Student name: Mai Minh Hieu

ID: 210049

# Dynamic Macroeconomics - Problem Set 1

Question 1:

a. Matrix form of the time series model:

$$\begin{bmatrix} a0 \\ B0 \\ 0 \end{bmatrix} = \begin{bmatrix} a1 & a2 & 0 \\ b3 & b1 & b2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ c_{t-1} \\ c_{t-2} \end{bmatrix} + \begin{bmatrix} \in_t \\ v_t \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

b. The model in AR (1) matrix form:

$$z_t = Az_{t-1} + C\xi_t$$

$$\Rightarrow \begin{bmatrix} a0 \\ b0 \\ 0 \end{bmatrix} = \begin{bmatrix} a1 & a2 & 0 \\ b3 & b1 & b2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ c_{t-1} \\ c_{t-2} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \in_t \\ v_t \end{bmatrix}$$

Question 2:

**a. Convert to Matrix T**

**From the diagram:**

In Subprime (S) state:

- 90% chance staying ($P_{S->S} = 0.9$)
- 10% chance of moving to average ($P_{S->A} = 0.1$)

In Average (A) state:

- 20% chance of moving to subprime ($P_{A->S} = 0.2$)
- 65% chance of staying average ($P_{A->A} = 0.65$)
- 15% chance of moving to prime ($P_{A->P} = 0.15$)

In Prime (P) state:

- 5% chance of moving to subprime ($P_{P->S} = 0.05$)
- 10% chance of moving to average ($P_{P->A} = 0.1$)
- 85% chance of staying prime ($P_{P->P} = 0.85$)

Thus, the transition matrix T is:

$$T = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0.65 & 0.2 & 0.15 \\ 0.85 & 0.15 & 0.05 \end{bmatrix}$$

- **Validity check:**

**1. All elements are probabilities (between 0 and 1)**

**2. Each row sums to 1, since:**

0.90 + 0.10 + 0.00 = 1.00

0.20 + 0.65 + 0.15 = 1.00

0.05 + 0.10 + 0.85 = 1.00

Since both conditions hold, this is a valid transition matrix.

b. Explain the diagram:

Imagine yourself as a borrower.

Your wealth levels will be classified by three levels:

- Subprime (High Risk) – Borrowers are struggling financially.
- Average (Medium Risk) – Borrowers are doing okay, but there's room for improvement.
- Prime (Low Risk) – Borrowers are financially stable and trustworthy.

Each period, the credit rating can change depending on how you manage your money:

- In the "Subprime" category (bad credit), there's a 90% chance you'll stay there next month, but a 10% chance you'll improve to "Average".
- If you're in the "Average" category, you have a 65% chance of staying there, a 20% chance of falling back to "Subprime", and a 15% chance of improving to "Prime".
- If you're in the "Prime" category (excellent credit), you have an 85% chance of staying there, but a 10% chance of dropping to "Average" and a 5% chance of falling all the way to "Subprime".

We can see that it's hard to move up but easy to fall. If you have bad credit, it's tough to escape. If you have great credit, it's easier to keep it, but not guaranteed as 100%.

Question 3:

a.

$$\pi_2 = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}$$

$$\pi_n = p\begin{bmatrix} \pi_{n-1} & 0 \\ 0 & 0 \end{bmatrix} + (1-p)\begin{bmatrix} 0 & \pi_{n-1} \\ 0 & 0 \end{bmatrix} + q\begin{bmatrix} 0 & 0 \\ 0 & \pi_{n-1} \end{bmatrix} + (1-q)\begin{bmatrix} 0 & 0 \\ \pi_{n-1} & 0 \end{bmatrix}$$

b. Solved on file code
c. Solved on file code
d. Solved on file code

Question 4:

a. The operation cost of a tank is given by $c(z_t)$, consist of maintenance cost $a(z_t)$ and breakdown cost $b(z_t)$.

$$c(z_t) = a(z_t) + b(z_t)$$

If the military buys a new tank, the cost is:

$$D_t + c(0)$$

Final Utility function (captures the military's per-period cost minimization problem):

$$U(z_t) = -\min\{a(z_t) + b(z_t), D_t + c(0)\}$$

b. We have:

