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Modeling and managing resilience and risk for interdependent networks

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ABSTRACT

Catastrophic events have the potential to cause major disruptions to interdependent networks such as supply chains, critical infrastructures and other networks that are vital for the functioning of our society. Addressing the resilience and risk of compromised networks is challenging due to a variety of factors. First, these networks are becoming increasingly interdependent, such that network recovery is contingent upon recovery in connected networks. Additionally, interdependent networks may have multiple functions, system users, owners, and stakeholders. There is a need for data-informed decision-making models that address resilience and risk for these interdependent infrastructure networks. We address these issues by: 1) Modeling interdependencies among infrastructure networks using a multi-layer interdependent network framework, 2) Quantifying the relationship between network resilience and the inter-network connection structure, and 3) Supporting management and strategic decision-making for investment in inter-network connection structures, supporting risk mitigation needs and strategies. Our paper combines network optimization and network science methods to produce a unified framework to analyze resilience. We apply the methods of this paper to the interdependent infrastructure network of Puerto Rico following the 2017 Hurricane Maria. This paper supports decision-making for data-informed resilience management for interdependent infrastructure networks.

1. Introduction

Supply chain and infrastructure networks are becoming increasingly connected and interdependent. Energy, water, communications, healthcare, and other types of critical infrastructure (CI) networks often rely on (and support) the functionality of other networks to maintain, improve, and regain operations. These interdependencies can be beneficial for the overall system (understood as a collection of networks) while also introducing the potential for adverse consequences, such as cascading failures across interdependent networks. Empirical evidence shows that energy network failures can cause failures to the dependent communications infrastructures, which in turn can cause additional energy network failures leading to cascading failures [1]. However, systems can potentially leverage interdependencies to produce high-performance and resilient infrastructure systems, as described in the United Nations International Strategy for Disaster Reduction Hyogo Framework, which states that "… *increasing global interdependence, concerted international cooperation and an enabling international environment are required to stimulate and contribute to developing the knowledge, capacities and motivation needed for disaster risk reduction at all levels*" [2]. It is therefore important to direct system investments toward network assets that can mitigate the potential for cascading failures and promote efficiencies in system recovery for effective risk management of these interdependent multi-network systems.

As CI networks become increasingly interdependent, there is an urgent need to concurrently manage risk, resilience, and performance. While management of these systems has historically involved isolated or network-specific goals and investments, it is imperative for regions and nations to manage these systems collaboratively, or, at a minimum, with acknowledgment of interdependencies, such as among energy, transportation, water, healthcare, manufacturing and other CI systems [3]. Accordingly, the United States Department of Homeland Security announced that the management of cross-sector dependencies and cascading effects is a research priority of national significance [4].

While infrastructure network interdependency is not a new phenomenon, there is potential for even more catastrophic damage as the level of interdependency increases across global infrastructures. For example, advancements in autonomous vehicle technologies will be dependent on functioning and reliable communications networks [5]. These communications networks, in turn, are increasingly reliant on a functioning and reliable electricity grid. The result is that a major energy

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Nomenclature:

Sets and Indices

H	Set of networks indexed by h, l
N	Set of nodes indexed by i, j, k
A	Set of arcs (or edges) in a network indexed by (i, j)
E	Set of uncertain scenarios indexed by $e_1, \ldots, e_m$

Variables and Functions

$x \geq 0$	Activity in network (x_h) or arc (x_{ij})
$F(x_h)$	Self-dynamic function affecting activity x
$G(x_h, x_l)$	Cross-effect function affecting activity x between networks h and l
$P_h(t)$	Function for performance on network h, such that $0 \leq P_h(t) \leq 100, \forall h$
$R_h(t)$	Resilience for each performance on network h, such that $R_h(t) \geq 0, \forall h$

Parameters

I_{hl}	Incidence matrix between networks h and l
$c_{ij} > 0$	Cost of edge (i, j)
$v_{ij}, u_{ij} \geq 0$	Lower and upper bounds for flow in edge (i, j)
b_k	Net production (>0) or consumption (<0) in node k
p	Number of networks, i.e., $\lvert H \rvert$
m	Number of uncertain scenarios to consider
n	Number of observations taken between disruptive scenario and full recovery
t_0	Time of initial state
t_v	Time of disruptive event
t_d	Time of final disruptive state
t_s	Time of resilience action
t_f	Time of recovery
t_r	Time of resilience computation $R_h(t_r)$

grid failure has the potential to cause widespread societal and economic damage, given its connections to generators, nuclear power plants, traffic systems, wastewater systems, and information systems, among others. Massive disruptions to infrastructure networks can be caused by relatively minor disruptive events. For example, it has been noted that a blackout of the entire east coast of the United States could result from a disruption in only nine of the nation's 55,000 electric transmission substations [6].

