A Logical Introduction to "An Introduction to Non-Classical Logic" by Graham Priest, by Tommy Schupp

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For Professor Lansky, Chris Barcy, and others

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1 Introduction

My goal with this paper is to understand the ideas of semantic and deductive validity from "An Introduction to Non-Classical Logic" by *Graham Priest*. I hope that this paper is interesting and makes the reader excited about non-classical logic!

2 What is a logic?

2.1 Purpose of a Logic

"The point of logic," Priest argues, "is to give an account of the notion of validity: what follows from what" (1.1.3). However, there are different such accounts, so there are different logics.

Abstractly, a logic (or a *system of logic*) is a language-game that has a deductive system of synthesizing propositions and a semantic system of evaluating propositions[1]. Intuitively, a logic is a tool that people create to practically reason about their experiences. In this intuitive sense, a logic provides the rules to make new statements that are valid from old ones and to verify that new statements are valid.

The types of propositions that we make, and what we can deduce from them, are dependent on the logic that we are "playing" at a given moment.

2.2 Notions of validity

But what does it mean for a statement to follow from another? What is validity?

There are two notions of validity: *deductive* (or *proof-theoretic*) validity and *semantic* validity. These notions come from the systems that compose a logic.

[1]Technically, a logic only needs one of these systems.

2.2.1 Deductive Validity

Let's first take a look at deductive validity. As noted above, a logic has a system of synthesizing propositions, called a deductive system. The deductive system is the set of rules for manipulating operations between propositions. If a new proposition is deductively valid, it can be synthesized from existing propositions using the rules of the logic's deductive system. The symbol to show that a statement is deductively valid in a logic is '$\vdash$,' and should be read as "yields." We prefix '$\vdash$' with given propositions, and we suffix '$\vdash$' with the proposition we are trying to determine the validity of.

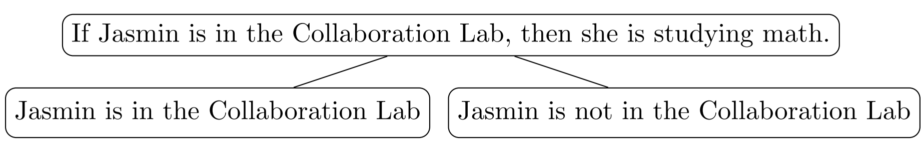
In summary, a statement is deductively valid in a logic if it can be proven using the rules of operation/symbol manipulation in that logic. As Priest notes, what distinguishes different logics in deductive validity is the *rules* they use for operation/symbol manipulation.

Without defining a logic (which would require going down a long tangent), we will look at an example of deductive validity in the natural logic of English. Suppose we know that *if Jasmin is in the Collaboration Lab, then she is studying math.* Furthermore, suppose we know that *Jasmin is in the Collaboration Lab.* We want to show that the statement *Jasmin is studying math* is deductively valid.

We represent this situation symbolically like this:
If Jasmin is in the Collaboration Lab, then she is studying math; Jasmin is in the Collaboration Lab $\vdash$ *Jasmin is studying math.*

And in English like this:
If Jasmin is in the Collaboration Lab, then she is studying math; Jasmin is in the Collaboration lab yields that *Jasmin is studying math.*

To show that this statement is deductively valid (that we are correct above in using $\vdash$ instead of $\nvdash$), we will use the intuitive English rules for manipulating propositions. From the first statement, *if Jasmin is in the Collaboration Lab, then she is studying math*, we can deduce that either Jasmin is outside the Collaboration Lab *or* she is inside the Collaboration lab and studying math.



Now consider the second statement, *Jasmin is in the Collaboration Lab.* English does not allow Jasmin to be both in the Collaboration Lab and not in the Collaboration Lab. Applying that *Jasmin is in the Collaboration Lab*, we can safely ignore the case where Jasmin is outside the Collaboration Lab. This leaves us with the case that Jasmin is inside the Collaboration Lab and studying math, so we deduce that Jasmin is studying math.

If Jasmin is in the Collaboration Lab, then she is studying math.

Jasmin is in the Collaboration Lab | ~~Jasmin is not in the Collaboration Lab~~

Jasmin is studying math.

It's important to note that in this example, the statement *Jasmin is studying math* is deductively valid *only because* we are able to obtain it using the applicable deductive system (English). That is, it is deductively valid only because it is *derivable*.

2.2.2 Semantic Validity

Now, let's look at semantic validity. A logic has a system of evaluating propositions, called a semantic system. If a new proposition is semantically valid, there is no combination of truth values of parameters of existing propositions that makes the existing propositions true, but the new proposition false. Priest refers to semantic validity as "truth preservation." Note that the existing propositions do not ever have to be true for an argument to be semantically valid. Intuitively, semantic validity means that if we toggle the facts of the matter in a way that preserves the truth of the larger situation, the consequence remains true. The symbol to show that a statement is semantically valid in a logic is '$\models$,' and should be read as "entails." We prefix '$\models$' with given propositions, and we suffix '$\models$' with the proposition we are trying to determine the validity of.

We will illustrate this, but it should be noted that the example will have many moving parts. If it were any simpler, we would risk not sufficiently elucidating semantic validity.

