Using Futhark for SIRT

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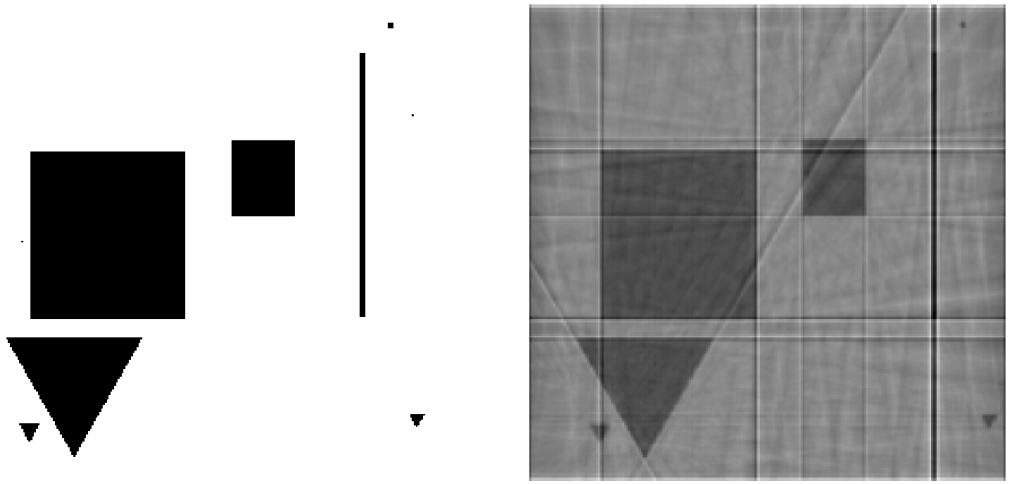
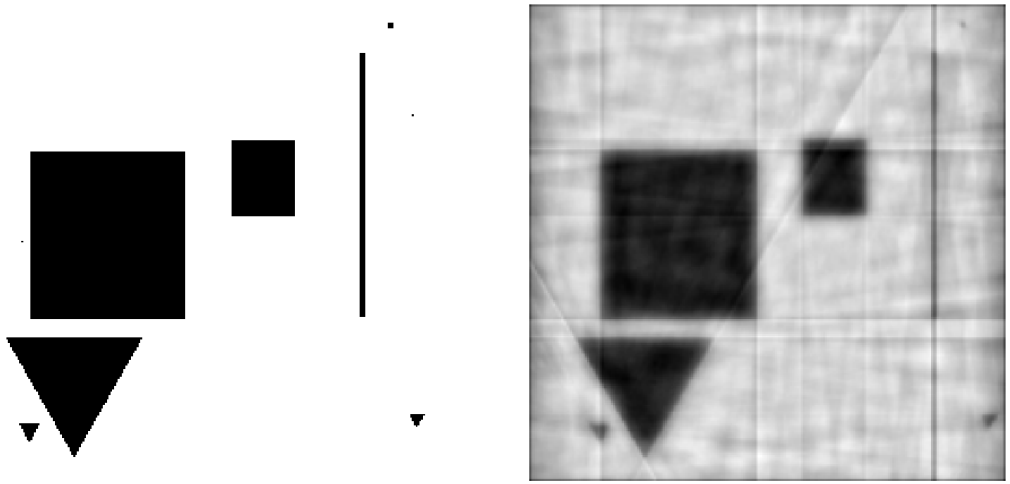
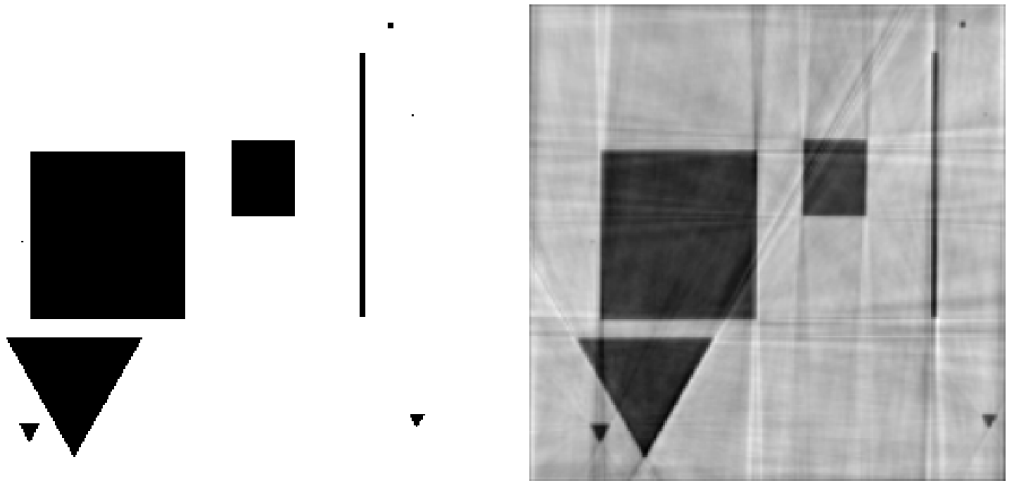
MSE: 0.21, SSIM: 0.60



