* Dot - Product

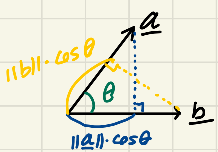
⟹ 두 vector가 얼마나 닮았는가?

$$\underset{a}{\begin{bmatrix}1\\3\end{bmatrix}} \cdot \underset{b}{\begin{bmatrix}5\\1\end{bmatrix}} = 8 = a^T \cdot b = \begin{bmatrix}1 & 3\end{bmatrix}\begin{bmatrix}5\\1\end{bmatrix}$$

$$a^Tb = \|a\| \cdot \|b\| \cdot \cos\theta$$ 두 vec의 사이각 θ

$$= \|a\| \cdot \cos\theta \cdot \|b\|$$

$$= \|b\| \cdot \cos\theta \cdot \|a\|$$

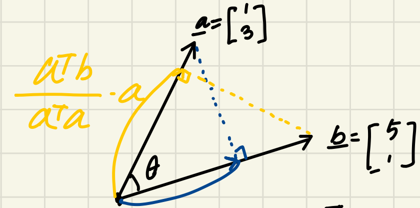


dot-product = projection.

내려서 곱한다.

- $a^Ta = \|a\|^2 = \sqrt{a^Ta}$
- Unit Vector = 크기가 1인 vector.

* projection



$$a^Tb = \underbrace{\|a\| \cdot \cos\theta}_{\|} \cdot \|b\|$$

$$\frac{a^Tb}{\|b\|} = \frac{a^Tb}{\sqrt{b^Tb}} \text{ (크기)}$$

$$\times \text{ 방향 } \frac{b}{\sqrt{b^T \cdot b}}$$

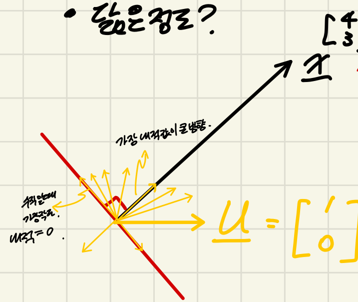
$$= \frac{a^Tb}{b^Tb} \cdot b$$

a와 방향은 같고 크기가 1인 vec

$$= \frac{a}{\sqrt{a^Ta}}$$

자기자신의 크기로 나눠주면
방향은 그대로 가진채
크기가 1인 vec으로 normalize.

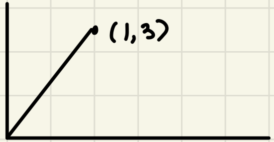
- 닮은정도?



x가 u 방향의 성분을 얼마나 많이 가지는가?

$$x^T \cdot u = 4.$$

* Vector norm



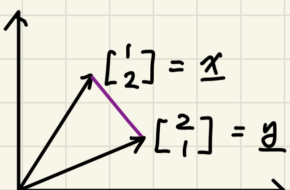
> 2-norm

$\left\|\begin{bmatrix}1\\3\end{bmatrix}\right\|_2 = \sqrt{1^2+3^2} = \sqrt{10}$.

$\|\underline{a}\|_2 = \sqrt{\underline{a}^T\underline{a}}$

$\underline{a} = \begin{bmatrix}1\\2\\3\end{bmatrix}$, $\|a\| = \sqrt{a^Ta} = \sqrt{1^2+2^2+3^2}$

$(1^2+2^2+3^2)^{\frac{1}{2}}$



$\underline{x} - \underline{y} = \begin{bmatrix}1\\2\end{bmatrix} - \begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}-1\\1\end{bmatrix}$

2-norm of $\underline{x}-\underline{y} = \sqrt{(1-2)^2+(2-1)^2}$

> 1-norm

1-norm of $\underline{b} = \begin{bmatrix}1\\2\\-3\end{bmatrix} = 1+2+3 = \|b\|_1$

> p-norm

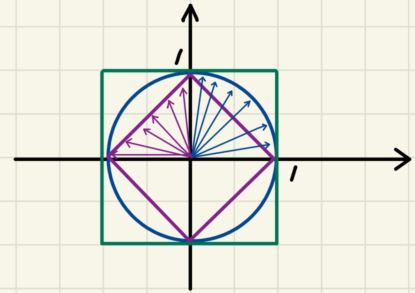
$\|\underline{x}\|_p = \left(|a|^p+|b|^p+|c|^p+\cdots\right)^{\frac{1}{p}}$

$\triangleq \left(\sum_i |x_i|^p\right)^{\frac{1}{p}}$ $(p \geq 1)$

> $p \to \infty$, infinity norm

$\|\underline{x}\|_\infty \triangleq \max_i |x_i|$

> Compare vector norms .



2-norm : $\begin{bmatrix}x\\y\end{bmatrix}$, $\sqrt{x^2+y^2} = 1$

1-norm : $|x|+|y| = 1$

∞-norm : $-|u|$

*** 행렬의 곱셈**

> 연립방정식 관점.

$$x_1+2y_1=4$$
$$2x_1+5y_1=9$$
$$x_2+y_2=3$$
$$2x_2+5y_2=7$$

$$\rightarrow \begin{bmatrix}1&1\\2&5\end{bmatrix}\begin{bmatrix}x_1&x_2\\y_1&y_2\end{bmatrix}=\begin{bmatrix}4&3\\9&7\end{bmatrix}$$

> 내적으로 바라보기.

$$A=\begin{bmatrix}\underline{a_1}^T\\\underline{a_2}^T\\\underline{a_3}^T\end{bmatrix},\quad AB=\begin{bmatrix}\underline{a_1}^T\\\underline{a_2}^T\\\underline{a_3}^T\end{bmatrix}[\underline{b_1},\underline{b_2},\underline{b_3}]=\begin{bmatrix}a_1^Tb_1 & a_1^Tb_2 & a_1^Tb_3\\ & \vdots & \end{bmatrix}$$

dot-product!!

