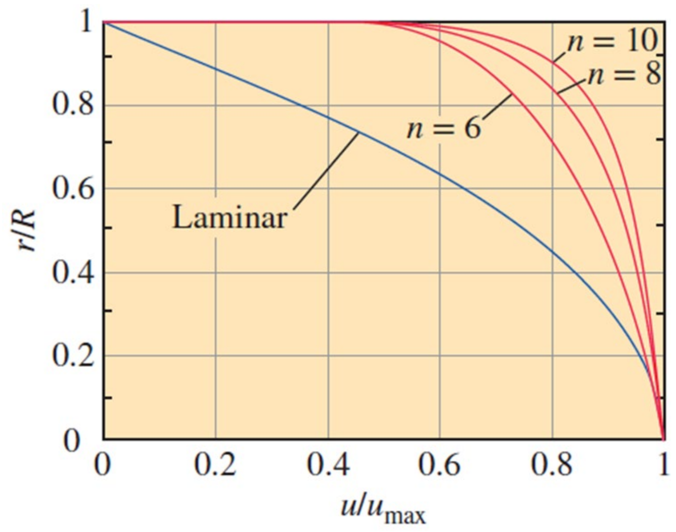
Comparison of the law of the wall and the logarithmic-law velocity profiles with experimental data for fully developed turbulent flow in a pipe.

Turbulent Flow in Pipes

Outer turbulent layer: $$\frac{u_{max} - u}{u^*} = 2.5 \ln \frac{R}{R - r}$$ Velocity defect law

The deviation of velocity magnitude from the centerline value $U_{max} - U$ is called the velocity defect.

Power-law velocity profile: $$\frac{u}{u_{max}} = \left(\frac{y}{R}\right)^{1/n} \quad \text{or} \quad \frac{u}{u_{max}} = \left(1 - \frac{r}{R}\right)^{1/n}$$



Power-law velocity profiles for fully developed turbulent flow in a pipe for different exponents, and its comparison with the laminar velocity profile.

The value $n = 7$ generally approximates many flows in practice, giving rise to the term *one-seventh power-law velocity profile.*

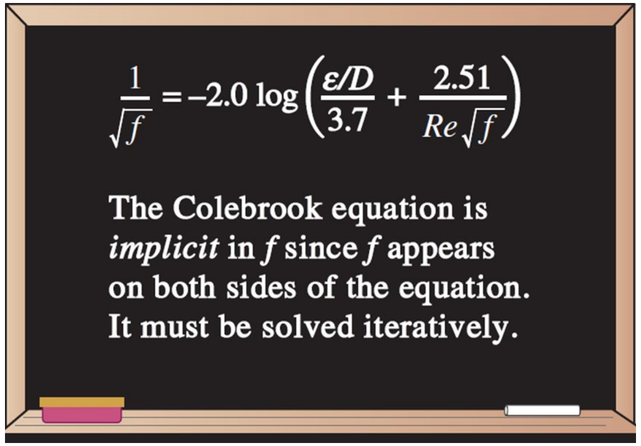
Turbulent Flow in Pipes

The Moody Chart and Its Associated Equations.

Colebrook equation (for smooth and rough pipes).

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \quad (\text{turbulent flow})$$

The friction factor in fully developed turbulent pipe flow depends on the Reynolds number and the **relative roughness** $\boldsymbol{\varepsilon / D}$.



Turbulent Flow in Pipes

Equivalent roughness values for new commercial pipes*

Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0 (Smooth)	0 (Smooth)
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

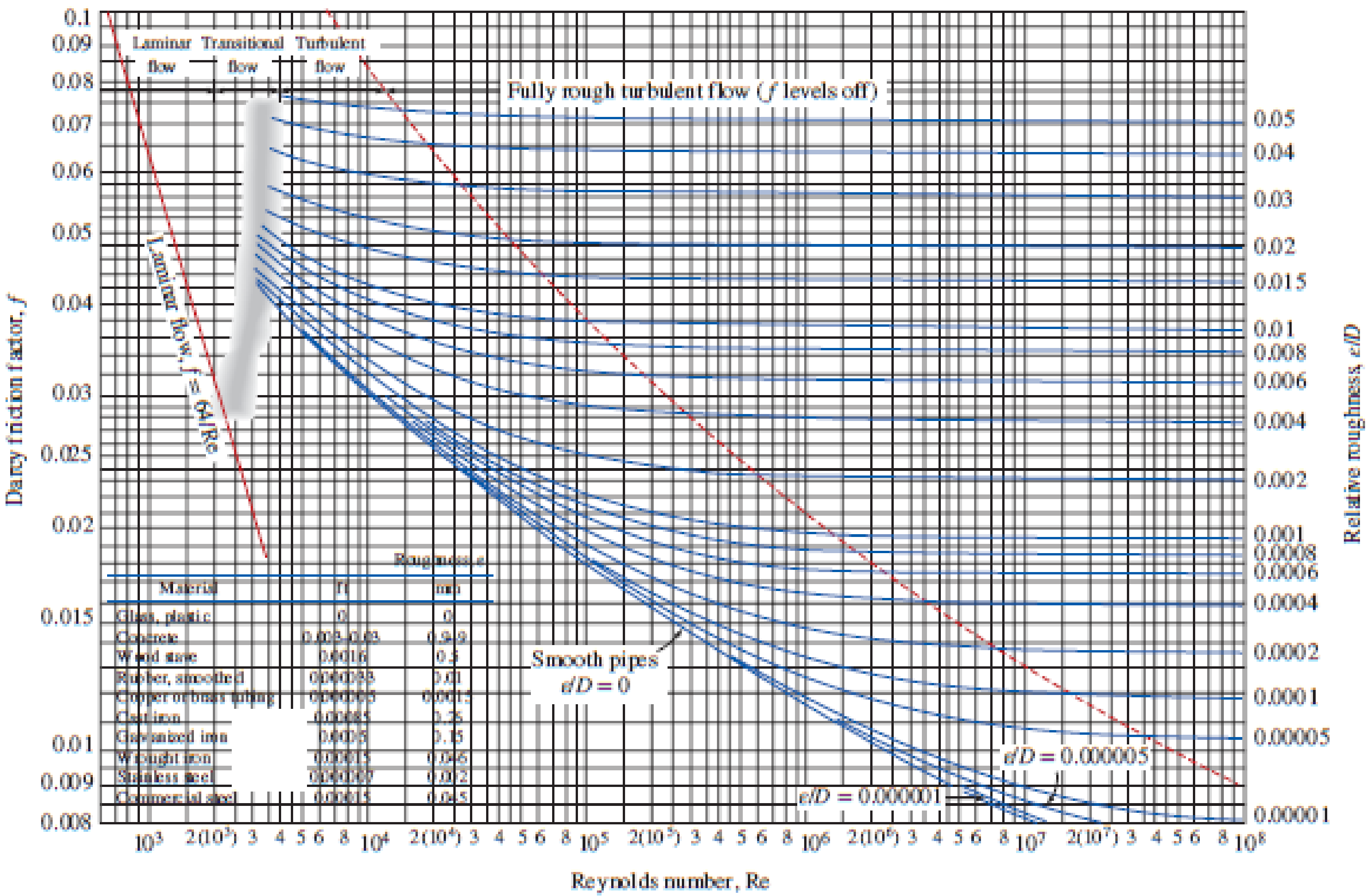
* The uncertainty in these values can be as much as ? 0 percent.

