Quanto Perpetual Units

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January 8, 2024

1 Intro

Quanto Perps uses different units to traditional perps. This document lays out those changes for the core variables involved in the system. The basis from this has been taken and adapted from Synthetix Perps - specifically SIP-80 [1].

For the sake of simplicity, we assume in all scenarios in this document that the base asset b is BTC and the quanto asset z is ETH.

2 Overview

There is one fundamental change in quanto perps market, and that is the unit for position size.

In traditional perps, position size is denominated in the base asset b or for our purposes BTC. In quanto perps, position size is denominated in the base asset b divided by the *quanto factor* p_z^e.

$$\frac{b}{p_z^e} = \frac{BTC}{\frac{USD}{ETH}} = \frac{BTC \cdot ETH}{USD}$$

$$\frac{z}{p_e} = \frac{ETH}{\frac{USD}{BTC}} = \frac{BTC \cdot ETH}{USD}$$

This means that position size $\frac{b}{p_z^e}$ is denominated in $\frac{BTC \cdot ETH}{USD}$ instead of BTC.

From this foundation, we can apply all the same classical perps maths, but we get a quanto payout instead of a classical one.

3 Margin and Notional Value

We can start with the implications of this change by looking at it's effect on margin.

Previously we defined margin in the following way:

$$m := \frac{v}{\lambda} = \frac{qp_e}{\lambda_e}$$

$$USD := USD = \frac{BTC \cdot USD}{BTC} = USD$$

This mathematical definition stays the same. However the units have now changed:

$$ETH := ETH = \frac{BTC \cdot ETH}{USD} \cdot \frac{USD}{BTC} = ETH$$

Everything is the same, except as position size is now denominated in $\frac{BTC \cdot ETH}{USD}$ instead of BTC, margin is now demoninated in ETH instead of USD.

So let's inspect the impact this has on notional value v:

$$v := qp = \frac{BTC \cdot ETH}{USD} \cdot \frac{USD}{BTC} = ETH$$

Whereas previously we had:

$$v := qp = BTC \cdot \frac{USD}{BTC} = USD$$

So now notional value is now denominated in ETH instead of USD.

4 Profit and Loss Example

I will now demonstrate with an example how making this simple change leads to a quanto payout.

Imagine a USD/BTC market with *TraderA*. It is Quanto, and the third currency is ETH. The pre-specified rate when entering the contract will be known as the *quanto factor* [2].

Assume the following trade:

$$TraderA \rightarrow long \rightarrow deposit\, 5\, ETH \rightarrow 2\times\, leverage \rightarrow \$20k/BTC \rightarrow \$2k/ETH$$

Let's calculate their position size q and notional value v:

$$q := \frac{m_e \lambda_e}{p_e} = \frac{5 \cdot 2}{20000} = \frac{10}{20000} = 0.0005$$

$$v = qp = 0.0005 \cdot 20000 = 10\, ETH$$

So the traders position size is $0.0005\, \frac{BTC \cdot ETH}{USD}$ and their notional value is $10\, ETH$.

Now we imagine that the price of BTC goes up to \$22k, and the price of ETH goes up to \$4k. Let's start by calculating the new notional value v:

$$v = qp = 0.0005 \cdot 22000 = 11\,ETH$$

Let's calculate the profit and loss r:

$$r = v - v_e = 11 - 10 = 1\,ETH$$

So the trader has made $1\,ETH$ profit. But what is the profit in USD? Let's calculate it:

$$r_{USD} = r \cdot p_z = 1 \cdot 4000 = \$4000$$

So the trader has made \$4000 profit, double what they would have made in a classic perps market, due to the increase in ETH price doubling their winnings.

5 Skew

The change in units for position size q has an impact on skew, the sum of all position sizes:

$$K := \sum_{c \in C} q^c$$

Skew is now denominated in $\frac{BTC \cdot ETH}{USD}$ instead of BTC. This is necessary for skew to work in a Quanto market.

