0.1 計算環境

MacBook Pro (13-inch, 2016, Two Thunderbolt 3 ports)

プロセッサ: 2 GHz Intel Core i5

メモリ: 8 GB 1867 MHz LPDDR3

ソフト: Jupyter notebook (5.4.0)

言語: Python 3.6.4

1 大問1

In [1]:
```
import numpy as np
```

In [2]:
```
# 初期化

A_1 = np.array([[1, 0, 0], [0, -5, 1], [0, 3, 0]])
A_2 = np.array([[1, 0, 0], [0, 0, -1], [0, 3, 0]])

v = np.array([[1], [1], [1]])
```

In [3]:
```
def power_method(A, x):
    tol = 1e-6
    error = tol + 1
    x_old = x

    while error > tol:
        y = np.dot(A, x_old)
        x = y / np.linalg.norm(y)
        error = min([np.linalg.norm(x - x_old), np.linalg.norm(x + x_old)]) / np.linalg.norm(x)
        print(x - x_old)
        x_old = x

    return x
```

In [4]:
```
def power_method(A, x):
    for i in range(5000):
        y = np.dot(A, x)
        x = y / np.linalg.norm(y)
        if i > 4990:
            print("==============")
            print(x)
```

1.1 (A1)

In [5]:
```
power_method(A_1, v)
```

```
==============
[[ 0.        ]
 [-0.87939705]
 [ 0.47608909]]
==============
[[ 0.        ]
 [ 0.87939705]
 [-0.47608909]]
==============
[[ 0.        ]
 [-0.87939705]
 [ 0.47608909]]
==============
[[ 0.        ]
 [ 0.87939705]
 [-0.47608909]]
==============
[[ 0.        ]
 [-0.87939705]
 [ 0.47608909]]
```

In [6]:
```
np.linalg.norm(np.array([[ 0.        ],  [ 0.87939705], [-0.47608909]]))
```

Out[6]:
```
0.9999999965828653
```

In [7]:
```
np.dot(A_1, np.array([[ 0.        ],  [ 0.87939705], [-0.47608909]])) / np.array([[ 0.        ],  [ 0.87939705], [-0.47608909]])
```

```
/Users/uedatomohiro/.pyenv/versions/anaconda3-5.1.0/lib/python3.6/site-packages/ipykernel_launcher.py:1: RuntimeWarning: invalid value encountered in true_divide
  """Entry point for launching an IPython kernel.
```

Out[7]:
```
array([[        nan],
       [-5.54138127],
       [-5.54138124]])
```

In [8]:
```
np.linalg.eig(A_1)
```

Out[8]:
```
(array([ 0.54138127, -5.54138127,  1.        ]),
 array([[ 0.        ,  0.        ,  1.        ],
        [ 0.17759186,  0.87939705,  0.        ],
        [ 0.98410423, -0.47608909,  0.        ]]))
```

kが十分大きいところでは上のように、互いに反対向きの大きさ1のベクトルを交互に繰り返している。

このベクトルは

np.linalg.eig(A_1)

や

np.dot(A_1, np.array([[0.], [0.87939705], [-0.47608909]])) / np.array([[0.], [0.87939705], [-0.47608909]]) = [[nan], [-5.54128127], [-5.54128127]]

となることからわかるように、A_1の固有ベクトルであり、その固有値は-5.54138127で、固有値の中で絶対値最大である。

1.2 A2

In [9]:
```
1 power_method(A_2, v)
```

```
[[0.       ]
 [0.70710678]
 [0.70710678]]
=============
[[ 0.       ]
 [-0.31622777]
 [ 0.9486833 ]]
=============
[[ 0.       ]
 [-0.70710678]
 [-0.70710678]]
=============
[[ 0.       ]
 [ 0.31622777]
 [-0.9486833 ]]
=============
[[0.       ]
 [0.70710678]
 [0.70710678]]
```

In [10]:
```
1 np.linalg.eig(A_2)
```

Out[10]:
```
(array([0.+1.73205081j, 0.-1.73205081j, 1.+0.       j]),
 array([[0.       +0. j, 0.       -0. j, 1.       +0. j],
        [0.       +0.5j, 0.       -0.5j, 0.       +0. j],
        [0.8660254+0. j, 0.8660254-0. j, 0.       +0. j]]))
```

紙に書きます