Relative Roughness, ε / D	FrictionFactor, f
0.0 *	0.0119
0.00001	0.0119
0.0001	0.0134
0.0005	0.0172
0.001	0.0199
0.005	0.0305
0.01	0.0380
0.05	0.0716

* Smooth surface. All values are for Re = 10^6 and are calculated from the Colebrook equation.

The friction factor is minimum for a smooth pipe and increases with roughness.

Turbulent Flow in Pipes



The Moody Chart

Turbulent Flow in Pipes

Observations from the Moody chart.

For laminar flow, the friction factor decreases with increasing Reynolds number, and it is independent of surface roughness.

The friction factor is a minimum for a smooth pipe and increases with roughness. The Colebrook equation in this case $(\varepsilon = 0)$ reduces to the **Prandtl equation.**

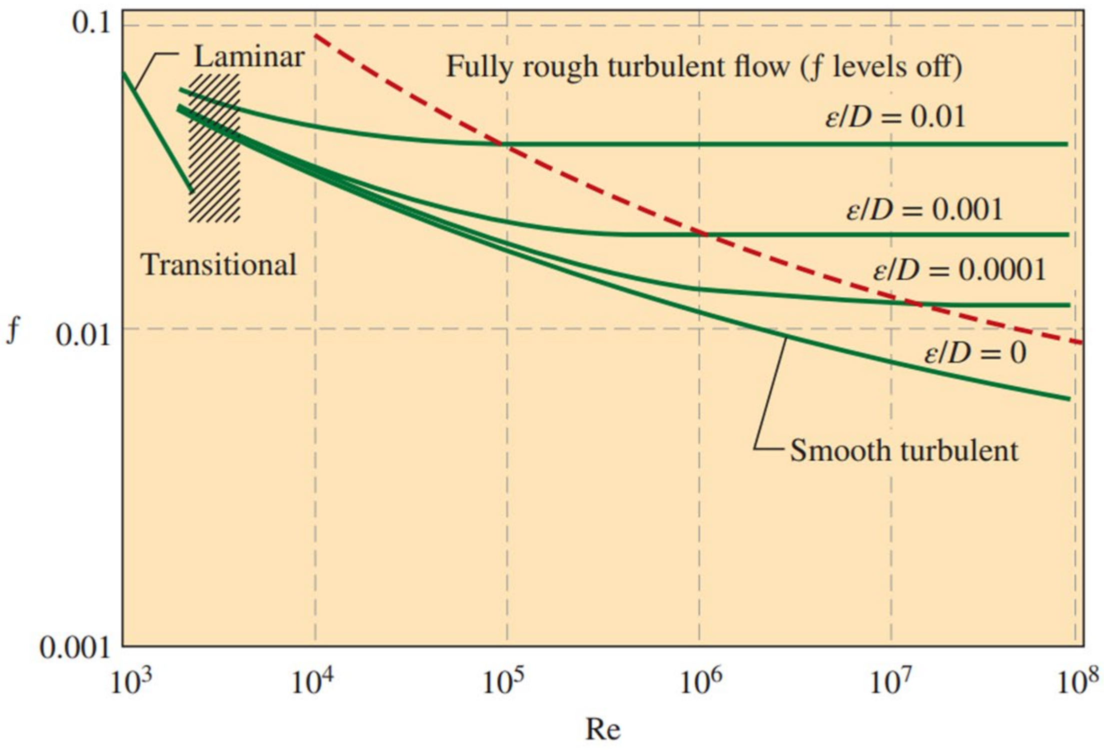
$$1/\sqrt{f} = 2.0\,\log(\mathrm{Re}\sqrt{f}) - 0.8$$

The transition region from the laminar to turbulent regime is indicated by the shaded area in the Moody chart. At small relative roughnesses, the friction factor increases in the transition region and approaches the value for smooth pipes.

At very large Reynolds numbers (to the right of the dashed line on the Moody chart) the friction factor curves corresponding to specified relative roughness curves are nearly horizontal, and thus the friction factors are independent of the Reynolds number. The flow in that region is called *fully rough turbulent flow* or just *fully rough flow* because the thickness of the viscous sublayer decreases with increasing Reynolds number, and it becomes so thin that it is negligibly small compared to the surface roughness height. The Colebrook equation in the *fully rough* zone reduces to the **von Kármán equation.**

$$1/\sqrt{f} = -2.0\,\log[(\varepsilon/D)/3.7]$$

Turbulent Flow in Pipes



At very large Reynolds numbers, the friction factor curves on the Moody chart are nearly horizontal, and thus the friction factors are independent of the Reynolds number. See Fig. A–12 for a full-page moody chart.

Turbulent Flow in Pipes

Standard sizes for Schedule 40 steel pipes.

Nominal Size, in	Actual Inside Diameter, in
$\frac{1}{8}$	0.269
$\frac{1}{4}$	0.364
$\frac{3}{8}$	0.493
$\frac{1}{2}$	0.622
$\frac{3}{4}$	0.824
1	1.049
$1\frac{1}{2}$	1.610
2	2.067
$2\frac{1}{2}$	2.469
3	3.068
5	5.047
10	10.02

In calculations, we should make sure that we use the actual internal diameter of the pipe, which may be different than the nominal diameter.

Turbulent Flow in Pipes

Types of Fluid Flow Problems.

1. Determining the **pressure drop** (or head loss) when the pipe length and diameter are given for a specified flow rate (or average speed).
2. Determining the **flow rate** when the pipe length and diameter are given for a specified pressure drop (or head loss).
3. Determining the **pipe diameter** when the pipe length and flow rate are given for a specified pressure drop (or head loss).

Problem type	Given	Find
1	$L, D, \dot{V}$	ΔP (or h_L)
2	$L, D, \Delta P$	$\dot{V}$
3	$L, \Delta P, \dot{V}$	D

The three types of problems encountered in pipe flow.

Turbulent Flow in Pipes

To avoid tedious iterations in head loss, flow rate, and diameter calculations, these explicit relations that are accurate to within 2 percent of the Moody chart may be used.

$$h_L = 1.07 \frac{\dot{V}^2 L}{g D^5} \left\{ \ln \left[\frac{\varepsilon}{3.7D} + ?.62 \left(\frac{\nu D}{\dot{V}} \right)^{0.9} \right] \right\}^{-2} \qquad \begin{array}{l} 10^{-6} < \varepsilon/D < 10^{-2} \\ 3000 < \mathrm{Re} < 3?\times\ 0^{8} \end{array}$$

$$\dot{V} = -0.965 \left(\frac{g D^5 h_L}{L} \right)^{0.5} \ln \left[\frac{\varepsilon}{3.7D} + \left(\frac{3.17 \nu^2 L}{g D^3 h_L} \right)^{0.5} \right] \qquad \mathrm{Re} > 2000$$

$$D = 0.66 \left[\varepsilon^{1.25} \left(\frac{L \dot{V}^2}{g h_L} \right)^{4.75} + \nu \dot{V}^{9.4} \left(\frac{L}{g h_L} \right)^{5.2} \right]^{0.04} \qquad \begin{array}{l} 10^{-6} < \varepsilon/D < 10^{-2} \\ 5000 < \mathrm{Re} < 3 \times 10^{8} \end{array}$$

All quantities are dimensional and the units simplify to the desired unit (for example, to m or ft in the last relation) when consistent units are used. Noting that the Moody chart is accurate to within 15 percent of experimental data, we should have no reservation in using these approximate relations in the design of piping systems.

