HW2 - Tommy Azzino

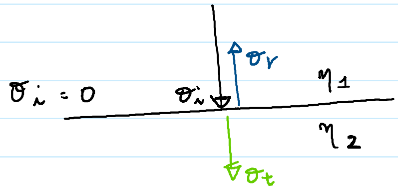
Tuesday, February 16, 2021 11:56 AM

Problem 3

(a) $\theta_r = \theta_i = 0°$

$\eta_1 \sin\theta_i = \eta_2 \sin\theta_t$

$\sin\theta_t = 0 \Rightarrow \theta_t = 0$



(b) $$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\Gamma_{\parallel} = \Gamma_{\perp} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

(c) $\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377\ \Omega \rightarrow$ free space characteristic impedance

$$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_{r2}}} = \frac{Z_0}{\underbrace{\sqrt{\varepsilon_{r2}}}_{\approx 4,5}} = \frac{377}{\sqrt{4,5}} \approx 178\ \Omega$$

$$|\Gamma|^2 = \left|\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right|^2 = \left|\frac{\cancel{377}\left(1 - \frac{1}{\sqrt{4,5}}\right)}{\cancel{377}\left(1 + \frac{1}{\sqrt{4,5}}\right)}\right|^2 = \left|\frac{1 - \frac{\sqrt{2}}{3}}{1 + \frac{\sqrt{2}}{3}}\right|^2 = \left|\frac{3-\sqrt{2}}{3+\sqrt{2}}\right|^2 \approx 0,129$$

PROBLEM 7

(a) $$PL_{max} = P_{tx} - N_0 - 10\log_{10}(B) - \text{SNR\_TARGET} = 20 - (-170) - 10\log_{10}(20 \cdot 10^6) - 10$$

$$\approx 106.98\ dB \approx 107\ dB$$

(b) $PL = PL_0 + \varepsilon$, $\quad \varepsilon \sim N(0, \sigma^2)$, $\sigma = 8\,dB$, $PL_0 = 100\,dB$

$$P_{out} = P_r(PL \geq PL_{MAX}) = P_r(x \geq PL_{max}) = \int_{PL_{MAX}}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x - PL_0)}{2\sigma^2}} dx$$

~ x is the lognormally distributed path loss.

IN MATLAB: $P_{out} = \text{qfunc}\left(\frac{PL_{MAX} - PL_0}{\sigma}\right) \approx \underline{0,1911}$ [~19%]

need to convert to standard normal

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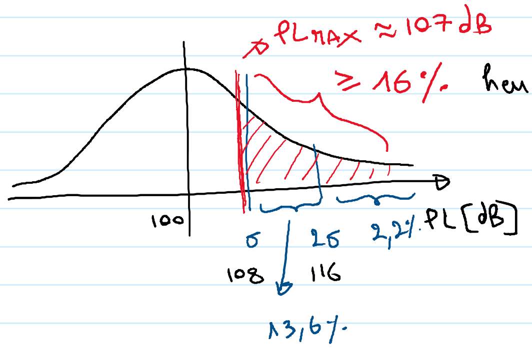


$\geq 16\%$ hence the obtained prob makes sense

(6)

$$PL = PL_0 + \mathcal{E} + \Delta N \quad , \quad \mathcal{E} \sim \mathcal{N}(0, \sigma^2)$$

$N =$ "# of walls" , $\Delta =$ "loss per wall" $= 7$ dB

$$P_{out} = P(PL \geq PL_{max}) = P(PL \geq PL_{max} \mid n = 0)\, P(n = 0) + P(PL \geq PL_{max} \mid n = 1)\, P(n = 1) + P(PL \geq PL_{max} \mid n = 2)\, P(n = 2)$$

TOTAL PROB. THEOREM

$$= P(PL \geq PL_{max} \mid n = 0) \cdot \frac{1}{2} + P(PL \geq PL_{max} \mid n = 1)\frac{3}{10} + P(PL \geq PL_{max} \mid n = 2) \cdot \frac{1}{5}$$

$$= 0{,}4076 \quad [\sim 41\%]$$

It makes sense that this prob is higher since with the path loss model of (c) we have that half of the time there's at least one wall introducing a 7 dB attenuation to the base path loss model of (b).