To understand why, we can look at an example with two traders, *TraderA* and *TraderB*, assuming the following trades:

$$TraderA \to long \to 2\,BTC \to \$30k/BTC \to \$1k/ETH$$

$$TraderB \to short \to 2\,BTC \to \$30k/BTC \to \$2k/ETH$$

Imagine that the price of BTC goes up to \$40k. We can calculate the profit and loss of each trader as follows:

$$r^z_{TraderA} = \frac{(2 \cdot \$40k) - (2 \cdot \$30k)}{\$1k/ETH} = \frac{\$80k - \$60k}{\$1k/ETH} = \frac{\$20k}{\$1k/ETH} = +20\,ETH$$

$$r^z_{TraderB} = \frac{(-2 \cdot \$40k) - (-2 \cdot \$30k)}{\$2k/ETH} = \frac{-\$80k + \$60k}{\$2k/ETH} = \frac{-\$20k}{\$2k/ETH} = -10\,ETH$$

Hence we have the net PnL of the market:

$$\text{NetPnL}_z := \sum_{c \in C} r^c_z = 20ETH - 10ETH = +10ETH$$

The net PnL is positive and traders have made money against LPs, 10 ETH in total. However, if we apply the perps skew formula using the old unit (the base unit b or BTC), we get a market skew of zero:

$$K \ := \ \sum_{c \in C} q^c \ = \ Q_L - Q_S = (2\,BTC) + (-2\,BTC) = 0$$

This is false, the skew cannot be zero and net PnL be positive. Traditional perps units do not take into account the *quanto factor*, they presume payout in USD, whereas Quanto Perps pays out in the quanto asset at each individual traders entry price.

The BTC position sizes on both sides of the market are balanced, but LPs are not delta neutral as the *quanto factor* for longs and shorts is different. For a market skew of zero to create a delta neutral environment for LPs, we must use the new units.

Recalculating the position sizes using our new units, we see the market skew is actually positive:

$$q_{TraderA} = \frac{v_e}{p_e} = \frac{60}{30,000} = 0.002$$

$$q_{TraderB} = \frac{v_e}{p_e} = \frac{-30}{30,000} = -0.001$$

$$K \ := \ \sum_{c \in C} q^c \ = \ Q_L - Q_S = 0.002 - 0.001 = 0.001$$

This is why LPs have lost money as the price of BTC went up.

6 Summary of all Unit Changes

To make the changes in units clearer, I have summarised them all below in tables.

6.1 Market and Position Parameters

Symbol	Formula	Unit	Quanto Unit	Description
q	$q := \frac{m_e \lambda_e}{p_e}$	BTC	$\frac{BTC \cdot ETH}{USD}$	Position Size
p	-	$\frac{USD}{BTC}$	$\frac{USD}{BTC}$	Base asset spot price
p_e	-	$\frac{USD}{BTC}$	$\frac{USD}{BTC}$	Base asset spot price at position entry time
p_z	-	n/a	$\frac{USD}{ETH}$	Quanto asset spot price
v	$v := qp$	USD	ETH	Notional Value
r	$r := v - v_e$	USD	ETH	Profit and Loss
C	-	-	-	The set of all positions in a market
b	-	BTC	BTC	The base asset which price p refers to
z	-	n/a	ETH	The quanto asset which price p_z refers to
Q	$Q := \sum_{c \in C} \lvert q^c \rvert$	BTC	$\frac{BTC \cdot ETH}{USD}$	Market Size
V_{max}	$p\ Q_{L\&S} \leq V_{max}$	USD	ETH	Open Interest Cap
K	$K := \sum_{c \in C} q^c$	BTC	$\frac{BTC \cdot ETH}{USD}$	Market Skew
λ_{max}	-	-	-	Maximum Leverage
$skewScale$	-	-	-	Skew scaling denominator constant

