Homework 1

Biological Neural Computation

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This homework is completely my own work.

1.

a)

The Nernst Equation for cross-membrane potential can be written as:

$$E = \frac{RT}{zF}\ln\frac{concentration\ of\ ion\ inside}{concentration\ of\ ion\ outside}$$

$$R = 8.314\ JK^{-1}mol^{-1}$$

$$T = 310.15\ K\ (given\ by\ question)$$

$$z = ionic\ charge$$

$$F = 9.649 * 10^{4} Cmol^{-1}$$

Using the Nernst Equation, we get the following reversal potentials for the ions and 37 degrees Celsius.

Ion	**Reversal Potential (4 s.f.)**
K+	-0.0801 V
Na+	0.0615 V
Ca2+	0.1231 V
Cl-	-0.0654 V

b)

Using the Goldman Equation for two ions (positive monovalent) and the parameters given in the question, we can re-summarize the question as:

$$-0.065 = \frac{8.314 * 310.15}{9.649 * 10^{4}} ln(\frac{P_K[K]_{out} + P_{Na}[Na]_{out}}{P_K[K]_{in} + P_{Na}[Na]_{out}})$$

Plugging in the numbers for concentrations given in 1 a), it gives the equation:

$$-0.065 = \frac{8.314 * 310.15}{9.649 * 10^{4}} ln(\frac{P_K * 5 + P_{Na} * 150}{P_K * 100 + P_{Na} * 15})$$

Rearrange as shown below:

$$-0.065 / \frac{8.314 * 310.15}{9.649 * 10^{4}} = ln(\frac{P_K * 5 + P_{Na} * 150}{P_K * 100 + P_{Na} * 15})$$

$$-2.432 = ln(\frac{P_K * 5 + P_{Na} * 150}{P_K * 100 + P_{Na} * 15})$$

$$e^{-2.432} = (\frac{P_K * 5 + P_{Na} * 150}{P_K * 100 + P_{Na} * 15})$$

$$0.08786 * (P_K * 100 + P_{Na} * 15) = P_K * 5 + P_{Na} * 150$$

$$0.08786 * (P_K * 100) - P_K * 5 = -0.08786 * (P_{Na} * 15) + P_{Na} * 150$$

$$P_K * 3.786 = P_{Na} * 148.68$$

Finally, the relative permeability of potassium to sodium can be written as:

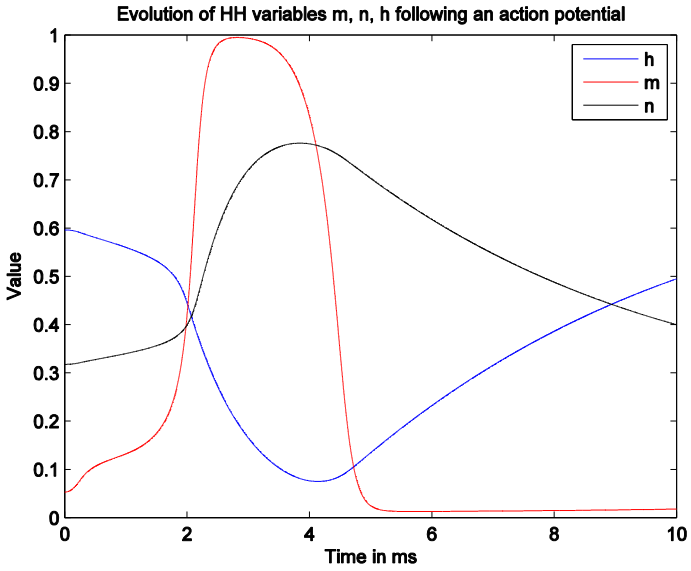
$$\frac{P_K}{P_{Na}} = \frac{148.68}{3.786} = 39.27$$

2.

a)

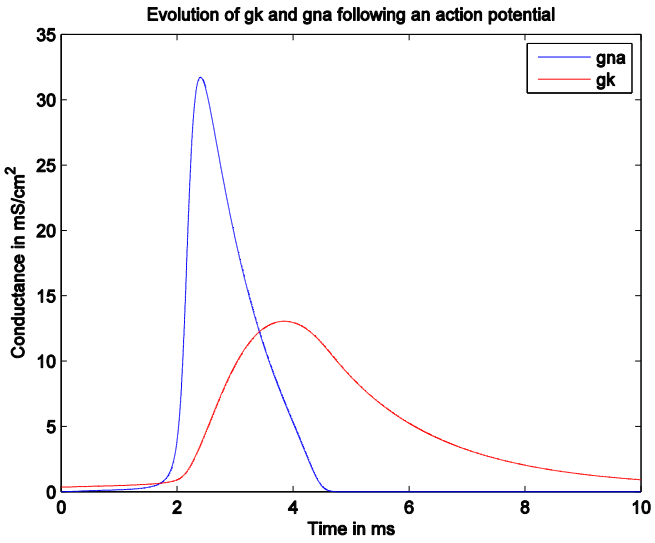
i)

After implementing the HH equations and solving through Euler's method, an input current of 0.25ms in length was used to excite the system. The following is the evolution of the m, n, h variables found in the HH equations. It should be noted that these are similar in behavior when compared to those shown in general literature for typical neuron conditions (Koch).



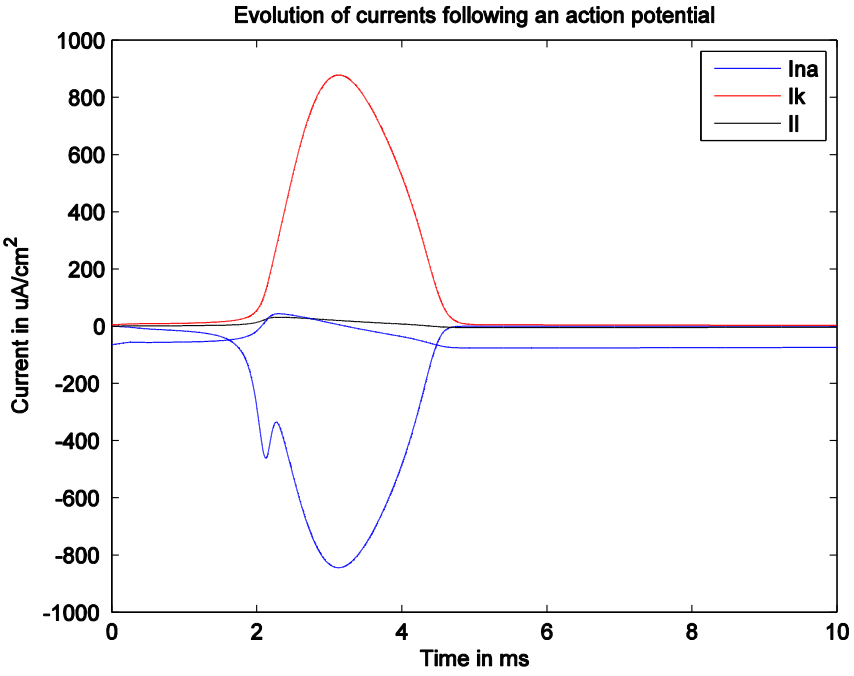
ii)

The following is the model output for conductances of sodium and potassium. This is similar to the plots found in general literature for typical neuron conditions (Koch).



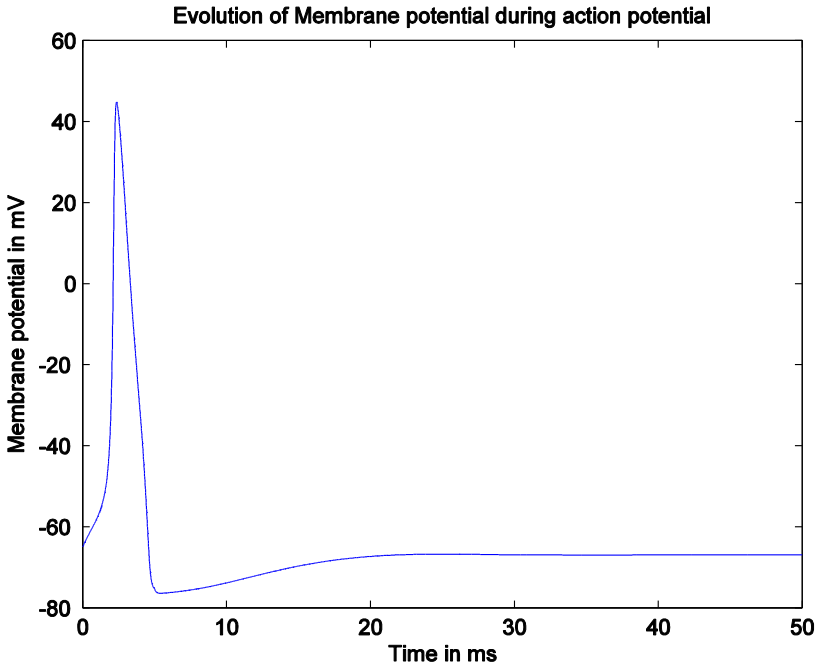
iii)

The shape of the curves produced by the model is correct, with the slight bump that can be seen on the Ina curve (Koch).

