Problem Solutions: Antennas and Free-Space Propagation
EL-GY 6023. Wireless Communications

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In all the problems below, unless specified otherwise, ϕ is the azimuth angle and θ is elevation angle.

1. *EM wave*: Suppose an EM plane wave has an E-field

$$\mathbf{E}(x, y, z, t) = E_0 \mathbf{e}_y \cos(2\pi f t - kx).$$

   (a) What is direction of motion?
   (b) If the average power flux density is $10^{-8}\,\mathrm{mW/m^2}$, what is E_0? Assume the characteristic impedance is $\eta_0 = 377\,\Omega$.
   (c) If the frequency is $f = 1.5\,\mathrm{GHz}$, what is k? What are the units of k?

2. *dBm to linear conversions:*

   (a) Convert the following to mW: 17 dBm, -73 dBm, -97 dBW.
   (b) Convert the following to dBm: 250 mW, $8(10)^{-8}$ W, $5(10)^{-6}$ mW

3. *Spherical-cartesian conversions:* When a transmitter is at the origin, its E-field in the far field can often be represented as,

$$\mathbf{E} = E_\theta \mathbf{e}_\theta + E_\phi \mathbf{e}_\phi,$$

   where $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$ are the basis vectors in elevation and azimuth direction. Complete the following MATLAB function that takes a 1×3 position vector `pos` and $n \times 1$ values of E_θ and E_ϕ and returns the an $n \times 3$ matrix `E` representing the E-field values in cartesian coordinates. You may use any built in MATLAB functions. Be careful whether the methods use degrees or radians.

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function E = convert(Etheta, Ephi, pos)
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4. *Rotation matrices*: In wireless systems, we often need to consider antennas that can be in arbitrary rotations. One way of specifying the orientation of an object is through its so-called *Euler* angles (α, β, γ) or *yaw, pitch* and *roll.* Let $R(\alpha, \beta, \gamma)$ be the rotation matrix for a given set of Euler angles. You can find the formulae for $R(\alpha, \beta, \gamma)$ in any reference such as wikipedia.

(a) Given elevation and azimuth angles (θ, ϕ) find (α, β, γ) with $\gamma = 0$ that rotates the x-axis to point in (θ, ϕ).

(b) Is $R(\alpha, 0, 0)^{-1} = R(-\alpha, 0, 0)$? Explain.

(c) Is $R(\alpha, \beta, 0)^{-1} = R(-\alpha, -\beta, 0)$? Explain.

5. *Angular areas:* Find the angular area in steradians of following sets of angles where ϕ is the azimuth angle and θ is the elevation angles in degrees:

   (a) $A_1 = \{(\phi, \theta) \mid \phi \in [-30°, 30°],\ \theta \in [-90°, 90°]\}$

   (b) $A_2 = \{(\phi, \theta) \mid \phi \in [-30°, 30°],\ \theta \in [-45°, 45°]\}$

6. *Directivity:* Suppose an antenna radiates power uniformly in the angular beam $\phi \in [-30°, 30°]$, and $\theta \in [-45°, 45°]$, and radiates no power at other angles. What is the maximum directivity of the antenna in dBi? You can use the results from the previous problem.

7. *Radiation intensity:* A 170 cm $\times$ 40 cm object (roughly the size of a human) is 800 m from a base station. If the base station antenna transmits 250 mW isotropically, how much power reaches the human? Use reasonable approximations that the human is far from the transmitter.

8. *Radiation integration:* Suppose the radiation intensity is

$$U(\phi, \theta) = A \cos^2(\theta), \quad A = 10\,\text{mW/sr},$$

   where (ϕ, θ) are the azimuth and elevation angles. find the total radiated power in dBm and maximum directivity in dBi. You can look up any integrals you need.

9. *Numerically integrating patterns:* Suppose we are given the radiation intensity $U(\theta, \phi)$ at discrete points, (θ_i, ϕ_j) where θ_i, $i = 1, \ldots, M$ is uniformly spaced on $[-\pi/2, \pi/2]$ and ϕ_j, $j = 1, \ldots, N$ is uniformly spaced on $[-\pi, \pi]$. Assume (θ, ϕ) are elevation and azimuth angles. Write a short MATLAB function to compute the radiated power P_{rad} and directivity $D(\theta_i, \phi_j)$ from a matrix of values $U(\theta_i, \phi_j)$.

10. *Friis' Law*: A transmitter radiates 15 dBm at a carrier $f_c = 2.1$ GHz with a directional gain of $G_t = 9$ dBi. Suppose the receiver is $d = 200$ m from the transmitter and the path is free space. What is the received power in dBm if:

    (a) The effective received aperture is 1 cm^2.

    (b) The receiver gain is $G_r = 5$ dBi.