

In [1]:

```
1 import numpy as np
2 import pandas as pd
3 import matplotlib.pyplot as plt
4 import math
5 % matplotlib inline
```

$$P = \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}$$
$$D = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

とすると、

$$P^{-1}JP = D$$

と対角化できる。

ここで、

$$x = \begin{bmatrix} p \\ q \end{bmatrix}$$

とし、

$$x = Pu$$

と置くと、

$$\begin{aligned} \dot{u} &= P^{-1}\dot{x} \\ &= P^{-1}Jx \\ &= P^{-1}JPP^{-1}x \\ &= Du \end{aligned}$$

つまり、

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

として、

$$\dot{u}_1 = iu_1$$

$$\dot{u}_2 = -iu_2$$

$$u_1(0) = \frac{1}{2}$$

$$u_2(0) = \frac{1}{2}$$

と書ける。

よって、uについて数値的に考察すれば、

$$x = Pu$$

を用いて

$$x = \begin{bmatrix} p \\ q \end{bmatrix}$$

の挙動を数値的に考えていくことができる。

また、この方程式を解くと、解は

$$x = \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix}$$

となる。

In [2]:

```

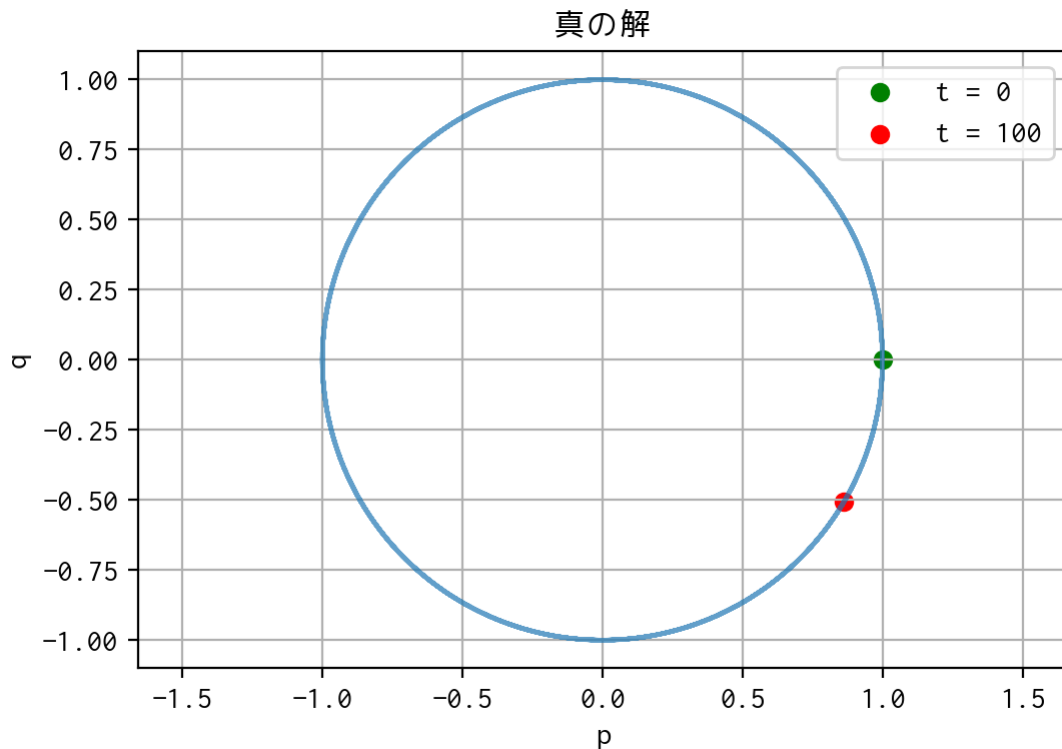
1 P = np.array([[1, 1], [-1j, 1j]])
2
3 def u2x(u_1_array, u_2_array):
4     return np.array([np.dot(P, np.array([u_1], [u_2])) for u_1, u_2 in zip(u_1_array, u_2_array)])
5
6 def plot_x(x, title):
7     plt.scatter(x[0, 0], x[0, 1], color='green', label='t = 0')
8     plt.scatter(x[-1, 0], x[-1, 1], color='red', label='t = 100')
9     plt.plot(x[:, 0], x[:, 1], alpha = 0.7)
10
11     plt.legend()
12     plt.title(title)
13     plt.xlabel('p')
14     plt.ylabel('q')
15     plt.grid()
16     plt.axis('equal')
17     plt.show()