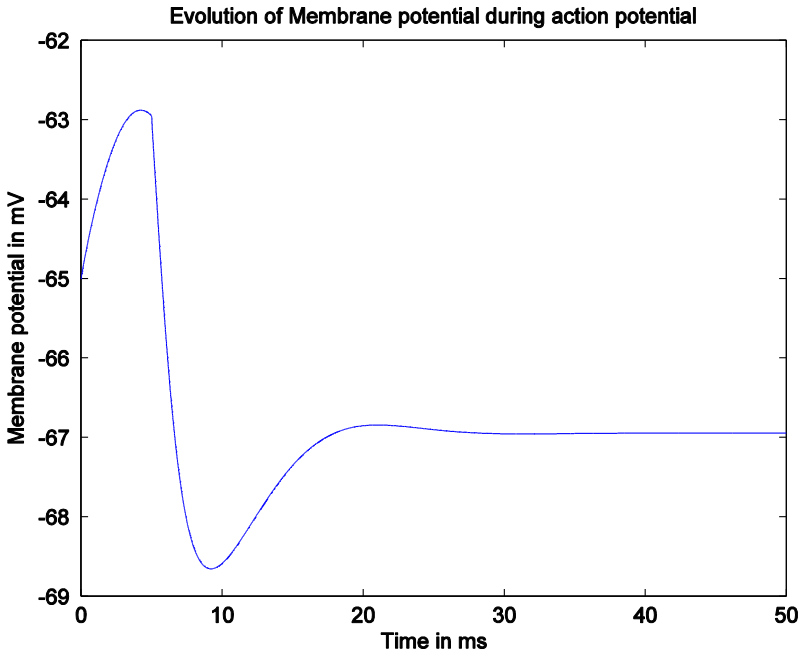


Iv)

The following plot is an action potential produced by an input of 1nA for 5ms. This is above threshold.



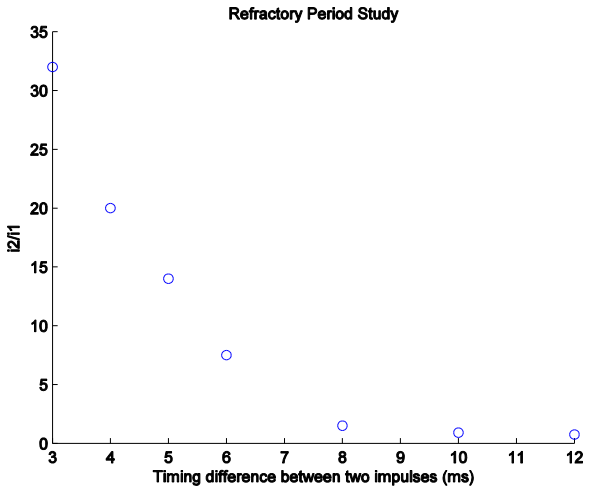
For below threshold behavior, the following is the membrane potential seen after a current input of 0.2nA for 5ms. This is below threshold, note that there is not a spike (the y-range is only between -63mV and -69mV).



The following data was collected by hand, the first input is an impulse 0.5nA for 0.5ms, while the second impulse was changed to be different timing differences away from the first. The recorded current values are the ones required to create a second action potential of the same peak height as the first.

Timing difference (ms)	i1 (nA)	i2 (Na)	i2/i1
12	0.50	0.38	0.76
10	0.50	0.46	0.92
8	0.50	0.75	1.5
6	0.50	3.75	7.5
5	0.50	7.00	14
4	0.50	10.00	20
3	0.50	16.00	32

The data can then be plotted using MatLab to give the following curve. Which is qualitatively the same relationship shown by the diagram in Koch 6.7.

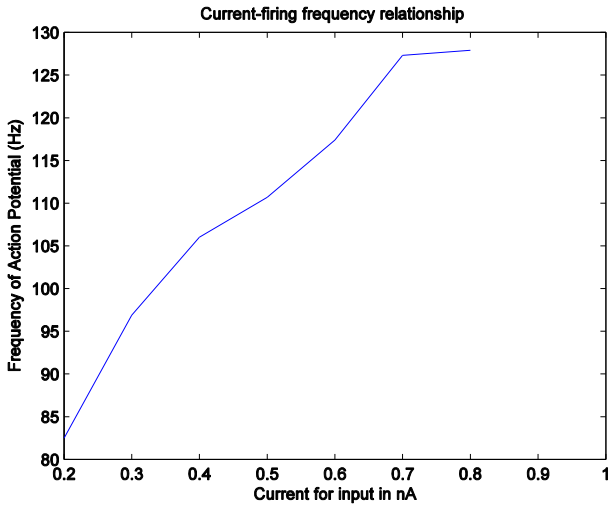


vi)

The following data was collected with different strength (nA) of 50ms constant inputs. The frequency of action potentials was calculated by sight using the MatLab plot tool and data cursor tool.

Current (nA)	Frequency (Hz)
0.2	82.5
0.3	96.9
0.4	106
0.5	110.7
0.6	117.4
0.7	127.3
0.8	127.9

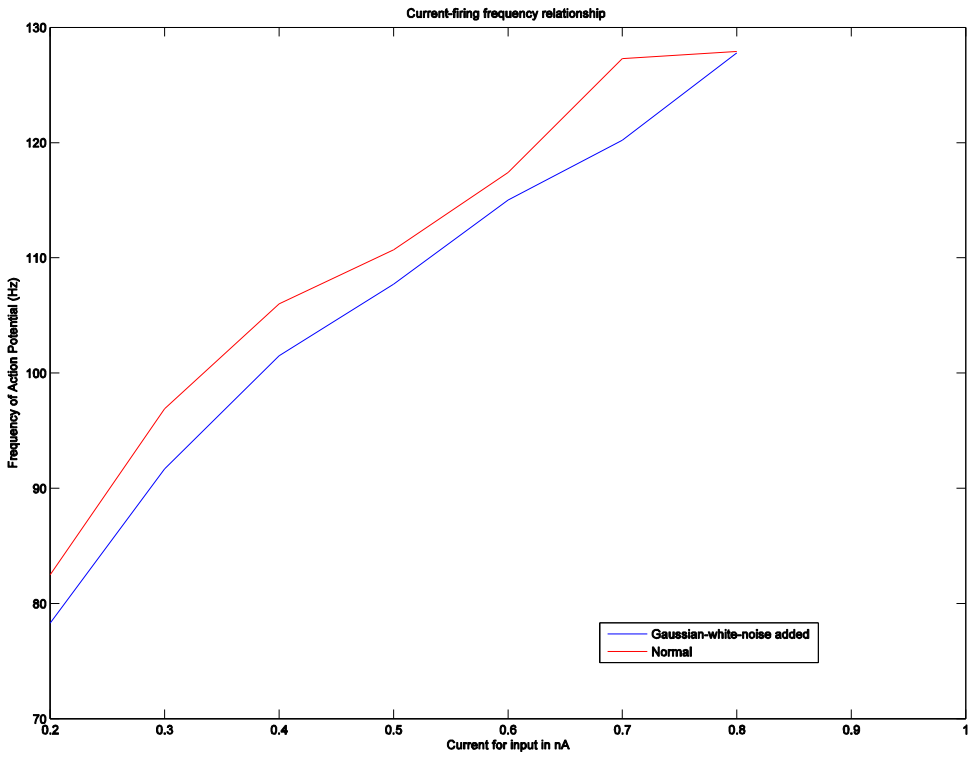
This can then be plotted as following. The curve is qualitatively and quantitatively very similar that found in Koch 6.10 (solid line).



The following data was collected after Gaussian white noise has been added to the whole input sequence.

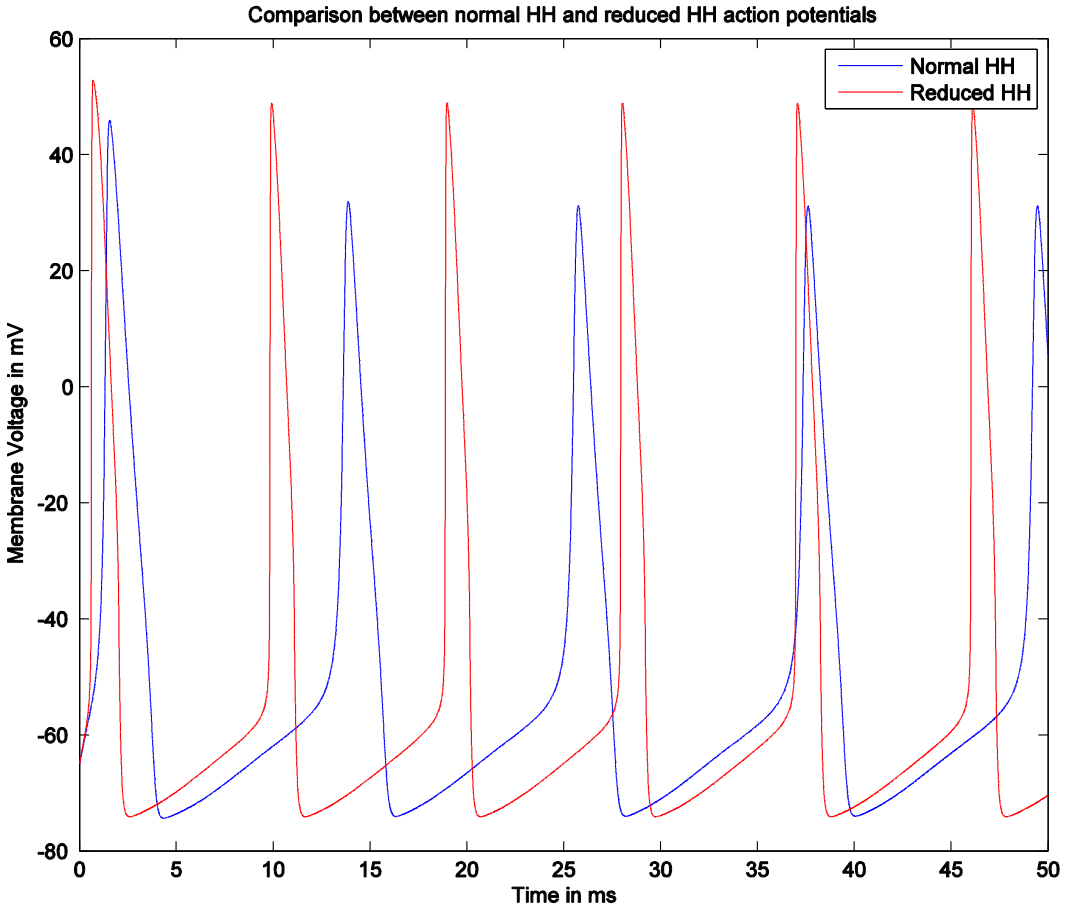
Current (nA)	Frequency (Hz)
0.2	78.3
0.3	91.7
0.4	101.5
0.5	107.7
0.6	115
0.7	120.2
0.8	127.8