> rank-1 matrix의 합.

$$AB=[\underline{a_1}\ \underline{a_2}\ \underline{a_3}]\begin{bmatrix}\underline{b_1}^T\\\underline{b_2}^T\\\underline{b_3}^T\end{bmatrix}=\underbrace{\underline{a_1}\,\underline{b_1}^T+\underline{a_2}\,\underline{b_2}^T+\underline{a_3}\,\underline{b_3}^T}_{\text{rank-1 matrixs}}$$

> Column space 관점.

$$A\underline{x}=[\underline{a_1}\ \underline{a_2}\ \underline{a_3}]\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\underline{a_1}x_1+\underline{a_2}x_2+\underline{a_3}x_3$$

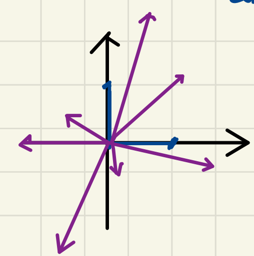
스칼라배.

스칼라배가 나타내는 결과의 집합이 나타내는 공간 = Column Space.

$\downarrow$

if A

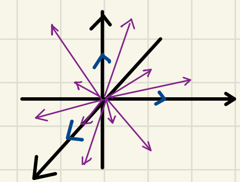
$$\begin{bmatrix}1&0\\0&1\end{bmatrix}=$$



2차원 공간 전체.

if A

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=$$



3차원 공간 전체.

* Span & Column space.

> Linear Combination

$$a_1 \underline{V_1} + a_2 \underline{V_2} + a_3 \underline{V_3}$$

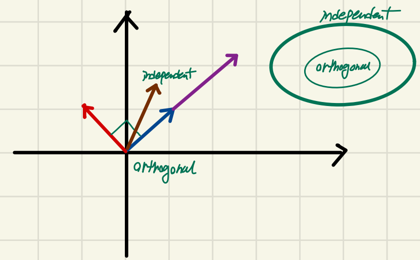
→ 각 벡터를 얼마나씩씩 조합할까?

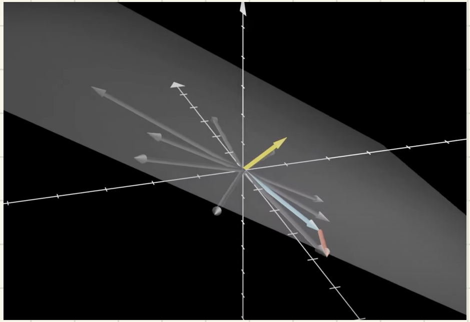
↓

Span = Vector 3 표현 가능한 영역의 범위

* Linearly independent & basis

더 많은 span을 나타내줄 수 있는





linearly independent 한 yellow

⇒ 3개의 independent vector?

= 3차원 span 전체.

n개의 linearly independent vectors.

⇒ n차원 전체.

* basis.

: 어떤 공간을 이루는 필수 구성요소.

ex) 2 dim span basis?

$\begin{bmatrix}1\\0\end{bmatrix}$ $\begin{bmatrix}0\\1\end{bmatrix}$ independent & Orthogonal & basis

$\begin{bmatrix}1\\0\end{bmatrix}$ $\begin{bmatrix}1\\1\end{bmatrix}$ independent & basis

$\begin{bmatrix}1\\0\end{bmatrix}$ $\begin{bmatrix}2\\0\end{bmatrix}$ 1 Dim을 전부 나타내지면

not independent ⇒ basis (X)

* Identity / inverse / diagonal / orthogonal matrix

> identity matrix (항등행렬)

$$A \times I = A$$

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ p×p 정사각행렬.

> inverse matrix (역행렬)

$$A \times A^{-1} = I$$

if exist, A is invertable.

$$A^{-1} \cdot A\underline{x} = A^{-1} \cdot B$$

$$\underline{x} = A^{-1} \cdot B$$

> diagonal matrix (대각행렬)

→ 대각성분 이외 0으로

$$\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

rectangular diagonic.

> orthogonal matrix (직교행렬)

$$Q \begin{bmatrix} \big(& \big(& \big(\end{bmatrix}$$ ortho normal vectors.

서로 수직하면서 크기는 각각 1인.

각각서로 dot product 0.

$$Q^{-1} = Q^T$$ "

ex) $I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, 크기 각각 1.

→ 3×3 $Q \times Q^{-1} = I_3 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\quad Q^T \cdot Q = I_3$.

$$\underset{Q^T}{\begin{pmatrix} q_1^T \\ q_2^T \\ q_3^T \end{pmatrix}} \times \underset{Q}{\begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

orth

orth

orth

* Rank .

: 행렬이 가지는 independent한 column의 수.

= column space 의 dimension. (= row space의 dim)

↳ * independent한 column의 수 = independent row의 수.

rank(A) = rank(A^T)

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$: rank = 1, $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, rank = 2

rank-deficient
2X3 : full-row rank

2X3 "

- 3X2, rank = 2 ⇒ full-column rank
- 3X3, rank = 3 ⇒ full rank
- 3X3, rank = 2, ⇒ rank-deficient

* Null space

$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

: $A\underline{x} = 0$ 만족하는 집합.

$$A\underline{x} = x_1\begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2\begin{bmatrix} 0 \\ 1 \end{bmatrix} + x_3\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \underline{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix} \cdots$$

$$\underline{x}_n = c \times \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

- A가 m×n, $\dim(N(A)) = n - r$
  A의 nullspace

$\dim(N(A)) + \dim(R(A)) = n$
A의 row space