# Using Futhark for a fast, parallel implementation of forward and back projection in algebraic reconstruction methods - A pre-study

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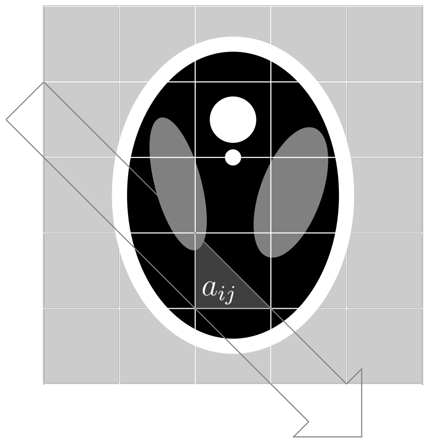



Figure 1: Example of a weighting $a_{ij}$.

# 1 Introduction

Computed tomography is the inverse problem of reconstructing an image or volume from its x-ray projections. The x-ray source spins around the object to be analyzed and sends x-rays that hit a detector on the oposite side. The detector shows how much of each x-ray was attenuated when passing through the volume. In this report we will focus on the bottlenecks of an algebraic reconstruction algorithm called the Simulataneous Iterative Reconstruction Algorithm (SIRT), namely forward projection and back projection and investigate how we may use the high level data-parallel language Futhark for implememting a fast version of this algorithm. The reason for choosing this algorithm is that it provides good reconstruction quality under non-optimal circumstances, but unfortunately has very poor performance compared to techniques such as filtered backprojection.
In algebraic reconstruction we solve the problem as a linear system of equations. The main idea is that the process can be modelled as a linear transformation by discretizing the object to be reconstructed into $N$ pixels. First we place a coordinate system with origo at the center of the object to be reconstructed and denote by $\boldsymbol{\theta}$ the vector of angles between the positions of the source and origo. For each angle several x-rays are cast from the source. We denote by the vector $\boldsymbol{\rho}$ the signed distances from each line to origo. The data produced by the process is called the sinogram. Then the sinogram values $p_i$ for each $(\theta_k, \rho_l)$ are a weighted sum of the attenuations at each pixel $f_j$ that the $(\theta_k, \rho_l)$ ray passes through:

$$\sum_{j=1}^{N} a_{ij} f_j = p_i \tag{1}$$

Where $a_{ij}$ are the weights, corresponding to the fraction of the pixel $j$ that the ray $i$ covers.

Writing all the projections as a column vector $\boldsymbol{p}$ and the attenuation values to be reconstructed as a column vector $\boldsymbol{f}$ the weightings are represented as an $M \times N$ matrix $\boldsymbol{A}$, we obtain a linear system of the form:

$$\boldsymbol{p} = \boldsymbol{A}\boldsymbol{f} \tag{2}$$

These systems of equations may easily be solved under the right circumstances, where $M = N$. However this is rarely the case. In most real cases $M > N$, i.e. the number of projections is larger than the number of pixels to be reconstructed and the size of the matrix is very large - more about this in the next section.
However, the algebraic reconstruction methods also have some advantages. Since the model closely relates to the real world scenario the weightings can be refactored to take irregularities in the setup, such as differences in beam energies or irregular geometries and missing data into account. Furthermore these methods generally give better image quality than analytic methods when the data is sparse.

A linear system like this is typically solved by minimizing some norm:

$$\|\boldsymbol{A}\boldsymbol{f} - \boldsymbol{p}\| \tag{3}$$

An example is the SIRT algorithm. The action of the matrix $\boldsymbol{A}$ is called the *forward projection*, and the matrix itself is called the *system matrix,projection matrix* or *radon matrix*. Each row of $\boldsymbol{A}$ represents the coefficients of the equation for one ray. The action of the transposed projection matrix $\boldsymbol{A}^T$ on the sinogram vector is called the *backprojection*, and can be vizualized as smearing the projection values across the reconstruction in the direction of the ray, or equivalently summing up the contrubitions of each ray for a given pixel. The idea behind the SIRT algorithm is to forward project the current reconstruction, then subtract this from the original projection data and do a weighted backprojection resulting in a correction factor which can be added to the current reconstruction. The update equeation is:

$$\boldsymbol{f}^n = \boldsymbol{f}^{(n-1)} + \boldsymbol{C}\boldsymbol{A}^T\boldsymbol{R}(\boldsymbol{p} - \boldsymbol{A}\boldsymbol{f}^{(n-1)}), \tag{4}$$

where $\boldsymbol{C}$ and $\boldsymbol{R}$ are the diagonal matrices containing the inverse column and row sums of the system matrix respectively.

It can be shown that this iterative scheme solves the problem:

$$\boldsymbol{f}^* = argmin_{\boldsymbol{f}} \|\boldsymbol{p} - \boldsymbol{A}\boldsymbol{f}\|_{\boldsymbol{R}}, \tag{5}$$

where $\|\boldsymbol{x}\|_{\boldsymbol{R}} = \boldsymbol{x}^T\boldsymbol{R}\boldsymbol{x}$.

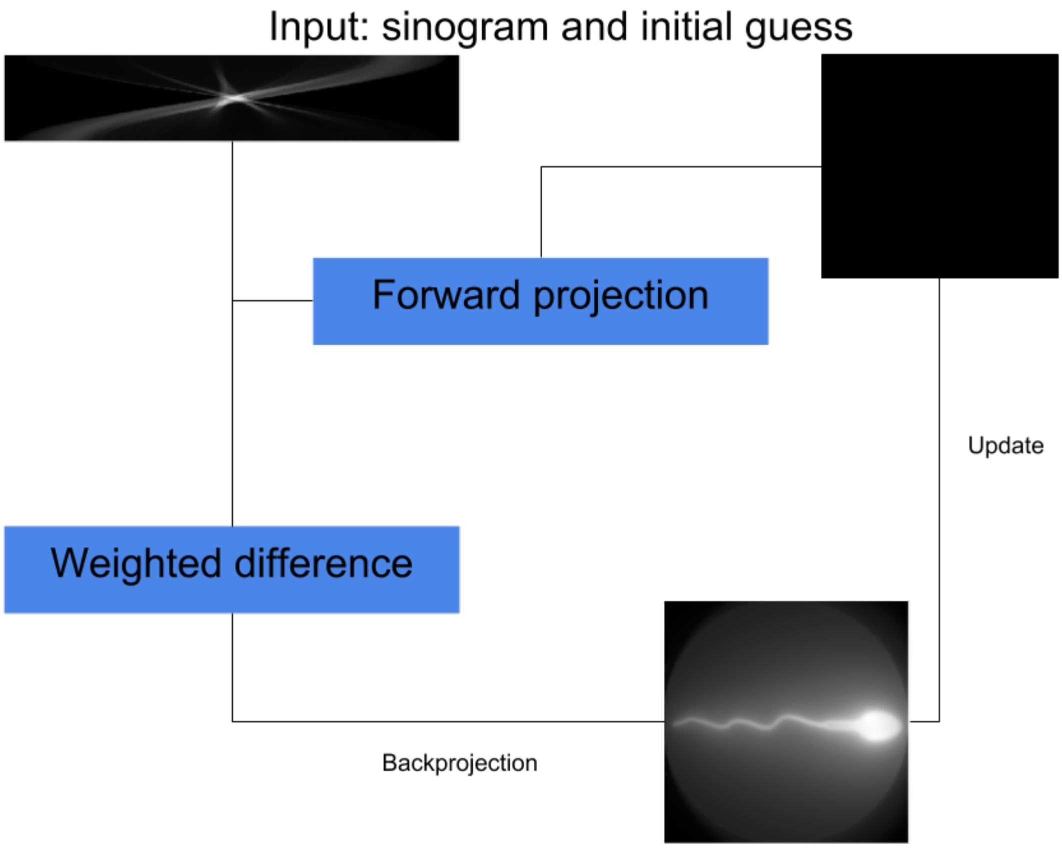



Figure 2: An illustration of the SIRT algorithm.

The backprojection and foward projection operations are standard operations in many iterative algebraic reconstruction methods and are the bottle necks of the algorithms [2]. Therefore, our main focus has been on optimizing these operations.
Different beam geometries exist, such as parallel beams, fan beams and cone beams. The geometry must be considered when constructing the system matrix. However as this study is intended for applications with synchrotron data only parallel beam geometries will be considered. Furthermore, we will only consider reconstructions of 2D images, since this is simply a pre-study and it makes the problem conceptually easier to follow.