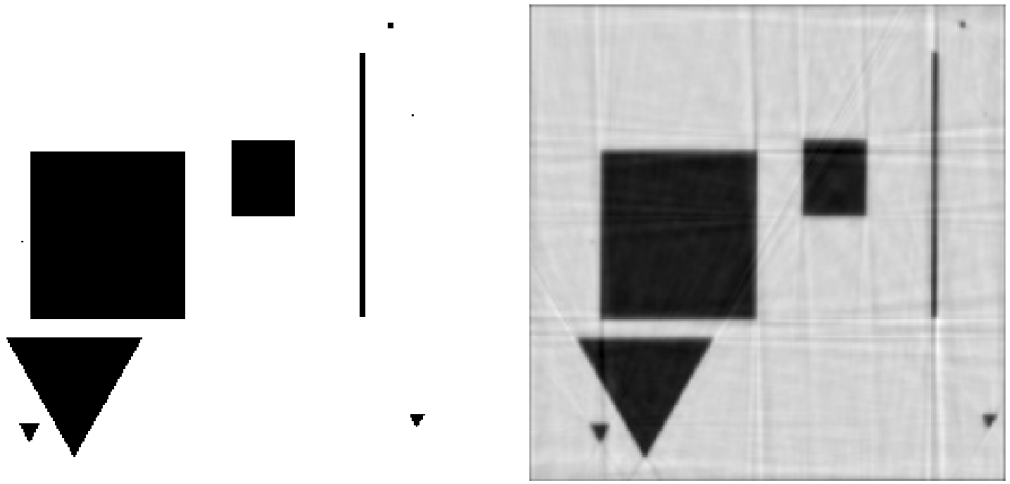
MSE: 0.08, SSIM: 0.69



MSE: 0.09, SSIM: 0.66



MSE: 0.05, SSIM: 0.73



SIRT

Solve the problem:

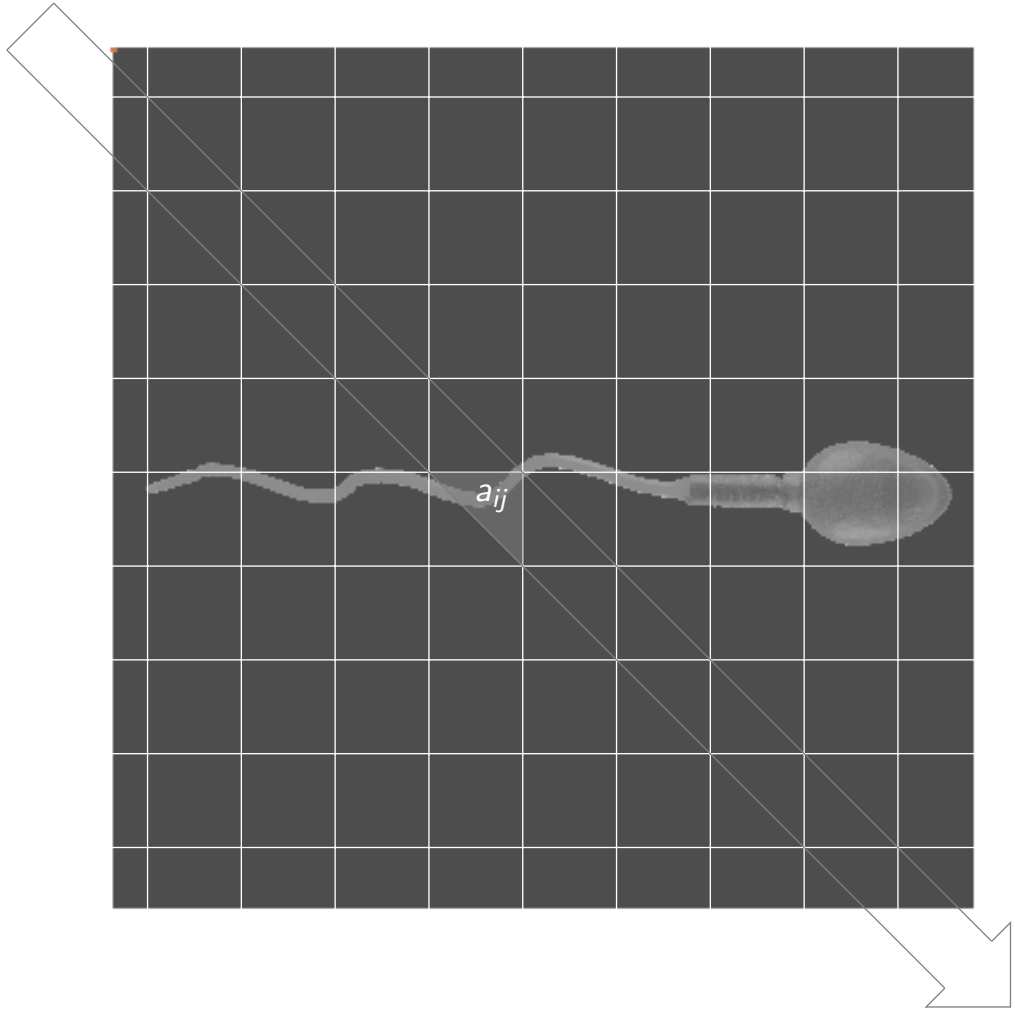
$$\boldsymbol{f}^* = argmin_{\boldsymbol{f}} \|\boldsymbol{p} - \boldsymbol{A}\boldsymbol{f}\| \tag{1}$$