- State variable: $z_t$: Age of tank at the time *t.*
- Decisions: the military can choose whether to:
  i. Keep the tank and incur operational costs.
  ii. Buy a new tank, resetting age *t* to 0.
- Exogenous shock ($\epsilon_t$): represents random cost variations (e.g., economic conditions affecting tank prices or maintenance costs). We are assuming $\epsilon_t$ is independent over time.
  - The military's new per-period cost function is:

$$c(z_t) = a(z_t) + b(z_t) + \epsilon_t$$

- If the military buys a new tank, the cost is:

$$D_t + c(0) + \epsilon_t$$

We have the Bellman Equation represents the minimum expected discounted cost:

$$V(z_t) = \min\{a(z_t) + b(z_t) + \beta E[V(z_{t+1})], D_t + c(0) + \beta E[V(0)]\}$$

Where:

- $V(z_t)$: Value function, representing the total expected cost.
- $\beta$: Discount factor

- First term: Keep the tank, pay maintenance/breakdown costs, and move to $z_{t+1}$
- Second term: Buy a new tank at cost $D_t$ and reset $z_{t+1}$ to 0.

c. Transition Probabilities of $z_t$

The transition probabilities describe the likelihood of moving from one state (tank age) to another state (tank age $z_t+1$) in the next period. We need to consider how the military's decision (control) influences the transition of the state variable $z_t$:

1. If the military maintains the tank:
   As the age increases by 1 each period, so:

$$P(z_{t+1}) = z_t+1 \mid \text{maintain}) = 1$$

Transition probability: Age of the tank simply increases by 1.

2. If the military buys a new tank:

The age resets to 0:

$$P\,(z_{t+1} = 0|\text{buy new tank}) =1$$

Transition probability: The age is reset to 0.

**Do the Transition Probabilities Depend on the States, Controls, or Both?**

States (Age $z_t$): The state $z_t$ represents the age of the tank, and it influences the maintenance decision, but it does not directly affect the probability of transition itself. The transition only depends on whether the military decisions: maintain or replace the tank.

Controls (maintain or replace): The decision (whether to maintain or replace the tank) directly determines the transition probabilities:

- Maintain: The state moves from $z_t$ to $z_t+1$
- Replace: The state $z_t$ become 0

**If a tank is worn down**

If the tank is old and may be worn down (increasing breakdown costs), the military may opt to buy a new tank. The probability of transitioning to a new tank (i.e., age resetting to 0) will increase if the cost of maintenance and breakdown exceeds the cost of a new tank.

Question 5:

a. Background of the chosen economy (Ho Chi Minh City):

**GDP Contribution:** HCMC contributes about 20% to Vietnam's total GDP. It has one of the highest GDP per capita rates in the country.

The city has a strong industrial base, including textiles, food processing, and electronics. The industrial sector has experienced significant growth, especially in export-oriented manufacturing. HCMC is also home to a thriving services sector, including finance, retail, and tourism. The city's financial sector has grown rapidly, with numerous banks, financial institutions, and international companies establishing operations.

The history of the development of HCMC can be divided into three periods:

- Pre-Reform Era: The period before the Đổi Mới (Renovation) reforms in 1986, when HCMC had a predominantly state-run economy. Growth was slow, and the focus was on heavy industries and collective agriculture.

- Post-Reform Era (1986 onwards): HCMC transitioned to a more industrialized, export-oriented economy as the economy shifted to market-oriented reforms in the 1980s. The city became the hub for foreign direct investment (FDI) and economic growth in Vietnam.

- Modern HCMC: Today, the city is a dynamic metropolis, contributing to about 30-35% of Vietnam's GDP. It has a diverse economy with strong sectors in manufacturing, services, technology, and trade.

b. The Role of Foreign Direct Investment (FDI) in Economic Growth in Ho Chi Minh City (HCMC)
Introduction:
Foreign Direct Investment (FDI) plays a crucial role in the economic development of Ho Chi Minh City (HCMC), acting as a major driver of growth, industrialization, and modernization. FDI not only provides capital but also brings in technology, management expertise, and market access, which contributes to the expansion of both the industrial and services sectors in HCMC.

**There are several key factors that affect growth through FDI:**

- **Capital Formation:** FDI is an essential source of capital in HCMC, by directly contributing to the growth of physical capital in the economy. Many multinational companies (MNCs) have invested in industries such as electronics, automobile manufacturing, and real estate development.

  **Technology Adopted:** FDI often leads technology transfer, where local firms can learn from the advanced technology and practices implemented by foreign companies.

  **Labor Productivity and Skill Development:** The inflow of FDI also boosts labor productivity, by introducing new technologies and innovations. Moreover, the FDI oriented industries often require skilled labor, enhancing the demand and motivation for the workforce to continually upskilled to meet the needs of foreign investors.

