Dummy title

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Abstract

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2012 ACM Subject Classification Replace ccsdesc macro with valid one

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2 Overview

Bichromatic closest pair was first introduced in [1] where Pankaj et al. give a reduction of euclidean minimum spannig tree to two colored bichromatic closest pair and compare their hardness. They also provide a randomized algorithm for computing the bichromatic closest pair which runs in expected $O((nm)^{1-\frac{1}{\lceil\frac{d}{2}\rceil+1}+\varepsilon} + m\log n + n\log m)$ for any $\varepsilon > 0$ and n the size of the first color's set and m the size of the second color's set. Later Aggrawal et al. [2] give an optimal algorithm to solve the k-colored bichromatic all nearest neighbors problem in plane. Their aproach uses a Voronoi diagaram to compute the neighbors and has a running time of $O(n\log n)$ for any number of colors. This in turn gives a $O(n\log n)$ solution to the bichromatic closest pair. Khuller and Matias [13] give a randomized algorithm for the closest pair problem that runs in expected $O(n)$ time which, they argue, can be extended to the bichromatic closest pair problem in higher dimensions with a running time still in expected $O(n)$. In the same year Eppstein [10] gives important algorithms which solve the dynamic Euclidean minimum spanning tree and dynamic bichromatic closest pair. The running time of maintaining the Euclidean minimum spannig tree and two colored bichromatic closest pair is $O(n^{1-\varepsilon})$ where $\varepsilon > 0$ depends on the dimension d. For rectilinear metrics, the MST can be maintained in time $O(\sqrt{n}\log^d n)$. Dumitrescu and Guha [9] gave an MST approach for the k-colored bichromatic closest pair in plane with running time $O(n\log n)$ and reduced the k-colored bichromatic closest pair problem to $\lceil\log k\rceil$ two-colored bichromatric closest pair problems. They also give an algorithm that maintains a bichromatic closest pair under color change with running time $O(\log n)$ per update in any dimension. In the same manner, Borgelt and Borglet [4] give an algorithm that maintains in plain a k-colored bichromatic all nearest neighbors with running time $O(n\log n)$ to construct a Voronoi diagram and amortized $O(\log n)$ time to update it. With Impagliazzo nad Paturi [12] stating the Strong Exponential Time Hypothesis (SETH) we have conditional lower-bounds on some problems. Williams [18] proved that SETH implies Orthogonal Vectors Hypothesis (OVH). O'Donnell et al. [14] gave strict bounds on the locality sensitive hashing approach. Alman et al. [3] give a randomized $(1+\varepsilon)$-approximate all nearest neighbor with expected running time $O(dn + n^{2-1/\Omega(\sqrt[3]{\varepsilon}/\log\frac{1}{\varepsilon})})$ for l_1 and Euclidean metrics. With Bringmann's introduction of Fine-grained complexity theory [5, 6] bichromatic closest pair was researched from a purely theoretical standpoint. Improving on to Alman et al. [3], Rubinstein [16] proved a better conditional lower-bound on the $(1+\varepsilon)$-approximate bichromatic closest pair with time $O(n^{2-\varepsilon})$, which in turn implies better conditional lower-bounds for approximate bichromatic closest pair queries and approximate bichromatic all nearest neighbors. He also pointed out similar results for Hamming distnace and Fréchte distance, whereas for l_1 and l_∞ exist subquadratic algorithms. Pagh et al. [15] proved similar results for $(1+\varepsilon)$-approximate bichromatic closest pair with Jaccard similarity. In a different approach, Flores-Velazco and Mount [11] studied the classification and give a datastructure that can query the most occuring color of the k ε-nearest neighbors of a given point. Horst et al. [17] also provide a datastructure that can answer the most occuring color of the exact k nearest neigbors of a given point.

3 Reduction proof

▶ **Theorem 1.** *The k-colored bichromatic closest pair problem can be reduced to $\lceil\log k\rceil$ 2-colored bichromatic closest pair problems.*

Proof. Let k be the number of different colors and let A_1, A_2, …, A_k be sets of points coresponding to different colors. To make the proof cleaner, let us relable the sets to A_0, A_1, …, A_{k-1}. Now construct $\lceil\log k\rceil$ pairs p_i with

$$p_i := \left(\bigcup_{j=0..(k-1)\wedge i\text{-th bit of } j \text{ is } 0} A_j, \bigcup_{j=0..(k-1)\wedge i\text{-th bit of } j \text{ is } 1} A_j \right).$$

For $i > \lceil \log k \rceil$, pairs will be of form $(\bigcup A_j, \emptyset)$ and so they are not needed. By computing the 2-colored bichromatic closest pair between the left and right set of each pair p_i and taking the minimal solution, we get a solution to the k-colored bichromatic closest pair. To prove this, assume the k-colored bichromatic closest pair is achieved between sets A_{r_1} and A_{r_2}. Since $r_1 \neq r_2$ there has to exist a bit in which r_1 and r_2 differ. Let that be the s-th bit. Since they differ in the s-th bit, they are split appart in the pair p_s. Now the 2-colored bichromatic pair on this pair p_s will return this solution (or one equally good). And the minimum over all of the pairs should be as good as this solution.