The following plot compares the two data sets. It seems that the white noise somewhat linearizes the data.



b)

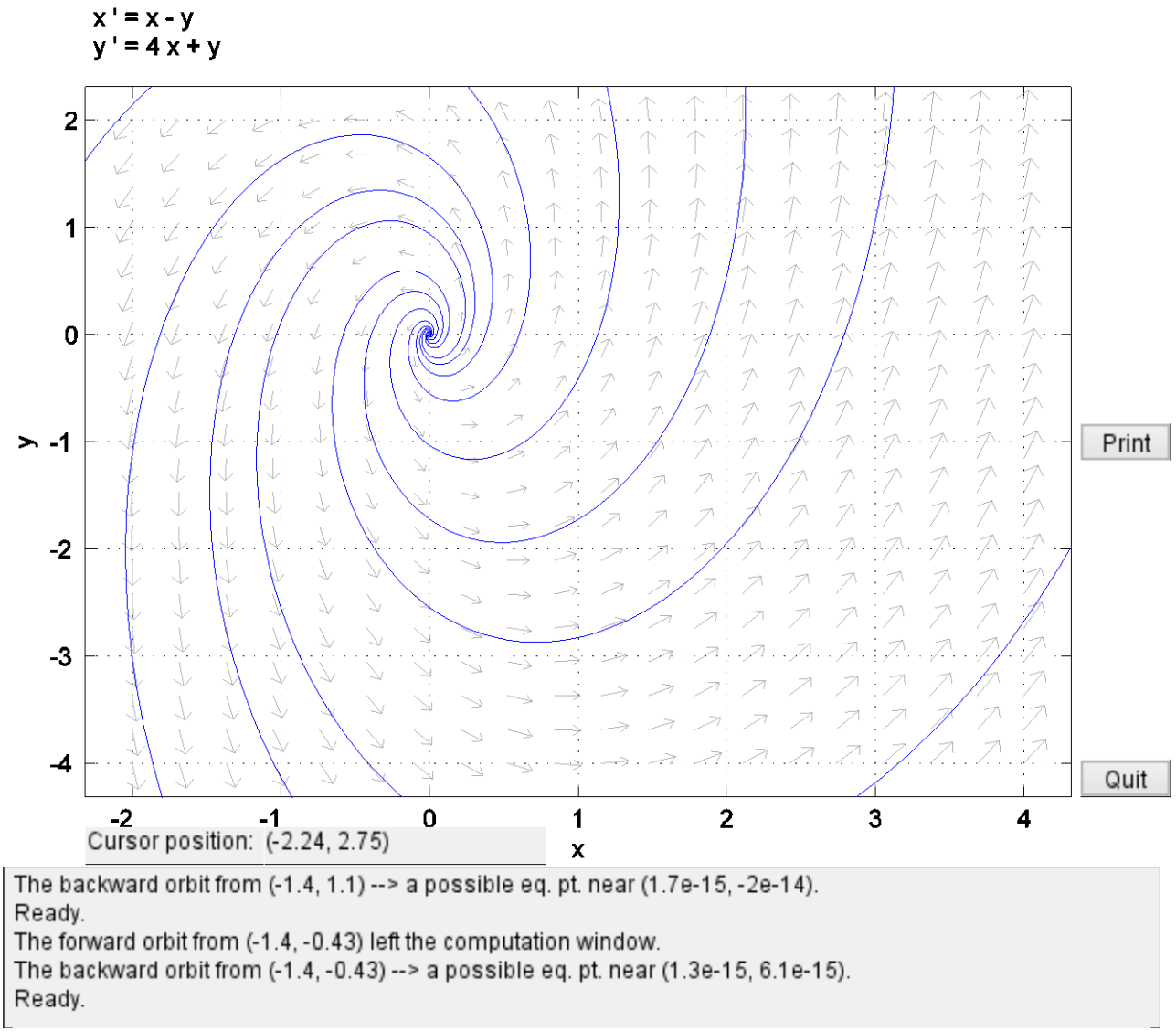
The following plot shows the difference between the action potentials modelled by the normal HH model and the reduced HH model. Both models were given the same input. The shapes of the action potential are qualitatively similar, but the membrane potential generated by the two different models are quantitatively different.



3.

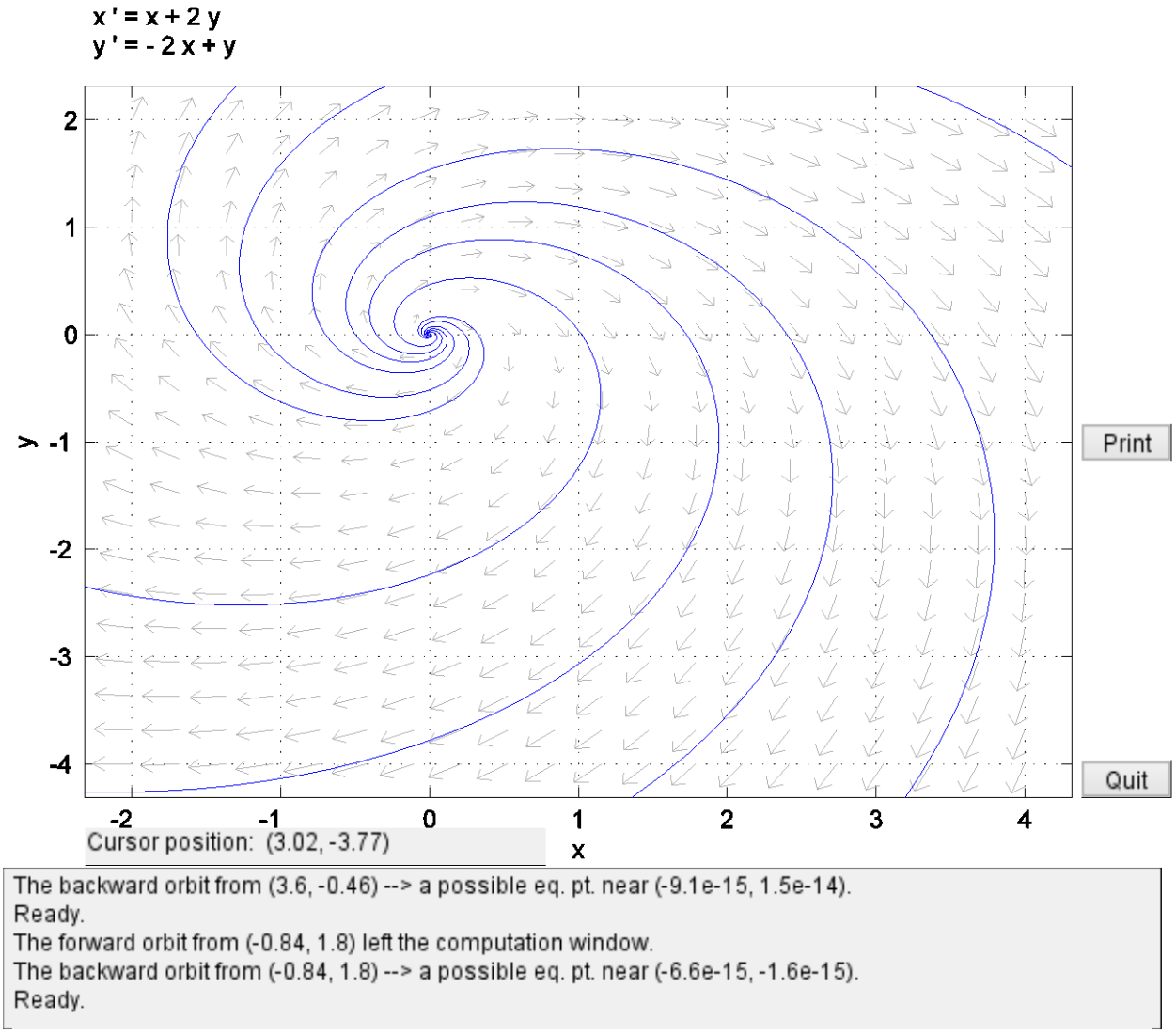
The following sketches have been made using the pplane8 tool for MatLab, made by John C. Polking from Rice University. This option was explored briefly in class. In the following plots, $x_1 = x;\ x_2 = y$.