Turbulent Flow in Pipes

Explicit Haaland equation

$$\frac{1}{\sqrt{f}} \cong -1.8\log\left[\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

The results obtained from this relation are within 2 percent of those obtained from the Colebrook equation.

An equation was generated by Churchill (1997) that is not only explicit, but is also useful for any Re and any roughness, even for laminar flow, and even in the fuzzy transitional region between laminar and turbulent flow.

$$f = 8\left[\left(\frac{8}{\mathrm{Re}}\right)^{12} + (A+B)^{-1.5}\right]^{\frac{1}{12}}$$

Explicit Churchill equation

$$A = \left\{-2.457\ln\left[\left(\frac{7}{\mathrm{Re}}\right)^{0.9} + 0.27\frac{\varepsilon}{D}\right]\right\}^{16}$$ and $$B = \left(\frac{37{,}530}{\mathrm{Re}}\right)^{16}$$

The difference between the Colebrook and Churchill equations is less than one percent.

Minor Losses

The fluid in a typical piping system passes through various fittings, valves, bends, elbows, tees, inlets, exits, enlargements, and contractions in addition to the pipes.

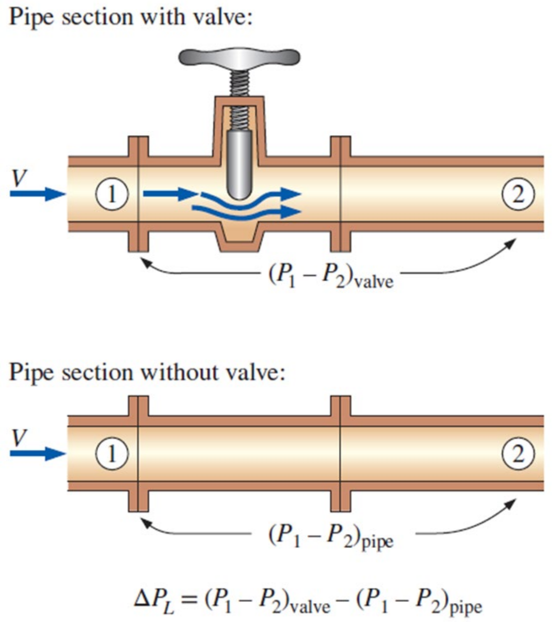
These components interrupt the smooth flow of the fluid and cause additional losses because of the flow separation and mixing they induce.

In a typical system with long pipes, these losses are minor compared to the total head loss in the pipes (the **major losses**) and are called **minor losses**.

Minor losses are usually expressed in terms of the **loss coefficient** K_L.

$$\textit{Loss coefficient}: \quad K_L = \frac{h_L}{V^2/(2g)}$$

$$h_L = \Delta P_L/\rho g$$ Head loss due to component



For a constant-diameter section of a pipe with a minor loss component, the loss coefficient of the component (such as the gate valve shown) is determined by measuring the additional pressure loss it causes and dividing it by the dynamic pressure in the pipe.

Minor Losses

When the inlet diameter equals outlet diameter, the loss coefficient of a component can also be determined by measuring the pressure loss across the component and dividing it by the dynamic pressure:

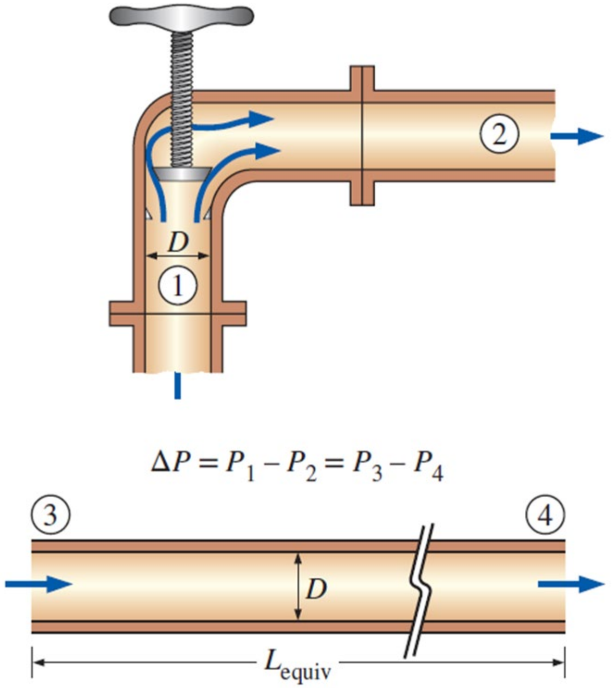
$$K_L = \Delta P_L / \left(pV^2 / 2 \right)$$

When the loss coefficient for a component is available, the head loss for that component is,

$$\textit{Minor loss}: \quad h_L = K_L \frac{V^2}{2g}$$

Minor losses are also expressed in terms of the **equivalent length** L_{equiv}

$$h_L = K_L \frac{V^2}{2g} = f \frac{L_{\text{equiv}}}{D} \frac{V^2}{2g} \quad \rightarrow \quad L_{\text{equiv}} = \frac{D}{f} K_L$$



The head loss caused by a component (such as the angle valve shown) is equivalent to the head loss caused by a section of the pipe whose length is the equivalent length.

Minor Losses

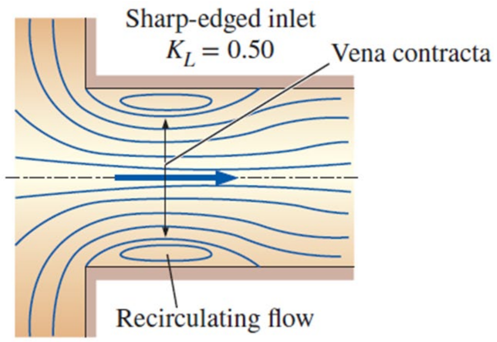
Total head loss (general)

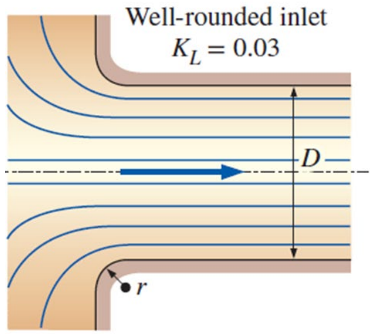
$$h_{L,\,total} = h_{L,\,major} + h_{L,\,minor}$$

$$= \sum_i f_i \frac{L_i}{D_i}\frac{V_i^2}{2g} + \sum_j K_{L,j}\frac{V_j^2}{2g}$$