Wireless Communications EL-GY 6023

Homework 2 - Tommy Azzino (ta1731)

Problem 5

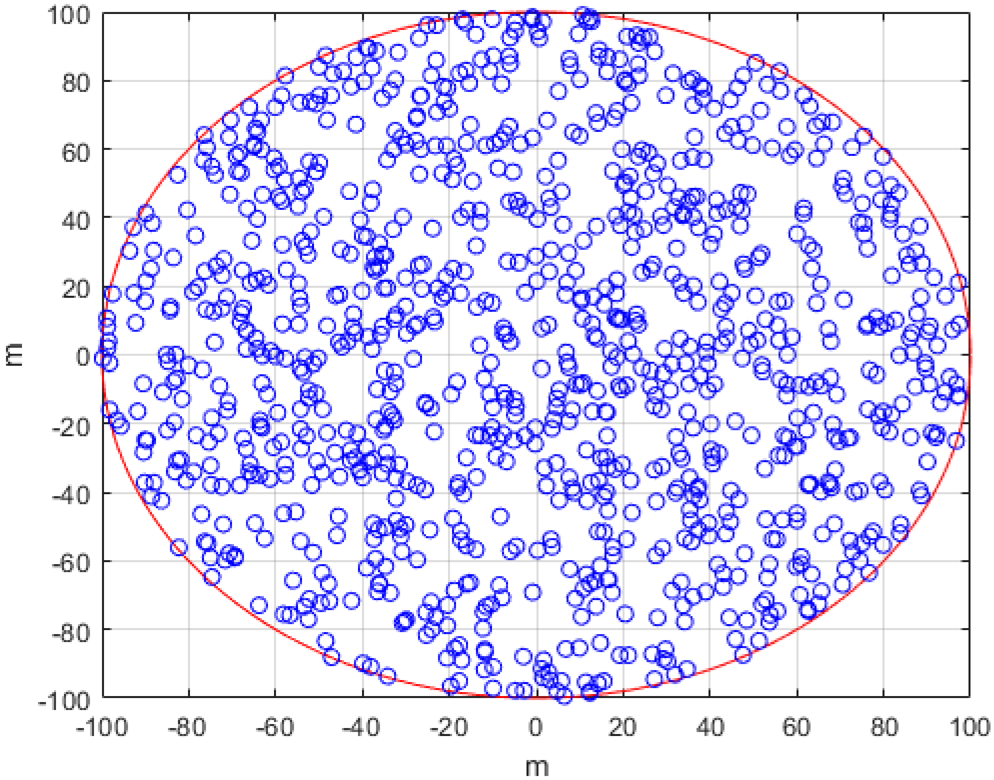
Suppose nrx=1000 RX locations are randomly located uniformly in a circle of radius

rmax = 100 m from the origin. Generate a random vector dist2 representing the

random distances from the origin of the RX locations.

```
nrx = 1000;
rmax = 100;
% coordinates are generated with respect to the origin
x0 = 0; y0 = 0;
% generating random angular directions
a = rand(nrx,1)*2*pi;
% generating random rays
r = sqrt(rand(nrx,1))*rmax; % need to take the sqrt to get uniformly distributed points in the
x = x0 + r.*cos(a);
y = y0 + r.*sin(a);

theta = 0:0.01:2*pi;
limit_x = x0 + rmax*cos(theta);
limit_y = y0 + rmax*sin(theta);
figure;
plot(limit_x, limit_y, 'r');
hold on;
plot(x,y,'bo');
hold off;
xlabel("m");
ylabel("m");
grid on;
```




```
% 2D distance of the random RX points from the origin (x0=0,y0=0)
dist2 = sqrt((x-x0).^2 + (y-y0).^2);
```