```

1 真の解

In [3]:

```
1 T = [i * 0.1 for i in range(1001)]
2 x = np.array([[math.cos(t)], [math.sin(t)]] for t in T])
3
4 plot_x(x, '真の解')
```



2 陽的Euler法

In [4]:

```
1 def f_1(x):
2     return 1j * x
3
4 def f_2(x):
5     return -1j * x
```

In [5]:

```
1 def explicit_euler_method(f, x_0, h = 0.1, T = 100):
2     t = 0
3     xs = [x_0]
4     while t < T:
5         xs.append(h * f(xs[-1]) + xs[-1])
6         t += h
7     return xs
```

In [6]:

```
1 u_1 = explicit_euler_method(f_1, 0.5)
2 u_2 = explicit_euler_method(f_2, 0.5)
```

In [7]:

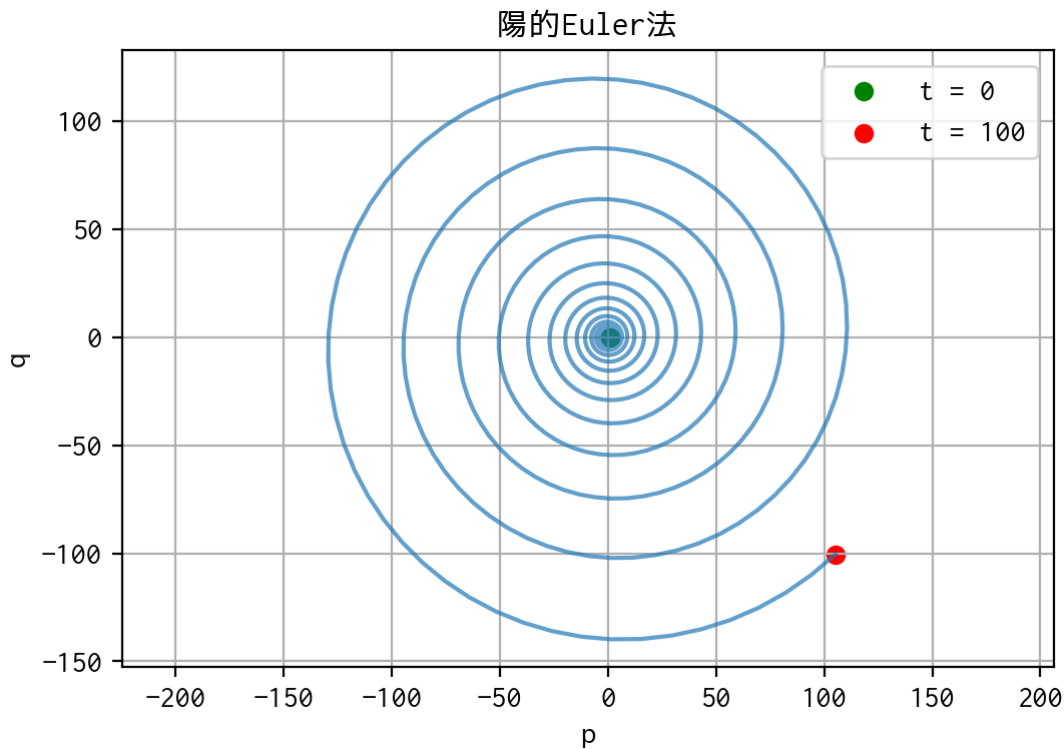
```
1 x = u2x(u_1, u_2)
2
3 plot_x(x, '陽的Euler法')
```

/Users/uedatomohiro/.pyenv/versions/anaconda3-5.1.0/lib/python3.6/site-packages/numpy/core/numeric.py:544: ComplexWarning: Casting complex values to real discards the imaginary part

```
return array(a, dtype, copy=False, order=order, subok=True)
```

/Users/uedatomohiro/.pyenv/versions/anaconda3-5.1.0/lib/python3.6/site-packages/numpy/core/numeric.py:492: ComplexWarning: Casting complex values to real discards the imaginary part

```
return array(a, dtype, copy=False, order=order)
```



3 陰的Euler法

In [8]:

```
1 def u_1_implicit_euler_method(x_0, h = 0.1, T = 100):
2     t = 0
3     xs = [x_0]
4     while t < T:
5         xs.append(xs[-1] / (1 - 1j * h))
6         t += h
7     return xs
```

In [9]:

```
1 def u_2_implicit_euler_method(x_0, h = 0.1, T = 100):
2     t = 0
3     xs = [x_0]
4     while t < T:
5         xs.append(xs[-1] / (1 + 1j * h))
6         t += h
7     return xs
```