iteratively using this update step:

$$\boldsymbol{f}^n = \boldsymbol{f}^{(n-1)} + \boldsymbol{C}\boldsymbol{A}^T\boldsymbol{R}(\boldsymbol{p} - \boldsymbol{A}\boldsymbol{f}^{(n-1)}), \tag{2}$$

where $\boldsymbol{C}$ and $\boldsymbol{R}$ are the diagonal matrices containing the inverse column and row sums of the system matrix respectively.

The system matrix



The problem is in the size

- Consider reconstructing a single slice of a volume from a detector of size $n \times n$
- The number of rays is n
- The number of angles is $\frac{n \cdot \pi}{2}$
- A typical value for n is 2048
- In semi sparse format the matrix will take up $2 \cdot 4 \cdot 2048 \cdot \lceil \frac{2048 \cdot \pi}{2} \rceil \cdot (2 \cdot 2048 - 1) \approx 216 GB$

System matrix computation

```
for ray in rays
  for i=-halfsize; i < halfsize; i++
        (l1,pixel1) = intersection1 ray i
        (l2,pixel2) = intersection2 ray i
        A[ray][pixel1] = l1
        A[ray][pixel2] = l2
```

Figure: $W(r, n) = O(r \cdot n)$, $D(r, n) = O(1)$

```
for pixel in pixels:
  for angle in angles:
    rays = rays that are within pixel
    for ray in rays:
        l = intersection ray pixel
```

