HW1 - Tommy Azzino (ta1731)

Monday, February 8, 2021 10:12 PM

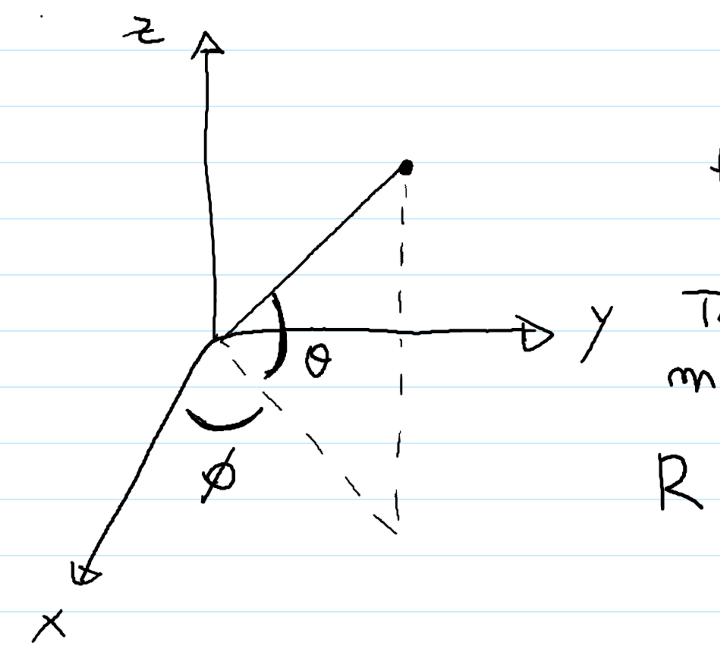
PROBLEM 4

(a) A rotation matrix can be obtained by:

$$R = R_z(\alpha) R_y(\beta) R_x(\gamma)$$

$$= \underbrace{\begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{YAW}} \underbrace{\begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix}}_{\text{PITCH}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix}}_{\text{ROLL}}$$

$$= \begin{pmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{pmatrix}$$

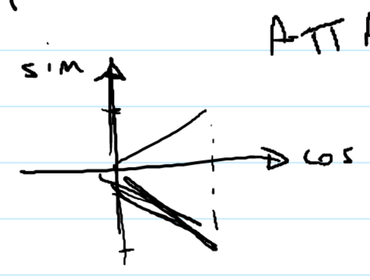


With this definition of rotation matrix we perform rotation about the z, y and x axes in the given order

To rotate the x-axis to point in (θ,ϕ) the rotation matrix is:

$$R = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(-\theta) & 0 & \sin(-\theta) \\ 0 & 1 & 0 \\ -\sin(-\theta) & 0 & \cos(-\theta) \end{pmatrix} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{I\ [\gamma=0]}$$

Therefore, we have $[\alpha = \phi,\ \beta = -\theta,\ \gamma = 0]$ (CHECK ALSO MATLAB CODE ATTACHED)



(b) Is $R(\alpha,0,0)^{-1} = R(-\alpha,0,0)$? [YES]

$$R(\alpha,0,0) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad R(-\alpha,0,0) = \begin{pmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$R(\alpha,0,0)^{-1} = R(\alpha,0,0)^T$ since a rotation matrix is orthonormal.

$$R(\alpha,0,0)^T = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} = R(-\alpha,0,0) \quad \checkmark$$

(c) $R(\alpha,\beta,0)^{-1} \overset{?}{=} R(-\alpha,-\beta,0)$ [NO]

$$R(\alpha,\beta,0) = \begin{pmatrix} \cos\alpha\cos\beta & -\sin\alpha & \cos\alpha\sin\beta \\ \sin\alpha\cos\beta & \cos\alpha & \sin\alpha\sin\beta \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} \qquad R(-\alpha,-\beta,0) = \begin{pmatrix} \cos\alpha\cos\beta & \sin\alpha & -\cos\alpha\sin\beta \\ -\sin\alpha\cos\beta & \cos\alpha & \sin\alpha\sin\beta \\ \sin\beta & 0 & \cos\beta \end{pmatrix}$$

We have that $R(\alpha,\beta,0)^T \neq R(-\alpha,-\beta,0)$, so $R(\alpha,\beta,0)^{-1} \neq R(-\alpha,-\beta,0)$ ✓

