

$$\begin{aligned}
 |A| &= (\sigma_1^2 + \bar{x}_1^2)(\sigma_2^2 + \bar{x}_2^2) + \bar{x}_1 \bar{x}_2 (\sigma_{12} + \bar{x}_1 \bar{x}_2) + \bar{x}_1 \bar{x}_2 (\sigma_{12} + \bar{x}_1 \bar{x}_2) \\
 &\quad - \bar{x}_1^2 (\sigma_1^2 + \bar{x}_1^2) - \bar{x}_2^2 (\sigma_2^2 + \bar{x}_2^2) - (\sigma_{12} + \bar{x}_1 \bar{x}_2)^2 \\
 &= \sigma_1^2 \sigma_2^2 + \cancel{\sigma_1^2 \bar{x}_2^2} + \cancel{\bar{x}_1^2 \sigma_2^2} + \cancel{(\bar{x}_1 \bar{x}_2)^2} + \cancel{\bar{x}_1 \bar{x}_2 \sigma_{12}} + \cancel{(\bar{x}_1 \bar{x}_2)^2} + \cancel{\bar{x}_1 \bar{x}_2 \sigma_{12}} + \cancel{(\bar{x}_1 \bar{x}_2)^2} \\
 &\quad - \cancel{\bar{x}_1^2 \sigma_1^2} - \cancel{\bar{x}_1^2 \bar{x}_1^2} - \cancel{\bar{x}_2^2 \sigma_2^2} - \cancel{(\bar{x}_1 \bar{x}_2)^2} - \sigma_{12}^2 - \cancel{2 \sigma_{12} \bar{x}_1 \bar{x}_2} - \cancel{(\bar{x}_1 \bar{x}_2)^2} \\
 &= \sigma_1^2 \sigma_2^2 - \sigma_{12}^2
 \end{aligned}$$