変数が2コある時つづき

$$\begin{pmatrix} n & \sum_{i=1}^{n} x_{i1} & \sum_{i=1}^{n} x_{i2} \\ \sum_{i=1}^{n} x_{i1} & \sum_{i=1}^{n} x_{i1}^2 & \sum_{i=1}^{n} x_{i1}x_{i2} \\ \sum_{i=1}^{n} x_{i2} & \sum_{i=1}^{n} x_{i2}x_{i1} & \sum_{i=1}^{n} x_{i2}^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_{i1} y_i \\ \sum_{i=1}^{n} x_{i2} y_i \end{pmatrix}$$

$$\cancel{n} \begin{pmatrix} 1 & \overbrace{\frac{1}{n}\sum_{i=1}^{n} x_{i1}}^{\bar{x}_1} & \frac{1}{n}\sum_{i=1}^{n} x_{i2} \\ \frac{1}{n}\sum_{i=1}^{n} x_{i1} & \frac{1}{n}\sum_{i=1}^{n} x_{i1}^2 & \frac{1}{n}\sum_{i=1}^{n} x_{i1}x_{i2} \\ \underbrace{\frac{1}{n}\sum_{i=1}^{n} x_{i2}}_{\bar{x}_2} & \frac{1}{n}\sum_{i=1}^{n} x_{i2}x_{i1} & \frac{1}{n}\sum_{i=1}^{n} x_{i2}^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} = \cancel{n} \begin{pmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_{i1} y_i \\ \sum_{i=1}^{n} x_{i2} y_i \end{pmatrix}$$

$$\sum_{i=1}^{n} x_{i1}^2 = \sum_{i=1}^{n}(x_{i1}^2 - 2x_{i1}\bar{x}_1 + \bar{x}_1^2 + 2x_{i1}\bar{x}_1 - \bar{x}_1)$$
$$= \sum(x_{i1} - \bar{x}_1)^2 + 2\bar{x}_1 \sum x_{i1} - n\bar{x}_1^2$$
$$= n\sigma_1^2 + 2n\bar{x}_1^2 - n\bar{x}_1^2$$

分散

$$= n(\sigma_1^2 + 2\bar{x}_1^2 - \bar{x}_1^2)$$
$$= n(\sigma_1^2 + \bar{x}_1^2)$$
$$\frac{1}{n}\sum x_{i1}^2 = \sigma_1^2 + \bar{x}_1^2$$

同様に、

$$\frac{1}{n}\sum x_{i2}^2 = \sigma_2^2 + \bar{x}_2^2$$

$$x_{i1}x_{i2} - x_{i1}\bar{x}_2 - \bar{x}_1 x_{i2} + \bar{x}_1\bar{x}_2$$

$$\sum x_{i2}x_{i1} = \sum\{(x_{i1}-\bar{x}_1)(x_{i2}-\bar{x}_2) + x_{i1}\bar{x}_2 + x_{i2}\bar{x}_1 - \bar{x}_1\cdot\bar{x}_2\}$$
$$= \sum(x_{i1}-\bar{x}_1)(x_{i2}-\bar{x}_2) + \bar{x}_2\cdot\sum x_{i1} + \bar{x}_1\sum x_{i2} - n\bar{x}_1\cdot\bar{x}_2$$
$$= n\sigma_{12} + \cancel{\bar{x}_2\cdot n\bar{x}_1} + \bar{x}_1 n\bar{x}_2 - \cancel{n\bar{x}_1\cdot\bar{x}_2}$$

共分散

$$= n(\sigma_{12} + \bar{x}_1\bar{x}_2)$$
$$\frac{1}{n}\sum x_{i2}x_{i1} = \sigma_{12} + \bar{x}_1\cdot\bar{x}_2$$

こちらも同様に

$$\begin{pmatrix} 1 & \bar{x}_1 & \bar{x}_2 \\ \bar{x}_1 & \sigma_1^2+\bar{x}_1^2 & \sigma_{12}+\bar{x}_1\bar{x}_2 \\ \bar{x}_2 & \sigma_{12}+\bar{x}_1\bar{x}_2 & \sigma_2^2+\bar{x}_2^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} \bar{y} \\ \bar{y}\bar{x}_1 + \sigma_{x_1 y} \\ \bar{y}\bar{x}_2 + \sigma_{x_2 y} \end{pmatrix}$$

$$\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & \bar{x}_1 & \bar{x}_2 \\ \bar{x}_1 & \sigma_1^2+\bar{x}_1^2 & \sigma_{12}+\bar{x}_1\bar{x}_2 \\ \bar{x}_2 & \sigma_{12}+\bar{x}_1\bar{x}_2 & \sigma_2^2+\bar{x}_2^2 \end{pmatrix}^{-1}}_{A} \begin{pmatrix} \bar{y} \\ \bar{y}\bar{x}_1 + \sigma_{x_1 y} \\ \bar{y}\bar{x}_2 + \sigma_{x_2 y} \end{pmatrix}$$