# 2 Computing the projection matrix

One of the main problems when parallelizing the algorithm is that the amount of data in real applications is huge. Images from synchrotrons are generated with detectors of sizes up to about $4000 \times 4000$, with a typical detector size being about $2048 \times 2048$. To accurately generate 3D reconstructions it has been shown that approximately $\frac{\pi \cdot N}{2}$ vieweing angles are needed, where $N$ is the size of the detector in one direction [3].
We benchmarked our algorithms by using sizes $N$ ranging from 128 to 4096 and using $\lceil \frac{\pi \cdot N}{2} \rceil$ angles and $N$ lines for each of these sizes to make the problem sizes realistic. However, in the future it might be worth investigating how the reconstructions look when using fewer angles, since one of the strengths of the SIRT algorithm is that it produces good results with few angles.
To solve the large problems it is not possible to store the whole system matrix on the GPU. Lets take as an example the amount of space used to store the matrix for a 3D problem with a $N = 4000$ detector, with $\lceil \frac{\pi \cdot N}{2} \rceil$ angles, and 4000 rays per angle. The matrix is sparse having at most $2 \cdot N - 1$ entries in each row, so instead of storing the whole matrix with all the zeroes, we will consider a semi-sparse representation with one 2 dimensional array containing the data as floats in arrays on each row of size $2 \cdot N - 1$, and one matching 2 dimensional array containing the column indexes of the datapoints. This is also the representation we have used in our implementations. For the example sketched above the number of rows of each matrix will be $\lceil \frac{\pi \cdot 4000}{2} \rceil \cdot 4000$ and the number of columns is $2 \cdot 4000 - 1$. Assuming ints and floats both take up 4 bytes each then storing the matrix requires $4 \cdot 2 \cdot \lceil \frac{\pi \cdot 4000}{2} \rceil \cdot 4000 \cdot (2 \cdot 4000 - 1) \approx 1TB$. This is of course one of the largest problems, but even for a problem with $N = 512$ we're looking at the matrix taking up around $3 - 4GB$. Besides the matrix we also need to store the sinogram data which will take approximately $420MB$, and the reconstruction taking up around $270MB$. Hence even for relatively small problems we're looking at an estimated $5GB$ of data. GPU memory is expensive, and is typically in the range 2-8GB for consumer end cards, and 10-48GB for high end cards. Thus, the small problems are not able to fit in a standard consumer card, and large problems won't fit on any cards. When we tested our algorithms, we were not able to compute the system matrix on the universitys GPU's for sizes larger than $256 \times 256$. There are several ways of getting around this problem. Some problems exhibit symmetries

in the system matrix so that we only have to store part of the matrix. This is already somewhat accounted for in the above calculations. In the example we only considered the matrix for one slice of a 3D reconstruction since it will be equivalent for other slices when using parallel beam geometry. Thus for cone beam geometries, the memory needed would be approximately $N$ times greater. Other symetries involve angle symmetries, where for example a scan from a 45° or 135° angle would contain the same values for mirrored pixels. However these are obviously not generally applicable since they will depend on which angles are used in the reconstruction.

Another more flexible approach is to calculate the system matrix in chunks which is what we have done. For this we started by using the code from a bachelor project in which they did an implementation of finding the system matrix using futhark. We had two different parallel implementations in futhark and a sequential version in python which we compared. The sequential version in python was too slow (in the order of hours) for realistic problem sizes to be worth considering. When we compared the two bachelor projects one seemed to outperform the other both in terms of memory usage and computation time. Since one of the implementations ran in to problems with memory for some of the smallest problems, we decided to keep working with the most promising of the two. We call this algorithm *projectionmatrix_jh*.

The system matrix may be adequately approximated by considering the x-rays as being thin lines and calculating the length of intersections with each pixel in a grid. For simplicity we have only considered square grids with isotropic pixels.

```
for ray = 0; ray < numberofrays; ray++ //parallel
  for pixel = 0; pixel < pixels; pixel++ //parallel
    if ray intersects pixel:
        (p1,p2) = intersectionpoints pixel ray
        A[ray][pixel] = distance p1 p2
```

Figure 3: A looped version of the projection matrix algorithm.

The computation of the system matrix contains two levels of possible parallelization. The implementation from the bachelor project we used had parallelized over the outer loop - i.e for each ray. To compute the intersection lengths a sequential loop was run following each line. The number of rays is very high (6.586.368 for a grid of size $2048 * 2048$), and larger than the number of pixels, this seems a good choice. By following each line, the inner loop only runs $2 * N - 1$ times. From their report it seems they tried a naive implementation of computing the intersection with each pixel in parallel (akin to the pseudocode in 2) but found it to be too slow to consider.

However, when we had an implementation of the foward projection and analyzed the compiled code, we could clearly see that the most time was spend in the kernel consisting of the sequential loop doing grid line intersections. Therefore we decided to spend some more time on the system matrix computations. Futhark does a lot of optimizations, but can not merge code across a loop in a map so our first approach was to attempt to change

the inner loop to a map. However, since the inner loop is inherently sequential because of cross loop dependencies, we did not completely succeed. We seperated out the part of the code which computes the intersection point, and did the distance calculations as a map. We call this algorithm *projectionmatrix_map*. Another issue with the projectionmatrix_jh is a lot of branches in the inner loop. We also tried to limit the computations in these branches as much as possible in projectionmatrix_map.
Since we did not manage to convert the inner loop to a map in projectionmatrix_jh, we also attemted a different approach, explained in [1]. It is based on the observation that each line with slope at most 1 intersects at most two rows of the grid in each column of the grid, and opposite for the lines with slope greater than 1. Hence there is potential for exploiting a second level of parallelism by parrallelizing over the columns or rows of the grid and finding the nontrivial intersections. They show a method for doing this in time $O(1)$. Unfortunately there is a small bug somewhere our implementation which seems to be related to the transition between lines with slope $> 1$ and those with slope $< 1$. We did not manage to find the error, but have decided to include the algorithm in the benchmarking anyway, since we believe it is a small error somewhere with no significant influence of the speed of the algorithm. We call this algorithm *projectionmatrix_ doubleparallel*.

We benchmarked the algorithms by generating the input, saving it to files and running with futhatk-bench. We used the opencl compiler, and did 10 runs. From the plot 2 we can see that the version where we exploit inner parallelism is indeed fastest. For the maximum size we were able to run, i.e 256 we got a speedup of about 1.2 compared to projectionmatrix_jh.

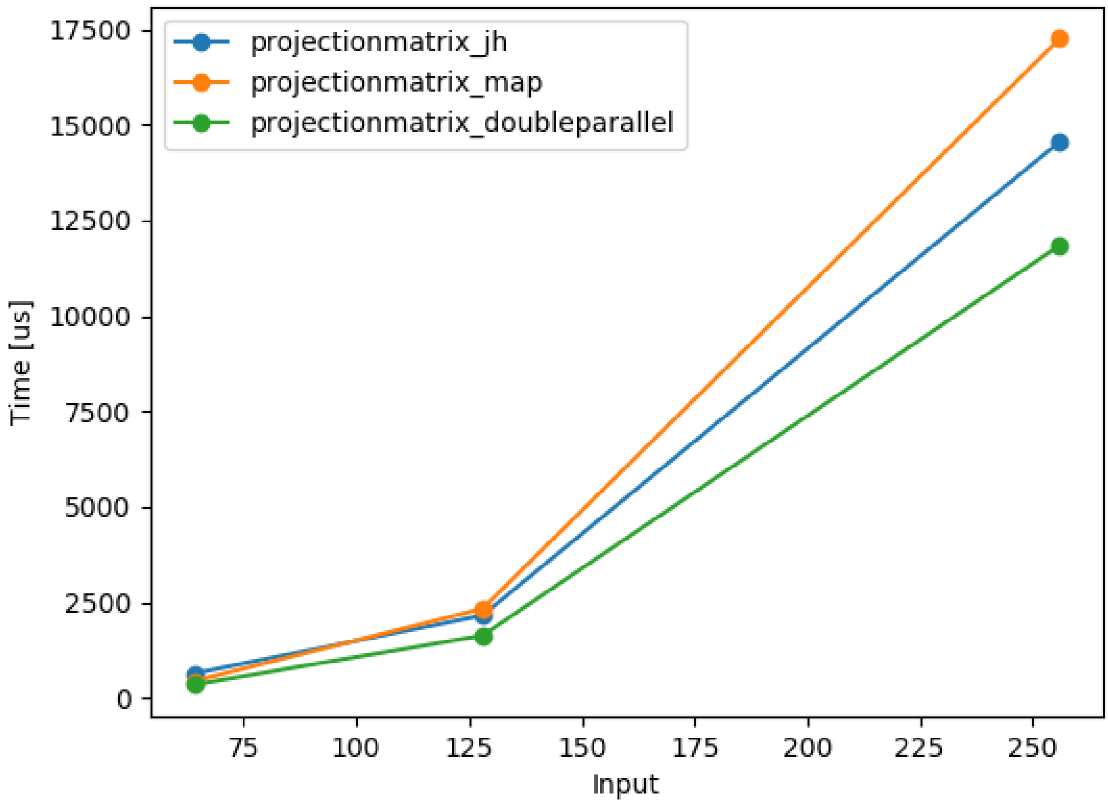



Figure 4: A lcomparison of the projection matrix algorithms run for gridsizes 64 to 256. Since the universities GPUs only have about 3GB of memory this was the largest size we could run for without chunking up the matrix.

