Using Futhark for a fast, parallel implementation of forward and back projection in algebraic reconstruction methods - A pre-study

Lærke Pedersen and Mette Bjerg Lindhøj

University of Copenhagen

08/11/2018

SIRT

Solve the problem:

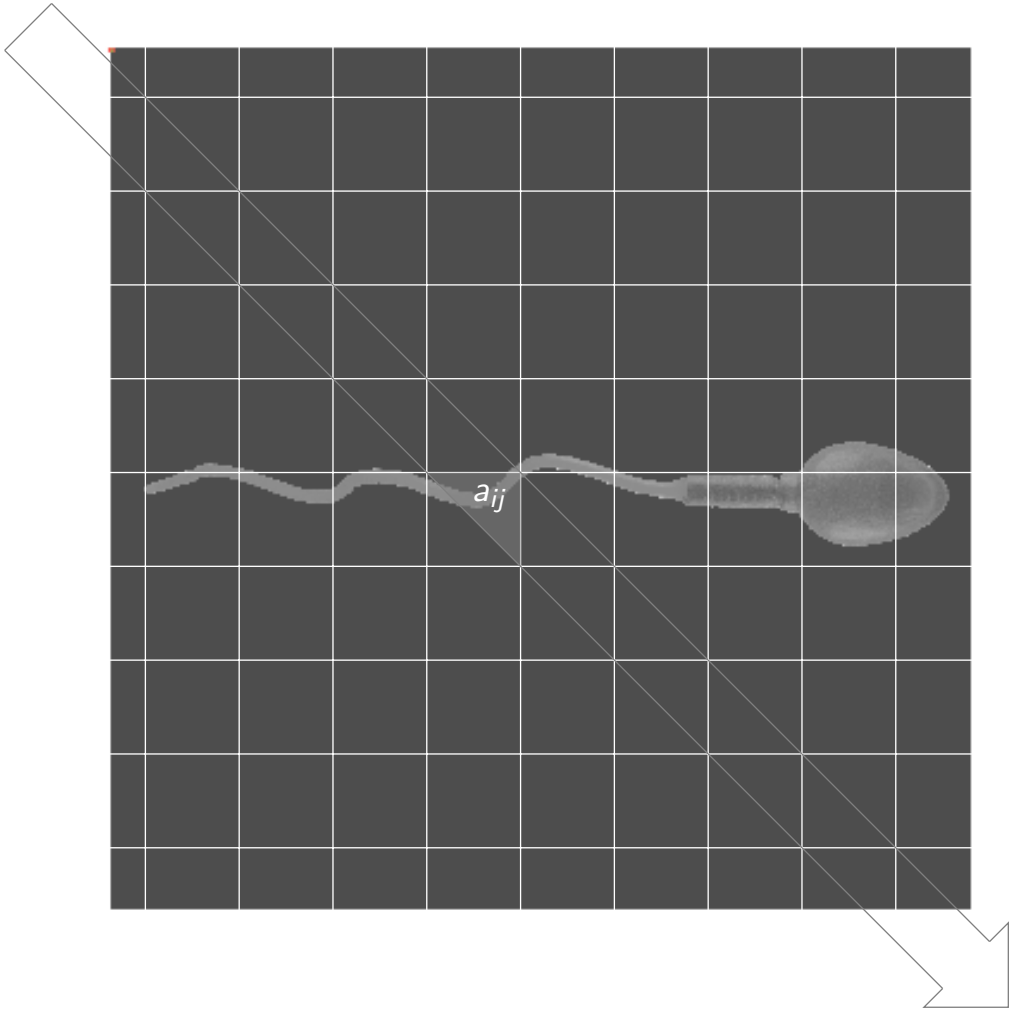
$$\boldsymbol{f}^* = argmin_{\boldsymbol{f}} \|\boldsymbol{p} - \boldsymbol{A}\boldsymbol{f}\| \tag{1}$$

iteratively using this update step:

$$\boldsymbol{f}^n = \boldsymbol{f}^{(n-1)} + \boldsymbol{C}\boldsymbol{A}^T\boldsymbol{R}(\boldsymbol{p} - \boldsymbol{A}\boldsymbol{f}^{(n-1)}), \tag{2}$$

where $\boldsymbol{C}$ and $\boldsymbol{R}$ are the diagonal matrices containing the inverse column and row sums of the system matrix respectively.