i)



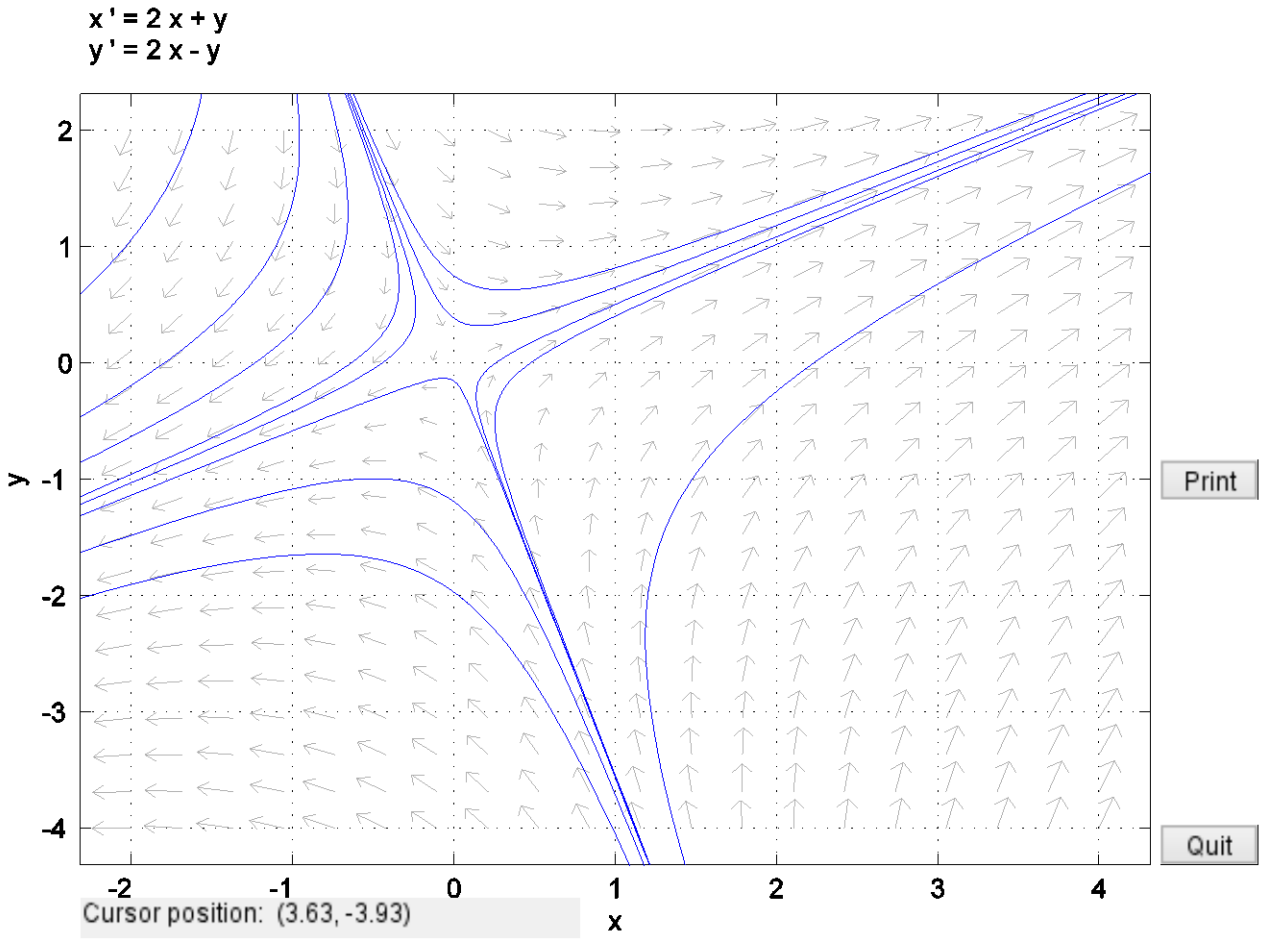
The spiral system is not stable. As t increases, the system diverges, shown by the arrows pointing outwards.

ii)



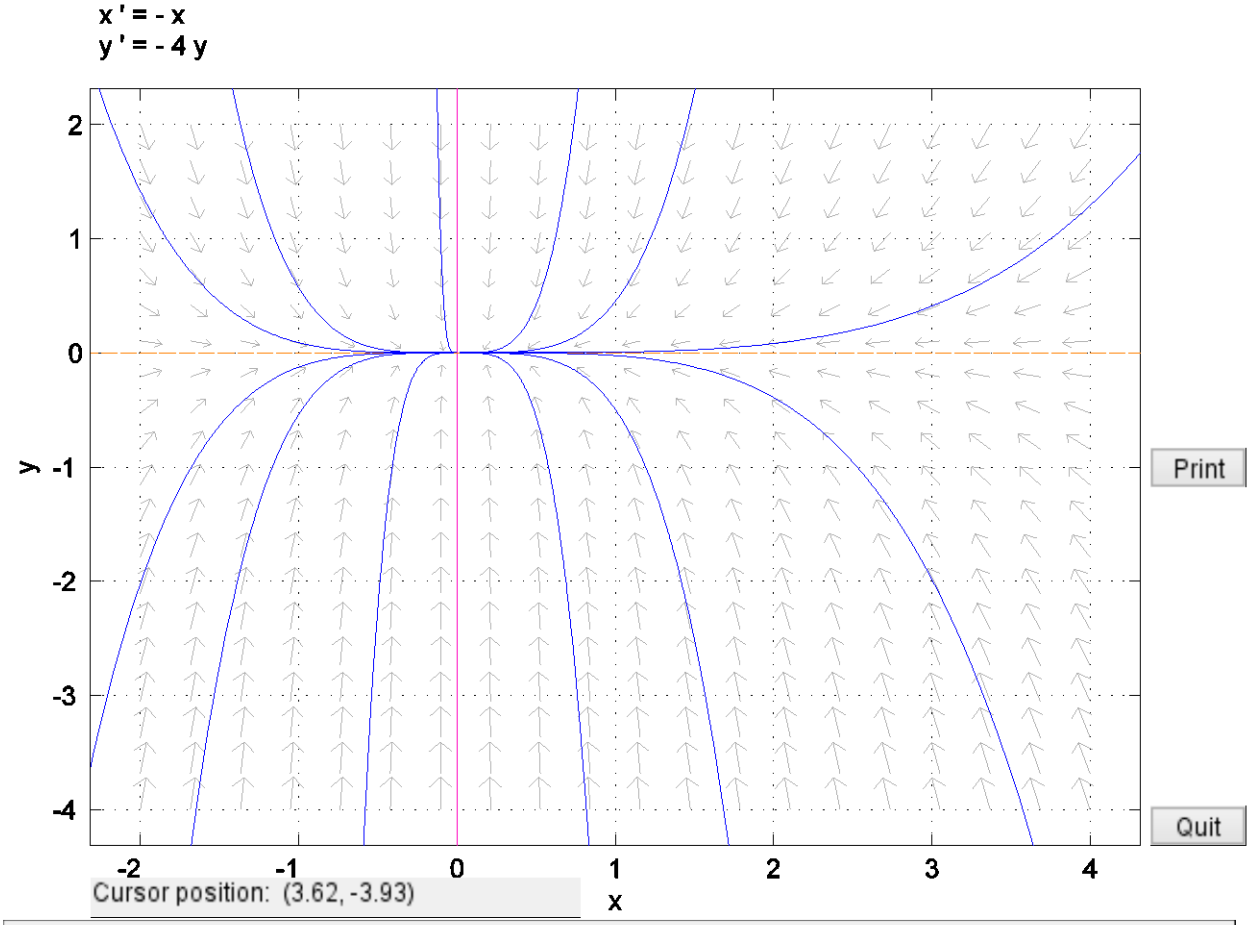
This spiral system is also not stable around the equilibrium point. As t increases, the arrows point outwards and the system diverges.

iii)



A saddle occurs, the system is unstable (except along the exact eigenvectors, as discussed in class). In general, as t increases, the system asymptotically diverges and never reaches a node.

iv)



