

# Problems: Adaptive Modulation and Coding

## ECE-GY 6023. Wireless Communications

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1. *MCS selection:* A system has four MCS selections with minimum required SNRs as shown:

MCS	1	2	3	4
Min SNR [dB]	0	4	8	12

- (a) Suppose that the SNR in dB,  $\gamma$ , is unknown and can be modeled as Gaussian variable with mean 6 dB and standard deviation of 2 dB. What is the probability that  $\gamma > 8$  dB, the required SNR for MCS 3.
- (b) The TX attempts MCS 3 and it fails, meaning  $\gamma \leq 8$  dB. If it now attempts MCS 2, what is the probability that it will pass assuming the channel has not changed.
2. *Markovian errors:* In this problem, we will show how to use Markov processes to model error probabilities on correlated channels. As a simple example, suppose that a channel in time slot  $k$  can be modeled as being in one of two states: a good state ( $X_k = 1$ ), and a bad state ( $X_k = 0$ ). Assume  $X_k$  is Markov with transition probability matrix

$$P = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}.$$

A transmitter sends packets in each time slot. Let  $Y_k = 0$  or  $1$  be the indicator that a packet fails or passes in time slot  $k$ . Assume,

$$P(Y_k = 1|X_k = 1) = 0.8, \quad P(Y_k = 1|X_k = 0) = 0.4.$$

Assume that, given the  $X_k$ 's, the  $Y_k$ 's are independent.

- (a) Let  $\alpha_k(i) = P(X_k = i)$ . Find the recursion for the values  $\alpha_{k+1}(i)$  in terms of the values  $\alpha_k(j)$ .
- (b) Let

$$\alpha_k^0(i) = P(X_k = i | Y_0 = 0, \dots, Y_{k-1} = 0).$$

That is,  $\alpha_k^0(i)$  is the probability that  $X_k = i$  given that the previous  $k - 1$  transmissions have failed. Find the recursion for  $\alpha_{k+1}^0(i)$  in terms of the values  $\alpha_k^0(i)$ .

- (c) Let  $T$  be the time,

$$T = \min \{ k \mid Y_k = 1 \},$$

which is the index of the first slot that the packet passes. Suppose that  $X_0 = 0$ . Write a simple MATLAB program to compute  $P(T = k)$  for  $k = 0, 1, \dots, 9$  using the above recursions.

3. *Multi-Process ARQ timeline:* Suppose that a gNB wants to send  $N = 10$  packet data units (PDUs). The PDUs are indexed  $n = 0, 1, \dots, N - 1$ . In each slot, it attempts to send one PDU beginning in slot  $k = 0$  starting with PDU 0. There are  $K = 4$  parallel HARQ processes. Suppose that the transmissions fails in slots  $k = 5, 6$  and  $7$  and passes in all other slots.
- For each PDU, indicate the first slot it is correctly decoded at the receiver.
  - Suppose the receiver only releases decoded the PDUs in order to the higher layer. So, for example, it holds PDU 3 back until it receives PDU 2. Also, there is a fixed delay of 3 slots from the time of transmission to the PDU being available at the receiver for higher layers. When do the PDUs arrive at the higher layer?
4. *TB size:* Suppose that a 64 kbps voice over IP (VoIP) system transmits frames once every 20 ms. Each voice frame also requires a 20 B IP header, 20 B UDP header, and 24 bits CRC.
- How many bits are in each voice frame?
  - Suppose the data is transmitted in an NR-like system with 14 OFDM symbols and 12 sub-carriers per RB. In each RB, 14 REs are used for overhead. At a spectral efficiency of 2 bits / RE, how many RBs are needed to transmit the voice packet?
  - If the system has 51 RBs in bandwidth with one slot every 0.5 ms, what is the fraction of RBs used by the VoIP application?
5. *HARQ Errors:* For each of the following events, state what will occur:
- The PDU can eventually recovered through HARQ, or
  - The PDU cannot be recovered through HARQ and will need to be recovered from a high-layer ARQ protocol (e.g. at the RLC or TCP layer).
- A DL PDCCH for an initial transmission is not seen by the UE, so it does not even know that there is a DL data transmission.
  - The UE decodes the DL data and sends an ACK to the gNB. But, the gNB mistakes the ACK for a NACK.
  - The UE fails to decode the DL data and sends a NACK to the gNB. But, the gNB mistakes the NACK for an ACK.
6. *Power and SNR estimation:* Suppose that we have two groups of reference symbols:

- Zero-power RS that contain noise only,

$$r_k = w_k, \quad w_k \sim \mathcal{CN}(0, N).$$

On these symbols, we compute a noise estimate,

$$\hat{N} = \frac{1}{K} \sum_{k=1}^K |r_k|^2,$$

where  $K$  is the number of symbols over which we average.