  **Infrastructure Development:** To effectively attract FDI, Government often improves physical infrastructure, such as roads, ports, and telecommunications, to facilitate foreign companies' operations. This infrastructure development has a multiplier effect, benefiting other businesses and boosting overall productivity.

  **Potential mechanism for Policymakers to stimulate or maintain growth:**

- Improving business environment: Including reform regulations, simplifying business processes, reducing bureaucracy to encourage and provide legal protection for companies to operate without excessive restrictions.
- Investment in infrastructure: Increase capacity of production for businesses, enhance the city's competitiveness and appeal to both domestic and foreign investors.
- Investment in Education and Skill Development: Including education and vocational training programs, to make the current workforce meet the needs of advanced industries, make the human capital availability more attractive to foreign enterprises.
- Expanding trade agreements: Ensuring that foreign investors have better access to regional and global markets can incentivize further FDI inflows.

c. We are incorporating the **education investment** mechanism into the stochastic growth model:

   **Explanation of the mechanism on growth model:**

- Investment in education $E_t$ directly enhances human capital $h_t$, which increases labor productivity and therefore output. The more investment in education, the higher the human capital stock, leading to greater output per worker
- As human capital improves, the labor force becomes more productive, facilitating the transition to more knowledge-intensive industries, ensuring growth, and making the economy foster long-term growth.

**State variables:**
$k_t$: Capital stock at time $t$
$h_t$: Human capital stock at time $t$, influenced by investments in education.
$A_t$: Total factor productivity (TFP) at time $t$, subject to stochastic shocks

**Decision Variables:**
$I_t$: Investment in physical capital at time $t$
$E_t$: Investment in human capital (education) at time $t$, which improves $h_t$

**Human capital accumulation:**
Human capital $h_t$ evolves according to the following equation:

$$h_{t+1} = (1 - \delta h)h_t + \eta E_t$$

Where:
$\delta_h$: The depreciation rate of human capital.
$\eta$ : The parameter indicating the productivity of education investment in enhancing human capital.
This equation shows how human capital builds over time due to investment in education, making the labor force more productive.

**Capital Accumulation:**
Capital accumulation remains the same as in the standard model:

$$k_{t+1} = (1-\delta)k_t + I_t$$

Where:
$\delta$ is the depreciation rate of capital.
**Production Function**
Output $Y_t$ is produced using Cobb-Douglas production function:

$$Y_t = A_t\, k_t^a\, h_t^B$$

Where:
α and $\beta$ represent the output elasticities of capital and human capital.

**Productivity Shock**
The productivity $At$ follows a stochastic process, typically modeled as an i.i.d. random variable:

$$A_t = A_0\,(1 + \epsilon_t)$$

**Objective Function (Lifetime Utility)**
The central planner maximizes lifetime utility, which is the sum of logarithmic utility over consumption $c_t$

$$V(k_t, h_t, A_t) = \max_{I_t, E_t} E\left[\sum_{t=0}^{\infty} \beta^t \ln(c_t)\right]$$

**Note: I used Latex (Overleaf) for this equation because I cannot add it on Word's Equation*

**Recursive Bellman Equation**

The recursive form of the Bellman equation is:

$$V(k_t, h_t, A_t) = \max_{I_t, E_t}\left[\ln(A_t k_t^{\alpha} h_t^{\beta} - I_t - E_t) + \beta E[V(k_{t+1}, h_{t+1}, A_{t+1})]\right]$$

**Note: I used Latex (Overleaf) for this equation because I cannot add it on Word's Equation*

Where:

The state evolves as:

$$k_{t+1} = (1-\delta\ )k_t + I_t$$

$$ht+1 = (1-\delta\ _h)h_t+\eta E_t$$

The expectation $E\ [V\ (k_{t+1},\ h_{t+1},\ A_{t+1})]$ accounts for the future value, incorporating the stochastic shock $A_t$

*d.* Solved using code

e. Specifically, we will compare the baseline results with a policy scenario where the investment in education is **doubled**. This will help us understand how an increase in education investment can affect economic growth, capital accumulation, and human capital accumulation. The baseline model assumes a fixed investment ratio in education of $\eta = 0.1$. In the counterfactual policy, we will double the investment in education, setting $\eta$=0.2. Our expected outcome is the higher level of education investment will improve the quality of labor, therefore increasing human capital and leading to higher economic output. [Solved using code]