Q3

- 変数が1個ある場合

$$\mathbb{X} = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{pmatrix}$$

$$\mathbb{X}^T \mathbb{X} \hat{\beta} = \mathbb{X}^T y$$

$$\begin{pmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{pmatrix} \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

$$\begin{pmatrix} 1 \times n & x_1 + x_2 + \cdots + x_n \\ x_1 + x_2 + \cdots x_n & x_1^2 + x_2^2 + \cdots + x_n^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} y_1 + y_2 + \cdots + y_n \\ x_1 y_1 + x_2 y_2 + \cdots + x_n y_n \end{pmatrix}$$

$$\begin{pmatrix} n & \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i & \sum_{i=1}^n x_i^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n x_i y_i \end{pmatrix}$$

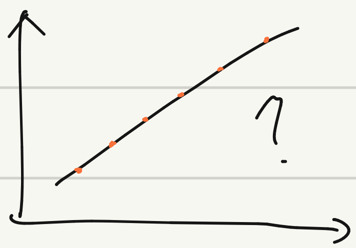
$$\therefore \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} = \underline{\begin{pmatrix} n & \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i & \sum_{i=1}^n x_i^2 \end{pmatrix}^{-1}} \underline{\begin{pmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n x_i y_i \end{pmatrix}} \equiv \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\hat{\beta} = \underline{(\mathbb{X}^T \mathbb{X})^{-1}} \underline{\mathbb{X}^T y}$$

$$\begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{pmatrix} = \frac{1}{\frac{1}{n} \sum_i x_i^2 - \cancel{\bar{x}^2}} \begin{pmatrix} \frac{1}{n} \sum x_i & -\bar{x} \\ -\bar{x} & 1 \end{pmatrix} \begin{pmatrix} \bar{y} \\ \frac{1}{n} \sum_i x_i y_i \end{pmatrix}$$

分散 $\sigma^2 - \cancel{\bar{x}^2}$



- 変数が2個ある場合

$$\mathbb{X} = \begin{pmatrix} 1 & x_{11} & x_{12} \\ 1 & x_{21} & x_{22} \\ \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} \end{pmatrix}$$

説明変数が2つ

2人目のデータ

n人目のデータ

$$\mathbb{X}^T \mathbb{X} \beta = \mathbb{X}^T y$$

$$\begin{pmatrix} 1 & 1 & \cdots & 1 \\ x_{11} & x_{21} & \cdots & x_{n1} \\ x_{12} & x_{22} & \cdots & x_{n2} \end{pmatrix} \begin{pmatrix} 1 & x_{11} & x_{12} \\ 1 & x_{21} & x_{22} \\ \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ x_{11} & x_{21} & \cdots & x_{n1} \\ x_{12} & x_{22} & \cdots & x_{n2} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

$$\begin{pmatrix} 1 \times n & x_{11} + x_{21} + \cdots + x_{n1} & x_{12} + x_{22} + \cdots + x_{n2} \\ x_{11} + x_{21} + \cdots + x_{n1} & x_{11}^2 + x_{21}^2 + \cdots + x_{n1}^2 & x_{11} x_{12} + x_{21} x_{22} + \cdots + x_{n1} x_{n2} \\ x_{12} + x_{22} + \cdots x_{n2} & x_{12} x_{11} + \cdots + x_{n2} x_{n1} & x_{12}^2 x_{22}^2 + \cdots + x_{n2}^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} y_1 + y_2 + \cdots + y_n \\ x_{11} y_1 + x_{21} y_2 + \cdots + x_{n1} y_n \\ x_{12} y_1 + x_{22} y_2 + \cdots + x_{n2} y_n \end{pmatrix}$$

$$\begin{pmatrix} n & \sum_{i=1}^n x_{i1} & \sum_{i=1}^n x_{i2} \\ \sum_{i=1}^n x_{i1} & \sum_{i=1}^n x_{i1}^2 & \sum_{i=1}^n x_{i1} x_{i2} \\ \sum_{i=1}^n x_{i2} & \sum_{i=1}^n x_{i2} x_{i1} & \sum_{i=1}^n x_{i2}^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n x_{i1} y_i \\ \sum_{i=1}^n x_{i2} y_i \end{pmatrix}$$

$$\therefore \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} = \underline{\begin{pmatrix} n & \sum_{i=1}^n x_{i1} & \sum_{i=1}^n x_{i2} \\ \sum_{i=1}^n x_{i1} & \sum_{i=1}^n x_{i1}^2 & \sum_{i=1}^n x_{i1} x_{i2} \\ \sum_{i=1}^n x_{i2} & \sum_{i=1}^n x_{i2} x_{i1} & \sum_{i=1}^n x_{i2}^2 \end{pmatrix}^{-1}} \underline{\begin{pmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n x_{i1} y_i \\ \sum_{i=1}^n x_{i2} y_i \end{pmatrix}} \equiv \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \underbrace{\tilde{A}}_{\text{余因子行列}} = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$A_{11} \to (-1)^{1+1} \begin{vmatrix} \sum_{i=1}^n x_{i1}^2 & \sum_{i=1}^n x_{i1} x_{i2} \\ \sum_{i=1}^n x_{i2} x_{i1} & \sum_{i=1}^n x_{i2}^2 \end{vmatrix}$$

$$\hat{\beta} = \underline{(\mathbb{X}^T \mathbb{X})^{-1}} \underline{\mathbb{X}^T y}$$

↳ 変数が3個以上も同様