PROBLEM 5

a) $A_1 = \{(\underset{\text{azimuth}}{\phi},\underset{\text{elevation}}{\theta}) \mid \phi \in [-30°, 30°],\ \theta \in [-90, 90°]\}$

$$A = \iint d\Omega = \iint \sin\theta\, d\theta\, d\phi \qquad \theta_{inc} = \frac{\pi}{2} - \theta_{el}$$

inclination

$$= \int_{-\pi/6}^{\pi/6} \int_0^{\pi} \sin\theta\, d\theta\, d\phi = \frac{\pi}{3}\left[-\cos\theta\Big|_0^{\pi}\right] = \frac{\pi}{3}\left[1-(-1)\right] = \frac{2\pi}{3}$$

b)
$$A = \int_{-\pi/6}^{\pi/6} \int_{\frac{\pi}{4}}^{\frac{3}{4}\pi} \sin\theta\, d\theta\, d\phi = \frac{\pi}{3}\left[-\cos\theta\Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\right] \qquad \theta_{inc} = \begin{cases} \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4} \\ \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4} \end{cases}$$

$$= \frac{\pi}{3}\left[\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\right] = \frac{\sqrt{2}\,\pi}{3}$$

PROBLEM 6

$$D(\theta,\phi) = \frac{4\pi\, U(\theta,\phi)}{P_{rad}} = \frac{4\pi\, \cancel{U(\theta,\phi)}}{\int_{-\frac{\pi}{6}}^{\pi/6}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cancel{U(\theta,\phi)} \cos\theta\, d\theta\, d\phi} = \frac{4\pi}{\int_{-\frac{\pi}{6}}^{\pi/6}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos\theta\, d\theta\, d\phi} =$$

uniform
does not depend on θ and ϕ

$$= \frac{4\pi}{\frac{\pi}{3}\int_{-\frac{\pi}{4}}^{\pi/4} \cos\theta\, d\theta} = \frac{4\cancel{\pi}}{\frac{\sqrt{2}\,\cancel{\pi}}{3}} = \frac{12}{\sqrt{2}} = \frac{12\cdot\sqrt{2}}{\cancel{\sqrt{2}}\cdot\sqrt{2}} = 6\sqrt{2} \Rightarrow 9{,}2867\ \text{dBi}$$

PROBLEM 8

1)
$$P_{rad} = \int_{-\pi}^{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} U(\theta,\phi) \cos\theta\, d\theta\, d\phi = \int_{-\pi}^{\pi}\int_{-\pi/2}^{\pi/2} A\cos^3(\theta)\, d\theta\, d\phi =$$

$$= 2\pi A \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3(\theta)\, d\theta = 2\pi A\,\frac{4}{3} = \frac{8}{3}\pi A = \frac{8}{3}\pi\, 10 = \frac{80}{3}\pi\ \text{mW}$$

$$P_{rad}[\text{dBm}] = 10\log_{10}\left(\overset{\text{"mW"}}{\frac{80}{3}\pi}\right) \approx 19{,}23\ \text{dBm}$$

2)
$$D(\theta,\phi) = \frac{4\pi\, U(\theta,\phi)}{P_{rad}} = \frac{4\pi\, A\cos^2(\theta)}{\frac{80}{3}\pi}$$

$$\max D(\theta,\phi) = \frac{40\pi}{\frac{80\pi}{3}} = \frac{3}{2} \Rightarrow 1{,}76\ \text{dBi}$$

PROBLEM 9

ATTACHED MATLAB CODE

Wireless Communications EL-GY 6023

Homework 1 - Tommy Azzino (ta1731)

Problem 4.a

```
theta = pi/4;
phi = pi/4;
eul = [phi -theta 0];
rotmZYX = eul2rotm(eul,'ZYX');
xVec = [1 0 0]; % vector on the x-axis
newVec = rotmZYX*xVec.'
```

```
newVec = 3×1
    0.5000
    0.5000
    0.7071
```

Problem 4.b

```
rotm = eul2rotm([pi/4 0 0],'ZYX');
rotm_n = eul2rotm([-pi/4 0 0],'ZYX');
disp(inv(rotm));
```

```
    0.7071    0.7071         0
   -0.7071    0.7071         0
         0         0    1.0000
```

```
disp(rotm_n);
```

```
    0.7071    0.7071         0
   -0.7071    0.7071         0
         0         0    1.0000
```

```
% the two rotation matrixes are equal
```