# 3 Nested parallel forward and back projctions

A first approach at forward and backprojection was to do a nested parallel version and using computed chunks of the system matrix.
A looped version of the forward projection with the system matrix cut in *steps* chunks looks like this:

```
for step = 0; step < steps; step++
  A = getRays(raysperstep)
  for ray = 0; ray < raysperstep; row++
    acc = 0.0
    for p = 0; p<numpixels; p++
      acc+= A[ray][p]*image[p]
    FP[step*raysperstep+ray] = acc

this can be written in futhark like pseudo code as:

loop (output, step, raysperstep)
  let A = getRays raysperstep step
  let partresult = map (\row -> reduce (+) 0 <| map ( \i -> row[i]*
     vector[i] ) (iota (length row)) ) A
  in (output++partresult, step, raysperstep)
```

Figure 5: A looped version of the forward projection, where the raysperstep should be chosen such that the computations fit in the memory. $step * raysperstep$ should equal the total number of rows.

```
for step = 0; step < steps; step++
  A = getRays(raysperstep)
  AT = A.transpose()
  for p = 0; p<numpixels; p++
    acc = 0.0
    for ray = 0; ray<raysperstep; ray++
      acc+= AT[p][ray]*sinogram[ray]
    BP[p] += acc

this can be written in futhark like pseudo code as:

loop (output, step, raysperstep)
  let A = getRays raysperstep step
  let AT = transpose A
  let partresult = map (\row -> reduce (+) 0 <| map2 (*) row vect )
     AT
  let result = map2 (+) partresult output
  in (result, step, raysperstep)
```

Figure 6: A looped version of the back projection, where the raysperstep should be the largest number possible such that the computations fit in the memory. $step * raysperstep$ should equal the total number of rows.

# 4 Optimizations

Exploiting nested parallelism, is difficult on GPU hardware since the hardware is organized on one, or maybe two, levels that allow threads to comminucate via shared scratchpad memory. Hence mapping the application level parallelism to the GPU requires a choice of which level to paralellize, since both level cannot be directly mapped.
One way to get around this problem is to use a flattening transformation. The problem with this is that it will require even more memory usage, and may prevent opportunities for locality optimizations. We tried to do a flattening of our forawrd and backward rpoejctions, since we have multiple levels of paralellism. The idea was to compute the matrix, the transform it to a sparse and flat version and use our sparse matrix vector multiplication from a previous assignment. However, it turned out to be quite complex, and a lot of code was required to transform the matrix to the correct sparse format. Since it later turned out that most of the time spend during the computations was during the matrix computation and we had many issues with running out of memory, this probably wasn't the best approach and we did not pursue it to the end. We report result of semiflat versions.
A much more promising approach seems to be feeding the data directly to the matrix computations, and not have to save the matrix data at all but only the end result. We managed to finish an implementation of this approach for forward projection by feeding the data directly to the projectiomatrix_doubleparallel version. Unfortunately, it ran out of resources so we need to investigate this further.

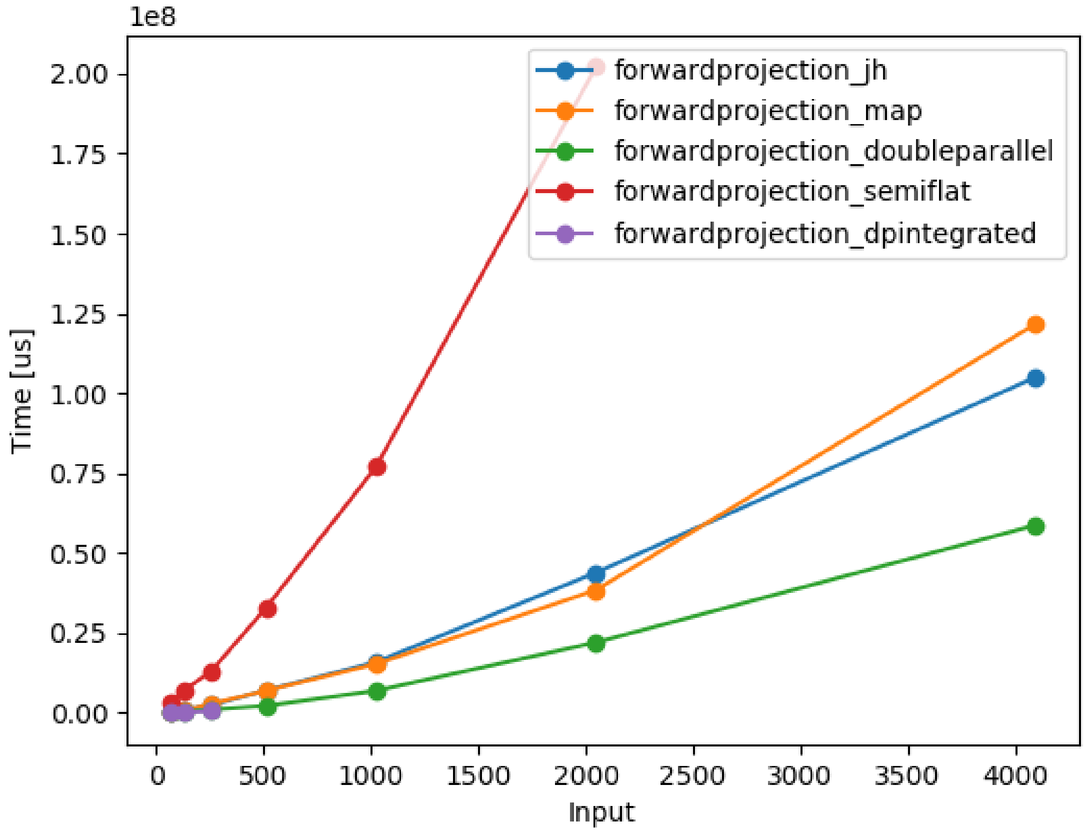



Figure 7: A comparison of the forward projections run with different matrix implementations and the version where the forward projection is integrated in projectiomatrix_doubleparallel. The chunk size was 32. We chose this number to fit a CUDA warp. As expected the semiflat version performed rather badly. The implementation using the double parallel version performed best. The rest of the algorithms are nested, using different projection matrix algorithms inside. We had to run our algorithms for only 30 angles as we got memory errors otherwise. This requires further investigation. In our initial tests our stripmined algorithms ran for all sizes, but perhaps we were competing for space with other groups.

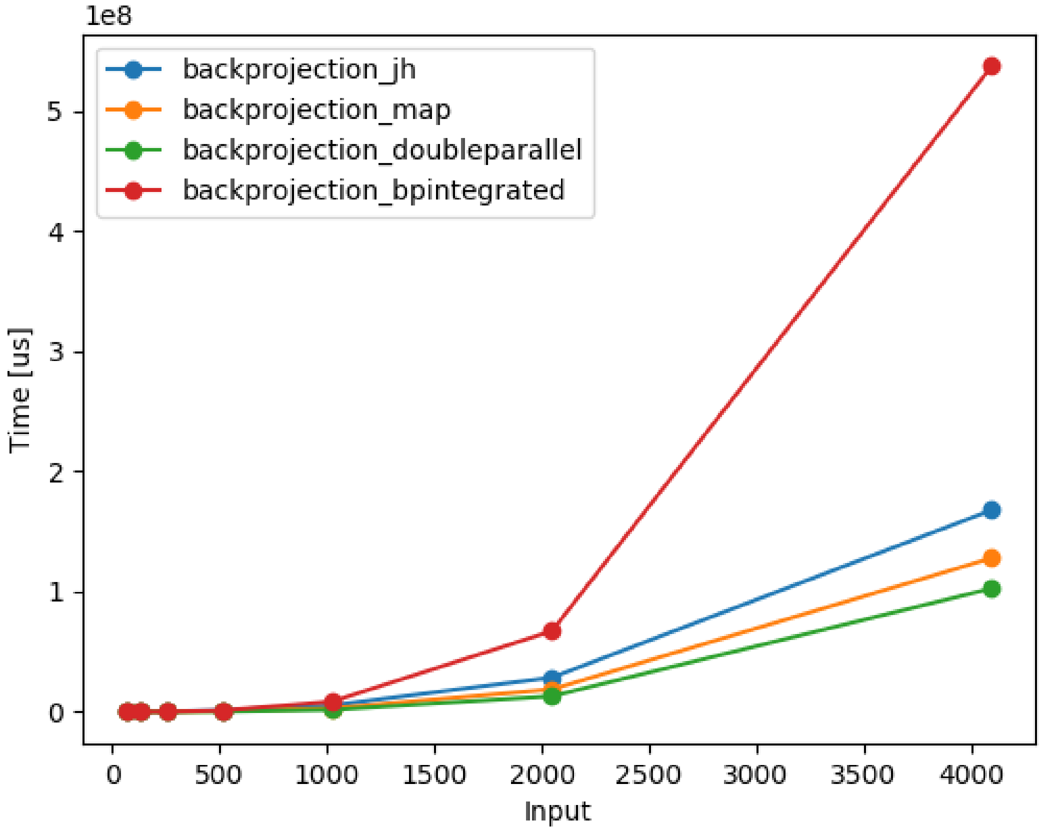



Figure 8: A comparison of the backprojections projections run with different matrix implementations and the version where the forward projection is integrated in projectiomatrix_doubleparallel. The chunk size was 32. The implementation using the double parallel version of the matrix performed best again as expected. The integrated version performed very badly, and it would be interesting to investigate why. We had to run our algorithms for only 30 angles as we got memory errors otherwise.

