# (Smaller) Grid Search Results

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We compare results from optimizations using two sets of initial values for  $(\log(k_0), k_1)$ : = (-12, 0.02) and  $(-12, 0.07\overline{3})$ .

We define convergence as a relative tolerance less than 0.001 for all parameters, where the relative tolerance between two values x and y is defined as

$$reltol(x,y) = |x - y|/\min(|x|, |y|). \tag{1}$$

For the algorithm to converge on iteration m, all 24 parameters k must have  $reltol(k^{(m)}, k^{(m-1)}) < 0.001$ .

## 1 Results

We ran each optimization to convergence. The minimum optimal value of the objective function was 0.3314327, and the larger optimal value was 0.3315864. The smaller optimal value actually had not reached convergence yet, after 40000 iterations. The larger optimal value was reached after 15117 iterations.

#### 1.1 Optimal parameters

The set of optimal parameters, corresponding to the optimization with the smallest optimal value of the objective function, is shown in Table 1.

Table 1: Optimal parameter values

| '   | Lk0     | k1    |
|-----|---------|-------|
| r12 | -10.295 | 0.088 |
| r16 | -8.599  | 0.073 |
| r18 | -7.209  | 0.041 |
| r23 | -10.121 | 0.083 |
| r29 | -10.288 | 0.102 |
| r34 | -4.000  | 0.040 |
| r63 | -7.725  | 0.081 |
| r67 | -9.786  | 0.043 |
| r74 | -6.303  | 0.054 |
| r87 | -7.183  | 0.054 |
| r89 | -12.911 | 0.037 |
| r94 | -8.246  | 0.068 |

# 1.2 Transitions

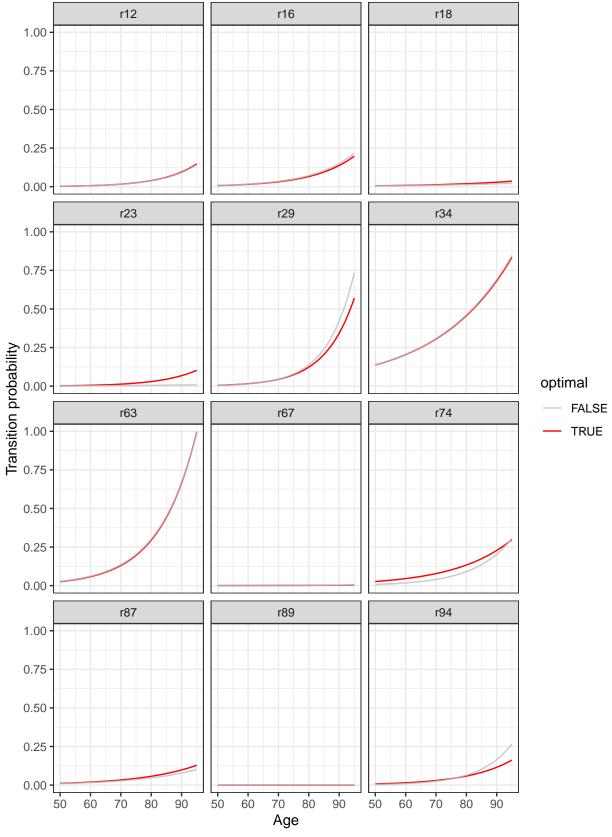
The figures below show the transition probabilities for every transition  $r_{ij}(a)$  for ages  $a \in (50, ..., 95)$ . Figure 1 has all y-axes fixed from 0 to 1, while Figure 2 has scales unique to each panel. The red line in each panel corresponds to transition probabilities for the optimal parameters in Table 1.

Figure 1: Transition rates after optimization using different initial values

r12

r16

r18



r12 r16 r18 0.15 0.20 0.03 0.15 0.10 0.02 0.10 0.05 0.05 0.01 -0.00 0.00 -70 70 50 60 80 60 70 80 50 60 r23 r29 r34 0.100 0.8 0.6 0.075 0.6 0.4 0.050 0.4 Lansition probability
0.000
1.00
1.00
0.75 0.2 0.2 0.0 optimal 60 70 80 90 70 80 90 60 70 80 90 50 60 **FALSE** r63 r67 r74 TRUE 0.005 0.3 0.004 0.75 0.2 0.003 0.50 0.002 0.1 0.25 0.001 0.00 0.0 70 80 50 60 70 80 90 60 60 70 80 50 r87 r89 r94 8e-04 0.10 0.2 6e-04 4e-04 0.1 0.05 2e-04 0.0 0e+00 50 60 70 80 70 80 90 60 70 80 90 50 60

Figure 2: Transition rates after optimization using different initial values

Age

## 1.3 Fit to Incidence and Prevalence data

Figures 3 and 4 below show the fit of optimized rates to the incidence data and prevalence data. In each plot, the dashed line represents the data from either the systematic review (for incidence) or Jack's 2017 paper (for prevalence), and the solid lines represent optimized estimates. The red line represents the most optimal solution.