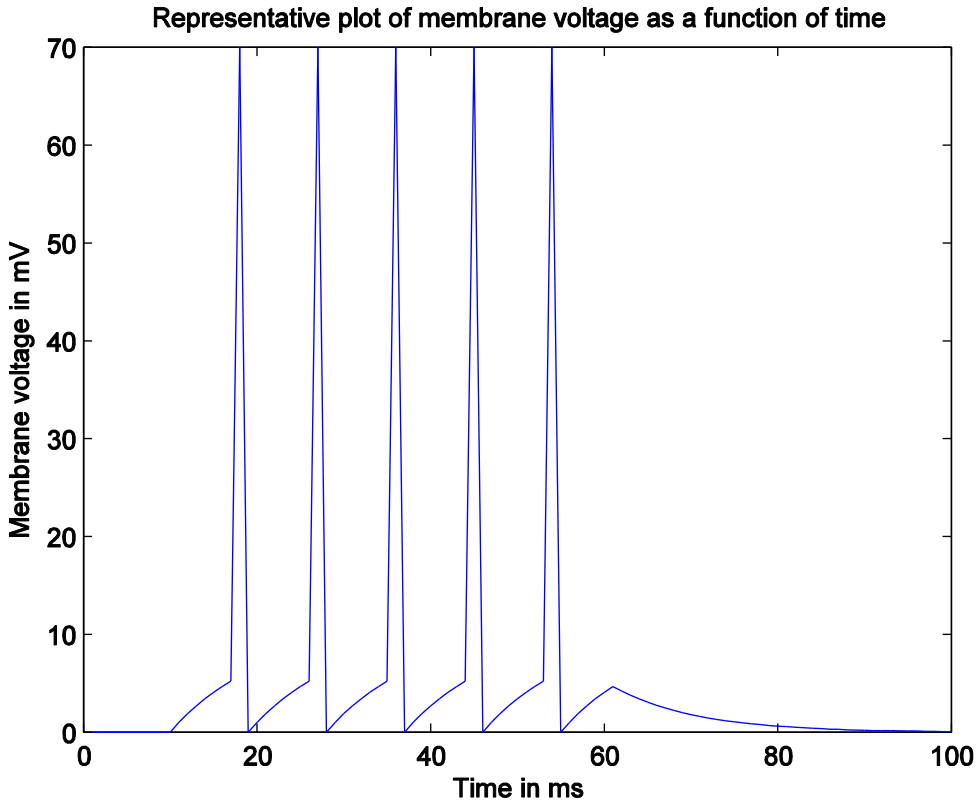

```
The backward orbit from (-0.42, -1.1) left the computation window.
Ready.
The forward orbit from (0.68, -1.9) --> a possible eq. pt. near (2.6e-14, 8.5e-22).
The backward orbit from (0.68, -1.9) left the computation window.
Ready.
```

The system is stable as t increases, this is indicated by the arrows all pointing asymptotically towards the equilibrium point.

4.

a)

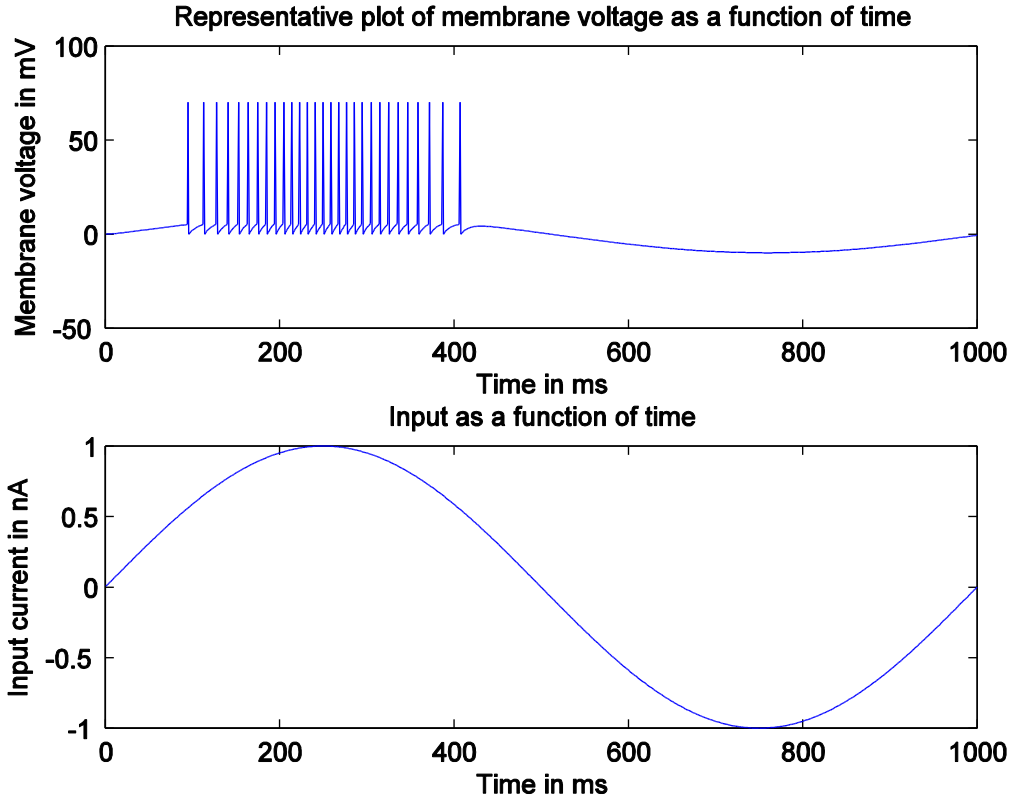
The following is the representative plot of the membrane voltage as a function of time, there are a total of 5 spikes. The input current was delivered from 10 to 60ms.

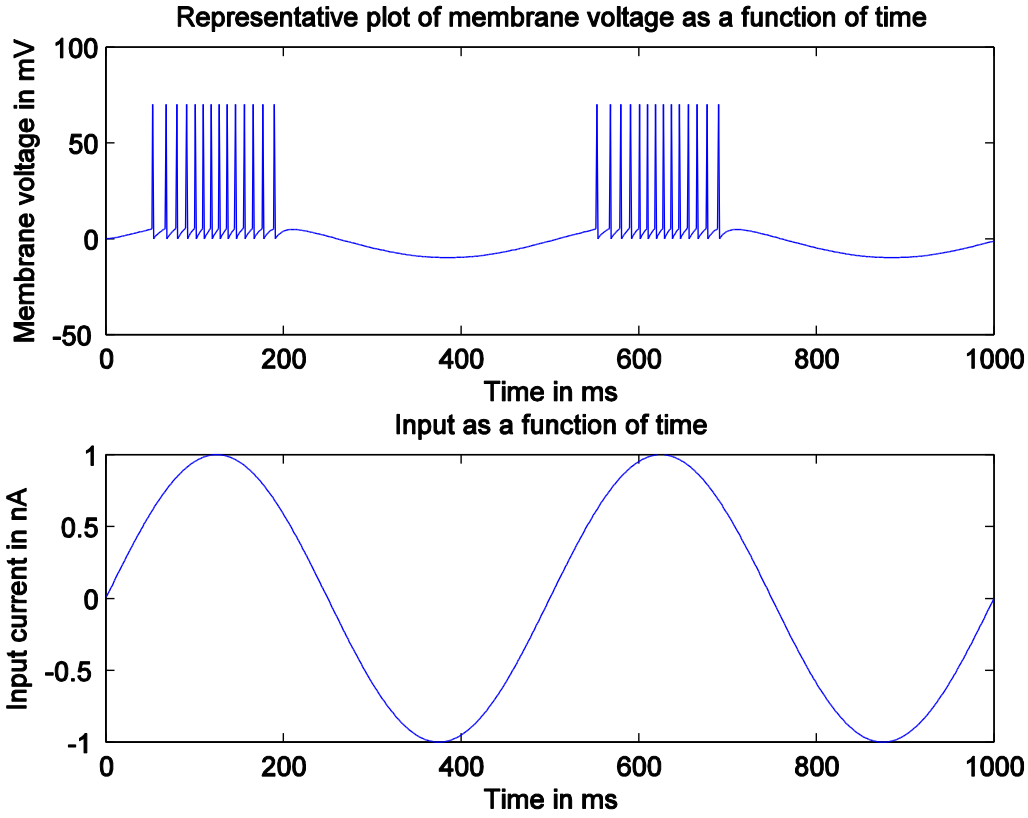


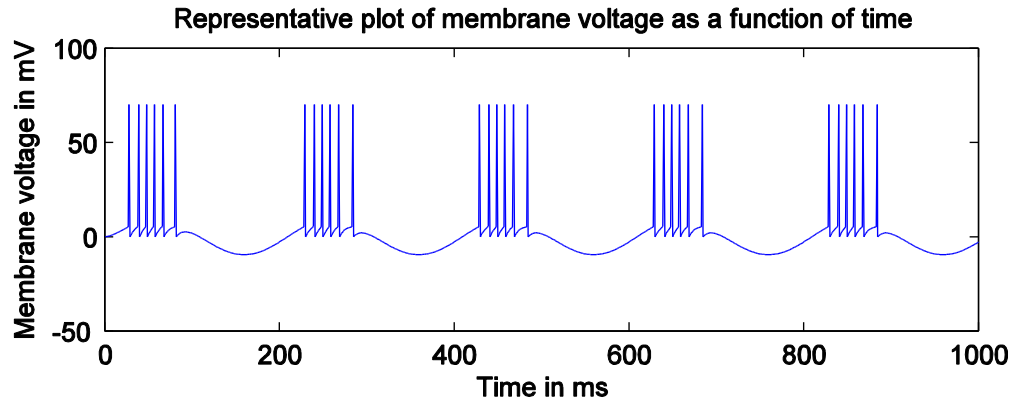
The model's response to sinusoidal stimulation was tested by supplying sinusoidal inputs of different frequencies and observing the spike counts within 1 second. Spike counts were calculated by observing how many 70mV values were within the membrane voltage matrix output from the model.

