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PROBLEM 3

$P_{TX} = 15$ dBm ; $B = 18$ MHz $T = -165$ dBm/Hz

(a)

At least 95% of the users are in Rooms: 1, 2, and 3

[SEE MATLAB] The SNR that can be guaranteed to at least 95% of the users is the SNR of Room 3 (i.e. the lowest one) which is 17,4473 dB

(b) Rayleigh fading

Expression for the CDF of the SNR including variation in both location and fading

$$F_\Gamma(\gamma) = P(\Gamma \le \gamma) = \int_{-\infty}^{\gamma} f_\Gamma(\gamma)\,d\gamma = 1 - e^{-\gamma/\bar\gamma} \quad \Big/ \quad P(\Gamma > \gamma) = e^{-\gamma/\bar\gamma}$$

In this case,

$$F_{\Gamma_{RL}}(\gamma) = P(\Gamma_{RL} \le \gamma) = \sum_{i=1}^{4} P(\Gamma \le \gamma \mid Room = i)\, P(Room = i)$$

$$= 0{,}6\left(1 - e^{-\gamma/\bar\gamma_1}\right) + 0{,}3\left(1 - e^{-\gamma/\bar\gamma_2}\right) + 0{,}06\left(1 - e^{-\gamma/\bar\gamma_3}\right) + 0{,}04\left(1 - e^{-\gamma/\bar\gamma_4}\right)$$, where $\bar\gamma_i$ = avg SNR in Room i

(c) SNR guaranteed to at least 95% of the people with slow fading

$0{,}05 = F_{\Gamma_{RL}}(\gamma) = P(\Gamma_{RL} \le \gamma) \Rightarrow$ [SEE MATLAB]

value we want to find

The SNR value we can guarantee to at least 95% of the users is $\approx 10{,}2086$ dB

5% of users will have less than this value / 95% will have greater SNR than this

PROBLEM 4

(a) Average SNR?

$$h(f,0) = \sqrt{P_1}\, e^{-2\pi j f \tau_1} + \sqrt{P_2}\, e^{-2\pi j f \tau_2} \Rightarrow |h|^2 = P_1 + P_2$$

Hence, $SNR_{avg} = \dfrac{P_1 + P_2}{W N_0}$

(b)

Ergodic Capacity $\Rightarrow C_E = E[\log(1+\gamma)]$

We know that $E[\log(1+\gamma)] \le \log(1 + E[\gamma]) = \log(1 + SNR_{Avg}) \Rightarrow C_E \le \log(1 + SNR_{Avg})$

AWGN capacity

linear scale

(c)

$\dfrac{P_1}{W N_0} = 8\text{dB} \xrightarrow{\text{linear}} 6{,}31$ / $\dfrac{P_2}{W N_0} = 5\text{dB} \xrightarrow{\text{linear}} 3{,}16$

$SNR_{avg} \simeq 9{,}76$ dB / $C_E \le \log_2(1 + 9{,}4624) \cong 3{,}387$

PROBLEM 5

$$L(r) = \log \frac{p(r \mid c=1)}{p(r \mid c=0)}$$

(a) Real-valued binary channel with fading

$$r = A_x + w, \quad w \sim N\left(0, \frac{N_0}{2}\right), \quad x = \begin{cases} \sqrt{E_x/2} & \text{if } c=1 \\ -\sqrt{E_x/2} & \text{if } c=0 \end{cases}$$

$$A_x = \begin{cases} A\sqrt{\frac{E_x}{2}} & \text{if } c=1 \\ -A\sqrt{\frac{E_x}{2}} & \text{if } c=0 \end{cases} \qquad w \sim N\left(0, \frac{N_0}{2}\right)$$

$$p(r \mid c=1) = \frac{1}{\sqrt{\pi N_0}}\, e^{-\frac{\left(r - A\sqrt{\frac{E_x}{2}}\right)^2}{N_0}}$$

$$p(r \mid c=0) = \frac{1}{\sqrt{\pi N_0}}\, e^{-\frac{\left(r + A\sqrt{\frac{E_x}{2}}\right)^2}{N_0}}$$

$\Big| -A\sqrt{\frac{E_x}{2}}$ if $c=0$ $\qquad\qquad \Big| \; p(r|c=0) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r+A\sqrt{\frac{E_x}{2}})^2}{N_0}}$

$$L(r) = \log \frac{p(r|c=1)}{p(r|c=0)} = \frac{-\left(r - A\sqrt{\frac{E_x}{2}}\right)^2}{N_0} + \frac{\left(r + A\sqrt{\frac{E_x}{2}}\right)^2}{N_0}$$

$$= \frac{\cancel{-r^2} + 2rA\sqrt{\frac{E_x}{2}} - \cancel{\left(A\sqrt{\frac{E_x}{2}}\right)^2} \cancel{+r^2} + 2rA\sqrt{\frac{E_x}{2}} + \cancel{\left(A\sqrt{\frac{E_x}{2}}\right)^2}}{N_0}$$

$$= \frac{4rA\frac{\sqrt{E_x}}{\sqrt{2}}}{N_0} = \frac{2\sqrt{2}\,rA\sqrt{E_x}}{N_0} = \frac{2rA\sqrt{2E_x}}{N_0}$$