6.2 Leverage and Margins

Symbol	Formula	Unit	Quanto Unit	Description
λ	$\lambda := \frac{v}{m}$	-	-	Leverage
m_e	$m_e := \frac{v_e}{\lambda_e} = \frac{q\ p_e}{\lambda_e}$	USD	ETH	Initial Margin
m	$m := max(m_e + r + f, 0)$	USD	ETH	Remaining Margin

6.3 Exchange Fees

Symbol	Formula	Unit	Quanto Unit	Description
ϕ_t	-	-	-	Taker fee rate
ϕ_m	-	-	-	Maker fee rate

6.4 Skew Funding Rate

Symbol	Formula	Unit	Quanto Unit	Description
W	$W \ := \ \frac{K}{skewScale}$	BTC	$\frac{BTC \cdot ETH}{USD}$	Proportional skew
W_{max}	-	BTC	$\frac{BTC \cdot ETH}{USD}$	Max funding skew threshold
i_{max}	-	-	-	Maximum funding rate
i	$i \ := \ clamp(\frac{-W}{W_{max}}, -1, 1) \ i_{max}$	% per day	% per day	Instantaneous funding rate
t_{last}	-	$secs$	$secs$	Skew last modified
F	$F_0 \ := \ 0$	$\frac{USD}{BTC}$	$\frac{USD}{BTC}$	Cumulative funding sequence
u	$u \ := \ i \ (now - t_{last})$	-	-	Unrecorded base funding
F_{now}	$F_{now} \ := F_n + p \ u$	$\frac{USD}{BTC}$	$\frac{USD}{BTC}$	Unrecorded cumulative funding
j	$j \leftarrow 0$	-	-	Last-modified index
f	$f^c \ := \begin{cases} 0 & \text{if opening } c \\ q^c(F_{now} - F_{j^c}) & \text{otherwise} \end{cases}$	USD	ETH	Accrued position funding

6.5 Aggregate Debt Calculation

Symbol	Formula	Unit	Quanto Unit	Description
Δ_e	$\Delta_e := \sum_{c \in C} m_e^c - v_e^c - q_e^c F_{j^c}$	USD	ETH	Aggregate position entry debt correction
D	$D := max(K\ (p + F_{now}) + \Delta_e, 0)$	USD	ETH	Market Debt

6.6 Liquidations and Keepers

Symbol	Formula	Unit	Quanto Unit	Description
$liqMargin$	$max(q \cdot p \cdot rFee, D) + q \cdot p \cdot rBuffer$	USD	ETH	Liquidation margin
$rBuffer$	-	-	-	Liquidation buffer ratio
$rFee$	-	-	-	Liquidation fee ratio
D	-	USD	?	Minimal keeper incentive
$p_{liqApprox}$	$p_e - \frac{m_e - liqMargin}{q} - (F_n - F_j)$	$\frac{USD}{BTC}$	$\frac{USD}{BTC}$	Approximate liquidation price

7 Conclusion

There are no changes in math required in Quanto Perps, everything works exactly the same as in classical perps, however by changing the units for different variables, we get a different payout graph.

The key change is the change in unit for position size q from the base asset b to the base asset b divided by the quanto factor p_z^e. This means that position size is now denominated in $\frac{BTC \cdot ETH}{USD}$ instead of BTC.

This neatly expresses how each position is now exposed to three assets instead of two - the base asset b, the quanto asset z and the currency USD. Before a user's position only exposed them to the base asset b and the currency USD.

References

[1] Anton Jurisevic, Jackson Chan, Kain Warwick, and Arthur Deygin. Sip-80: Synthetic futures. *Synthetix Improvement Proposals*, 2020. URL `https://sips.synthetix.io/sips/sip-80/`.

[2] Uwe Wystup. Quanto options. *MathFinance AG*, 2008. URL https://mathfinance.com/wp-content/uploads/2017/06/wystup_quanto_eqf.pdf.