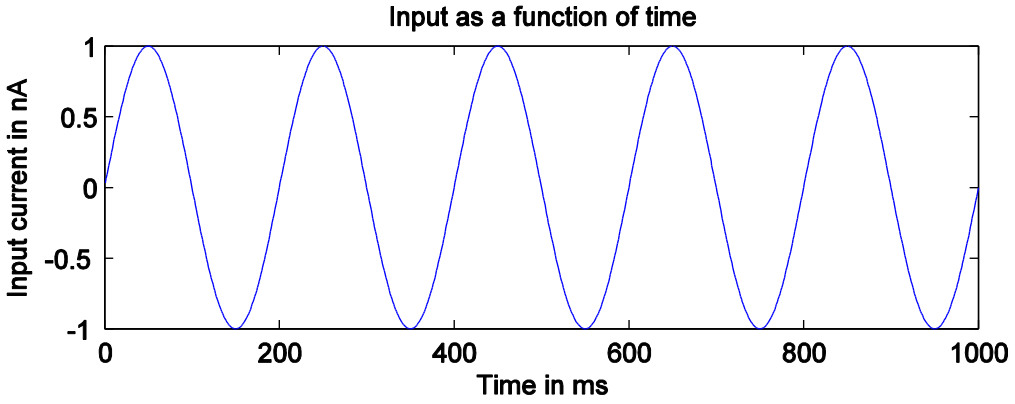
Sinusoid input frequency (Hz)	1	2	5	10	20	50	100
Spike count in 1 sec	29	28	30	30	20	0	0

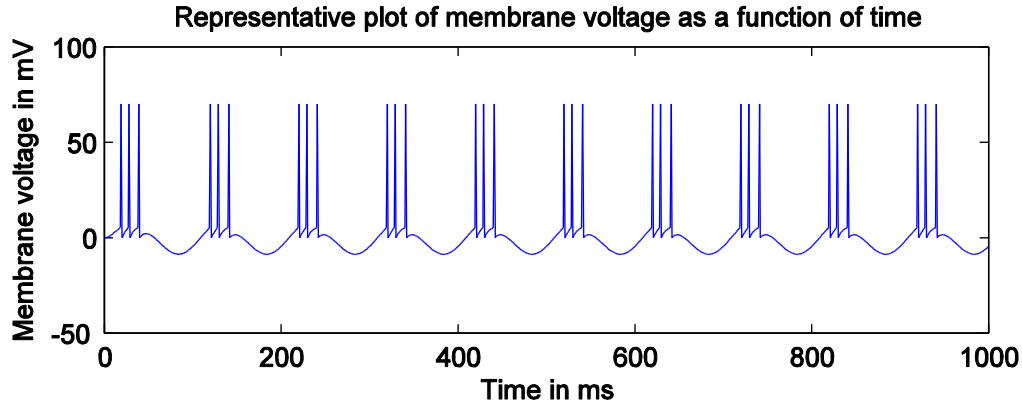
The following plots demonstrate the behavior of the membrane potential generated by the model, along with the sinusoidal inputs that were used as inputs.

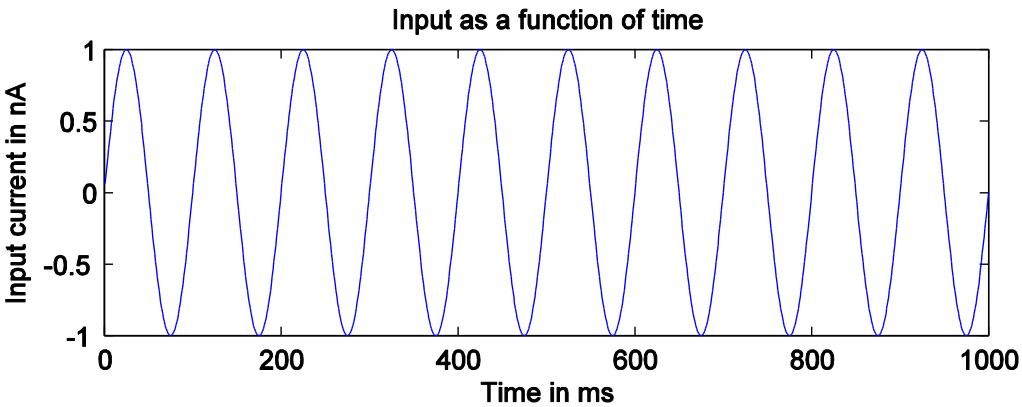

Representative plot of membrane voltage as a function of time
Membrane voltage in mV
100
50
0
-50
0
200
400
600
800
1000
Time in ms
Input as a function of time
Input current in nA
1
0.5
0
-0.5
-1
0
200
400
600
800
1000
Time in ms

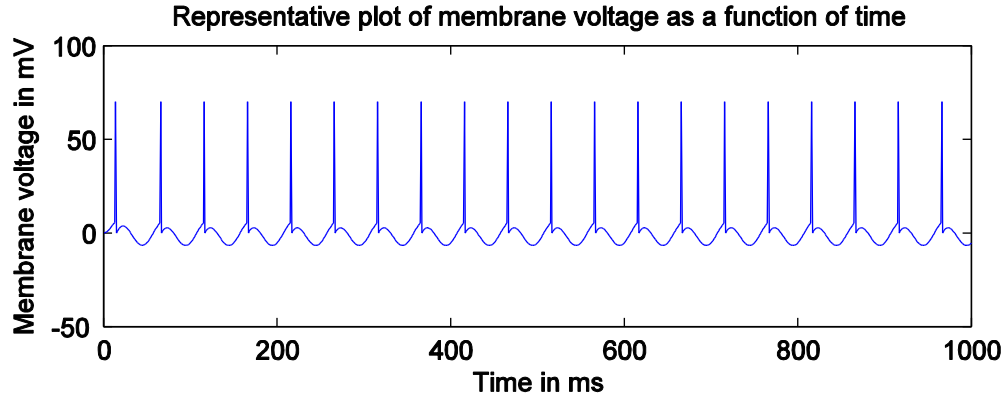

Representative plot of membrane voltage as a function of time
Membrane voltage in mV
100
50
0
-50
0
200
400
600
800
1000
Time in ms
Input as a function of time
Input current in nA
1
0.5
0
-0.5
-1
0
200
400
600
800
1000
Time in ms

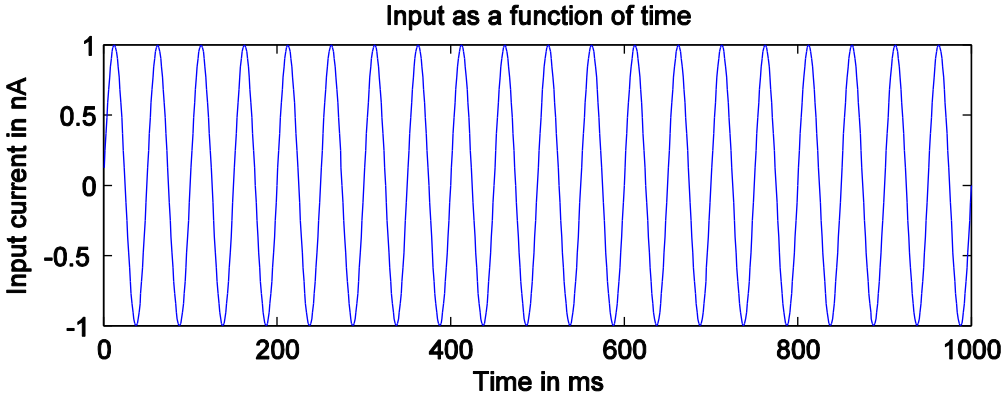

Representative plot of membrane voltage as a function of time
Membrane voltage in mV
100
50
0
-50
0
200
400
600
800
1000
Time in ms

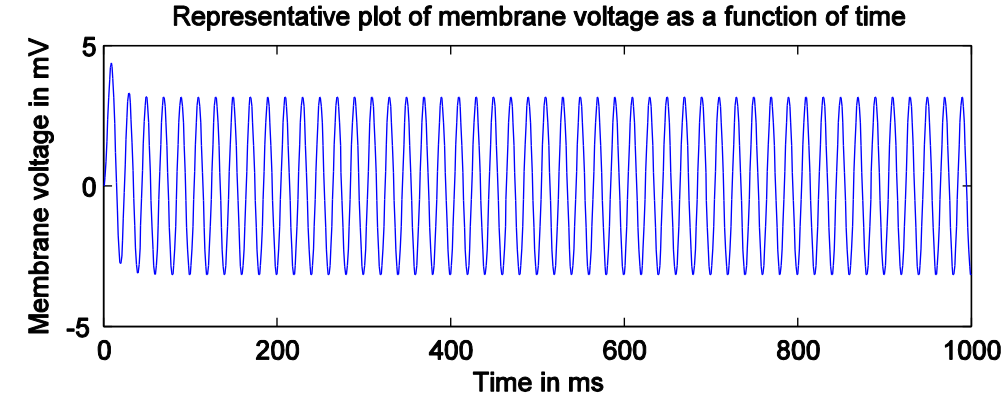

Input as a function of time
Input current in nA
1
0.5
0
-0.5
-1
0
200
400
600
800
1000
Time in ms

