1 Formal definition of a Turing Machine

A Turing Machine is a 7-tuple (Q, Σ, Γ, δ, q_0, q_{accept}, q_{reject}):

- Q is the set of states
- Σ is the input alphabet *not* containing special blank symbol $\sqcup$
- Γ is the tape alphabet satisfying $\Sigma \subset \Gamma$ and $\sqcup \in \Gamma$
- $\delta : Q \times \Gamma \to Q \times \Gamma \times L, R$ is the transition function
- $q_0 \in Q$ is the start state
- $q_{accept} \in Q$ is the accept state
- $q_{reject} \in Q$ is the reject state, $q_{reject} \neq q_{accept}$

2 Turing Machine computation

Tape content is unbounded but always finite, and the first (leftmost) blank symbol marks the end of tape content.

A configuration C_1 yields the configuration C_2 if the Turing Machine can legally go from C_1 to C_2 in a single step. A **configuration** consists of:

- The current state
- The tape content
- The head location

The **start configuration** on an input $w \in \Sigma^*$ consists of start state q_0, w as the tape content, and the head location is the first (leftmost) position of the tape.

A configuration is **accepting** if its state is q_{accept}. A configuration is **rejecting** if its state is q_{reject}.

Accepting and rejecting configurations are **halting** configurations.

A Turing Machine M **accepts** an input w if there is a sequence of configurations $C_1, C_2, ..., C_k$ such that:

1. C_1 is the start configuration of M on input w
2. C_i yields C_{i+1} for $1 \leq i \leq k-1$
3. C_k is an accepting configuration

The **language** of M, denoted $L(M)$, is the set of strings accepted by M.

3 Turing-recognisable languages

A language L is **Turing-recognisable** if there is a Turing Machine M that recognises it, i.e. L is the language of M.
Turing-recognisable means the same thing as **semi-decidable** and **recursively enumerable**.

If M recognises L, it may or may not halt on words not in L.

4 Turing-decidable languages

A language L is **Turing-decidable** if there is a Turing Machine M that accepts every $w \in L$ and rejects every $w \notin L$.
Turing-decidable means the same thing as **recursive**.

If M decides L, it always halts.