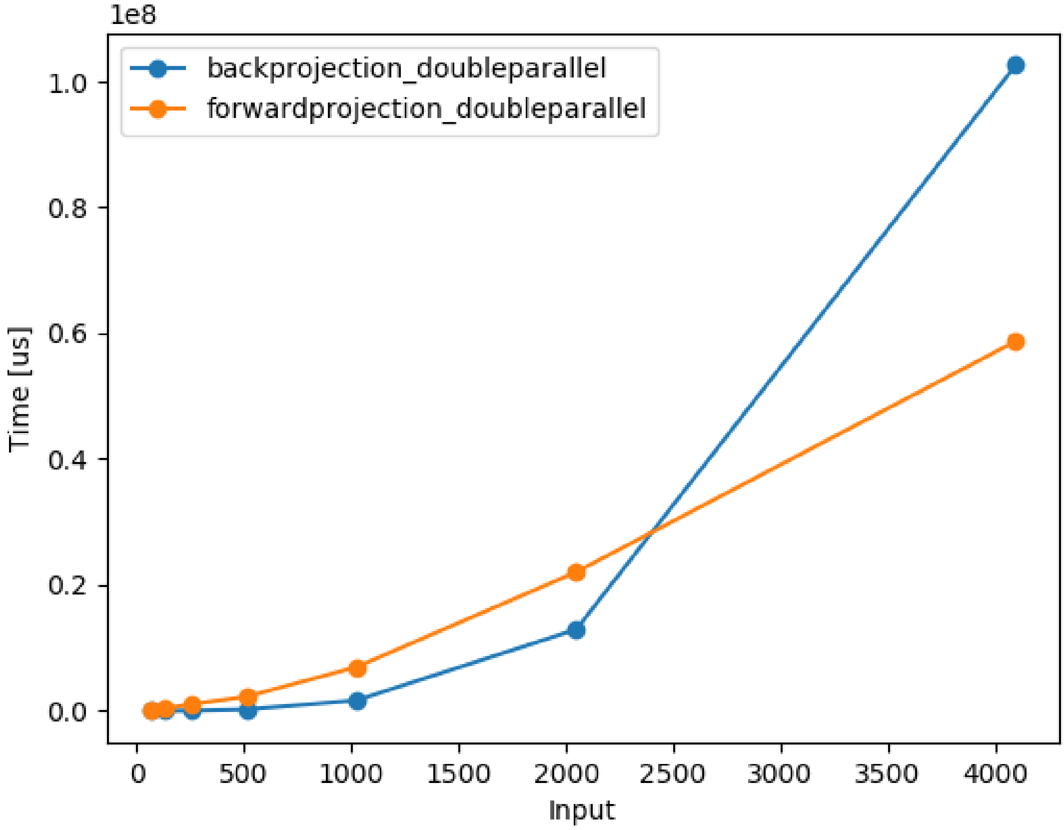
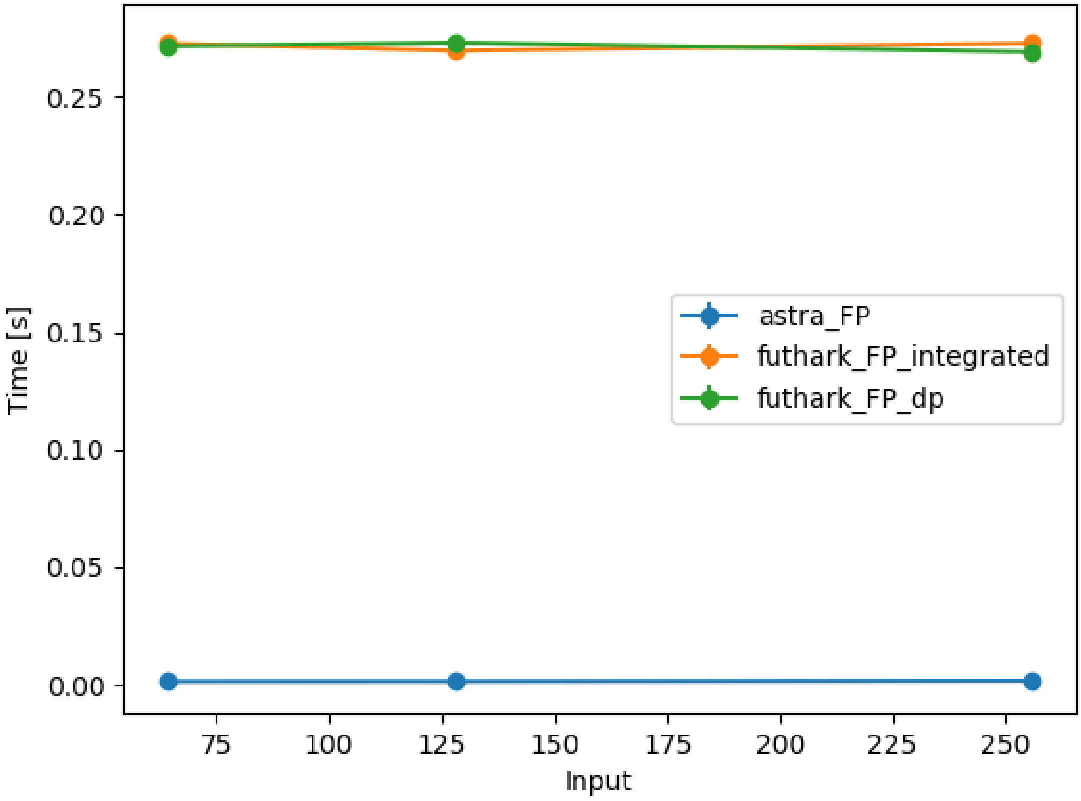



Figure 9: A comparison of backprojection and forward projection. Backprojection seems to be faster for small problem sizes, but scale worse.

# 5 Comparison to a CUDA implementation

We compared our implementation of forward projection and back projection to implementations from the astra toolbox. However we only compared the algorithms through python, by compiling our algorithms with pyopencl and running them through python and doing the same with the forward and backward rpojection from the atsra toolbox. Ideally the functions should have been compared without the memory overhead.



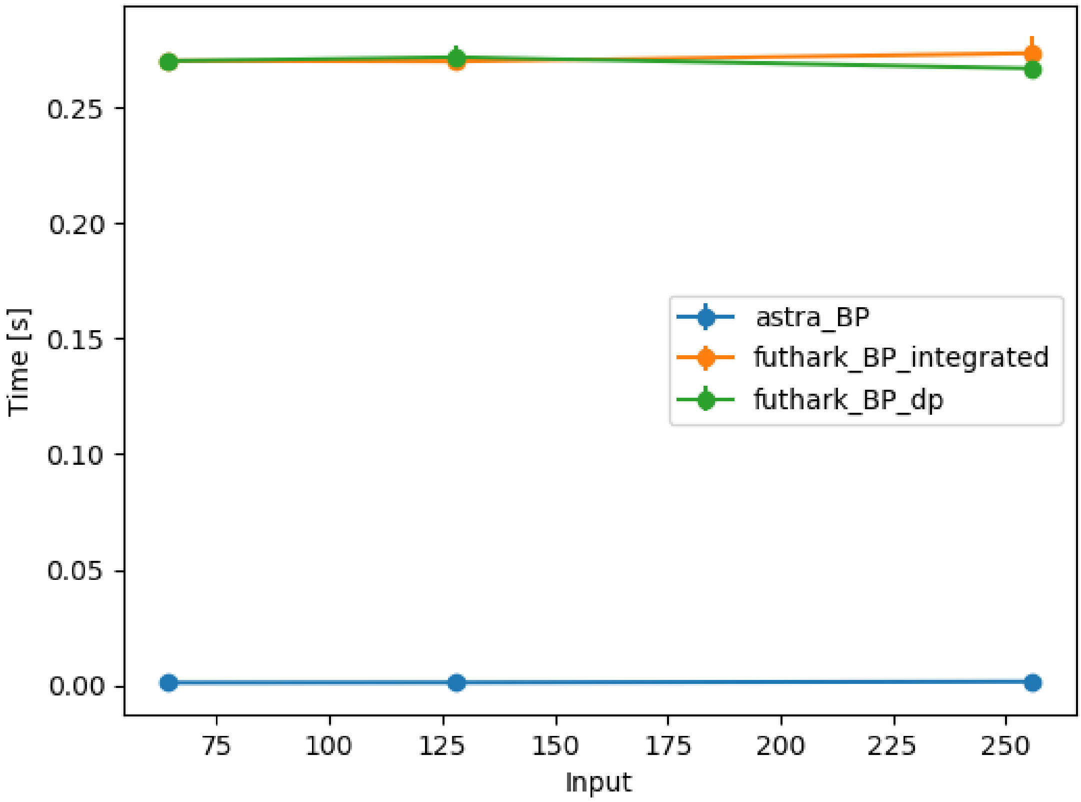



Figure 10: A comparison of the back projections from astra and our fastest futhark algorithms. The futhark algorithms perform much worse, approximately 128 times worse for both forward and backprojection. It would be worth comparing our code with the implemented version from the astra toolbox, to see how they get such a good running time.