In [10]:

```
1 u_1 = u_1_implicit_euler_method(0.5)
2 u_2 = u_2_implicit_euler_method(0.5)
```

In [11]:

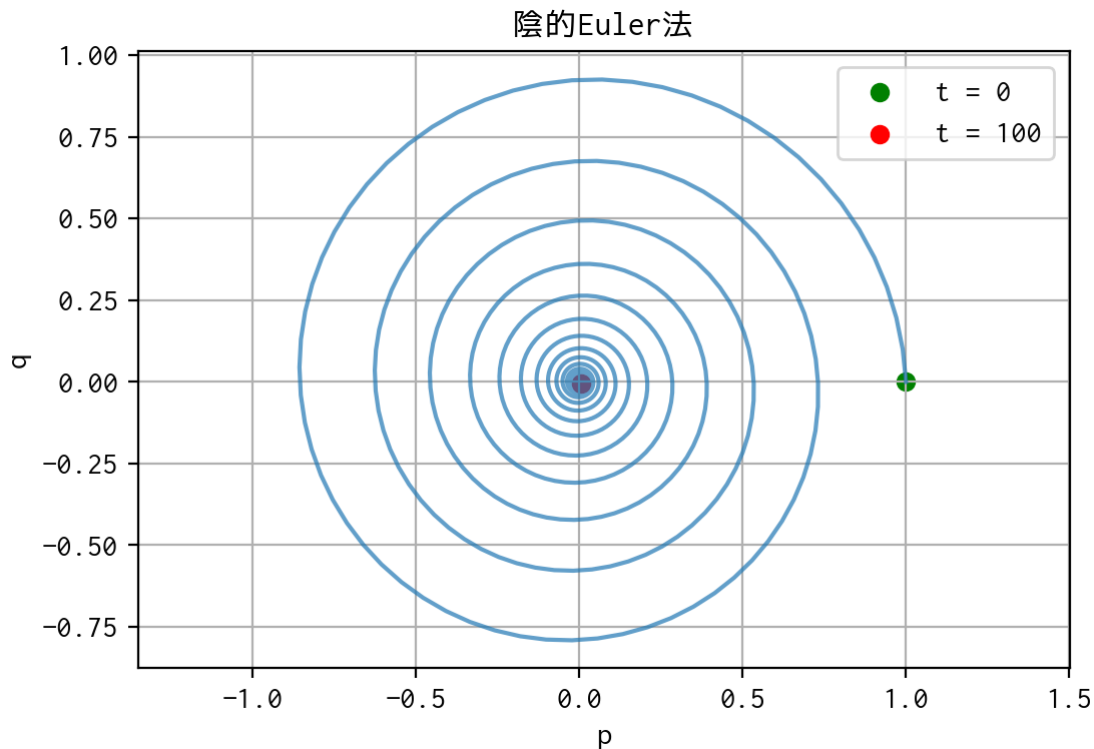
```
1 x = u2x(u_1, u_2)
2
3 plot_x(x, '陰的Euler法')
```

/Users/uedatomohiro/.pyenv/versions/anaconda3-5.1.0/lib/python3.6/site-packages/numpy/core/numeric.py:544: ComplexWarning: Casting complex values to real discards the imaginary part

return array(a, dtype, copy=False, order=order, subok=True)

/Users/uedatomohiro/.pyenv/versions/anaconda3-5.1.0/lib/python3.6/site-packages/numpy/core/numeric.py:492: ComplexWarning: Casting complex values to real discards the imaginary part

return array(a, dtype, copy=False, order=order)



4 台形法

In [12]:

```
1 def u_1_trapezoidal_method(x_0, h = 0.1, T = 100):
2     t = 0
3     xs = [x_0]
4     while t < T:
5         xs.append((2 + h * 1j) / (2 - h * 1j) * xs[-1])
6         t += h
7     return xs
```

In [13]:

```
1 def u_2_trapezoidal_method(x_0, h = 0.1, T = 100):
2     t = 0
3     xs = [x_0]
4     while t < T:
5         xs.append((2 - h * 1j) / (2 + h * 1j) * xs[-1])
6         t += h
7     return xs
```

In [14]:

```
1 u_1 = u_1_trapezoidal_method(0.5)
2 u_2 = u_2_trapezoidal_method(0.5)
```

In [15]:

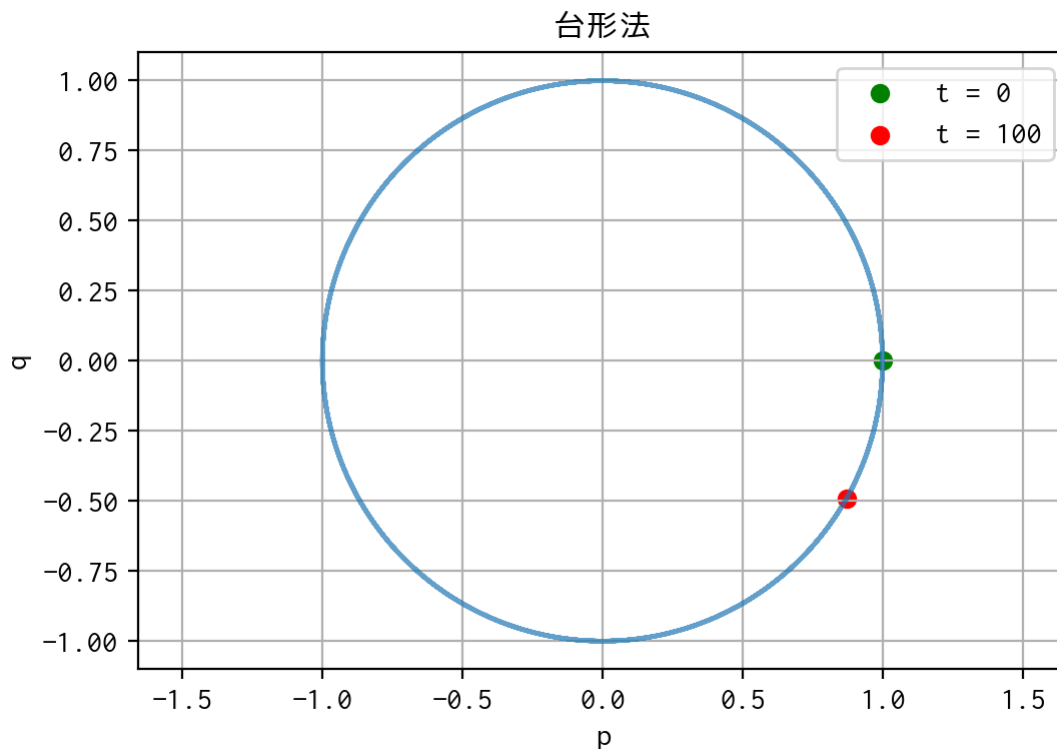
```
1 x = u2x(u_1, u_2)
2
3 plot_x(x, '台形法')
```

/Users/uedatomohiro/.pyenv/versions/anaconda3-5.1.0/lib/python3.6/site-packages/numpy/core/numeric.py:544: ComplexWarning: Casting complex values to real discards the imaginary part

```
return array(a, dtype, copy=False, order=order, subok=True)
```

/Users/uedatomohiro/.pyenv/versions/anaconda3-5.1.0/lib/python3.6/site-packages/numpy/core/numeric.py:492: ComplexWarning: Casting complex values to real discards the imaginary part

```
return array(a, dtype, copy=False, order=order)
```



5 Runge-Kutta法

In [16]:

```
1 def runge_kutta_method(f, x_0, h = 0.1, T = 100):
2     t = 0
3     xs = [x_0]
4     while t < T:
5         y_n = xs[-1]
6         k_1 = f(y_n)
7         k_2 = f(y_n + h / 2 * k_1)
8         k_3 = f(y_n + h / 2 * k_2)
9         k_4 = f(y_n + h * k_3)
10        xs.append(xs[-1] + h * (k_1 / 6 + k_2 / 3 + k_3 / 3 + k_4 / 6))
11        t += h
12    return xs
```

In [17]:

```
1 u_1 = runge_kutta_method(f_1, 0.5)
2 u_2 = runge_kutta_method(f_2, 0.5)
```


In [18]:

```
1 x = u2x(u_1, u_2)
2
3 plot_x(x, 'Runge-Kutta法')
```

/Users/uedatomohiro/.pyenv/versions/anaconda3-5.1.0/lib/python3.6/site-packages/numpy/core/numeric.py:544: ComplexWarning: Casting complex values to real discards the imaginary part

```
return array(a, dtype, copy=False, order=order, subok=True)
```

/Users/uedatomohiro/.pyenv/versions/anaconda3-5.1.0/lib/python3.6/site-packages/numpy/core/numeric.py:492: ComplexWarning: Casting complex values to real discards the imaginary part

```
return array(a, dtype, copy=False, order=order)
```