- Non zero-power RS with signal and noise,

$$r_k = h_k x_k + w_k, \quad h_k \sim \mathcal{CN}(0, E_s), \quad w_k \sim \mathcal{CN}(0, N),$$

and  $|x_k| = 1$  is known to the receiver. On these symbols, we compute a signal power estimate

$$\hat{S} = \frac{1}{M} \sum_{k=1}^M |r_k|^2,$$

where  $M$  is the number of symbols over which we average.

- Show that  $\hat{N}$  and  $\hat{S}$  can be written as a scaled chi-squared distributions with a certain number of degrees of freedom. You can look up this distribution in any source.
- Show that the ratio  $\hat{S}/\hat{N}$  can be written as a scaled  $F$ -distribution distribution with a certain number of degrees of freedom. You can look up this distribution in any source.
- Suppose we use

$$\hat{\gamma} = \max \left\{ 0, \frac{\hat{S}}{\hat{N}} - 1 \right\}$$

as the estimate of the true SNR  $\gamma = E_s/N$ . Plot the probability that the SNR is accurate within 0.5 dB as a function of  $K$  with  $K = M$ . You can use the MATLAB function `fcdf`. Assume the true SNR is,  $\gamma = 3$  dB.

- CSI estimation bias:** Suppose that in a group of  $K$  symbols, reference symbols  $x_k$  are received as

$$r_k = h x_k + w_k, \quad w_k \sim \mathcal{CN}(0, N), \quad |h x_k|^2 = E_s,$$

where  $h$  is an unknown channel,  $N$  is the noise power, and  $E_s$  is the received signal energy. We channel and noise estimates via

$$\hat{h} = \frac{\sum_{k=1}^K x_k^* r_k}{\sum_{k=1}^K |x_k|^2}, \quad \hat{N} = \frac{\alpha}{K} \sum_{k=1}^K |r_k - \hat{h} x_k|^2.$$

- Find the constant  $\alpha$  such that the noise estimate is unbiased. That is,

$$\mathbb{E} [\hat{N}] = N.$$

- Suppose that you obtain an accurate estimate of the noise  $\hat{N} = N$  (for example, by averaging over large numbers of groups). How would you get an unbiased estimate of  $E_s$ ?
- Rate matching:** Suppose you send 200 bits with a rate 1/2 convolutional code with constraint length  $K = 7$ .
    - How many coded bits are output from the convolutional encoder? Remember to include the tail bits.
    - Suppose you want to send the data on 150 QPSK symbols. How many bits should be punctured or repeated?

9. *Comparing Chase and IR:* Suppose that a TX can create mother codes that, on an AWGN channel, require an SNR  $\gamma$  and provide a rate per symbol of

$$R(\gamma) = \min\{\rho_{\max}, \alpha \log_2(1 + \gamma)\},$$

where  $\rho_{\max}$  is the maximum spectral efficiency and  $\alpha$  is the fraction that the code achieves within the Shannon rate. Now suppose we use this code for HARQ with  $K$  transmissions. Suppose that all the symbols in transmission  $k$  experience some SNR  $\gamma_k$ ,  $k = 1, \dots, K$ .

- (a) Suppose, we use Chase combining where we create a packet from the mother code and retransmit it in each of the  $K$  transmissions. For each target  $\gamma$ , find the condition on  $\gamma_1, \dots, \gamma_K$  that the packet will pass. Also, find the rate,  $R_{\text{chase}}(\gamma)$  that will be achieved if the packet passes after  $K$  transmissions.
- (b) Next, suppose we use IR where we create a longer packet and transmit a fraction  $1/K$  symbols in each transmission. For each target  $\gamma$ , find the condition on  $\gamma_1, \dots, \gamma_K$  that the packet will pass. Also, find the rate,  $R_{\text{IR}}(\gamma)$  that will be achieved if the packet passes after  $K$  transmissions.
- (c) Set  $K = 3$  and generate random i.i.d.  $\gamma_k$  that are exponentially distributed with an average of 3 dB (i.e. independent Rayleigh fading). Generate  $n = 1000$  instances of this channel using MATLAB. By varying the target SNR  $\gamma$ , plot the probability that the packet the packet passes after  $K$  transmissions, vs. the rates  $R_{\text{chase}}(\gamma)$  and  $R_{\text{IR}}(\gamma)$ , for both Chase and IR combining.