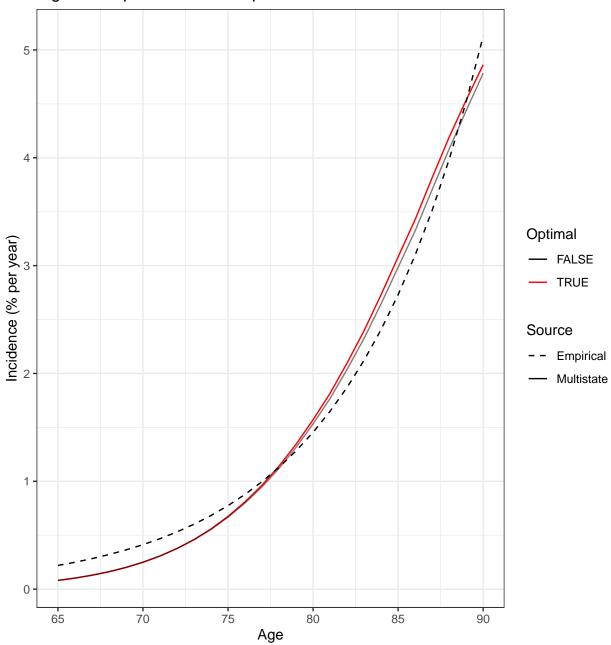


Figure 3: Optimized and Empirical Incidence Rates

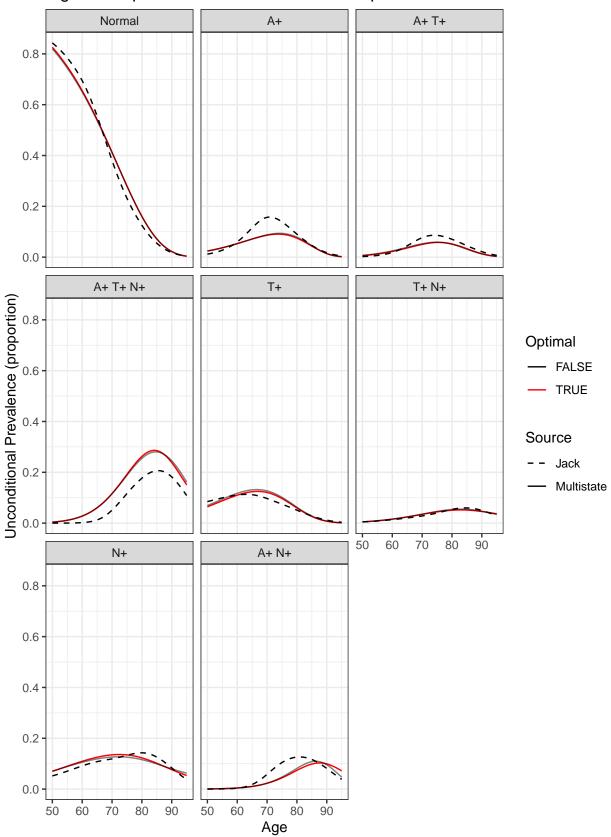


Figure 4: Optimized vs Jack unconditional prevalence rates

# 1.4 Lifetime risks

Tables 2 and 3 show the lifetime risks for ages  $a \in (60, 65, \dots, 90)$  for males and females using the transition parameters in Table 1.

Table 2: Lifetime risks for females

| Age | Normal  | A       | A+T     | A+T+N   | A+T+N+MCI | Т       | T+N     | N       | A+N     |
|-----|---------|---------|---------|---------|-----------|---------|---------|---------|---------|
| 60  | 17.91~% | 18.83 % | 54.33~% | 58.4 %  | 95.63~%   | 42.22~% | 41.17~% | 16.94~% | 26.14 % |
| 65  | 16.52~% | 17.71~% | 51.45~% | 56.21~% | 93.57~%   | 39.79~% | 38.24~% | 15.01~% | 24.35 % |
| 70  | 14.63~% | 16.06~% | 47.29~% | 52.82~% | 90.06 %   | 36.25~% | 34.29~% | 12.67~% | 21.89~% |
| 75  | 12.22~% | 13.81~% | 41.56~% | 47.89 % | 84.71 %   | 31.41~% | 29.22~% | 10.01~% | 18.71 % |
| 80  | 9.37~%  | 10.97~% | 33.95~% | 40.91~% | 76.19~%   | 25.1~%  | 22.99~% | 7.21~%  | 14.81~% |
| 85  | 6.51~%  | 7.88~%  | 24.73 % | 31.79 % | 63.81~%   | 17.78~% | 16.17~% | 4.7~%   | 10.57~% |
| 90  | 4.49~%  | 5.44~%  | 15.83~% | 21.73 % | 46.77~%   | 11.44~% | 10.39 % | 3.02~%  | 7.06~%  |

Table 3: Lifetime risks for males

| Age | Normal  | A       | A+T     | A+T+N   | A+T+N+MCI | Т       | T+N     | N       | A+N     |
|-----|---------|---------|---------|---------|-----------|---------|---------|---------|---------|
| 60  | 12.06~% | 12.97~% | 44.85~% | 49.4~%  | 92.93~%   | 32.54~% | 32.05~% | 11.61~% | 19.23 % |
| 65  | 11.08~% | 12.2~%  | 42.27~% | 47.5~%  | 90.37~%   | 30.57~% | 29.61~% | 10.19~% | 17.86~% |
| 70  | 9.69~%  | 10.96~% | 38.37~% | 44.28~% | 85.99 %   | 27.53~% | 26.21~% | 8.45~%  | 15.9~%  |
| 75  | 7.91~%  | 9.27~%  | 33.07~% | 39.61~% | 79.54~%   | 23.39~% | 21.89~% | 6.49~%  | 13.36~% |
| 80  | 5.84~%  | 7.14~%  | 26.19~% | 33.05~% | 69.91 %   | 18.11~% | 16.7~%  | 4.47~%  | 10.28~% |
| 85  | 3.81~%  | 4.88~%  | 18.3~%  | 24.88~% | 56.66~%   | 12.25~% | 11.22~% | 2.72~%  | 7.02~%  |
| 90  | 2.44~%  | 3.14~%  | 10.89~% | 16.17~% | 40.19~%   | 7.27~%  | 6.68~%  | 1.61~%  | 4.36~%  |