The system matrix



The problem is in the size

- Consider reconstructing a single slize of a volume from a detector of size $n \times n$
- The number of rays is n
- The number of angles is $\frac{n \cdot \pi}{2}$
- A typical value for n is 2048
- In semi sparse format the matrix will take up $2 \cdot 4 \cdot 2048 \cdot \lceil \frac{2048 \cdot \pi}{2} \rceil \cdot (2 \cdot 2048 - 1) \approx 216 GB$

System matrix computation

```
1 for ray = 0; ray < numberofrays; ray++ //parallel
2   for pixel = 0; pixel < pixels; pixel++ //parallel
3     if ray intersects pixel:
4         (p1,p2) = intersectionpoints pixel ray
5         A[ray][pixel] = distance p1 p2
```

Figure: $W(r, n) = O(r \cdot n^2)$, $D(r, n) = O(1)$

```
1 for ray in rays //parallel
2   while (isingrid focuspoint) //seq
3         pixel = calculatepixel focuspoint ray
4         nextpoint = findnextpoint focuspoint ray
5         A[ray][pixel] = distance nextpoint focuspoint
6         focuspoint = nextpoint
```

Figure: $W(r, n) = O(r \cdot n)$, $D(r, n) = O(n)$

Flattening

Utilizing nested parallelism is notoriously difficult, flattening the code can give marked improvements. However, flattening is also a way to systematically reason about nested code, as well as transforming it. Our project is currently a mix of flattened and nested code.

I will look into flattening our code in an example of one of our primary work areas, forward projection

Flattening

Forward projection is handled as a matrix vector multiplication. We utilize both a nested version and a flattened version. The nested version will, for each row, multiply the row and vector and get the sum.

```
1     map (\row ->
2       reduce (+) 0 <| map (\(v, ind) ->
3         unsafe (if ind == -1 then 0.0 else v*vect[ind])
4       ) row
5     ) mat_vals
```

Flattening this will require a flattened matrix, (calculating) a flag array,

Flattening

```
1    map (\row -> reduce (+) 0 <| map (\(v, ind) -> unsafe
         (if ind == -1 then 0.0 else v*vect[ind])) row )
       mat_vals
```

This translates to, still nested;

```
1    map(\row ->
2      let (ts, fs) = partition2 (\(_, ind) -> != -1) row
           Work(n), depth(log(n)) --if-then-else
3      let multiplied = map (\(v, ind) -> v*vect[ind]) ts
           work(n)
4      in reduce (+) 0 multiplied Work(n), depth(log(n))
5    ) mat_vals
```

Flattening

```
1     map (\row -> reduce (+) 0 <| map (\(v, ind) -> unsafe
          (if ind == -1 then 0.0 else v*vect[ind])) row )
        mat_vals
```

This translates to, still nested;

```
1     make flag array
```

Flattening

```
1    map (\row -> reduce (+) 0 <| map (\(v, ind) -> unsafe
         (if ind == -1 then 0.0 else v*vect[ind])) row )
       mat_vals
```

- A flattened matrix enforces coalesced reads from memory
- In SIMD a if-then-else is executed by evaluating the if, executing the then brach while the rest of the cores wait, then the same for the else branch. Partition means coalesced reads going forward

Flattening

```
1    map (\row -> reduce (+) 0 <| map (\(v, ind) -> unsafe
         (if ind == -1 then 0.0 else v*vect[ind])) row )
       mat_vals
```

- First step is to make a flag array, based on the matrix shape, and flatten the matrix
- Then

Code transformations

- By dumping the compiled code with the –dump command and getting the time spend in different kernels with the -D option we found that most of the time was spend on calculating the system matrix
- Futhark can not merge a map with a loop
- The code had a lot of branching
- We tried to mitigate this by removing as much from the loop as possible, and reorganising branches.

Changing the algorithm

```
1  for ray in rays //parallel
2    for i=-halfsize; i < halfsize; i++ //parallel
3          (l1,pixel1) = intersection1 ray i
4          (l2,pixel2) = intersection2 ray i
5          A[ray][pixel1] = l1
6          A[ray][pixel2] = l2
```

Figure: $W(r, n) = O(r \cdot n)$, $D(r, n) = O(1)$

Flattening the new algorithm

Show them how its done ;-)


Hao Gao.
Fast parallel algorithms for the x-ray transform and its adjoint.
Medical physics, 39(23127102):7110–7120, November 2012.


Y. Long, J. A. Fessler, and J. M. Balter.
3d forward and back-projection for x-ray ct using separable footprints.
IEEE Transactions on Medical Imaging, 29(11):1839–1850, Nov 2010.


F. Natterer.
The Mathematics of Computerized Tomography.
Society for Industrial and Applied Mathematics, 2001.


Z. Xue, L. Zhang, and J. Pan.
A new algorithm for calculating the radiological path in ct image reconstruction.
In *Proceedings of 2011 International Conference on Electronic Mechanical Engineering and Information Technology*, volume 9, pages 4527–4530, Aug 2011.