Total head loss (D = constant)

$$h_{L,\,total} = \left(f\frac{L}{D} + \sum K_L \right)\frac{V^2}{2g}$$

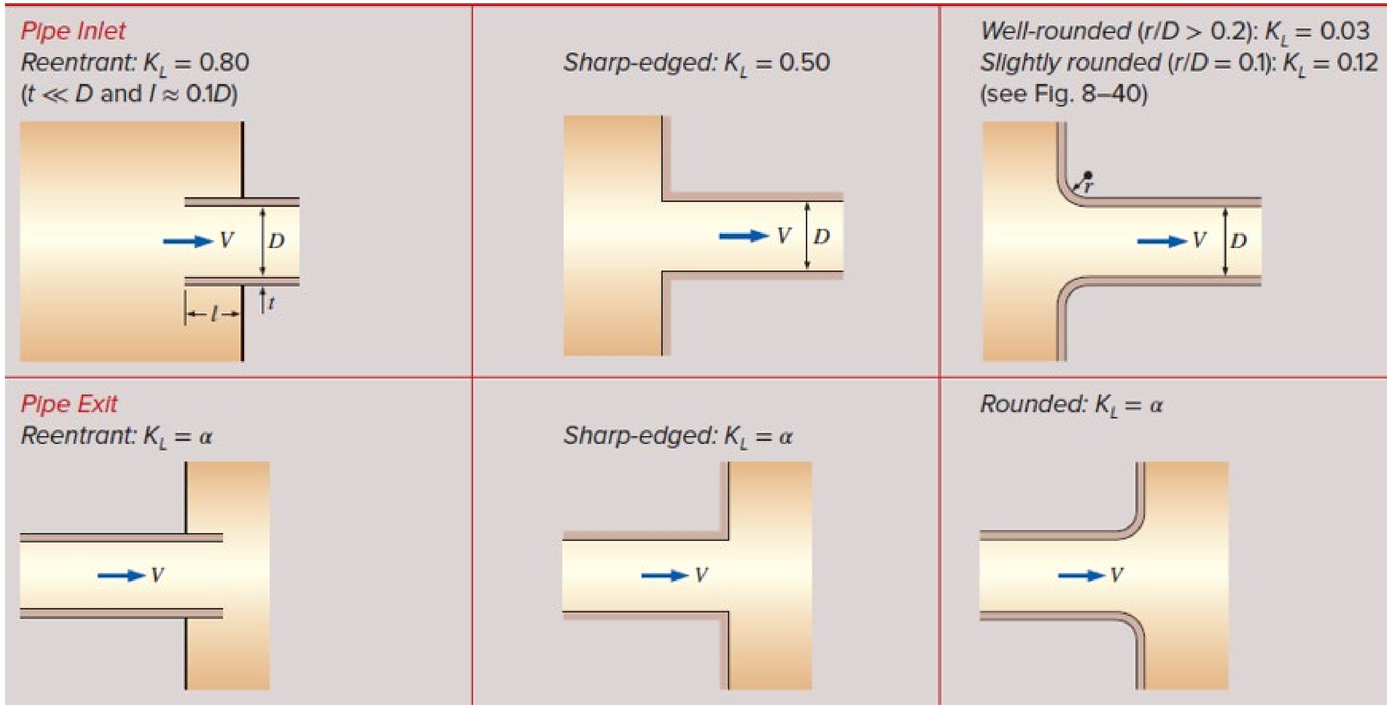




The head loss at the inlet of a pipe is almost negligible for well-rounded inlets $(K_L = 0.03 \text{ for } r/D > 0.2)$ but increases to about 0.50 for sharp-edged inlets.

Minor Losses

Loss coefficients K_L of various pipe components for turbulent flow (for use in the relation $h_L = K_L V^2 / (2g)$, where V is the average velocity in the pipe that contains the component)*

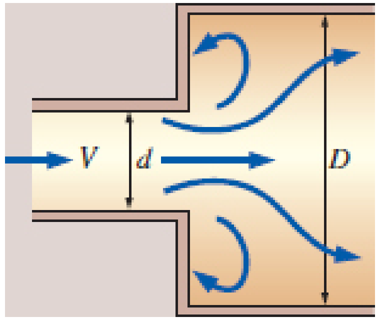


Note: The kinetic energy correction factor is $\alpha = 2$ for fully developed laminar flow, and $\alpha \approx 1.05$ for fully developed turbulent flow.

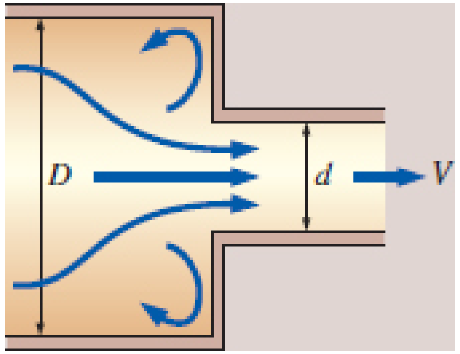
Minor Losses

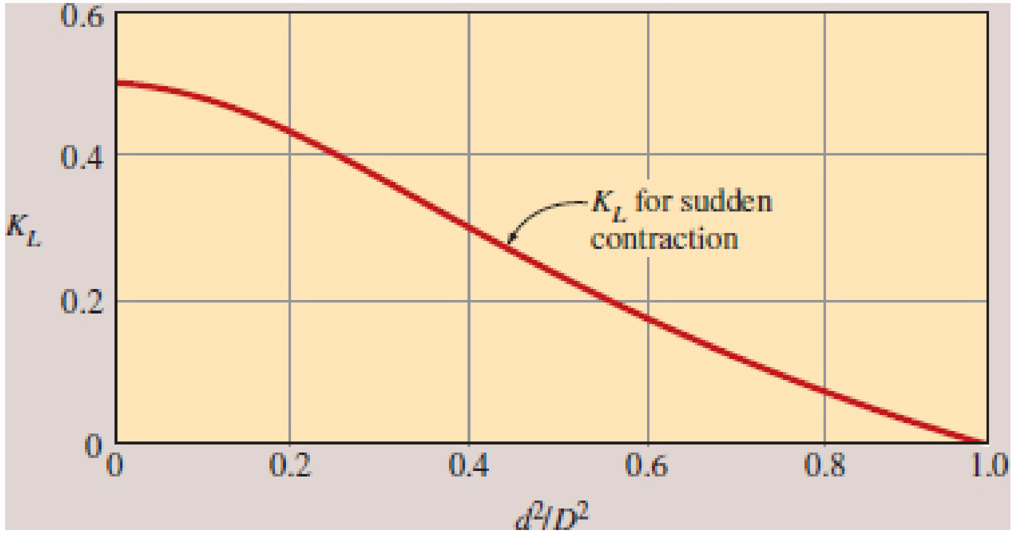
Sudden Expansion and Contraction (based on the velocity In the smaller-diameter pipe).

$$\textit{Sudden expansion}: K_L = \alpha\left(1-\frac{d^2}{D^2}\right)^2$$



Sudden contraction: See chart.