◀

4 Bichromatic closest pair

▶ **Definition 2** (bichromatic closest pair). *For a given dimension d, and two sets of points $A, B \subset \mathbb{R}^d$ where A are red colored points and B are blue colored points, compute the*

$$\min_{p \in A,\ q \in B} d_2(p, q)$$

where $d_2(p, q)$ is the Euclidean distance.

▶ **Definition 3** (bichromatic closest pair with k-colors). *For a given dimension d, number of colors k, and sets $A_1, A_2, \ldots A_k \subset \mathbb{R}^d$, compute*

$$\min_{p \in S_i,\ q \in S_j,\ i \neq j} d_2(q, p).$$

Approximate bichromatic nearest neighbor

Let $A_1, A_2, \ldots, A_t \subset \mathbb{R}^d$. In *all bichromatic approximate nearest neighbor* problem, for each $i = 1 \ldots t$ and each $q \in A_i$, we want to find a point $z \in \bigcup\limits_{i \neq j} A_j$ such that $d_2(q, p) \leq d_2(q, z) \leq (1 + \varepsilon) d_2(q, p)$, where $p \in \arg\min\limits_{p \in \bigcup\limits_{i \neq j} A_j} d_2(q, p)$. We will show that all bichromatic approximate nearest neighbor problem can be computed in $O(n \log n)$ time with the help of the well-separated pair decomposition, a well-know geometric decomposition introduced in a seminar work of Callahan and Kosaraju in 1995 [7, 8].

4.0.0.1 Well-separated pair decomposition.

Let S be a set of n points in $\mathbb{R}^d$. For any $A \subseteq S$, let $R(A)$ denote the minimum enclosing axis-aligned box of A. Let C_A be the minimum enclosing ball of $R(A)$, and let $r(A)$ denote the radius of C_A. Let $C^{r(A)}$ be the ball with the same center as C_A, but with radius $r(A)$. Furthermore, for two sets A, $B \subseteq S$, let $r = \max(r(A), r(B))$, and let $d(A, B)$ denote the minimum distance between C_A^r and C_B^r. For example, if the C_A intersects C_B, then $d(A, B) = 0$.

▶ **Definition 4.** *A pair of sets A and B are said to be well-separated if $d(A, B) > s \cdot r$, for any given separation constant $s > 0$ and $r = \max\{r(A), r(B)\}$.*

▶ **Definition 5** (WSPD). *A well-separated pair decomposition of $S \subset \mathbb{R}^d$, for a given $s > 0$, is a sequence $(A_1, B_1), \ldots, (A_k, B_k)$, where $A_i, B_i \subseteq S$, such that*

1. *A_i, B_i are well-separated with respect to separation constant s, for all $i = 1, \ldots, k$;*

2. *for all* $p \neq q \in S$ *there exists a unique pair* (A_i, B_i) *such that* $p \in A_i, q \in B_i$ *or* $q \in A_i, p \in B_i$.

Note that WSPD always exists since one could use all singleton pairs $(\{p\}, \{q\})$, for all pairs $p, q \in S$. However, this would yield a sequence of dumbbells of size $k = \Theta(n^2)$. The question is whether one could do better than that. The answer to that question was given by the following theorem.

▶ **Theorem 6** ([8]). *Given a set* S *of* n *points in* $\mathbb{R}^d$ *and a separation constant* $s > 0$, *a WSPD of* S *with* $O(s^d d^{d/2} n)$ *many dumbbells can be computed in* $O(dn \log n + s^d d^{d/2} n)$.

We will choose the separation constant s and modify the construction of WSPD such that it can be reused in the context of our problem.

Let (A_i, B_i), $i = 1, \ldots, k$, denote the WSPD for some set of points $S \subset \mathbb{R}^d$. For $a, a' \in A_i$ and $b, b' \in B_i$ for some dumbbell i, we make the following observations:

1. Points within the sets A_i and B_i can be made 'arbitrarily close' as compared to points in the opposite sets by choosing the appropriate separation $s > 0$, i.e.

$$d(a, a') \leq 2r < \frac{2}{s} d(A_i, B_i) \leq \frac{2}{s} d(a, b). \tag{1}$$

2. Distances between points in the opposite sets can be made 'almost equal', by choosing the appropriate $s > 0$, i.e.

$$d(a', b') \leq d(a, a') + d(a, b) + d(b, b') < (1 + \frac{4}{s}) d(a, b). \tag{2}$$

Thus, for $s = \frac{4}{\varepsilon}$, we have that $d(a', b') \leq (1 + \varepsilon) d(a, b)$, for any $\varepsilon > 0$.

