ON MINIMIZING ODD-EVEN POLYNOMIAL PAIR DIFFERENCES TO CONSTRUCT STEERABLE NEAR-CONJUGATE FILTER PAIRS

A PREPRINT

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ABSTRACT

For the construction of axially symmetric three-dimensional filters pairs to approximately represent the real and imaginary parts of a quadrature filter - giving the analytic representation of the impulse response of a filter - it is necessary to construct a pair of polynomials - one odd, one even - that differ minimally on the unit interval. This minimization is convex and therefore straightforward to implement. However, it is not clear that such a pair of polynomials would necessarily exhibit other desirable properties of this minimization, such as positivity, concentration at one, or that this difference would vanish as we allow our maximum bandwidth N to increase. We demonstrate that under the constraints that the polynomials vanish at the origin and are normalized so that the coefficients add up to one, we may in fact guarantee (1) the optimally similar polynomial pairs have positive coefficients, (2) the objective function loss vanishes as the maximum degree N allowed approaches infinity and (3) these polynomials converge pointwise to the indicator function at 1 on the relevant unit interval. In the process, we explore various properties of the closely related Hilbert matrices and their inversion.

***K*eywords** steerability · quadrature · filter · quadratic programming · hilbert matrices

1 Introduction

In this section we give the application motives for our results. We introduce the need to approximately represent quadrature filters using near-conjugate filters and explore briefly certain sufficient conditions for steerability. We precisely define our optimization problem in terms of an odd and even polynomial pair under a bandwidth N, and finally reformulate our problem in vectorized form using generalized Hilbert matrices, which is the line of thinking we will adopt for the rest of the paper.

1.1 Motivation

On minimizing conjugacy loss of odd and even polynomial filter pairs to represent steerable near quadrature filters On steerable near-conjugate polynomial filters pairs to approximate quadrature filters On minimizing odd-even polynomial pair differences to construct steerable near-conjugate filter pairs [GIVE BACKGROUND ON QUADRATURE FILTERS, NEAR GABOR REPRESENTATIONS, STEERABILITY.... MOTIVATE THE NEED TO FIND SUCH POLYNOMIAL PAIRS] Quadrature filters are analytic representations of the impulse response of a real-valued filter - such as a Gabor filter. Since nonzero quadrature filters are complex, they are typically represented as two real-valued filters corresponding to the real and imaginary part of the filter. Therefore it is desirable to construct a pair of functions - one even, one

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odd - respectively, to represent the filter. In many applications of such filters, it is computationally desirable that such functions are exactly steerable. However, this requires that

1.2 The quadratic optimization problem

We now formulate the problem precisely.

1.3 Reformulation with Generalized Hilbert Matrix

Definition 1.1. A *generalized Hilbert matrix with order p*

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2 Important Properties of the Generalized Hilbert Matrix

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[1, 2] and see [3].

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Of note is the command `\citet`, which produces citations appropriate for use in inline text. For example,

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2.1 Definitions and Notations

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2.2 Exact expression for entries and sum of entries

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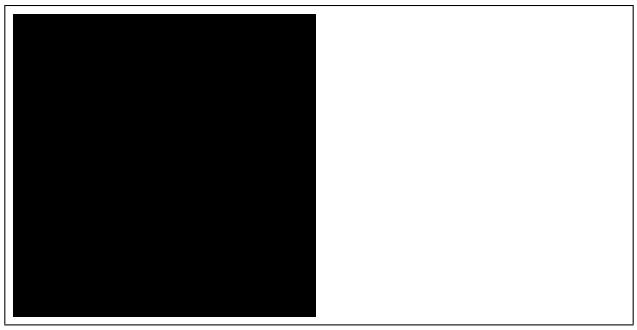

Figure 1: Sample figure caption.

Table 1: Sample table title

	Part	
Name	Description	Size (μm)
Dendrite	Input terminal	$\sim$100
Axon	Output terminal	$\sim$10
Soma	Cell body	up to 10^6

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See awesome Table 1.

2.3 Notes on magnitudes and signs of important sums

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3 Theorems and Results

3.1 The positivity of optimal polynomial pair coefficients

3.2 Asymptotic behavior of polynomial pairs

References

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