Tommy Frank, 08/2016

CONJECTURE

Let $f(x)$ be a polynomial of degree n, and let $\mathbf{v}_i$ be i-dimensional vectors such that $v_{ij} = v_{(i-1)(j+1)} - v_{(i-1)j}$. I conjecture that if

$$v_{n+2} = \begin{pmatrix} f(a) \\ f(a+d) \\ f(a+2d) \\ \vdots \\ f(a+nd) \\ 0 \end{pmatrix}$$

for any a and d, then $v_{11} = -f(a+(n+1)d)$.

Thus, for any polynomial $f(x)$ of degree n:

> Given n+1 initial values for $f(x)$ that are separated by a common difference d (d can be any value), there is an algorithm that will provide all subsequent values of $f(x)$ that are separated by d, which uses only subtraction and has no knowledge of $f(x)$ itself.

As an example, let polynomial $f(x)$ be: $3x^3 - 5x^2 + 2$, and let the chosen n+1 initial values of x be: 3, 7, 11, 15. Then $f(3) = 38, f(7) = 786, f(11) = 3390, f(15) = 9002$, and the subsequent values of $f(x)$ can be determined by using the following subtraction triangle:

```
38          786          3390          9002          0
    748          2604          5612          -9002
         1856          3008          -14614
              1152          -17622
                   -18774
```

And so, $f(19) = 18774$.

For an implementation in Java, go to:
`https://github.com/tommyfrank/math-programs`