4.0.0.2 Construction of WSPD with an additional color information.

For the construction of WSPD the split tree of a set S is computed by Algorithm 1. Even

Algorithm 1 SplitTree(S)

```
if size(S) = 1 then
    return leaf(S).
else
    Partition S into sets S_1 and S_2 by halving R(S) with hyperplane along its longest side.
    Return a node with children (SplitTree(S_1), SplitTree(S_2)).
end if
```

though such a tree might have linear depth and therefore a naive construction of the split tree by Algorithm 1 in the worst case takes quadratic time. However, the work of [8] showed how to compute such a binary tree in $O(n \log n)$ time. With every node u of that tree we can conceptually associate the set S_u of all points contained in its subtree. Node u is called colorful node if S_u contains points from two or more colors. Otherwise, we say that S_u is monochromatic.

▶ **Definition 7** (Colorful edge). *Let* u *and* w *denote two nodes in a split tree such that the corresponding set of points* (S_u, S_w) *for a well-separated pair. If either* S_u *or* S_w *is colorful, then the* (S_u, S_w) *is called colorful edge. If both* S_u *and* S_w *are monochromatic, but colored with different colors, then* (S_u, S_w) *is called colorful edge.*

Algorithm 2 FindColorfulEdges(v, w)

Require: Split tree T of S, separation constant $s > 0$
  if S_v and S_w are well-separated and (S_v, S_w) colorful **then**
    add colorful edge (v, w) to the tree T.
  else if $L_{\max}(S_v) > L_{\max}(S_w)$ **then**
    FindColorfulEdges$(v_l,\ w)$, FindColorfulEdges$(v_r,\ w)$
  else
    FindColorfulEdges$(v,\ w_l)$, FindColorfulEdges$(v,\ w_r)$
  end if

In order to compute the subset of WSPD consisting only of colorful edges, for each internal node u of the split tree T with children v, w and v_l, v_r and w_l, w_r denoting left and right child of v, w respectively, we invoke Algorithm 2. Let C_S denote the set of colorful edges computed by Algorithm 2. Note that C_S is a subset of WSPD of S. Furthermore, for any node u of the split tree T, let $C_{S_u} \subseteq C_S$ denote the set of all colorful edges such that $(S_w, S_v) \in C_S$, for nodes $v, w \in T$, and v ancestor of u. We consider u to be an ancestor of itself. Let $(S_{w'}, S_u) = \arg\min_{(S_w,S_v)\in C_{S_u}} d(S_w, S_v)$, i.e. $(S_{w'}, S_u)$ is the 'shortest' colorful edge in C_{S_u}. We save that information with every node $v \in T$.

▶ **Theorem 8.** *Given a set S of n points in R^d, AND k COLORS, the approximate all-bichromatic nearest neighbors problem can be solved in $O(n \log n)$ time.*

Proof. Let p by any point in S and let $q \in S$ be its bichromatic nearest neighbor. Let u denote the leaf in the split tree that stores q, and let $(S_{w'}, S_u$ is the shortest colorful edge saved with u. Then $p \in$ ◀

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A Styles of lists, enumerations, and descriptions

List of different predefined enumeration styles:

- `\begin{itemize}...\end{itemize}`
- ...
- ...

1. `\begin{enumerate}...\end{enumerate}`
2. ...
3. ...

(a) `\begin{alphaenumerate}...\end{alphaenumerate}`
(b) ...
(c) ...

(i) `\begin{romanenumerate}...\end{romanenumerate}`
(ii) ...
(iii) ...

(1) `\begin{bracketenumerate}...\end{bracketenumerate}`
(2) ...

(3) ...

Description 1 `\begin{description} \item[Description 1]  ...\end{description}`
Description 2 Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit amet neque.
Description 3 ...

Proposition 12 and Proposition 12 ...

B Theorem-like environments

List of different predefined enumeration styles:

▶ **Theorem 9.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit amet neque.*

▶ **Lemma 10.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit amet neque.*

▶ **Corollary 11.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit amet neque.*

▶ **Proposition 12.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit amet neque.*

▶ **Conjecture 13.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit amet neque.*

▶ **Observation 14.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit amet neque.*

▶ **Exercise 15.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit amet neque.*

▶ **Definition 16.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit amet neque.*

▶ **Example 17.** Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit amet neque.

▶ Note 18. Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit amet neque.

▶ Note. Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit amet neque.

▶ Remark 19. Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit amet neque.

▶ Remark. Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit amet neque.

▷ Claim 20. Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit amet neque.

▷ Claim. Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit amet neque.

Proof. Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit amet neque. ◀

Proof. Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit amet neque. ◁