Figure: $W(r, n) = O(r \cdot n)$, $D(r, n) = O(1)$

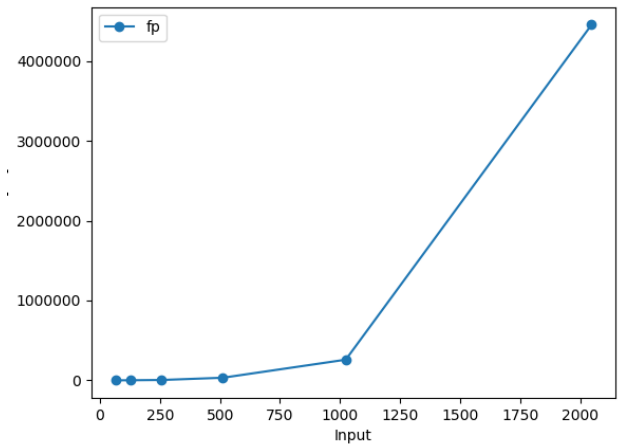
Benefits of Futhark

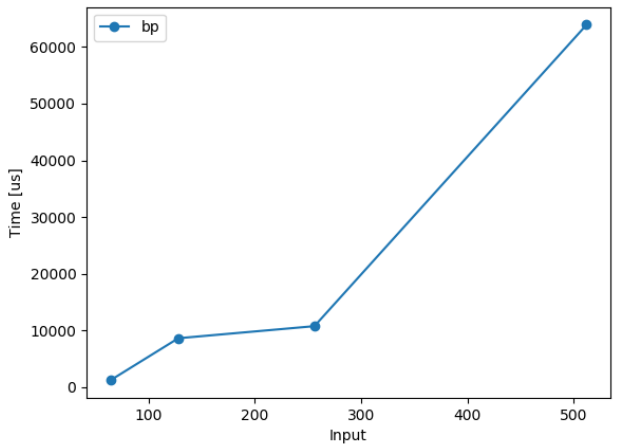
```
1  % Input: sparse system matrix A, data b.
2  % Output: SIRT reconstruction x.
3  x = zeros(d * d, 1);
4  [rows cols] = size(A);
5  C = sparse(1 : cols, 1 : cols, 1 ./ sum(A));
6  R = sparse(1 : rows, 1 : rows, 1 ./ sum(A'));
7  CATR = C * A' * R;
8  for i = 1 : 100
9    x = x + CATR * (b - A * x);
10 end
```

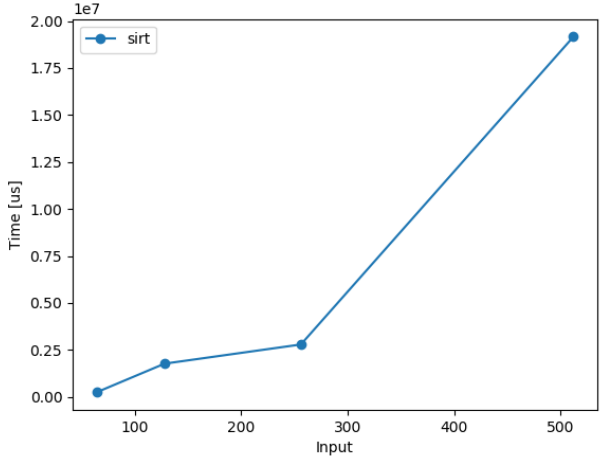
Figure: Example of high level implementation

```
1  unsigned int blockStart = 0;
2  unsigned int blockEnd = 0;
3  bool blockVertical = false;
4  for (unsigned int a = 0; a <= dims.iProjAngles; ++
       a) {
5    bool vertical = false;
6    if (a != dims.iProjAngles)
7      vertical = (fabsf(angles[a].fRayX) <= fabsf(
           angles[a].fRayY));
8    if (a == dims.iProjAngles || vertical !=
         blockVertical) {
9
10     blockEnd = a;
11     if (blockStart != blockEnd) {
12       dim3 dimGrid((blockEnd-blockStart+
             g_anglesPerBlock-1)/g_anglesPerBlock,
13                   (dims.iProjDets+g_detBlockSize
                       -1)/g_detBlockSize); //
                       angle blocks, detector
                       blocks...
```

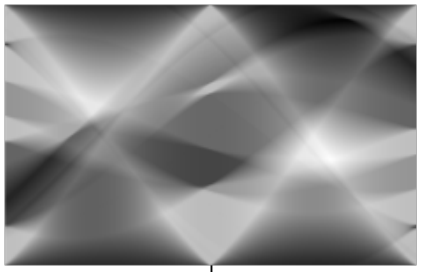
Figure: Example of CUDA implementation

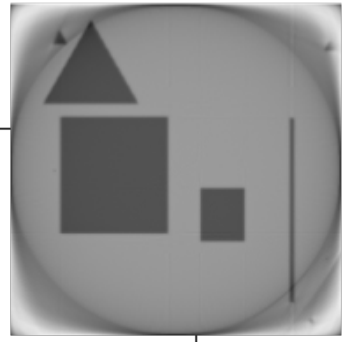

fp
4000000
3000000
2000000
1000000
0
0
250
500
750
1000
1250
1500
1750
2000
Input


bp
60000
50000
40000
30000
20000
10000
0
Time [us]
100
200
300
400
500
Input


1e7
sirt
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
Time [us]
100
200
300
400
500
Input

Input: sinogram and initial guess

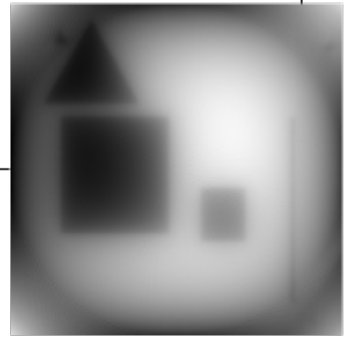




Forward projection

Update

Weighted difference



Backprojection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Fast parallel algorithms for the x-ray transform and its adjoint.
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IEEE Transactions on Medical Imaging, 29(11):1839–1850, Nov 2010.


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