Assuming the transmitter is at the origin at a height htx=2 m higher than the RX,

compute the distances dist to the RXs

```
htx = 2;
% distance of the RX points from the TX at the origin with height 2m
dist = sqrt(dist2.^2 + htx^2);
```

Assuming the given path loss model,

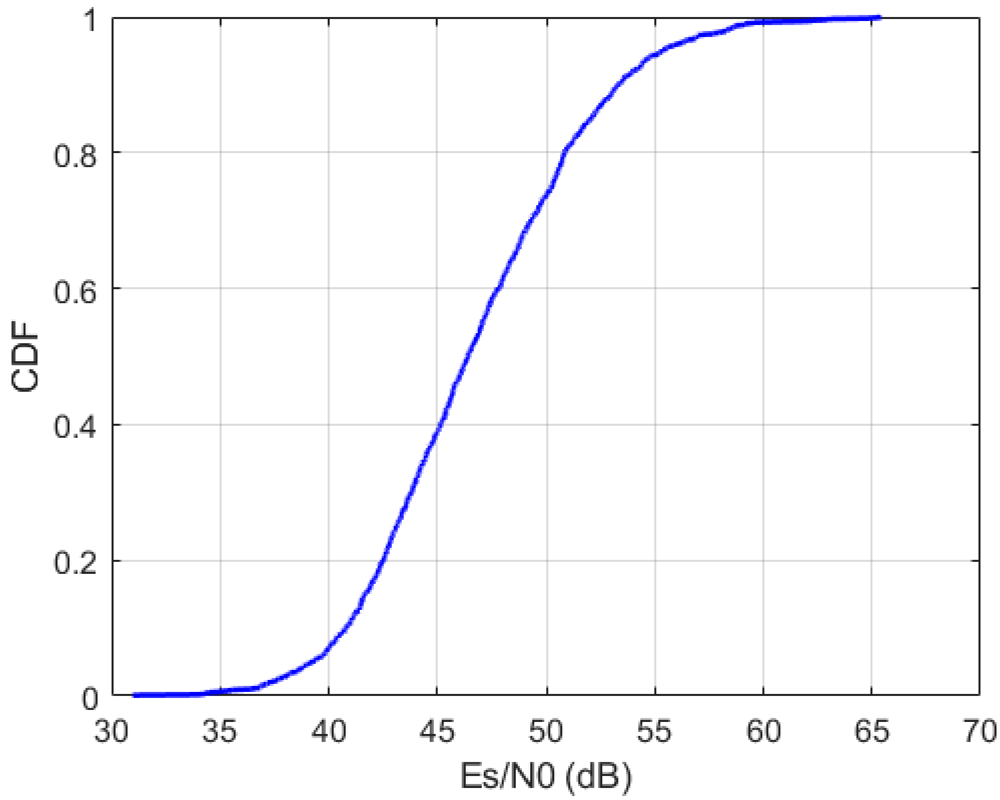
generate random path losses to the RXs. Assume $\sigma = 4$ dB and $f_c = 2.3$ GHz.

```
fc = 2.3e9;
sigma = 4;
% compute the PL according to the provided formula
PL = 32.4 + 14.3*log10(dist) + 20*log10(fc/1e9) + randn(size(dist)).*sigma;
```

Finally plot a CDF of Es/N0 with transmit power, Ptx = 15 dBm, bandwidth B = 20MHz and thermal noise N0= −170 dBm/Hz.

