On Minimizing Polynomial Pair Conjugacy Loss for Steerable Near-quadrature 3D Filters

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***Abstract*—For the construction of axially symmetric three-dimensional filters pairs to approximately represent the real and imaginary parts of a quadrature filter - giving the analytic representation of the impulse response of a filter - it is necessary to construct a pair of polynomials - one odd, one even - that differ minimally on the unit interval. This minimization is convex and therefore straightforward to implement. However, it is not clear that such a pair of polynomials would necessarily exhibit other desirable properties of this minimization, such as positivity, concentration at one, or that this difference would vanish as we allow our maximum bandwidth N to increase. We demonstrate that under the constraints that the polynomials vanish at the origin and are normalized so that the coefficients add up to one, we may in fact guarantee (1) the optimally similar polynomial pairs have positive coefficients, (2) the objective function loss vanishes as the maximum degree N allowed approaches infinity and (3) these polynomials converge pointwise to the indicator function at 1 on the relevant unit interval. In the process, we explore various properties of the closely related Hilbert matrices and their inversion.**

***Index Terms*—steerability quadrature filter quadratic programming hilbert matrices**

I. Introduction

ANALYTIC signals provide a powerful framework for the extraction of local properties of signals. Quadrature filters that give Hilbert transforms of signals are essential to these calculations, and several methods have been proposed to extend the Hilbert transform to higher dimensions [**?**]. Of these, methods for two-dimensional signal via the use of steerable filters to combine information about images at various orientations have been proposed to preserve the isotropy property, necessary to obtain invariance of the measure of the impulse of the local signal under rotation.
However, three-dimensional filters remain technically difficult. Since quadrature filters are often represented as two real filters - it is invaluable to be able to construct two filters that are harmonic conjugates of each other. In a three-dimensional sense, these filters must agree on a certain domain and differ by only a sign on the complement. Additionally, it would be computationally desirable to demand that such filter pairs are steerable.

A. Mathematical Formulation of the Problem

Subsection text here.

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1) Generalization of the Hilbert Transform: Subsubsection text here.

2) Three-dimensional Axially Symmetric Steerable Filters:

3) Optimization Problem:

II. Results

A. Solution to Quadratic Minimization

B. Properties of Generalized Hilbert Matrix Inverse and Notation

C. Theorem on Behavior of Minimal Solution

D. Asymptotic Results

E. Numerical Results

III. Proofs

IV. Conclusion

V. Conclusion

The conclusion goes here.

Appendix A
Proof of the First Zonklar Equation

Appendix one text goes here.

Appendix B

Appendix two text goes here.

Acknowledgment

The authors would like to thank...

References

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Michael Shell Biography text here.

John Doe Biography text here.

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