1 m-reduciblity

Definition. Let A and B be languages over alphabet Σ. A is many-to-one reducible to B, written $A \leq B$, if there is a Turing machine F that terminates on every input $u \in \Sigma^*$, and such that:

$$A = \{u \in \Sigma^* | F(u) \in B\}$$

Informally, this means that checking $u \in A$ is no harder than checking $w \in B$.

1.1 Properties

Proposition. Suppose $A \leq B$.

1. If B is Turing-decidable, so is A
2. If B is Turing-recognisable, so is A
3. If $A \leq B$ and $B \leq C$, then $A \leq C$

Denote $A \equiv B$ to mean that $A \leq B$ and $B \leq A$. Informally, this means that A and B are equally difficult.

2 m-completeness

Language A is **m-complete** if:

1. A is Turing-recognisable, and
2. for every Turing-recognisable language B, $B \leq A$.

Informally, if A is m-complete, then A is as hard as any other Turing-recognisable language

Corollary. If A is m-comnplete and $A \leq B$, then B is m-complete.

Definition. The Halting language H consists of the words