As a very unfortunate, but perhaps motivating example, consider the 2017 Hurricane Maria that devastated the United States territory of Puerto Rico. The impact of the hurricane was catastrophic, causing thousands of deaths [7] and an estimated \$90 billion in total losses [8]. This humanitarian crisis was deeply exacerbated by the design and structure of interdependent infrastructure networks in the region. Delays in repairing the electricity grid, combined with resource constraints and the unreliability of pre-existing infrastructure led to cascading and prolonged outages across the interdependent infrastructure networks, including hospitals, related healthcare facilities, manufacturing, water treatment and distribution, law enforcement, educational facilities, and others. The consequences were also global, as the debilitated manufacturing operations led to worldwide medicine shortages [9].

We address the issue of quantifying resilience and cascading recoveries across multiple interdependent infrastructure networks. More precisely, in this paper we: (1) develop a framework of study of interdependent networks composed by modeling individual networks, their interdependencies and appropriate metrics, (2) quantify resilience and the cascading recovery of interdependent networks that incorporate time in the recovery of the networks, and (3) offer guidance to identify critical design issues in management of CI and supply chain networks. To this effect, the paper briefly reviews the literature on network modeling and resilience in Section 2. Section 3 describes the main components, contributions, limitations, and applicability of the methodology while section 4 provides numerical experiments to validate the methodology. Results of the case study in Puerto Rico are described in Section 5. Conclusions and policy recommendations close the paper in its final section.

2. Literature review

Management of interdependent networks is becoming increasingly important for commerce (e.g., multimodal supply chains), basic services and development (e.g., water and power distribution) and communications (e.g., internet of things). This is particularly clear in the case of managing critical infrastructures (CIs). The United States Department of Homeland Security identifies 16 CI sectors that are vital to the nation's security, economy, health, and safety, including transportation, water, wastewater, energy, and critical manufacturing, among others [4]. Both the design and management of these CI networks represent large national costs. For example, the American Society of Civil Engineers estimates a \$2.0 trillion in CI maintenance needs, particularly in the areas of transportation, water, and energy in the United States [10]. With broad interdependencies, such maintenance insufficiencies and inefficiencies have the potential to compromise the functionality of entire networks. In this section we lay out the pertinent fields to our paper. We first describe the evolution of networks as an appropriate representation of infrastructure and how interdependencies can be modeled. Then we explain key concepts for understanding resilience and risk of interdependent networks.

2.1. Networks and interdependent networks

Networks have been used in multiple applications to our daily lives: shortest path methods [11,12] obtain directions for the fastest route from origin to destination; or minimum spanning trees [13–16] help determine the sequence of streets to open after a disaster [17]. In general terms, nodes represent intersections or sources/destinations of traffic, while arcs represent the streets and avenues connecting nodes. In energy generation and distribution, network models may represent power transmission and distribution as a network flow. Nodes represent generation points, transformers, substations, and consumers, while arcs represent the power lines connecting nodes. Furthermore, the need for communication and coordination among these networks fostered the rise of dependent and interdependent networks. Understanding and managing such relationships requires clear representation of how they couple together. A property of interdependent networks is that a failure of even a small fraction of nodes or links in one network can lead to a complete collapse and, even worse, the cascading collapse of the interconnected networks.

In recent years, infrastructure types have become increasingly interconnected. For example, energy grid operations have become even more vital to the operations of public utilities, cyber-systems, banking, and other CIs. As a result, exploited vulnerabilities of these infrastructures have disastrous consequences across potentially all infrastructure sectors. Modeling of these types of disruptions commonly involves a mathematical representation of interdependency, such as in input-output inoperability models [18], dynamic input-output inoperability models [19], simulation models [20], and other mathematical representations [21]. As a result of infrastructure network interconnectivity, disruptions can cascade throughout the network, causing widespread loss of performance. The 2003 Northeast Blackout in the

United States, which caused a loss of power for 50 million people, cost $6 billion, and contributed to 11 deaths, was caused by a single failure of a high-voltage power line, followed by additional failures, and a cascade of failures in other overtaxed supporting power lines [22]. In the 2003 electrical blackout in Italy, a power station failure affected the connected internet nodes, which caused additional power stations to fail in a cascade of failures that left the entire country without electricity [1].