# 6 Code Overview

Our code is primarily comprised of Futhark code, alongside a few python script for data generation, and shell scripts.

## 6.1 folder structure

- futhark contains the source code files, as well as the files generated during compilation.
- runscripts contains the shell scripts used to run tests and benchmarks
- root folder - the python scripts are located in the root folder, which makes it easier to navigate the folder structure from the python code.
- data and output - data is an, so far, empty folder to store the generated input data, while the output is the same for output data

## 6.2 Code structure

The actual Futhark code is located in the files projection_lib.fut, matrix_lib.fut and line_lib.fut. All the forwardprojection and backprojection files contains main functions to call the libraries for testing. The projection_lib contains the actual code for doing the forwardprojections and backprojections, as well as a few limited helper functions. The matrix_lib is focused on generating the system matrices, with code for each way of doing this. Finally the line_lib contains helper functions for the matrix_lib.

# 7 Conclusion

We set out to make a parallel implementation of forward and back projection for algebraic tomographic reconstruction. We found that most of the time spend in our initial nested versions, using code from a bachelor project for generating the system matrix, was spend on calculating the matrix. We therefore concentrated on improving the code for calcualting the matrix. We managed to get a slight speedup of about 1.3 times compared to the code from the bachelor project by exploiting a second level of parallelism. When we compare a c compiled version of our futhark code with an opencl compiled version we get a speedup of about 1.3. Strangely enough our c compiled code actually runs slightly faster than the opencl compiled version of the code from the bachelor project. It therefore seems that our new version is better both sequentially and in parallel.
We attempted to compare our implementations with those in a python library called astra toolbox with a backend implemented in CUDA. However, we did not time the kernels only, but timed the call from python which has significant overhead. Therefore, it is difficult to compare. The results we got differed on which function was run first, and generally seemed

to vary a lot. To properly time the functions, we would need to go to the astra code an time the kernel only. It seems to perform better than our versions though, and it would be nice to compare the CUDA code with the opencl compiled versions from futhark.
Some of our code may benefit from upcoming block and register tiling in futhark, or streaming possibilities as memory seems to be the biggest limitation. Other possibilities for optimizations could be to consider other datastructures for more sparse storage, or investigate how much precision is needed for good reconstructions. It will also be worth it to look at the branching in the kernels. Flattening does not seem very promising since we already have memory issues and our semiflat versions performed rather badly. Perhaps if the computation of the system matrix was also sparse and flat we would see an improvement.
Overall we were happy to get a slight speedup for the matrix computations, and there is still lots of work that could be done on this project. One thing that needs investigation is why chunking up the matrix does not seem to work as expected as we still ran into memory problems. We look forward to keep working on it with new upcoming features in futhark and to have time to scour the compiled code for further optimizations.