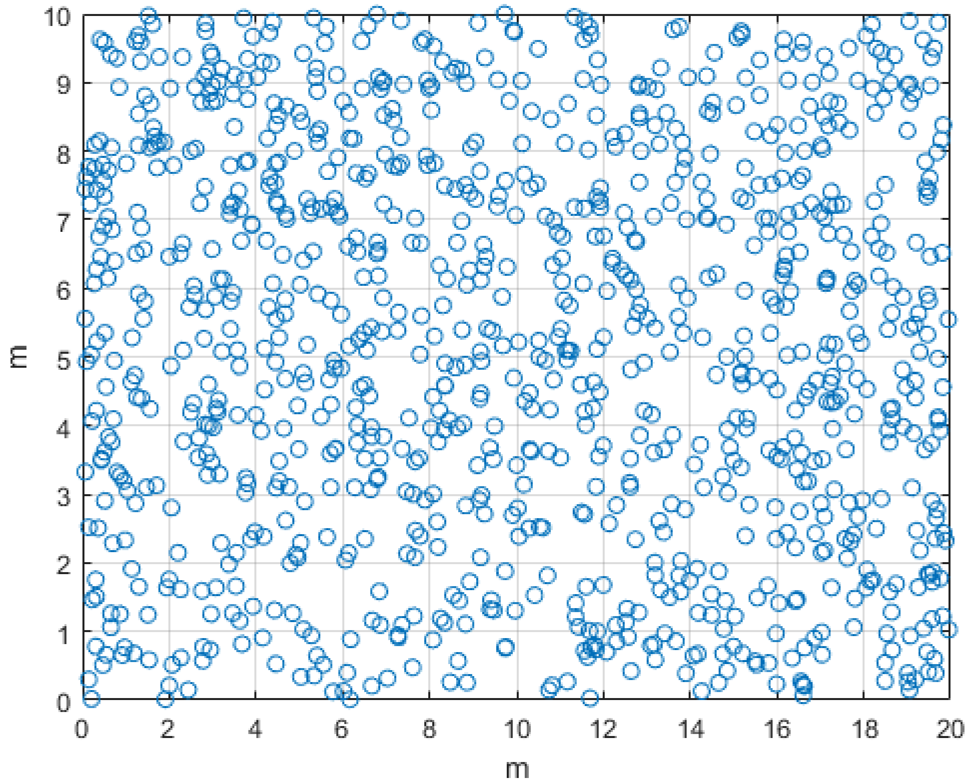
```
Ptx = 15; % power in dBm
B = 20e6; % bandwidth
N0 = -170; % termal noise in dBm/Hz
EsN0 = Ptx - PL - N0 - 10*log10(B); % computing Es/N0
```

```
plot(sort(EsN0), (1:nrx)/nrx,'b','Linewidth', 2);
grid on;
xlabel('Es/N0 (dB)');
ylabel('CDF');
set(gca, 'Fontsize', 12);
```



