Using Futhark for a fast, parallel implementation of forward and back projection in algebraic reconstruction methods - A pre-study

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SIRT

Solve the problem:

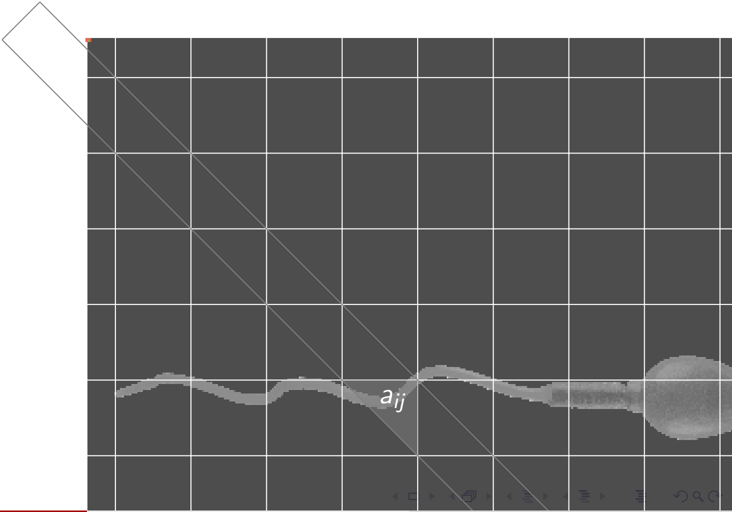
$$\boldsymbol{f}^* = argmin_{\boldsymbol{f}} \|\boldsymbol{p} - \boldsymbol{A}\boldsymbol{f}\| \tag{1}$$

iteratively using this update step:

$$\boldsymbol{f}^n = \boldsymbol{f}^{(n-1)} + \boldsymbol{C}\boldsymbol{A}^T\boldsymbol{R}(\boldsymbol{p} - \boldsymbol{A}\boldsymbol{f}^{(n-1)}), \tag{2}$$

where $\boldsymbol{C}$ and $\boldsymbol{R}$ are the diagonal matrices containing the inverse column and row sums of the system matrix respectively.

The system matrix



- Consider reconstructing a single slize of a volume from a detector of size $n \times n$
- The number of rays is n
- The number of angles is $\frac{n \cdot \pi}{2}$
- A typical value for n is 2048
- In semi sparse format the matrix will take up $2 \cdot 4 \cdot \lceil \frac{2048 \cdot \pi}{2} \rceil \cdot (2 * 2048 - 1) \approx 105MB$

System matrix computation

```
for ray = 0; ray < numberofrays; ray++ //parallel
  for pixel = 0; pixel < pixels; pixel++ //parallel
    if ray intersects pixel:
        (p1,p2) = intersectionpoints pixel ray
        A[ray][pixel] = distance p1 p2
```

Futhark

- High level data-parallel, and purely functional array language
- Comes with a heavily optimising ahead-of-time compiler
- Has performed well on several benchmarks
- Hardware-agnostic

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