Again, using the natural logic of English, we will demonstrate semantic validity. Suppose we know that *either Eli is a physics major, or he got an A on the calculus final*. Next, suppose we know that *either Eli got an A on the calculus final, or Eli is the vice-president of Not Math Club*. Finally, suppose we know that *either Eli is not a physics major, or Eli is not the vice-president of Not Math Club, or Eli got an A on the calculus final*. Note that all of these disjunctions are inclusive; that is, Eli could still be a physics major *and* have gotten an A on the calculus final. We want to show that the statement *Eli got an A on the calculus final* is semantically valid.

To do that, we will toggle the truth values of the parameters (Eli's major, his grade on the calculus final, and his position in Not Math Club) in a way that preserves the truth of the given statements. If there is no combination of truth values that make the given statements true, but that Eli does not have an A on the calculus final, then the statement is semantically valid.

We will construct a truth table for each of our given statements. Please excuse the abbreviated column titles.

Physics Major	A on Calc Final	(Physics Major OR A on Calc Final)
T	T	T
T	F	T
F	T	T
F	F	F

So, the possible facts of the matter that preserve the truth of the first given statement are:

- Eli is a physics major and got an A on the calculus final.
- Eli is a physics major and did not get an A on the calculus final.
- Eli is not a physics major and did get an A on the calculus final.

A on Calc Final	NMC VP	(A on Calc Final OR NMC VP)
T	T	T
T	F	T
F	T	T
F	F	F

So, the possible facts of the matter that preserve the truth of the second given statement are:

- Eli got an A on the calculus final and is the vice president of NMC.
- Eli got an A on the calculus final and is not the vice president of NMC.
- Eli did not get an A on the calculus final and is the vice president of NMC.

Phys. Maj.	NMC VP	A on Calc	(NOT Phys. Maj. OR NOT NMC VP, OR A on Calc)
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

So, the possible facts of the matter that preserve the truth of the last given statement are any *except* for if Eli is a physics major, the vice president of the math club, and did not get an A on the calculus final.

Combining the information from the three tables, the possible facts of the matter that preserve the truth of all three given statements are

Phys. Maj.	NMC VP	A on Calc
T	T	T
T	F	T
F	T	T
F	F	T

By simply looking at this table, we can see that there is no combination of truth values that make the given statements true, but that Eli does not have an A on the calculus final. The above four combinations of parameters are all of the combinations where the given statements are true, and in all of them, the statement we are trying to prove is true. Therefore, we have verified that the statement *Eli got an A on the calculus final* is semantically valid.

2.3 Soundness and Completeness

In the previous section, we saw that there are two ways for a statement to follow from another, that is, for a statement to be valid. But what if these notions of validity are the same?

We will now look at the soundness and completeness properties of a logic. If a logic is sound and complete, it means that its deductive validity and semantic validity are intrinsically tied to each other; they are two sides of the same coin.

2.3.1 Sound

No, not like *noise*. Rather, a logic being sound is used in much the same way that we could describe a table as being sound. For a logic to be sound, it means that every deductively valid statement is also semantically valid. That is, if a statement can be proved using $\vdash$, then it can be proved using $\models$.

Intuitively, sound means that we cannot deduce ourselves into a false statement, holding the given statements true.

2.3.2 Complete

For a logic to be complete, it means that every semantically valid statement is deductively valid. That is, if a statement can be proved using $\models$, then it can be proved using $\vdash$.

Intuitively, complete means that if we have a statement that is semantically valid, that we have the tools to synthesize it with our deductive system.

As put wonderfully in a Stack Exchange answer[2]: "soundness is the property of only being able to prove "true" things, and completeness is the property of being able to prove all true things."

3 Classical Logic

3.1 Interpretation

Instead of using the intuitive phrase 'facts of the matter,' let's use a more rigorous method for what we mean. Let ν be a function, called an *interpretation*, that accepts a proposition and assigns it True or False. Complex propositions in an interpretation, ν, are entirely dependent on the truth values of the propositional parameters. An interpretation is a dictionary of propositions: for a given

[2] https://philosophy.stackexchange.com/a/6993

key/parameter/fact of the matter, an interpretation gives its truth value. The rules for an interpretation in classical logic are as follows:

- $\nu(\neg A)$ is true when $\nu(A)$ is false, and false otherwise.
- $\nu(A \wedge B)$ is true when both $\nu(A)$ and $\nu(B)$ are true, and false otherwise.
- $\nu(A \vee B)$ is true when $\nu(A)$ is true or $\nu(B)$ is true, and false otherwise.
- $\nu(A \supset B)$ is true when $\nu(A)$ is false or $\nu(B)$ is true, and false otherwise.
- $\nu(A \equiv B)$ is true if $\nu(A) = \nu(B)$, and false otherwise.

To be clear, there are many different interpretations that all follow the same rules of interpretation. Any (consistent) combination of the truth values of the propositional parameters can be plugged into these rules of interpretation, and we will still have the same logic. That is, Eli getting an A on the calculus final or Eli getting an F on the calculus are different interpretations, but the interpretations will follow the same rules, and thus still be the same logic.

The rules of interpretation in a logic give the tools to determine semantic validity. There is another method, tableaux, that is used to determine deductive validity.

4 Conclusion: Can We Have One Without The Other?

Intuitively, it seems weird that we could have a logic that is complete but not sound, or vice versa. However, this is much more common than one would think. My goal is to write 2 or 3 more papers like this (not over winter break though!) that go into modal and many-valued logics, also that are more rigorous, and hopefully be able to fully explain something like:

4.1 Gödel's Incompleteness Theorem

Gödel proved that many logics, namely, logics strong enough to have information about the natural numbers in them, are incomplete yet consistent.

Thanks for reading!