# 8 Code

## 8.1 projection lib

```
import "futlib/array"
import "matrix_lib"
open Matrix

module Projection = {

    --segmented scan with (+) on floats:
    let sgmSumF32 [n]
         (flg : [n]i32)
         (arr : [n]f32) : [n]f32 =
             let flgs_vals =
              scan ( \ (f1, x1) (f2,x2) ->
                         let f = f1 | f2 in
                         if f2 > 0 then (f, x2)
                         else (f, x1 + x2) )
                      (0,0.0f32) (zip flg arr)
             let (_, vals) = unzip flgs_vals
             in vals

    -- segmented scan with (+) on ints:
    let sgmSumI32 [n]
         (flg : [n]i32)
         (arr : [n]i32) : [n]i32 =
             let flgs_vals =
```

```
                scan ( \ (i1, x1) (i2,x2) ->
                        let i = i1 | i2 in
                        if i2 > 0 then (i, x2)
                        else (i, x1 + x2) )
                     (0,0i32) (zip flg arr)
               let (_, vals) = unzip flgs_vals
               in vals

      -- unzip for 2D arrays
      let unzip_d2 (xs: [][](f32,i32)): ([][]f32,[][]i32) =
          xs |> map unzip
               |> unzip

      -- step in radix sort
      let rsort_step [n] (xs: [n](f32, i32, i32), bitn: i32): [n](f32, i32, i32) =
                let (data,rays,pixels) = unzip3 xs
                let unsigned = map(\p -> u32.i32 p) pixels
                let bits1 = map (\x -> (i32.u32 (x >> u32.i32 bitn)) & 1) unsigned
                let bits0 = map (1-) bits1
                let idxs0 = map2 (*) bits0 (scan (+) 0 bits0)
                let idxs1 = scan (+) 0 bits1
                let offs  = reduce (+) 0 bits0
                let idxs1 = map2 (*) bits1 (map (+offs) idxs1)
                let idxs  = map2 (+) idxs0 idxs1
                let idxs  = map (\x->x-1) idxs
                in scatter (copy xs) idxs xs

      -- Radix sort algorithm, ascending
      let rsort [n] (xs: [n](f32, i32, i32)): [n](f32,i32,i32) =
               loop (xs) for i < 32 do rsort_step(xs,i)

      -- sparse matrix vector multiplication
      let spMatVctMult [num_elms] [vct_len] [num_rows]
          (mat_val : [num_elms](i32,f32))
          (shp_scn : [num_rows]i32)
          (vct : [vct_len]f32) : [num_rows]f32 =
               let len = shp_scn[num_rows-1]
               let shp_inds =
                   map (\i -> if i==0 then 0
                     else unsafe shp_scn[i-1]
                   ) (iota num_rows)
               let flags = scatter ( replicate len 0)
                     shp_inds ( replicate num_rows 1)
               let prods = map (\(i,x) -> x*(unsafe vct[i])) mat_val
               let sums = sgmSumF32 flags prods
               let mat_inds = map (\i -> i-1) shp_scn
               in map (\i -> unsafe sums[i]) mat_inds

      let notSparseMatMult [num_rows] [num_cols]
```

```
                         (mat_vals : [num_rows][num_cols](f32,i32))
                         (vect : []f32) : [num_rows]f32 =
    map (\row -> reduce (+) 0 <| map (\(v, ind) -> unsafe (if ind == -1 then 0.0 else v*vect[ind]) ) row ) mat_vals

-- not sparse, nested, matrix vector multiplication for backprojection, uses a padded version of the matrix
let notSparseMatMult_back [num_rows] [num_cols]
                          (mat_vals : [num_rows][num_cols]f32)
                          (vect : []f32) : [num_rows]f32 =
    map (\row -> reduce (+) 0 <| map2 (*) row vect ) mat_vals

-- gets the shape of a matrix - i.e number of entries pr. row when mat is in the format [[(d,i),(d,i)...]] where d is data
-- and i is column index and -1 means no data
let getshp (matrix: [][](f32,i32)) : []i32 =
         let dp = unzip_d2 matrix
         let flagsforindexes = map(\r -> map(\p -> if p == -1 then 0 else 1)r)dp.2
         in map(\f -> reduce (+) 0 f)flagsforindexes

-- helper function to determine if we entered new segment of consecutive numbers in an array.
let isnewsegment (i: i32) (arr: []i32) : bool =
                   i!=0 && (unsafe arr[i]) != (unsafe arr[i-1])

let backprojection_semiflat  (rays : []f32)
                   (angles : []f32)
                   (projections : []f32)
                   (gridsize: i32) : []f32=
         let halfsize = r32(gridsize)/2
         let entrypoints = convert2entry angles rays halfsize
         let intersections = map (\(p,sc) -> (lengths gridsize sc.1 sc.2 p)) entrypoints
         --convert to triples (data,ray,pixel)
         let triples_tmp = flatten(map(\i -> map(\v -> (v.1, i, v.2))(unsafe intersections[i])) (iota (length intersections)))
         -- remove none values
         let triples = filter (\x -> x.3 != -1) triples_tmp
         -- sort by pixel indexes
         let pixelsorted = rsort triples
         -- slplit int three arrays in order to use pixels only for shp
         let (data,rays,pixels) = unzip3 pixelsorted
         let num_pixels = length pixels
         -- contains sum of values where a row ends since columns will be rows.
         let shp_scn_tmp = map (\i -> if (i == num_pixels || (isnewsegment i pixels)) then i else 0) (iota (num_pixels+1))
```

```
        let shp_scn = filter (\p -> p != 0) shp_scn_tmp
        let values = map(\x-> (x.2,x.1))pixelsorted
        in spMatVctMult values shp_scn projections

-- pads and transposes the matrix, nested, will perform better when tiling is improved in futhark
let trans_map [m] [n]
              (matrix : [m][n](f32, i32)) (gridsize: i32): [][]f32 =
        let rs = gridsize*gridsize
        let padded = map (\row -> let (vals, inds) = (unzip row) in scatter (replicate rs 0.0f32) inds vals ) matrix
        in transpose padded

-- backprojection nested map version.
let backprojection_map  (angles : []f32)
                   (rays : []f32)
                   (projections : []f32)
                   (gridsize: i32)
                   (stepSize : i32) : []f32=
        let halfsize = r32(gridsize)/2
        let entrypoints = convert2entry angles rays halfsize
        let totalLen = (length entrypoints)
        let runLen = (totalLen/stepSize)
        -- result array
        let backmat = replicate (gridsize*gridsize) 0.0f32
        -- stripmined, sequential outer loop, mapped inner
        let (backmat, _, _, _, _, _, _) =
            loop (output, run, runLen, stepSize, gridsize, entrypoints, totalLen) = (backmat, 0, runLen, stepSize, gridsize, entrypoints, totalLen)
            while ( run < runLen ) do
                -- if the number of entrypoints doesn't line perfectly up with the stepsize
                let step = if (run+1)*stepSize >= totalLen then totalLen - run*stepSize else stepSize
                -- calc part of matrix, stepSize rows
                let partmatrix = map (\s -> unsafe (lengths_map gridsize (entrypoints[run*stepSize + s].2).1 (entrypoints[run*stepSize + s].2).2 entrypoints[run*stepSize + s].1 )) (iota step)
                -- transpose
                let transmat = trans_map partmatrix gridsize
                -- mult
                let partresult = (notSparseMatMult_back transmat projections[(run*stepSize) : (run*stepSize + step)])
                -- add
                let result = (map2 (+) partresult output)
```

```
                    in (result, run+1, runLen, stepSize, gridsize,
                       entrypoints, totalLen)
            in backmat

    let backprojection_jh (angles : []f32)
                          (rays : []f32)
                          (projections : []f32)
                          (gridsize: i32)
                          (stepSize : i32) : []f32=
            let halfsize = r32(gridsize)/2
            let entrypoints = convert2entry angles rays halfsize
            let totalLen = (length entrypoints)
            let runLen = (totalLen/stepSize)
            -- result array
            let backmat = replicate (gridsize*gridsize) 0.0f32
            -- stripmined, sequential outer loop, mapped inner
            let (backmat, _, _, _, _, _, _) =
                loop (output, run, runLen, stepSize, gridsize,
                   entrypoints, totalLen) = (backmat, 0, runLen,
                   stepSize, gridsize, entrypoints, totalLen)
                while ( run < runLen ) do
                    -- if the number of entrypoints doesn't line
                       perfectly up with the stepsize
                    let step = if (run+1)*stepSize >= totalLen then
                       totalLen - run*stepSize else stepSize
                    -- calc part of matrix, stepSize rows
                    let partmatrix = map (\s -> unsafe (lengths
                       gridsize (entrypoints[run*stepSize + s].2).1
                       (entrypoints[run*stepSize + s].2).2
                       entrypoints[run*stepSize + s].1 )) (iota step
                       )
                    -- transpose
                    let transmat = trans_map partmatrix gridsize
                    -- mult
                    let partresult = (notSparseMatMult_back transmat
                        projections[(run*stepSize) : (run*stepSize +
                        step)])
                    -- add
                    let result = (map2 (+) partresult output)
                    in (result, run+1, runLen, stepSize, gridsize,
                       entrypoints, totalLen)
            in backmat

    let backprojection_doubleparallel  (angles : []f32)
                          (rays : []f32)
                          (projections : []f32)
                          (gridsize: i32)
                          (stepSize : i32) : []f32=
            let halfsize = r32(gridsize)/2
            let entryexitpoints =  convert2entryexit angles rays
               halfsize
```

```
        let totalLen = (length entryexitpoints)
        let runLen = (totalLen/stepSize)
        -- result array
        let backmat = replicate (gridsize*gridsize) 0.0f32
        -- stripmined, sequential outer loop, mapped inner
        let (backmat, _, _, _, _, _, _) =
            loop (output, run, runLen, stepSize, gridsize,
               entryexitpoints, totalLen) = (backmat, 0, runLen,
                stepSize, gridsize, entryexitpoints, totalLen)
            while ( run < runLen ) do
                -- if the number of entrypoints doesn't line
                   perfectly up with the stepsize