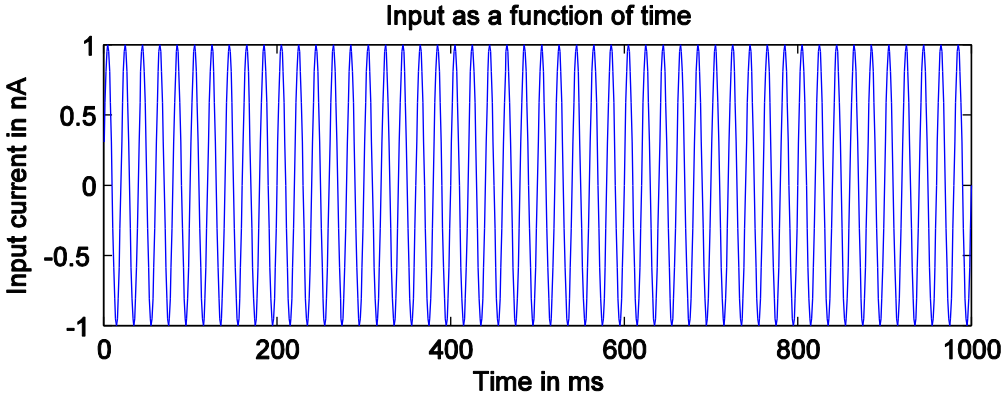

Representative plot of membrane voltage as a function of time
Membrane voltage in mV
100
50
0
-50
0
200
400
600
800
1000
Time in ms

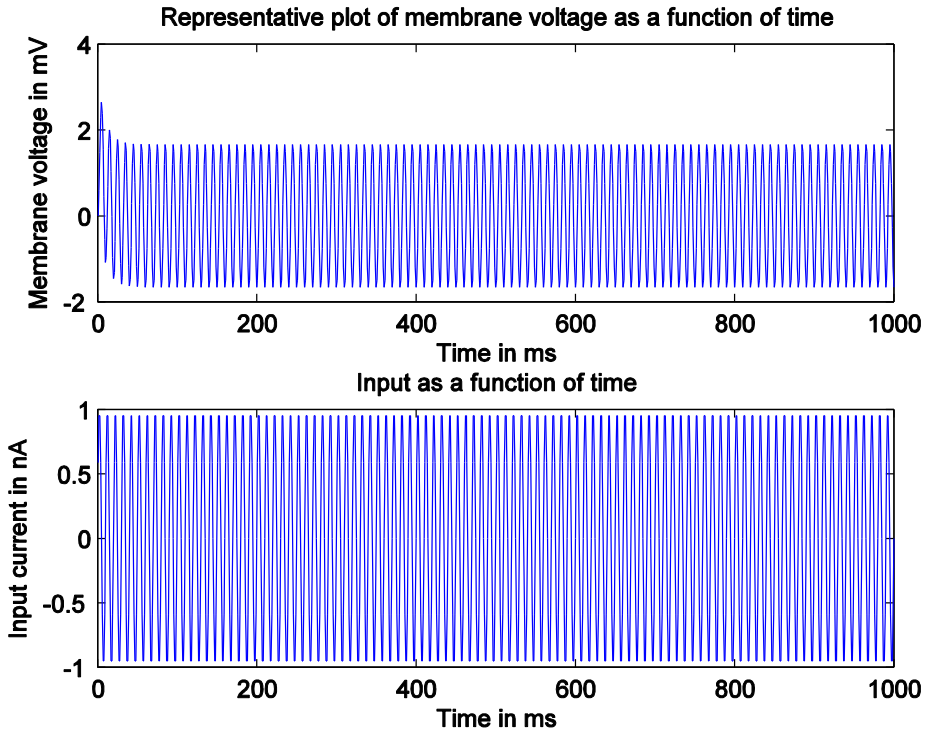

Input as a function of time
Input current in nA
1
0.5
0
-0.5
-1
0
200
400
600
800
1000
Time in ms


Representative plot of membrane voltage as a function of time
Membrane voltage in mV
100
50
0
-50
0
200
400
600
800
1000
Time in ms


Input as a function of time
Input current in nA
1
0.5
0
-0.5
-1
0
200
400
600
800
1000
Time in ms

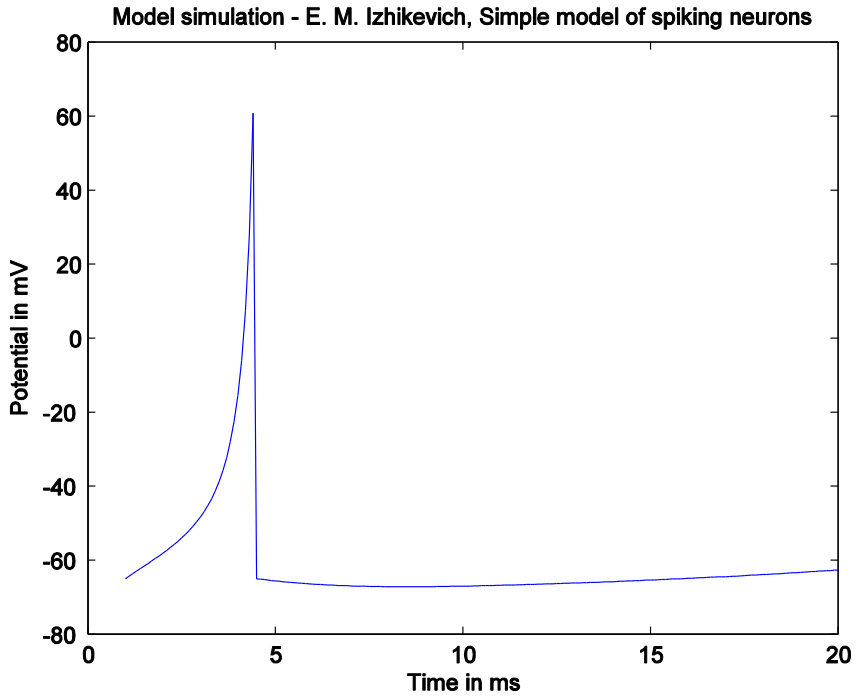

Representative plot of membrane voltage as a function of time
Membrane voltage in mV
5
0
-5
0
200
400
600
800
1000
Time in ms


Input as a function of time
Input current in nA
1
0.5
0
-0.5
-1
0
200
400
600
800
1000
Time in ms

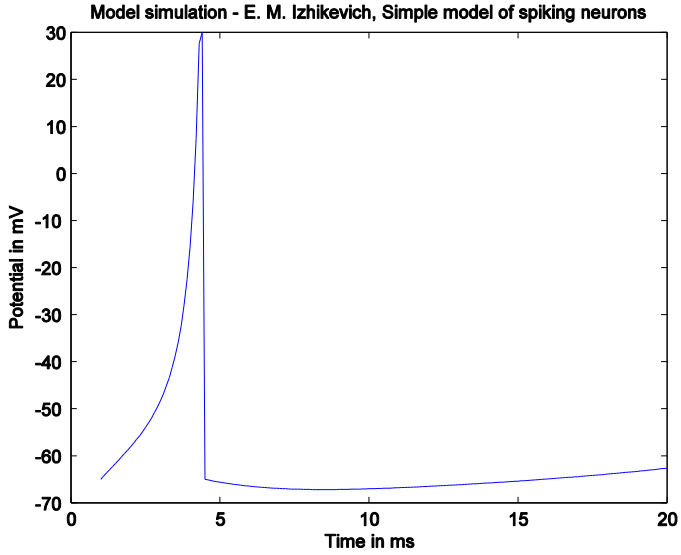


b)

The following is a series of action potentials generated by an step dc-current input (I = 10) over 20ms, as suggested by Figure 2 in the paper. (Izhikevich, 2003)

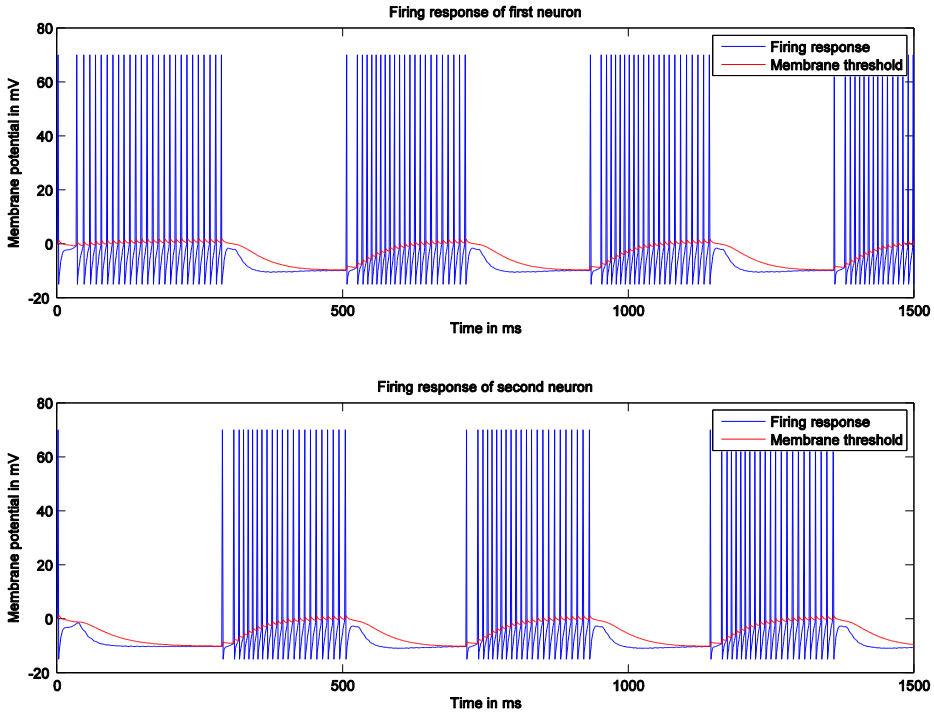


It should be noted that the spike goes to 60mV, but that is just a single value which is above 30, so the model can be modified so that the values above 30 are adjusted to 30. This results in the following plot, which is more in-line with that found in the paper.