There has been significant work on optimization modeling for resilience in CI networks. A wide range of literature explores network design interventions with regard to network flow [23,24] offer an approach to determine infrastructure damage using outage reports. They introduce a formulation with decision variables that represent the arc flows, with an objective function to maximize flow to demand nodes. They also introduce an interdiction-based approach that determines the order of component inspections in the network [25]. propose a defender model aimed at identifying defensive investments. The complementary attacker model focuses on identifying the set of possible single-link attacks. Other resilience frameworks incorporate modeling the coping capacity [26]. Many of these models involve a network-flow perspective that assumes that when attacked, the flow is completely stopped, which may or may not be the case in real-world natural or man-made disruptions.

Other exemplary literature includes [27], which introduces an interdependent network design problem that models network flows among interdependent networks. They incorporate elements of flow costs, and costs for reconstruction of network components, with decision variables representing the commodity flows, a binary variable representing whether an arc is to be reconstructed, and a binary variable representing whether a node needs to be functional. The model minimizes total cost.

Managing resilience for interdependent infrastructure systems is also fundamental. One significant contribution is from Ref. [28] who introduce a framework to understand universal resilience patterns mostly applied to ecological systems. They introduce resilience as measured by functionality by coupled nonlinear equations, representing interactions among network components and partners [29]. describe simulation models to understand the effects of various attack scenarios on network robustness against cascading failures on power grids. The results show the need for direct investments to improve network robustness. The formulation includes nonlinear functions representing dynamical laws that govern the system's components, and also a weighted connectivity matrix that captures interactions between nodes. Future research is needed to incorporate these findings into a risk-based decision-making framework.

2.2. Key concepts for understanding risk of critical interdependent infrastructures

The concept of risk has been studied across disciplines, involving the study of a future activity in relation to the consequences of that activity with respect to what humans value, in which at least one outcome is considered undesirable [30]. The risk of a particular activity can be evaluated by considering the likelihood and magnitude of the potential consequences. The work of [31] describes the relevant questions pertaining to risk: 1) What can go wrong? 2) What are the likelihoods? and 3) If it does happen, what would be the consequences? Haimes [32] suggests such additional considerations as: the relevant timeframe; the tradeoffs in terms of costs, benefits, and risks; and the impact of current decisions on future options. More recent work, including from ISO 31000 (ISO, 2018) instead defines risk as the *"effect of uncertainty on objectives,"* thereby highlighting the role of uncertainty instead of probabilities.

The modeling of risk is often accompanied by resilience analyses. Resilience has been studied in a wide variety of academic field applications, including ecosystems [33], engineering [34], and sociology [35]. Resilience applications associate improved resilience characteristics with reduced failure probabilities, reduced consequences from failures, and reduced recovery times [36]. In addition to quantifying resilience, there is growing interest in quantifying robustness, redundancy, resourcefulness, rapidity, and recovery [37].

Although established principles of infrastructure risk apply to multilayered interdependent networks, there are several challenges that need to be addressed. First, infrastructure assets are often owned and operated in the private sector, public sector, or in some version of public-private collaboration. With non-central management structures, coordinated investments in infrastructure maintenance or protection become complex tasks. In addition, these infrastructure types all have varying functions and related performance metrics. For example, the performance of a transportation network may commonly involve traffic volume, efficiencies, profits, safety, and other competing performance metrics, while performance for energy infrastructure may involve such metrics as grid reliability, safety, and energy loss. Third, although methods exist to quantify the resilience of each individual network, there are few methods and models that can evaluate the resilience of interdependent networks.

3. Methodology description

The operation of individual networks is modeled by a Minimum Cost Network Flow (MCNF) optimization model. Individual networks optimize their operations by minimizing cost while meeting network flow requirements. Next, the methodology integrates dependencies and interdependencies among the networks. For example, a node in network A is *dependent* on a node in network B if a failure of the node in B would make a node in A reduce its capacity or fail. When this relationship is mutual between networks, there is an *interdependency*. Identifying dependencies and interdependencies entails mapping infrastructure networks and operations. The next step is to identify the type(s) of relationship between networks: physical, cyber, geographic, or logical. In addition to identifying the type of dependency, it is important to quantify the level of such dependency. Some networks cannot operate without certain supporting networks (e.g., trucking when roads are impassable) while others are forced to operate at lower capacity (e.g., hospitals during a power outage use power generators). Finally, this methodology delivers metrics that allow for performance evaluations. It is of key importance in this topic to incorporate the time of disruption, impact on operation, time of recovery and the new state of the system after the recovery.