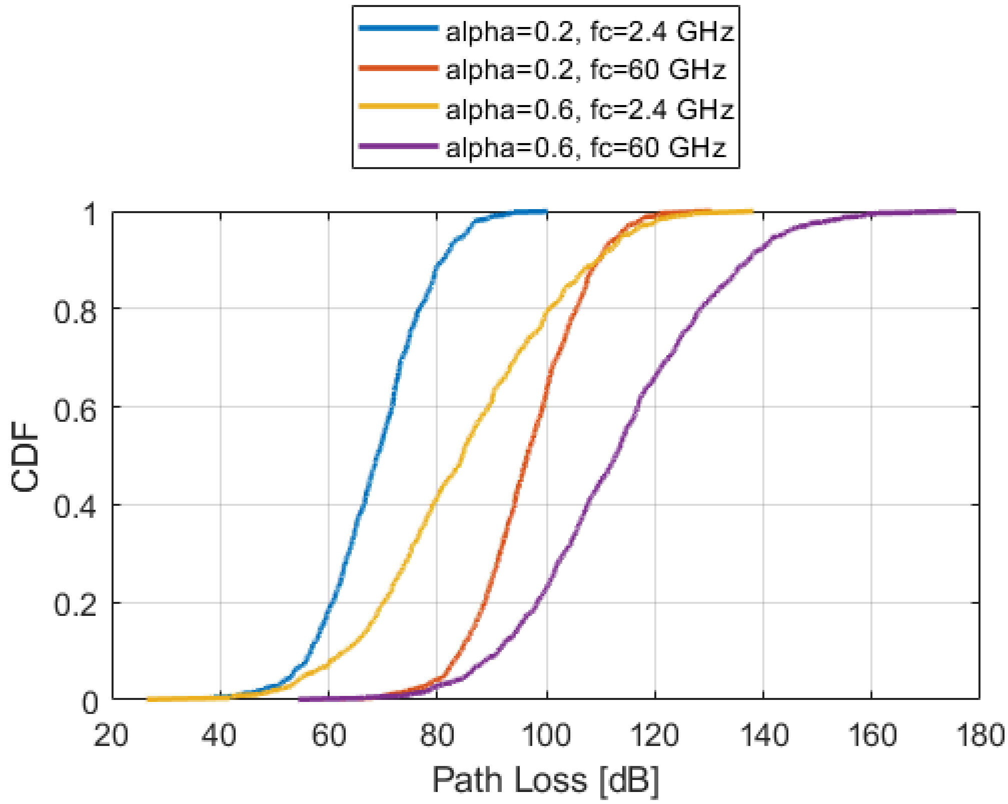
Problem 6

```
% Generating points in a squared region
% according to some suitable indoor scenario dimension
l = 20;  % length of region in m
w = 10;  % width in m
X = rand(nrx,2).*[l w];
dist = sqrt(sum(X.^2,2)); % distances from the centre (0,0)
% plot the random points
plot(X(:,1), X(:,2), 'o');
grid on;
xlabel("m");
ylabel("m");
```




```
L0 = 3.4; % number found in the literature
alphas = [0.2, 0.6];
fcs = [2.4e9, 60e9]; % frequencies
PL_s = zeros(length(alphas)*length(fcs), nrx);
k=1;
p = (1:nrx)/nrx;
figure;
for i=1:length(alphas)
    for j=1:length(fcs)
        PL = indoorPL(dist,fcs(j),L0,alphas(i));
        PL_s(k,:) = PL;
        plot(sort(PL), p, "LineWidth",2);
        hold on;
        k=k+1;
    end
end
entries = [];
for i=1:length(alphas)
    for j=1:length(fcs)
        entries= [entries, ("alpha="+string(alphas(i))+", fc="+string(fcs(j)/1e9)+" GHz")];
    end
end
legend(entries(1), entries(2), entries(3), entries(4), "Location","northoutside");
xlabel("Path Loss [dB]");
ylabel("CDF");
set(gca, 'Fontsize', 12);
hold off;
```

```
grid on;
```



Problem 7

```
Ptx = 20; % tx power in dBm
B = 20e6; % bandwidth
N0 = -170; % noise power density in dBm/Hz
```

(a) What is the maximum path loss, PLmax, that the link can have to meet an SNR target

of 10 dB?

```
snr_target = 10; % snr target in dB
% EsN0 = Ptx - PL - N0 - 10*log10(B)
PLmax = Ptx - N0 - 10*log10(B) - snr_target;
fprintf(1, 'PLmax value is %f\n',PLmax);
```

```
PLmax value is 106.989700
```

(b) Suppose that the path loss is lognormally distributed.

What is the outage probability Pout = Pr(PL >= PLmax) using the value PLmax from part (a)?

```
PL0 = 100; % dB
sigma = 8; % dB
Pout_b = qfunc((PLmax-PL0)/sigma);
```

```
fprintf(1, 'Pout value is %f\n',Pout_b);
```

```
Pout value is 0.191137
```

(c)

```
D = 7; % dB
Pout_c = Pout_b*0.5 + qfunc((PLmax-PL0-D)/sigma)*0.3 + qfunc((PLmax-PL0-2*D)/sigma)*0.2;
fprintf(1, 'Pout value is %f\n',Pout_c);
```

```
Pout value is 0.407635
```

```
function pl = indoorPL(d, fc, L0, alpha)
    % d is the vector of distances
    % fc is the carrier frequency
    % L0 is the loss factor associated to each wall
    % alpha is the average number of walls per meter of distance
    c = physconst('Lightspeed');
    lambda = c/fc;
    % compute path loss given the provided formula
    pl = fspl(d,lambda) + L0.*poissrnd(alpha*d);

end
```