(b) Binary Simmetric Channel

$r = c + w \pmod 2$ $\qquad w = \begin{cases} 1 & \text{with probablity } p \\ 0 & \text{with probablity } 1-p \end{cases}$

r can be $0,1$

$$L(r) = \log \frac{p(r|c=1)}{p(r|c=0)} \Rightarrow L(1) = \log\frac{1-p}{p} \;\Big/\; L(0) = \log\frac{p}{1-p}$$

Which can be written as: $L(r) = (2r-1)\log\frac{1-p}{p}$, for $r=0,1$

(c) Non-coherent channel

$r = \begin{cases} h+n & \text{when } c=1 \\ n & \text{when } c=0 \end{cases}$ $\qquad h \sim CN(0,E_s)$, $n \sim CN(0,N_0)$

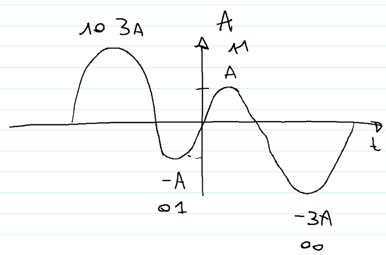
$L(r) = \log \frac{p(r|c=1)}{p(r|c=0)}$ $\qquad p(r|c=0) = \frac{1}{\pi N_0} e^{-|r|^2/N_0}$ $\qquad r = n \sim CN(0,N_0)$

$p(r|c=1) = \frac{1}{\pi(N_0+E_s)} e^{-|r|^2/(N_0+E_s)}$, $\quad r = h+n \sim CN(0, E_s+N_0)$
assuming h and n are independent

$$L(r) = \log \frac{\frac{1}{\cancel{\pi}(N_0+E_s)} e^{-|r|^2/(N_0+E_s)}}{\frac{1}{\cancel{\pi}N_0} e^{-|r|^2/N_0}} = \log\frac{N_0}{E_s+N_0} + \frac{-|r|^2}{(N_0+E_s)} + \frac{|r|^2}{N_0} = \log\frac{N_0}{E_s+N_0} + |r|^2\left(\frac{1}{N_0} - \frac{1}{E_s+N_0}\right)$$

PROBLEM 6

$r = x + n \qquad n \sim \left(0, \frac{N_0}{2}\right) \qquad x = \begin{cases} -3A & \text{if } c_0,c_1 = 00 \\ -A & \text{if } c_0,c_1 = 01 \\ A & \text{if } c_0,c_1 = 11 \\ 3A & \text{if } c_0,c_1 = 10 \end{cases}$



(a) average symbol energy

$$E|X|^2 = \frac{9A^2 + A^2 + A^2 + 9A^2}{4} = 5A^2$$

$$5A^2 = \frac{E_s}{2} \Rightarrow A = \sqrt{\frac{E_s}{10}}$$

(b) Bitwise LLR for c_0:

$$L_0(r) = \log \frac{p(r|c_0=1)}{p(r|c_0=0)} \qquad p(r|c_0) = \frac{1}{2}\left[p(r|c_0,c_1=1) + p(r|c_0,c_1=0)\right]$$

$$L_0(r) = \log\left[\frac{\overset{N(3A,N_0/2)}{p(r|c_0,c_1=1,0)} + \overset{N(A,N_0/2)}{p(r|c_0,c_1=1,1)}}{\underset{N(-3A,N_0/2)}{p(r|c_0,c_1=0,0)} + \underset{N(-A,N_0/2)}{p(r|c_0,c_1=0,1)}}\right]$$

$$= \log\left[\frac{\cancel{\frac{1}{\sqrt{\pi N_0}}}\, e^{-(r-3A)^2/N_0} + \cancel{\frac{1}{\sqrt{\pi N_0}}}\, e^{-(r-A)^2/N_0}}{\cancel{\frac{1}{\sqrt{\pi N_0}}}\, e^{-(r+3A)^2/N_0} + \cancel{\frac{1}{\sqrt{\pi N_0}}}\, e^{-(r+A)^2/N_0}}\right]$$

$$= \log\left[\frac{e^{-(r-3A)^2/N_0} + e^{-(r-A)^2/N_0}}{e^{-(r+3A)^2/N_0} + e^{-(r+A)^2/N_0}}\right]$$

$$L_1(r) = \log\left[\frac{p(r|c_0,c_1=0,1) + p(r|c_0,c_1=1,1)}{p(r|c_0,c_1=0,0) + p(r|c_0,c_1)=1,0)}\right]$$

$$= \log\left[\frac{e^{-(r+A)^2/N_0} + e^{-(r-A)^2/N_0}}{e^{-(r+3A)^2/N_0} + e^{-(r-3A)^2/N_0}}\right]$$