Minor Losses

Gradual Expansion and Contraction (based on the velocity In the smaller-diameter pipe).

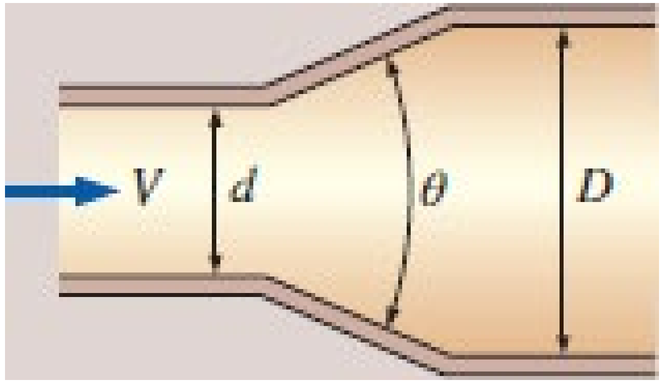
$Expansion\left(for\ \theta = 20°\right):$

$K_L = 0.30 \text{ for } d/D = 0.2$

$K_L = 0.25 \text{ for } d/D = 0.4$

$K_L = 0.15 \text{ for } d/D = 0.6$

$K_L = 0.10 \text{ for } d/D = 0.8$

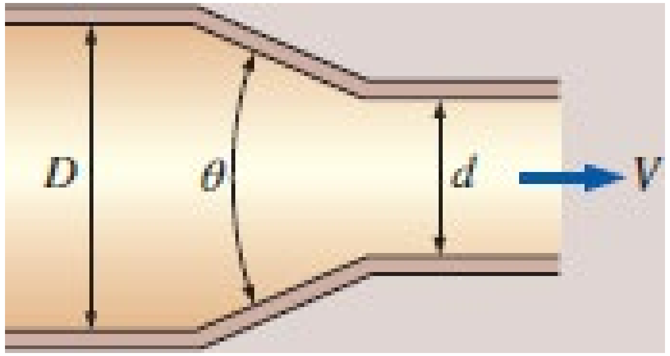


Contraction:

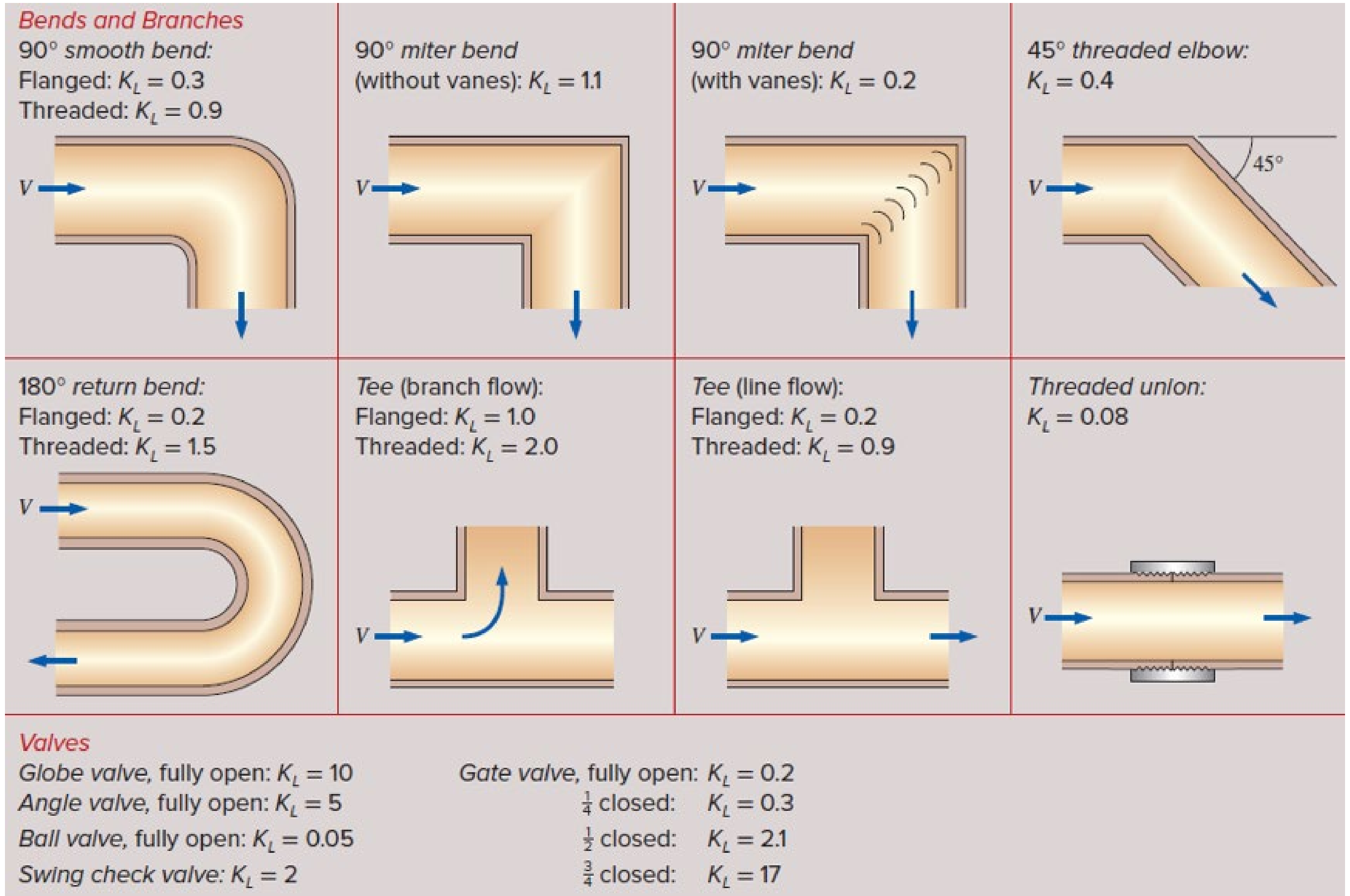
$K_L = 0.02 \text{ for } \theta = 30°$

$K_L = 0.04 \text{ for } \theta = 45°$

$K_L = 0.07 \text{ for } \theta = 60°$



Minor Losses



*These are representative values for loss coefficients. Actual values strongly depend on the design and manufacture of the components and may differ from the given values considerably (especially for valves). Actual manufacturer's data should be used In the final design.