Problem 4.c

```
rotm = eul2rotm([pi/4 pi/3 0],'ZYX');
rotm_n = eul2rotm([-pi/4 -pi/3 0],'ZYX');
disp(inv(rotm));
```

```
    0.3536    0.3536   -0.8660
   -0.7071    0.7071   -0.0000
    0.6124    0.6124    0.5000
```

```
disp(rotm_n);
```

```
    0.3536    0.7071   -0.6124
   -0.3536    0.7071    0.6124
    0.8660         0    0.5000
```

```
% the two rotation matrixes are different
```

Problem 9

```
% more points --> more accurate integral approximation
theta = linspace(-90,90,10000); % elevation angles
phi = linspace(-180,180,20000); % azimuth angles
% create radiation intesity of Problem 8
A = 10e-3;
U = (cos(deg2rad(theta)').^2 + cos(deg2rad(phi)).^2*0)*A;

% get the radiated power
Prad = get_radiated_power(U,theta,phi,180,360);
fprintf(1, 'Prad [dBm] is %f\n', pow2db(Prad*1e3));
```

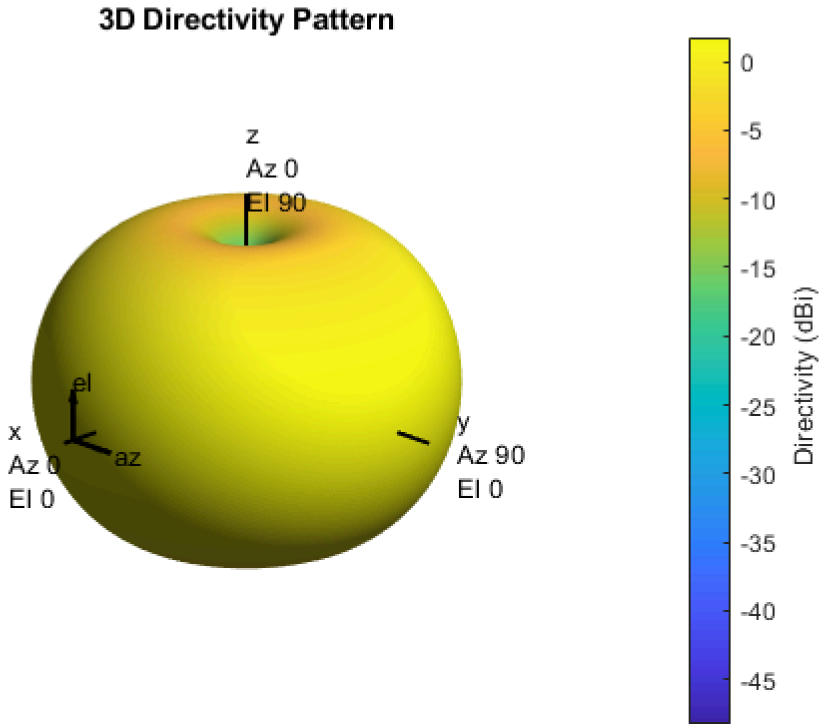
```
Prad [dBm] is 19.230752
```

```
dir = get_directivity(U,Prad);
maxDir = max(dir,[],'all');
fprintf(1, 'Max Directivity [dBi] is %f\n', pow2db(maxDir));
```

```
Max Directivity [dBi] is 1.761347
```

```
% same results as in Problem 8

% let's plot the directivity pattern of the antenna
phasePattern = zeros(size(dir));
ant = phased.CustomAntennaElement(...
    'AzimuthAngles', phi, 'ElevationAngles', theta, ...
    'MagnitudePattern', pow2db(dir), ...
    'PhasePattern', phasePattern);
fc = 28e9;
ant.pattern(fc);
```




```
function prad=get_radiated_power(U,theta,phi,theta_range,phi_range)
    theta_len = length(theta);
    phi_len = length(phi);
    U_times_cos = U'.*cos(deg2rad(theta));
    prad = sum(U_times_cos,'all')*deg2rad(theta_range)*deg2rad(phi_range)/(theta_len*phi_len);
end

function D=get_directivity(U, prad)
    D=U*4*pi/prad;
end
```