                let step = if (run+1)*stepSize >= totalLen then
                   totalLen - run*stepSize else stepSize
                -- calc part of matrix, stepSize rows
                let halfgridsize = gridsize/2

                let partmatrix = map(\(ent,ext) -> (flatten(map
                   (\i ->
                          calculate_weight ent ext i gridsize
                     )((-halfgridsize)...(halfgridsize-1))))) (
                        entryexitpoints[(run*stepSize) : (run*
                        stepSize + step)])
                -- transpose
                let transmat = trans_map partmatrix gridsize
                -- mult
                let partresult = (notSparseMatMult_back transmat
                    projections[(run*stepSize) : (run*stepSize +
                    step)])
                -- add
                let result = (map2 (+) partresult output)
                in (result, run+1, runLen, stepSize, gridsize,
                   entryexitpoints, totalLen)
        in backmat

let forwardprojection_doubleparallel (angles : []f32)
                   (rays : []f32)
                    (voxels : []f32)
                    (stepSize : i32) : []f32 =
        let gridsize = t32(f32.sqrt(r32((length voxels))))
        let halfgridsize = gridsize/2
        let entryexitpoints =  convert2entryexit angles rays (
           r32(halfgridsize))
        let totalLen =  (length entryexitpoints)
        let runLen = (totalLen/stepSize)
        let testmat = [0f32]
        let (testmat, _, _, _, _, _, _) =
            loop (output, run, runLen, stepSize, gridsize,
               entryexitpoints, totalLen) = (testmat, 0, runLen,
                stepSize, gridsize, entryexitpoints, totalLen)
            while ( run < runLen ) do
```

```
                        let step = if (run+1)*stepSize >= totalLen then totalLen - run*stepSize else stepSize
                        let partmatrix = map(\(ent,ext) -> (flatten(map (\i ->
                                   calculate_weight ent ext i gridsize
                          )(-halfgridsize...halfgridsize-1)))) (entryexitpoints[run*stepSize:run*stepSize+step])
                        let partresult = notSparseMatMult partmatrix voxels
                        in (output++partresult, run+1, runLen, stepSize, gridsize, entryexitpoints, totalLen)
               in (tail testmat)

    let forwardprojection_jh [r][a][n] (angles : [a]f32)
                             (rays : [r]f32)
                              (voxels : [n]f32)
                              (stepSize : i32) : []f32 =
               let gridsize = t32(f32.sqrt(r32((length voxels))))
               let halfsize = r32(gridsize)/2
               let entrypoints = convert2entry angles rays halfsize
               let totalLen = (length entrypoints)
               -- let runLen = if (totalLen/stepSize) == 0 then 1 else (totalLen/stepSize)
               let runLen = (totalLen/stepSize)
               let testmat = [0f32]
               let (testmat, _, _, _, _, _, _) =
                   loop (output, run, runLen, stepSize, gridsize, entrypoints, totalLen) = (testmat, 0, runLen, stepSize, gridsize, entrypoints, totalLen)
                   while ( run < runLen ) do
                        let step = if (run+1)*stepSize >= totalLen then totalLen - run*stepSize else stepSize
                        let partmatrix = map (\s -> unsafe (lengths gridsize (entrypoints[run*stepSize + s].2).1 (entrypoints[run*stepSize + s].2).2 entrypoints[run*stepSize + s].1 )) (iota step )
                        let partresult = notSparseMatMult partmatrix voxels
                        in (output++partresult, run+1, runLen, stepSize, gridsize, entrypoints, totalLen)
               in (tail testmat)

    let forwardprojection_map [r][a][n] (angles : [a]f32)
                             (rays : [r]f32)
                              (voxels : [n]f32)
                              (stepSize : i32) : []f32 =
               let gridsize = t32(f32.sqrt(r32((length voxels))))
               let halfsize = r32(gridsize)/2
               let entrypoints = convert2entry angles rays halfsize
```

```
             let totalLen = (length entrypoints)
             let runLen = (totalLen/stepSize)
             let testmat = [0f32]
             -- stripmined, sequential outer loop, mapped inner
             let (testmat, _, _, _, _, _, _) =
                 loop (output, run, runLen, stepSize, gridsize, entrypoints, totalLen) = (testmat, 0, runLen, stepSize, gridsize, entrypoints, totalLen)
                 while ( run < runLen ) do
                     -- if the number of entrypoints doesn't line perfectly up with the stepsize
                     let step = if (run+1)*stepSize >= totalLen then totalLen - run*stepSize else stepSize
                     -- calc part of matrix, stepSize rows
                     let partmatrix = map (\s -> unsafe (lengths_map gridsize (entrypoints[run*stepSize + s].2).1 (entrypoints[run*stepSize + s].2).2 entrypoints[run*stepSize + s].1 )) (iota step )
                     let partresult = notSparseMatMult partmatrix voxels
                     in (output++partresult, run+1, runLen, stepSize, gridsize, entrypoints, totalLen)
             in (tail testmat)

    let forwardprojection_semiflat [r][a][n] (angles : [a]f32)
                   (rays : [r]f32)
                    (voxels : [n]f32)
                    (stepSize : i32) : []f32 =
             let gridsize = t32(f32.sqrt(r32((length voxels))))
             let halfsize = r32(gridsize)/2
             let entrypoints = convert2entry angles rays halfsize
             let totalLen = (length entrypoints)
             -- let runLen = if (totalLen/stepSize) == 0 then 1 else (totalLen/stepSize)
             let runLen = (totalLen/stepSize)
             let testmat = [0f32]
             let (testmat, _, _, _, _, _, _) =
                 loop (output, run, runLen, stepSize, gridsize, entrypoints, totalLen) = (testmat, 0, runLen, stepSize, gridsize, entrypoints, totalLen)
                 while ( run < runLen ) do
                     let step = if (run+1)*stepSize >= totalLen then totalLen - run*stepSize else stepSize
                     let intersections = map (\(p,sc) -> (lengths gridsize sc.1 sc.2 p)) entrypoints[(run*stepSize) : (run*stepSize + step)]
                     let shp = getshp intersections
                     let shp_scn = scan (+) 0 shp
                     let values_tmp = flatten(map(\r -> map(\(d,p)->(p,d))r)intersections)
```

```
                        let values = filter (\x -> x.1 != -1) values_tmp
                        let partresult = spMatVctMult values shp_scn voxels
                        in (output++partresult, run+1, runLen, stepSize, gridsize, entrypoints, totalLen)
            in (tail testmat)

    let forwardprojection_integrated [r][a][n] (angles : [a]f32)
                        (rays : [r]f32)
                         (voxels : [n]f32)
                         (stepSize : i32) : []f32 =
            let gridsize = t32(f32.sqrt(r32((length voxels))))
            let halfsize = gridsize/2
            let entryexitpoints =  convert2entryexit angles rays (r32(halfsize))
            let totalLen = (length entryexitpoints)
            -- let runLen = if (totalLen/stepSize) == 0 then 1 else (totalLen/stepSize)
            let runLen = (totalLen/stepSize)
            let testmat = [0f32]
            let (testmat, _, _, _, _, _, _) =
                loop (output, run, runLen, stepSize, gridsize, entryexitpoints, totalLen) = (testmat, 0, runLen, stepSize, gridsize, entryexitpoints, totalLen)
                while ( run < runLen ) do
                        let step = if (run+1)*stepSize >= totalLen then totalLen - run*stepSize else stepSize
                        let partresult = map(\(ent,ext) -> (reduce (+) 0 (flatten(map (\i ->
                                        calculate_fp_val ent ext i gridsize voxels
                          )((-halfsize)...(halfsize-1)))))) entryexitpoints[run*stepSize:run*stepSize+step]
                        in (output++partresult, run+1, runLen, stepSize, gridsize, entryexitpoints, totalLen)
            in (tail testmat)


    let backprojection_integrated  (angles : []f32)
                        (rays : []f32)
                        (projections : []f32)
                        (gridsize: i32)
                        (stepSize : i32) : []f32=
            let halfsize = gridsize/2
            let entryexitpoints =  convert2entryexit angles rays (r32(halfsize))
            let totalLen = (length entryexitpoints)
            let runLen = (totalLen/stepSize)
            -- result array
            let backmat = replicate (gridsize*gridsize) 0.0f32
```

```
                -- stripmined, sequential outer loop, mapped inner
                let (backmat, _, _, _, _, _, _) =
                    loop (output, run, runLen, stepSize, gridsize,
                        entryexitpoints, totalLen) = (backmat, 0, runLen,
                         stepSize, gridsize, entryexitpoints, totalLen)
                    while ( run < runLen ) do
                        -- if the number of entrypoints doesn't line
                           perfectly up with the stepsize
                        let step = if (run+1)*stepSize >= totalLen then
                           totalLen - run*stepSize else stepSize
                        let partmatresult = map(\j ->
                             (flatten(map (\i ->
                                 calculate_bp_val (unsafe
                                    entryexitpoints[run*stepSize+j]).1 (
                                    unsafe entryexitpoints[run*stepSize+
                                    j]).2 i gridsize (unsafe projections
                                    [j])
                            )((-halfsize)...(halfsize-1))))) (iota step)
                        let transp = trans_map partmatresult gridsize
                        let partresult = map (\row -> reduce (+) 0 row)
                           transp
                        -- add
                        let result = (map2 (+) partresult output)
                        in (result, run+1, runLen, stepSize, gridsize,
                           entryexitpoints, totalLen)
                in backmat
}
```