Minor Losses

Values:

$$\textit{Globe value}, \text{ fully open}: K_L = 10$$

$$\textit{Angle value}, \text{ fully open}: K_L = 5$$

$$\textit{Ball value}, \text{ fully open}: K_L = 0.05$$

$$\textit{Swing check value}: K_L = 2$$

$$\textit{Gate value}, \text{ fully open}: K_L = 0.2$$

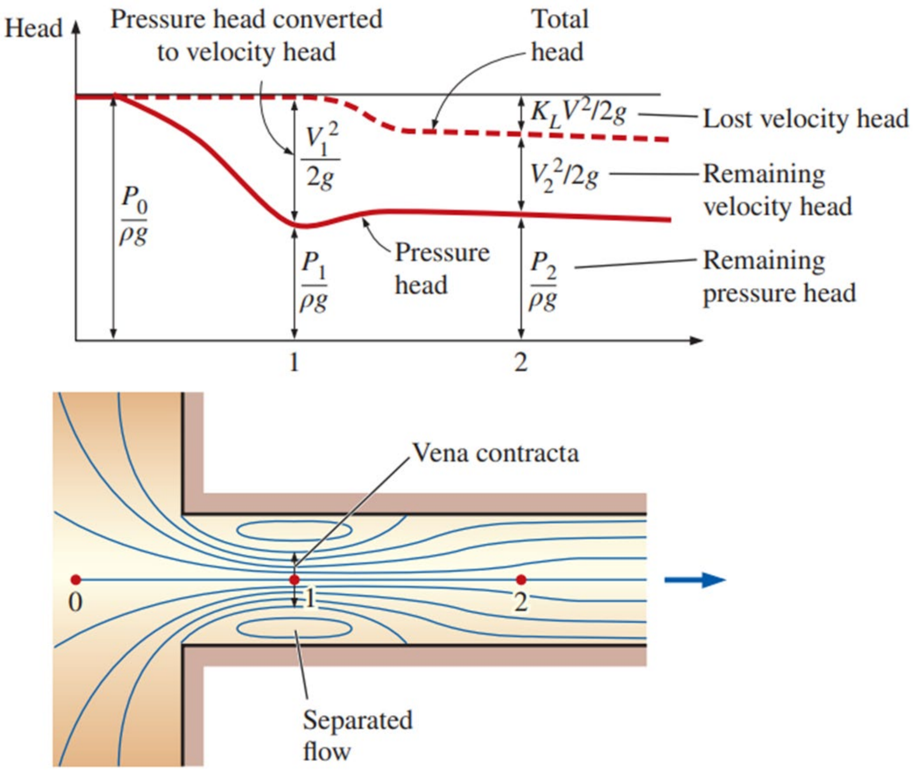
$$\frac{1}{4} \text{ closed}: \quad K_L = 0.2$$

$$\frac{1}{2} \text{ closed}: \quad K_L = 2.1$$

$$\frac{3}{4} \text{ closed}: \quad K_L = 17$$

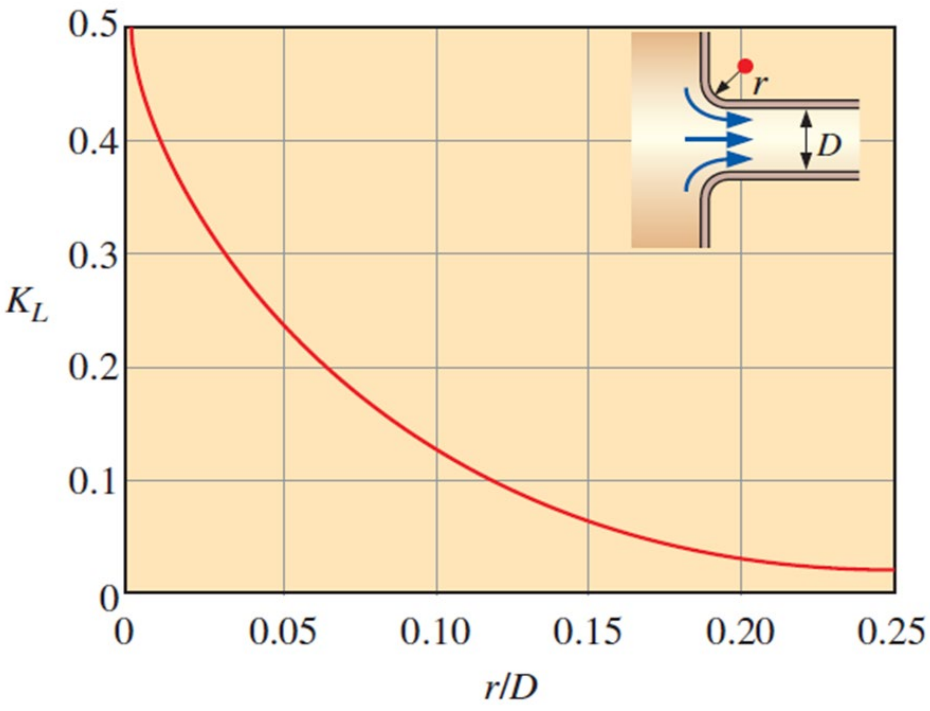
Minor Losses

$$K_L = \alpha\left(1 - \frac{A_{\text{small}}}{A_{\text{large}}}\right)^2 \qquad (\text{sudden expansion})$$



Graphical representation of flow contraction and the associated head loss at a sharp-edged pipe inlet.

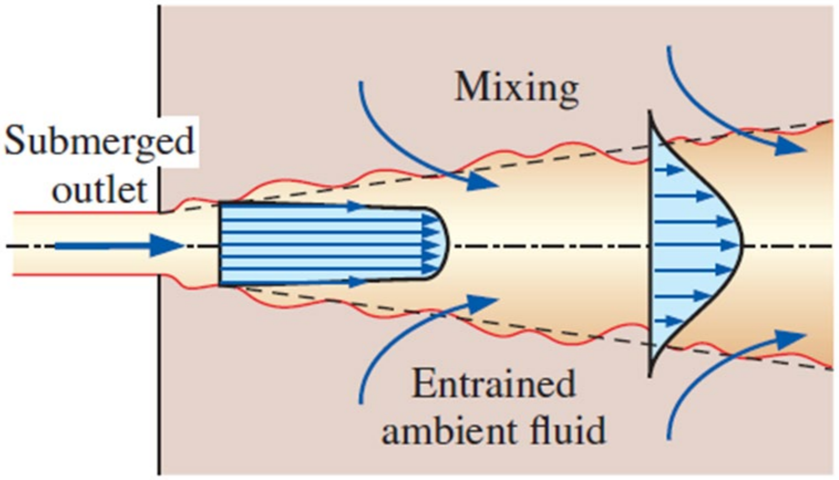
Minor Losses



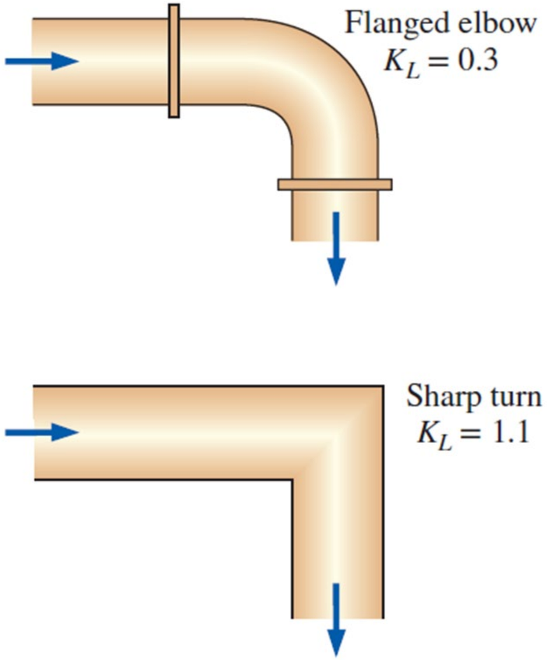
Data from ASHRAE Handbook of Fundamentals.

