SE 276A/MAE 232A Finite Elements in Solid Mechanics I **J. S. Chen**
Fall 2024

Final Project. Due December 6, 2024

Consider heat conduction in a 2-dimensional plate with geometry given in Fig.1. The *isotropic* thermal conductivity of the plate is $k=50$ *W/(m °C)*, the heat input per area is $Q=\sin\left(\frac{\pi x}{2}\right)\sin\left(\frac{\pi y}{2}\right)$ *W/m*^{3}, the boundaries Γ^3 and Γ^4 are insulated ($h=0$ *W/m*^{2}), and the boundaries Γ^1 and Γ^2 are prescribed with zero temperature ($u=0$ *°C*).

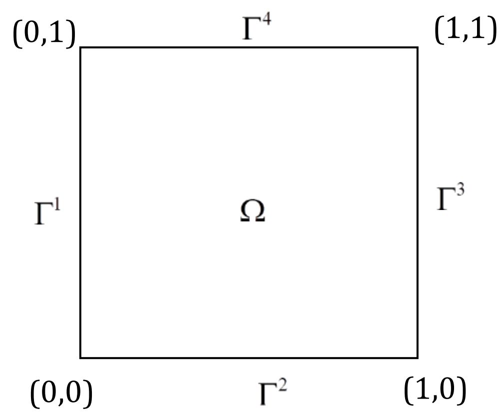

Figure 1. Problem Domain (unit: *m*)

The analytical solution of this problem is $u(x,y)=\frac{2}{50\pi^2}\sin\left(\frac{\pi x}{2}\right)\sin\left(\frac{\pi y}{2}\right)$.

(1) Obtain finite element solutions u^h <u>using 8x8, 16x16, and 32x32 4-node elements</u>, and compare the finite element solutions u^h with the analytical solution u along the line $x=0.5$ in a plot.

(2) Compare the finite element solutions $u^h_{,y}$ obtained by using the 3 discretizations with the analytical solution $u_{,y}$ along the line $x=0.5$ in a plot.

(3) Plot the errors of finite element solutions $e=\left[\int_\Omega \left(u-u^h\right)^2 d\Omega\right]^{1/2}$ vs element dimensions "*h*" in a log-log plot and obtain the rate of convergence of this error measure. Use 4x4 Gauss integration to integrate error *e*.

(4) Discuss how the fiunite element solution errors are reduced as the model is refined, and how the numerical rate of convergence compares with the theoretical rate of convergence.

The final report should contain problem statement, finite element formulation, numerical results, and detailed discussions. A copy of your program should be attached.