HW2 - Tommy Azzino

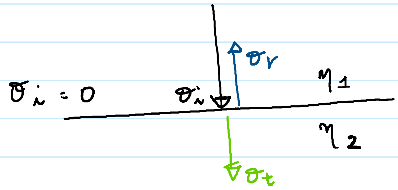
Tuesday, February 16, 2021 11:56 AM

Problem 3

(a) $\theta_r = \theta_i = 0°$

$\eta_1 \sin\theta_i = \eta_2 \sin\theta_t$

$\sin\theta_t = 0 \Rightarrow \theta_t = 0$



(b) $$\Gamma_{\parallel} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = \frac{\eta_2 - \eta_1}{\eta_2+\eta_1}$$

$$\Gamma_{\perp} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i - \eta_1\cos\theta_t} = \frac{\eta_2 - \eta_1}{\eta_2+\eta_1}$$

$$\Gamma_{\parallel} = \Gamma_{\perp} = \frac{\eta_2 - \eta_1}{\eta_2+\eta_1}$$

(c) $\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377\,\Omega \rightarrow$ free space characteristic impedance

$$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_{r2}}} = \frac{Z_0}{\underbrace{\sqrt{\varepsilon_{r2}}}_{\approx 4,5}} = \frac{377}{\sqrt{4,5}} \approx 178\,\Omega$$

$$|\Gamma|^2 = \left|\frac{\eta_2-\eta_1}{\eta_2+\eta_1}\right|^2 = \left|\frac{\cancel{377}\left(1-\frac{1}{\sqrt{4,5}}\right)}{\cancel{377}\left(1+\frac{1}{\sqrt{4,5}}\right)}\right|^2 = \left|\frac{1-\frac{\sqrt{2}}{3}}{1+\frac{\sqrt{2}}{3}}\right|^2 = \left|\frac{3-\sqrt{2}}{3+\sqrt{2}}\right|^2 \approx 0,129$$

PROBLEM 7

(a) $$PL_{max} = P_{tx} - N_0 - 10\log_{10}(B) - \text{SNR\_TARGET} = 20-(-170) - 10\log_{10}(20\cdot 10^6) - 10$$

$$\approx 106.98\text{ dB} \approx 107\text{ dB}$$

(b) $PL = PL_0 + \varepsilon$, $\quad \varepsilon \sim N(0,\sigma^2)$, $\sigma = 8$ dB, $PL_0 = 100$ dB

$$P_{out} = P_r(PL \geq PL_{MAX}) = P_r(x \geq PL_{max}) = \int_{PL_{MAX}}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-PL_0)}{2\sigma^2}}\,dx$$

~ x is the lognormally distributed path loss.

IN MATLAB: $P_{out} = \text{qfunc}\left(\frac{PL_{MAX} - PL_0}{\sigma}\right) \approx \underline{0,1911}$ [~19%]

need to convert to standard normal

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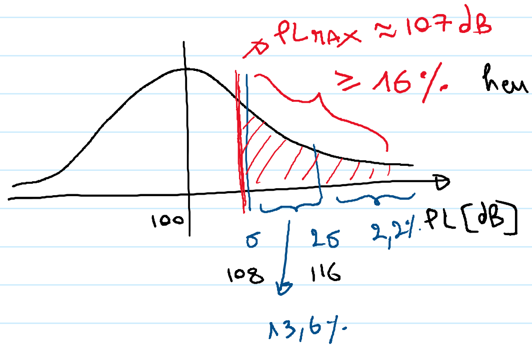

$\geq 16\%$ hence the obtained prob makes sense

(6)

$$PL = PL_0 + \varepsilon + \Delta N \quad , \quad \varepsilon \sim N(0, \sigma^2)$$

N = "# of walls" , Δ = "loss per wall" = 7 dB

$$P_{out} = P(PL \geq PL_{max}) = P(PL \geq PL_{MAX} \mid n=0)\,P(n=0) + P(PL \geq PL_{MAX} \mid n=1)\,P(n=1) + P(PL \geq PL_{MAX} \mid n=2)\,P(n=2)$$

TOTAL PROB. THEOREM

$$= P(PL \geq PL_{MAX} \mid n=0) \cdot \frac{1}{2} + P(PL \geq PL_{MAX} \mid n=1)\,\frac{3}{10} + P(PL \geq PL_{MAX} \mid n=2)\,\frac{1}{5}$$

$$= 0{,}4076 \quad [\sim 41\%]$$

It makes sense that this prob is higher since with the path loss model of (c) we have that half of the time there's at least one wall introducing a 7 dB attenuation to the base path loss model of (b).