The effect of rounding of a pipe inlet on the loss coefficient.

Minor Losses



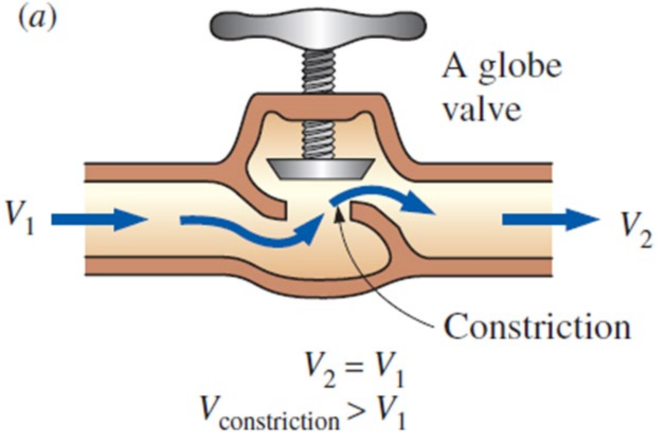
All the kinetic energy of the flow is “lost” (turned into thermal energy) through friction as the jet decelerates and mixes with ambient fluid downstream of a submerged outlet.



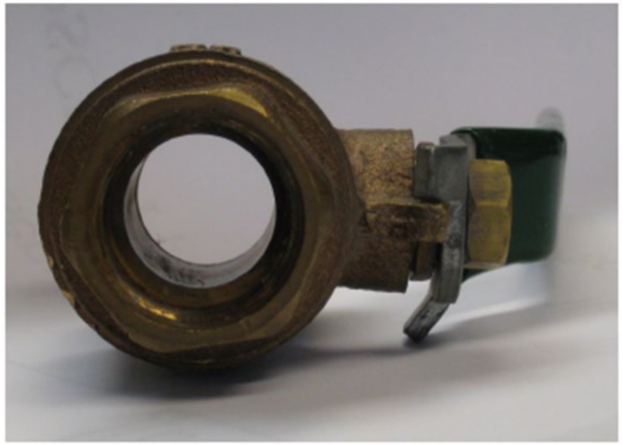
The losses during changes of direction can be minimized by making the turn “easy” on the fluid by using circular arcs instead of sharp turns.

Minor Losses

(*a*)



(*b*)

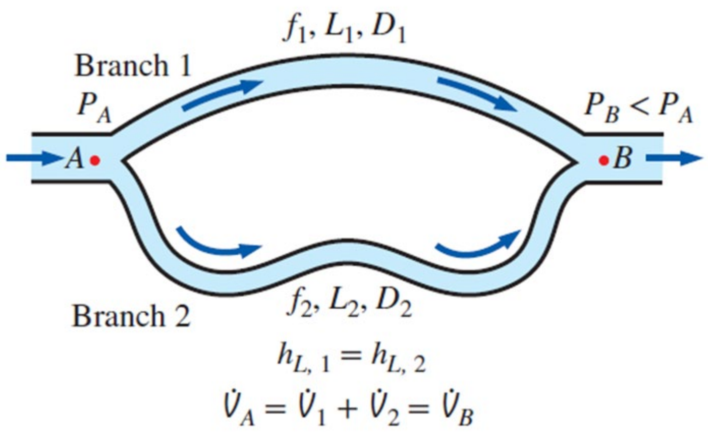


(*a*) The large head loss in a partially closed valve is due to irreversible deceleration, flow separation, and mixing of high-velocity fluid coming from the narrow valve passage.

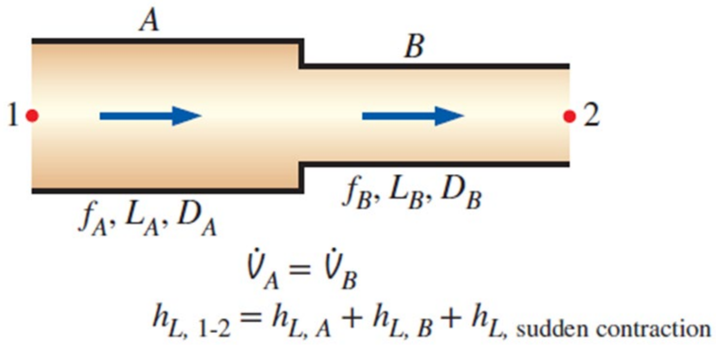
(*b*) The head loss through a fully-open ball valve, on the other hand, is quite small.

Piping Networks And Pump Selection

A piping network in an industrial facility.



For pipes *in parallel*, the head loss is the same in each pipe, and the total flow rate is the sum of the flow rates in individual pipes.



For pipes *in series*, the flow rate is the same in each pipe, and the total head loss is the sum of the head losses in individual pipes.

Piping Networks And Pump Selection

The relative flow rates in parallel pipes are established from the requirement that the head loss in each pipe be the same.

$$h_{L,?} = h_{L,?} \quad \rightarrow \quad f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g}$$

$$\frac{V_1}{V_2} = \left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{1/2} \quad \text{and} \quad \frac{\dot{V}_1}{\dot{V}_2} = \frac{A_{c,?}V_1}{A_{c,?}V_2} = \frac{D_1^2}{D_2^2}\left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{1/2}$$

The flow rate in one of the parallel branches is proportional to its diameter to the power $5/2$ and is inversely proportional to the square root of its length and friction factor.

The analysis of piping networks is based on two simple principles:

1. ***Conservation of mass throughout the system must be satisfied***. This is done by requiring the total flow into a junction to be equal to the total flow out of the junction for all junctions in the system.
2. ***Pressure drop (and thus head loss) between two junctions must be the same for all paths between the two junctions***. This is because pressure is a point function and it cannot have two values at a specified point. In practice this rule is used by requiring that the algebraic sum of head losses in a loop (for all loops) be equal to zero.