## 8.2 matrix lib

```
import "line_lib"
open Lines
module Matrix =
{
      let calculate_fp_val(ent: point)
                  (ext: point)
                  (i: i32)
                  (N: i32)
                  (pixels: []f32) : []f32 =
            let Nhalf = N/2
            -- handle all lines as slope < 1 reverse the others
            let slope = (ext.2 - ent.2)/(ext.1 - ent.1)
            let reverse = f32.abs(slope) > 1
            let gridentry = if reverse then (if slope < 0 then (-ent.2,
                ent.1) else (-ext.2,ext.1)) else ent
            let k = if reverse then (-1/slope) else slope

            --- calculate stuff
            let ymin = k*(r32(i) - gridentry.1) + gridentry.2 + r32(
                Nhalf)
            let yplus = k*(r32(i) + 1 - gridentry.1) + gridentry.2 +
```

```
r32(Nhalf)
     let Ypixmin = t32(f32.floor(ymin))
     let Ypixplus = t32(f32.floor(yplus))
     let baselength = f32.sqrt(1+k*k)
     -- in [(baselength,Ypixmin),(baselength,Ypixplus)]
     let Ypixmax = i32.max Ypixmin Ypixplus
     let ydiff = yplus - ymin
     let yminfact = (r32(Ypixmax) - ymin)/ydiff
     let yplusfact = (yplus - r32(Ypixmax))/ydiff
     let iindex = i+Nhalf
     -- index calculated wrong for reversed lines i think
     let pixmin = if reverse then (N-iindex-1)*N+Ypixmin else iindex+Ypixmin*N
     let pixplus = if reverse then (N-iindex-1)*N+Ypixplus else iindex+Ypixplus*N
     let lymin = yminfact*baselength
     let lyplus = yplusfact*baselength
     let min = if (pixmin >= 0 && pixmin < N ** 2) then
          (if Ypixmin == Ypixplus then (unsafe baselength*pixels[pixmin]) else (unsafe lymin*pixels[pixmin]))
          else 0
     let plus = if (pixplus >= 0 && pixplus < N ** 2) then
          (if Ypixmin == Ypixplus then 0 else (unsafe lyplus*pixels[pixmin]))
          else 0
     in [min,plus]

let calculate_bp_val(ent: point)
          (ext: point)
          (i: i32)
          (N: i32)
          (proj_val: f32) : [](f32,i32) =
     let Nhalf = N/2
     -- handle all lines as slope < 1 reverse the others
     let slope = (ext.2 - ent.2)/(ext.1 - ent.1)
     let reverse = f32.abs(slope) > 1
     let gridentry = if reverse then (if slope < 0 then (-ent.2,ent.1) else (-ext.2,ext.1)) else ent
     let k = if reverse then (-1/slope) else slope

     --- calculate stuff
     let ymin = k*(r32(i) - gridentry.1) + gridentry.2 + r32(Nhalf)
     let yplus = k*(r32(i) + 1 - gridentry.1) + gridentry.2 + r32(Nhalf)
     let Ypixmin = t32(f32.floor(ymin))
     let Ypixplus = t32(f32.floor(yplus))
     let baselength = f32.sqrt(1+k*k)
     -- in [(baselength,Ypixmin),(baselength,Ypixplus)]
     let Ypixmax = i32.max Ypixmin Ypixplus
     let ydiff = yplus - ymin
```

```
        let yminfact = (r32(Ypixmax) - ymin)/ydiff
        let yplusfact = (yplus - r32(Ypixmax))/ydiff
        let iindex = i+Nhalf
        -- index calculated wrong for reversed lines i think
        let pixmin = if reverse then (N-iindex-1)*N+Ypixmin else iindex+Ypixmin*N
        let pixplus = if reverse then (N-iindex-1)*N+Ypixplus else iindex+Ypixplus*N
        let lymin = yminfact*baselength
        let lyplus = yplusfact*baselength
        let min = if (pixmin >= 0 && pixmin < N ** 2) then
             (if Ypixmin == Ypixplus then ((baselength*proj_val), pixmin) else (lymin*proj_val,pixmin))
             else (-1f32,-1i32)
        let plus = if (pixplus >= 0 && pixplus < N ** 2) then
             (if Ypixmin == Ypixplus then (-1f32,-1i32) else ((lyplus*proj_val), pixmin))
             else (-1f32,-1i32)
        in [min,plus]


    --- DOUBLE PARALLEL
    -- function which computes the weight of pixels in grid_column for ray with entry/exit p
    let calculate_weight(ent: point)
             (ext: point)
             (i: i32)
             (N: i32) : [](f32,i32) =
        let Nhalf = N/2
        -- handle all lines as slope < 1 reverse the others
        let slope = (ext.2 - ent.2)/(ext.1 - ent.1)
        let reverse = f32.abs(slope) > 1
        let gridentry = if reverse then (if slope < 0 then (-ent.2,ent.1) else (-ext.2,ext.1)) else ent
        let k = if reverse then (-1/slope) else slope

        --- calculate stuff
        let ymin = k*(r32(i) - gridentry.1) + gridentry.2 + r32(Nhalf)
        let yplus = k*(r32(i) + 1 - gridentry.1) + gridentry.2 + r32(Nhalf)
        let Ypixmin = t32(f32.floor(ymin))
        let Ypixplus = t32(f32.floor(yplus))
        let baselength = f32.sqrt(1+k*k)
        -- in [(baselength,Ypixmin),(baselength,Ypixplus)]
        let Ypixmax = i32.max Ypixmin Ypixplus
        let ydiff = yplus - ymin
        let yminfact = (r32(Ypixmax) - ymin)/ydiff
        let yplusfact = (yplus - r32(Ypixmax))/ydiff
        let lymin = yminfact*baselength
        let lyplus = yplusfact*baselength
```

```
    let iindex = i+Nhalf
    -- index calculated wrong for reversed lines i think
    let pixmin = if reverse then (N-iindex-1)*N+Ypixmin else iindex+Ypixmin*N
    let pixplus = if reverse then (N-iindex-1)*N+Ypixplus else iindex+Ypixplus*N
    let min = if (pixmin >= 0 && pixmin < N ** 2) then
        (if Ypixmin == Ypixplus then (baselength,pixmin) else (lymin,pixmin))
        else (-1f32,-1i32)
    let plus = if (pixplus >= 0 && pixplus < N ** 2) then
        (if Ypixmin == Ypixplus then (-1f32,-1i32) else (lyplus,pixplus))
        else (-1f32,-1i32)
    in [min,plus]

-- assuming  gridsize even
let weights_doublepar(angles: []f32) (rays: []f32) (gridsize: i32): [][](f32,i32) =
    let halfgridsize = gridsize/2
    let entryexitpoints =  convert2entryexit angles rays (r32(halfgridsize))
    in map(\(ent,ext) -> (flatten(map (\i ->
            calculate_weight ent ext i gridsize
        )((-halfgridsize)...(halfgridsize-1))))) entryexitpoints

--- JH VERSION
let lengths    (grid_size: i32)
               (sint: f32)
               (cost: f32)
               (entry_point: point): [](f32, i32) =

    let horizontal = cost == 0
    let vertical = f32.abs(cost) == 1
    let slope = cost/(-sint) -- tan(x+90) = -cot(x) = slope since the angles ar ethe normals of the line

    let size = r32(grid_size)
    let halfsize = size/2.0f32

    let A = replicate (t32(size*2f32-1f32)) (-1f32, -1)

    let y_step_dir = if slope < 0f32 then -1f32 else 1f32
    let anchorX = f32.floor(entry_point.1) + 1f32
    let anchorY = if y_step_dir == -1f32
        then f32.ceil(entry_point.2) - 1f32
        else f32.floor(entry_point.2) + 1f32

   let (A, _, _, _, _) =
     loop (A, focusPoint, anchorX, anchorY, write_index) = (A,
```

```
   entry_point, anchorX, anchorY, 0)
while ( isInGrid halfsize y_step_dir focusPoint ) do
  --compute index of pixel in array by computing x component and y component if
  --center was at bottom left corner (add halfsize), and add them multiplying y_comp by size
  let y_floor = f32.floor(halfsize+focusPoint.2)
  let y_comp =
    if (y_step_dir == -1f32 && focusPoint.2 - f32.floor(focusPoint.2) == 0f32)
    then y_floor - 1f32
    else y_floor
  let x_comp= f32.floor(halfsize+focusPoint.1)
  let index = t32(x_comp+size*y_comp)

  --compute the distances using the difference travelled along an axis to the
  --next whole number and the slope or inverse slope
  let dy = if vertical then 1f32 else if horizontal then 0f32 else (anchorX-focusPoint.1)*slope
  let dx = if vertical then 0f32 else if horizontal then 1f32 else (anchorY-focusPoint.2)*(1/slope)
  let p_anchor_x = (anchorX, focusPoint.2+dy)
  let p_anchor_y = (focusPoint.1+dx, anchorY)

  let dist_p_x = distance focusPoint p_anchor_x
  let dist_p_y = distance focusPoint p_anchor_y

  in
    if horizontal then
      unsafe let A[write_index] = (dist_p_x, index)
      in (A, p_anchor_x, anchorX + 1f32, anchorY, write_index+1)
    else if vertical then
      unsafe let A[write_index] = (dist_p_y, index)
      in (A, p_anchor_y, anchorX, anchorY + y_step_dir, write_index+1)
    else
    if (f32.abs(dist_p_x - dist_p_y) > 0.000000001f32)
    then
      if ( dist_p_x < dist_p_y )
      then
        unsafe let A[write_index] = (dist_p_x, index)
        in (A, p_anchor_x, anchorX + 1f32, anchorY, write_index+1)
      else
        unsafe let A[write_index] = (dist_p_y, index)
        in (A, p_anchor_y, anchorX, anchorY + y_step_dir, write_index+1)
    else
        unsafe let A[write_index] = (dist_p_x, index)
```

```
                    in (A, p_anchor_x, anchorX + 1f32, anchorY + y_step_dir, write_index+1)
        in A

    let weights_jh   (angles: []f32)
                    (rays: []f32)
                    (gridsize: i32) : [][](f32,i32) =
        let halfsize = r32(gridsize)/2
        let entrypoints = convert2entry angles rays halfsize
        in map (\(p,sc) -> (lengths gridsize sc.1 sc.2 p)) entrypoints

    ------------------------MAP----------------------------------------------------------
    let index (focusPoint: point) (halfsize: f32) (y_step_dir): i32 =
         let y_floor = f32.floor(halfsize+focusPoint.2)
         let y_comp =
              if (y_step_dir == -1f32 && focusPoint.2 - f32.floor(focusPoint.2) == 0f32)
              then y_floor - 1f32
              else y_floor
         let x_comp= f32.floor(halfsize+focusPoint.1)
         in t32(x_comp+(halfsize*2f32)*y_comp)

      -- let nextpointonline (vertical: bool) (horizontal: bool) (anchorX: point) (anchorY: point) (slope: f32) (focusPoint: point): point or array of points and lengths
    let nextpointonline (slope: f32) (vertical: bool) (focusPoint: point): point =
         let y_step_dir = if slope < 0f32 then -1f32 else 1f32
         let anchorX = if vertical then focusPoint.1 else f32.floor(focusPoint.1) + 1f32
         let anchorY = if slope == 0 then focusPoint.2 else if y_step_dir == -1f32
            then f32.ceil(focusPoint.2) - 1f32
            else f32.floor(focusPoint.2) + 1f32
         let dy = if slope == 1 then 1f32 else if slope == 0 then 0f32 else (anchorX-focusPoint.1)*slope
         let dx = if slope == 1 then 0f32 else if slope == 0 then 1f32 else (anchorY-focusPoint.2)*(1/slope)
         let p_anchor_x = (anchorX, focusPoint.2+dy)
         let p_anchor_y = (focusPoint.1+dx, anchorY)
         in if p_anchor_x.1 < p_anchor_y.1 then p_anchor_x else p_anchor_y

    let getFocusPoints (entryPoint: point) slope vertical halfsize y_step_dir =
         let A = replicate (t32(2f32*halfsize*2f32-1f32)) (f32.lowest, f32.lowest)
         let (A, _, _) =
```

```
            loop (A, focusPoint, write_index) = (A, entryPoint, 0)
                while ( isInGrid halfsize y_step_dir focusPoint ) do
                let nextpoint = (nextpointonline slope vertical focusPoint)
                in unsafe let A[write_index] = focusPoint
                in (A, nextpoint, write_index+1)
            in A

        let lengths_map
            (grid_size: i32)
            (sint: f32)
            (cost: f32)
            (entryPoint: point): [](f32, i32) =

            let vertical = f32.abs(cost) == 1
            let slope =  cost/(-sint)

            let size = r32(grid_size)
            let halfsize = size/2.0f32

            let y_step_dir = if slope < 0f32 then -1f32 else 1f32
            let focuspoints = (getFocusPoints entryPoint slope vertical halfsize y_step_dir)
            --for all focuspoints, save index and distance
            let mf = map(\i ->
                let ind = if !(isInGrid halfsize y_step_dir (unsafe focuspoints[i])) then -1 else index (unsafe focuspoints[i]) halfsize y_step_dir
                let dist = if isInGrid halfsize y_step_dir (unsafe focuspoints[i+1]) then (unsafe (distance focuspoints[i] focuspoints[i+1])) else 0.0f32
                in (dist, ind)
             ) (iota ((length focuspoints)-1))
             in mf

        let weights_map   (angles: []f32)
                        (rays: []f32)
                        (gridsize: i32) : [][](f32,i32) =
            let halfsize = r32(gridsize)/2
            let entrypoints = convert2entry angles rays halfsize
            in map (\(p,sc) -> (lengths_map gridsize sc.1 sc.2 p)) entrypoints
}
```

# References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