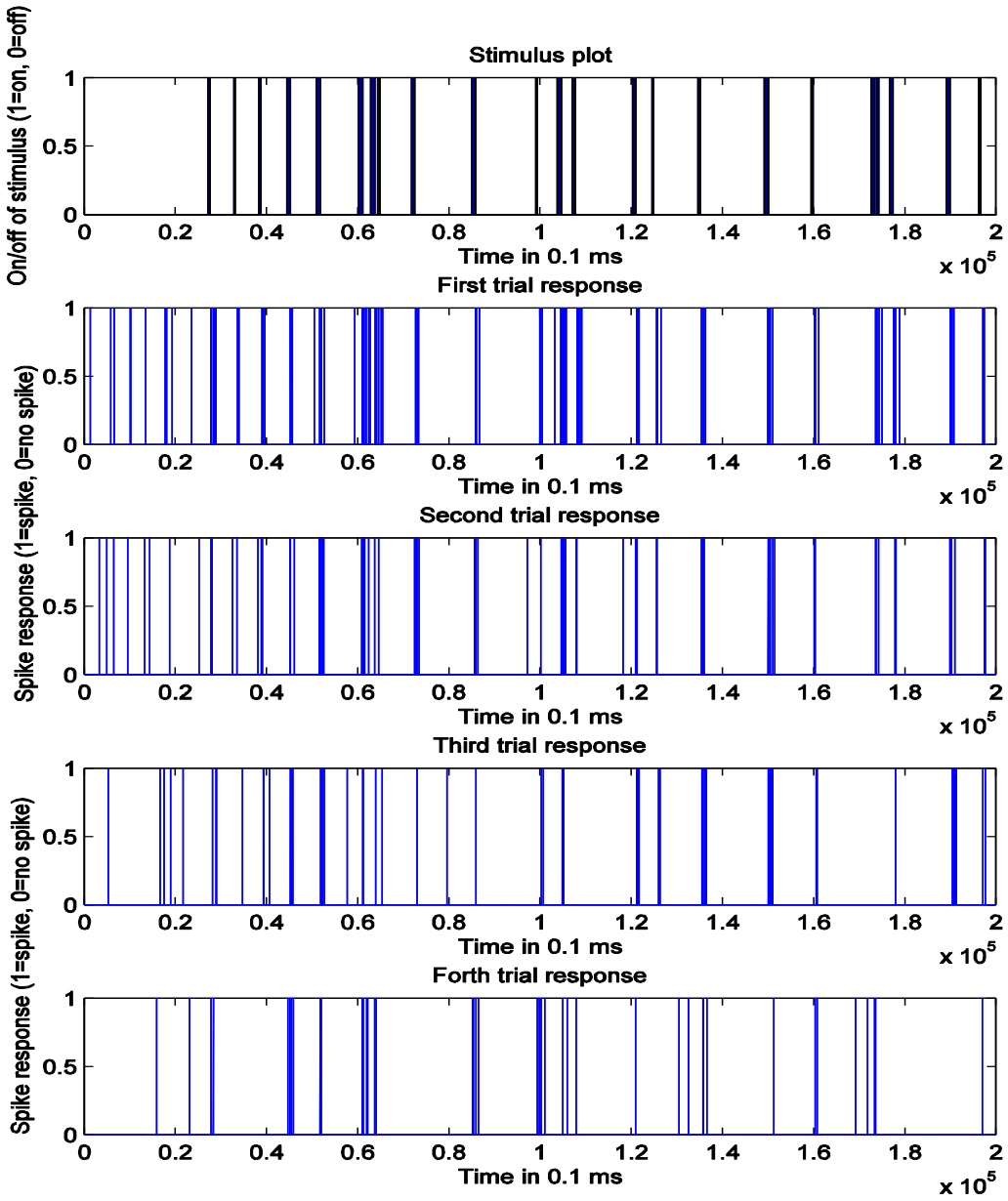
c)

The following plot is of the membrane potential and threshold of the two neurons. The inhibitory relationship between the two neurons can be observed by the neurons firing only when the other neuron is not.



5.

The data found on blackboard needs significant treatment before it becomes usable in plots. For stimulus.txt, it was assumed that the first column of the document indicated the on-times of the stimulus, while the second column indicated the off-times of the stimulus. For spikes.txt, it was assumed that the five trials were recorded immediately after each other, with the first trial being the first 20 seconds of the data, the second trial being between 20-40 secs in the data, and so forth. This enables some data manipulation, the normalized data can be plotted as shown below.



The plot above is very similar to that found within the homework set, therefore should be passable as the input data for the model analysis of the neuron. By combining the instructions given in the homework, the olfactory data analysis methods found online (Geffen et al., 2009), established methods of creating phenomenagical models (Chichilnisky, 2001), and the information given in lectures, following strategy can be used to construct the model.

With the definitions of the matrices as given in the homework, we must minimize:

$$(R - SW)^2$$

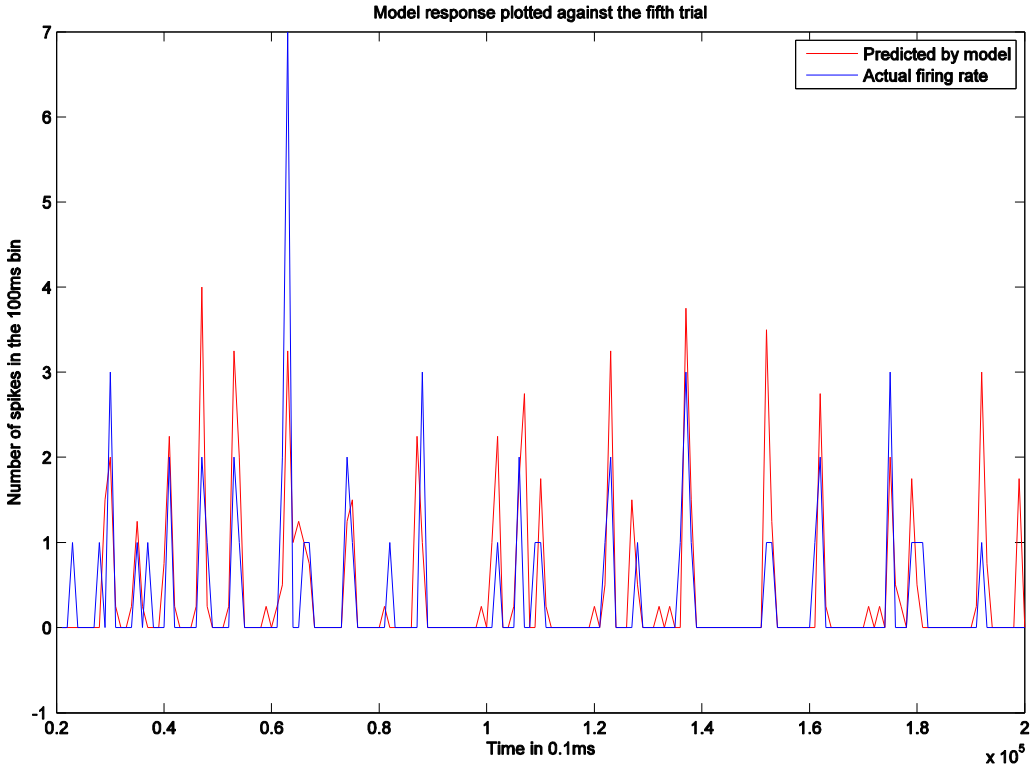
This can be achieved by returning the following:

$$W = psuedoinverse(S) * r$$

Where pseudoinverse is a pre-existing matlab function (as discussed during the lectures), and r is the spiking average between the first 4 trials. The prediction can then be found by doing the following calculation:

$$Predicted\ Response = SW$$

The following plot is the predicted response compared to the total number of spikes in 100ms bins for the 5th trial. The model can predict the spiking pattern.



Question 6 was not completed as it was not yet covered in class.

References

Koch, Christof. *Biophysics of Computation: Information Processing in Single Neurons*. New York: Oxford UP, 1999. Print.

Chichilnisky, E.j. "A Simple White Noise Analysis of Neuronal Light Responses." *Network Network: Computation in Neural Systems* 12.2 (2001): 199-213. Web.

Geffen, Maria N., Bede M. Broome, Gilles Laurent, and Markus Meister. "Neural Encoding of Rapidly Fluctuating Odors." *Neuron* 61.4 (2009): 570-86. Web.

Izhikevich, E.m. "Simple Model of Spiking Neurons." *IEEE Trans. Neural Netw. IEEE Transactions on Neural Networks* 14.6 (2003): 1569-572. Web.