Piping Networks And Pump Selection

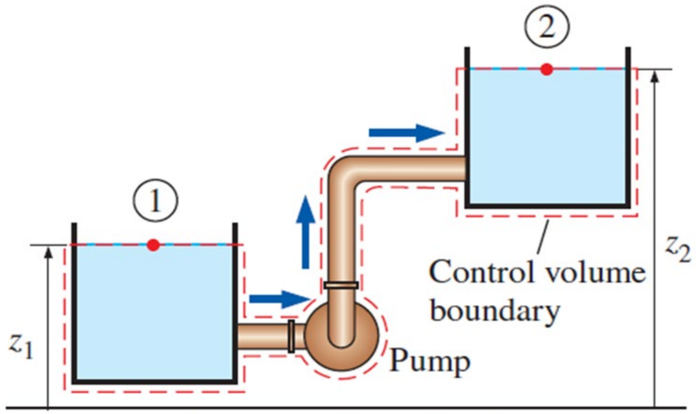
Piping Systems with Pumps and Turbines.

$$\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1 + w_{\text{pump, } u} = \frac{P_2}{\rho} + \alpha \frac{V_2^2}{2} + gz_2 + w_{\text{turbine, } e} + gh_L$$

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, } u} = \frac{P_2}{\rho g} + \alpha \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, } e} + h_L$$

$$h_{\text{pump,} u} = (z \;\; - z_1) + h_L$$

the steady-flow energy equation



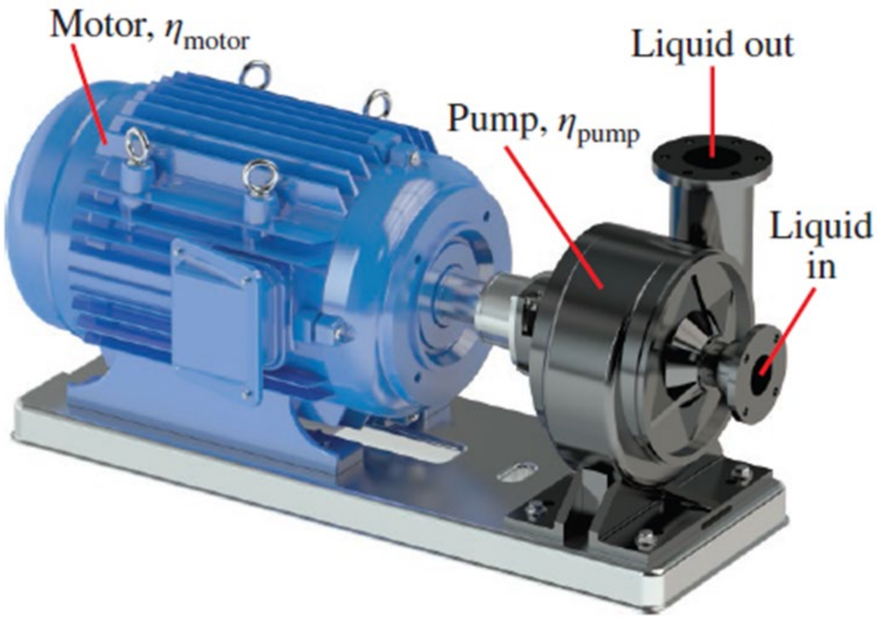
$$h_{\text{pump, } u} = (z_2 - z_1) + h_L$$
$$\dot{W}_{\text{pump, } u} = \rho \dot{V} g h_{\text{pump, } u}$$

When a pump moves a fluid from one reservoir to another, the useful pump head requirement is equal to the elevation difference between the two reservoirs plus the head loss.

Piping Networks And Pump Selection

$$\dot{W}_{\text{pump, shaft}} = \frac{\rho \dot{V} g h_{\text{pump, u}}}{\eta_{\text{pump}}}$$

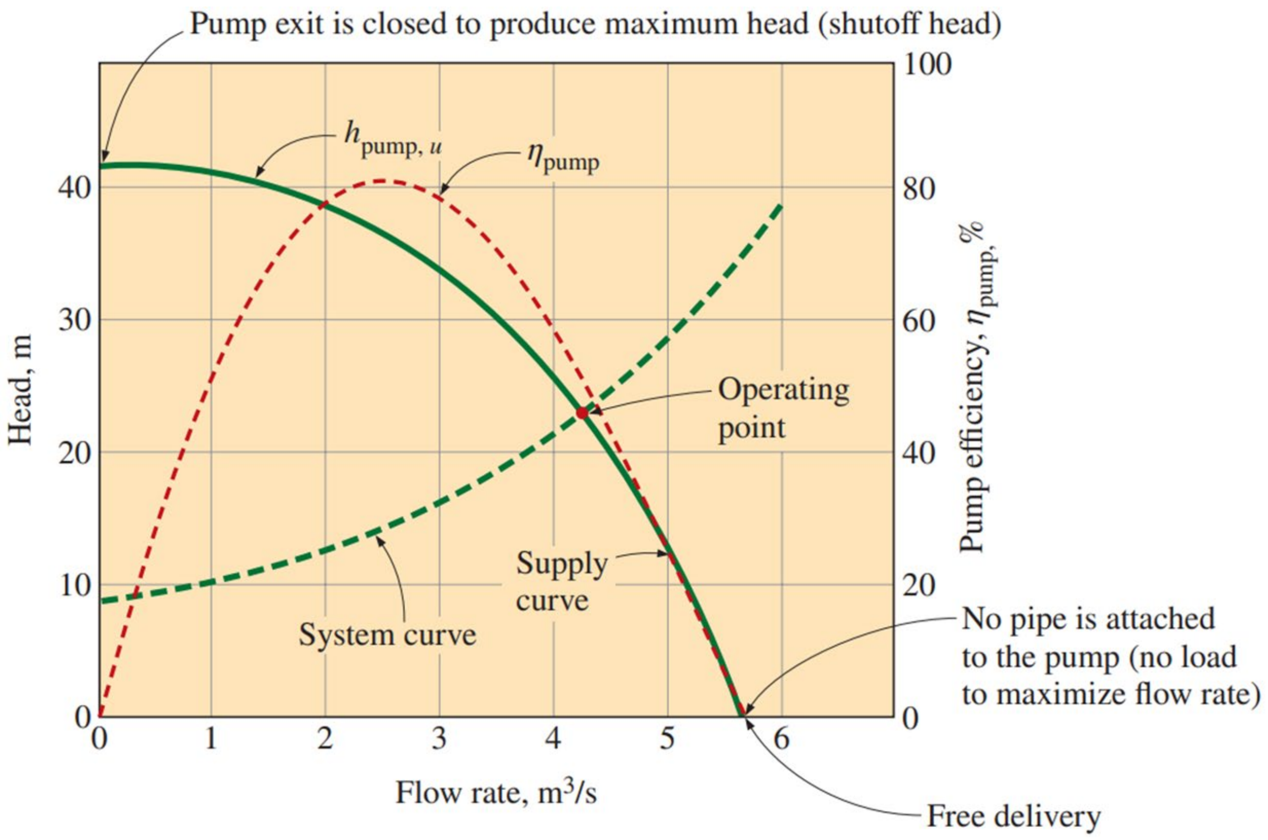
$$\dot{W}_{\text{elect}} = \frac{\rho \dot{V} g h_{\text{pump, u}}}{\eta_{\text{pump-motor}}}$$



$$\eta_{\text{pump-motor}} = \eta_{\text{pump}} \eta_{\text{motor}}$$

The efficiency of the pump–motor combination is the product of the pump and the motor efficiencies.

Piping Networks And Pump Selection



Characteristic pump curves for centrifugal pumps, the system curve for a piping system, and the operating point.

Piping Networks And Pump Selection

Flow rate of cold water through a shower may be affected significantly by the flushing of a nearby toilet.

Summary

- Laminar and Turbulent Flows.
- Reynolds Number, Entrance Region.
- Pressure Drop and Head Loss.
- Piping Networks and Pump Selection.