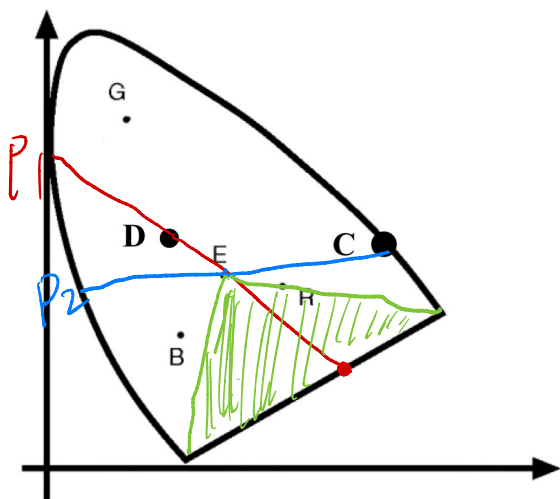


Q1



i. P1

ii. No, in green area, there is not corresponding dominant wave length. The color in this area is combination of other colors.

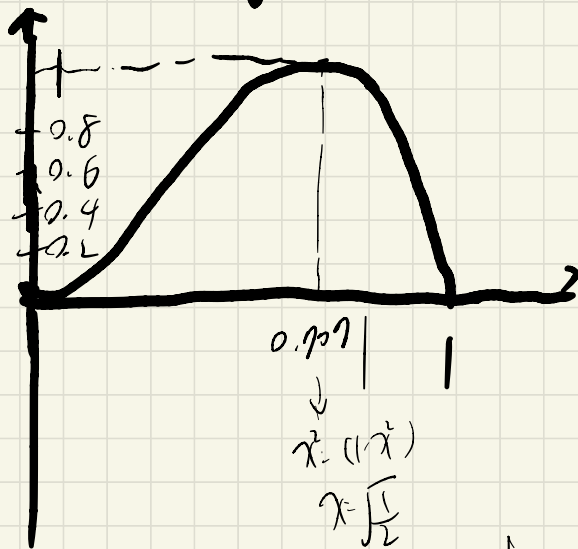
iii. P2

iv. white, black, grayscale

Q2:

$$1. \quad P(x) = x^2 \quad P(y) = 1 - x^2$$

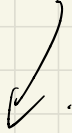
$$\begin{aligned} H(x) &= -\sum p_i \log p_i = -x^2 \log x^2 - (1-x^2) \log (1-x^2) \\ &= -2x^2 \log x - (1-x^2) \log (1-x^2) \end{aligned}$$



2. Entropy will be minimum at $x = 1, 0$
 \therefore Entropy minimum would happen when a symbol that are much more likely to appear than other symbols

3. happen at $P(x) = 1$ or $P(y) = 1$
 $x^k = 1$ or $1 - x^k = 1$
 $x = 1$ $x = 0$

4. $P(X) = P(Y)$ maximum entropy would happen at.
 $x^2 = 1 - x^2$ if all symbols are equally probable

5. $x = \frac{1}{\sqrt{2}}$ 

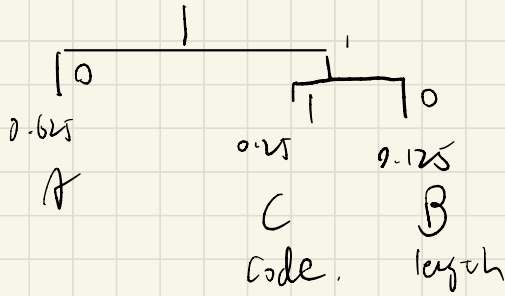
$$P(X) = P(Y)$$

$$1 - x^k = x^k$$

$$x^k = \frac{1}{2}$$

$$x = \frac{1}{\sqrt[4]{2}}$$

Q3



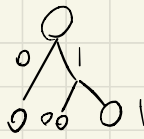
		code.	length
A	0.625	0	1
B	0.125	10	2
C	0.25	11	2

1. average length = $0.625 \times 1 + 0.125 \times 2 + 0.25 \times 2 = \underline{1.375}$ H

2. $2^2 = 4$

A	0	0	1	1
B	1	0	00	01
C	1	1	01	00

3. 2^{N-1}



A binary tree has $N-1$ edges for N nodes

each edge can represent as 0 or 1 in coding

$\therefore 2^{N-1} \rightarrow$ edges
 \downarrow
 0 or 1

4.

$$E(X) = -(0.625 \log_2 0.625 + 0.125 \log_2 0.125 + 0.25 \log_2 0.25) \\ = 1.298$$

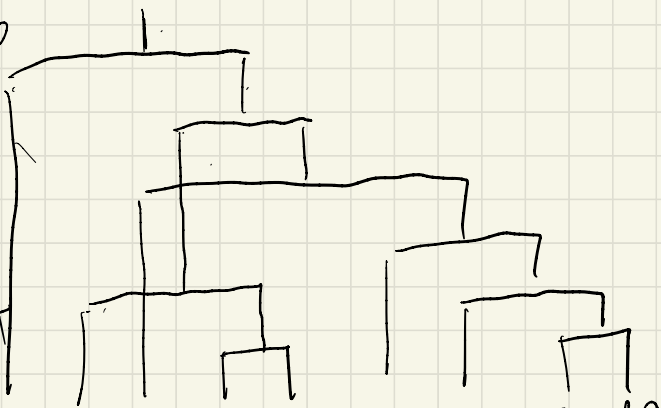
$\therefore E(X) \neq \text{average length}$

Better solution:

try to encode as two characters

$P(AA) = 0.39$	1
$P(AB) = 0.08$	4
$P(AC) = 0.15$	3
$P(CA) = 0.15$	4
$P(CB) = 0.02$	6
$P(CA) = 0.01$	5
$P(CA) = 0.15$	3
$P(CB) = 0.03$	6
$P(CC) = 0.06$	4

average length = 1.3203
 < 1.375



1.3203	AA	AC	CA	AB	BA	CC	BC	CB	BB
	0.39	0.15	0.15	0.08	0.08	0.06	0.03	0.03	0.02
			0.31					0.04	0.26