3.1. Individual network operation

The methodology assumes a set of infrastructure networks H where h denotes each network (e.g., power distribution). N^h and A^h denote the set of nodes and arcs in h. Five fundamental assumptions are made about network structure and operation, as follows:

1. Each network performs one activity (e.g., transportation) that is performed solely in that network;
2. Demand and supply are known;
3. There is known capacity for each arc in the networks;
4. Flow capacities are independent between different networks; and
5. There is a cost associated with the flows in each network.

The *MCNF* optimization problem, with flow conservation and supply/demand satisfaction, as shown below:

$$min \sum_{(i,j)\epsilon A^h} c_{ij}x_{ij} \tag{1}$$

Subject to:

$$\sum_{(i,k)\epsilon A^h} x_{ik} - \sum_{(k,j)\epsilon A^h} x_{kj} = b_k, \forall k \in N^h \tag{2}$$

Table 1
Types of interdependencies assumed to exist in our case study. Levels in parentheses denote the magnitude of interdependency, L = Low, M = Medium, H=High.

	Power	Water	Road	Communication
Power		Physical (H)	Physical (H), Geographic (H), Logical (L)	Physical (H), Cyber (H), Logical (L)
Water			Geographical (M) Logical (L)	Cyber (H)
Road				Cyber (M)
Communication				

$$v_{ij} \leq x_{ij} \leq u_{ij}, \forall (i,j) \in A^h \tag{3}$$

The *MCNF* model can represent a variety of different network operations [38]. In the case of a supply chain network, x_{ij} represents the amount of cargo being transported in $(i,j) \in A$ from points of production $(b_k < 0)$ to consumption $(b_k > 0)$, and going through transshipment nodes in the network $(b_k = 0)$. The cost c_{ij} is the transportation cost from i to j. If the purpose is to model a communications network, MCNF evaluates connectivity. To do this, c_{ij} becomes a unitary cost, $b_k = -1$ for all nodes that receive communication, and supply and demand are the same. In this case, what matters is the fraction of total nodes in the network connected to a communication tower. A similar situation occurs when modeling power and water distribution networks, where the purpose is to evaluate whether population centers are connected to sources of power or water, respectively. The application of network flow models to water and power, in our case, is limited to connectivity. It is important to point out that the power distribution exhibits a rather complex behavior. Mainly, power distribution networks are ruled by physics laws and controlled at the substation levels by both computer control systems and manual control [39]. Therefore, modeling power distribution as an optimization model that finds the best path from generation to consumption has inherent simplifications and shortcomings.

3.2. Dependencies and interdependencies

Interdependencies occur when nodes, links or networks rely on each other to operate. For example, power substations require control equipment that relies on the communications infrastructure to operate (e.g., the internet), as described by Refs. [1]. A power outage can disrupt the communications network, which can cause other power stations to fail. A worst-case scenario occurs when these interdependencies produce a cascading effect leading to a complete outage of the power and/or communications network. Modeling these interactions is challenging due to the wide variety of dependencies and interdependencies that exist. Barzel and Barabasi [40] provide a modeling framework to represent these interactions that was later utilized by Ref. [28] in the context of ecological systems. These publications describe the self-dynamics of the system, and the interactions between the system components with other interacting partners using a set of nonlinear equations. We have adopted this methodology as shown in Equation (4). $F(x_h)$ captures the "self-dynamic" behavior, I_{hl} captures the existence of interaction between networks and $G(x_h, x_l)$ describes crossed effects. More precisely, $F(x_h)$ represents the impact on the network due to disruptions in the same network (e.g., capacity reductions). $G(x_h, x_l)$ captures the interdependency impacts between coupled networks and I_{hl} denotes a weighted incidence matrix.

$$\frac{dx_h}{dt} = F(x_h) + \sum_{l=1\ldots|H|} I_{hl}\, G(x_h, x_l) \tag{4}$$

Mapping I_{hl} requires a classification of interdependencies between networks. Table 1 provides an example of the types of interdependencies [41]. The high, medium, and low interdependencies relate to the Rinaldi and Perrow designation of loose or tight coupling of interdependent networks, such that tight coupling implies little slack in the cascading of failures which would correspond to high interdependency.

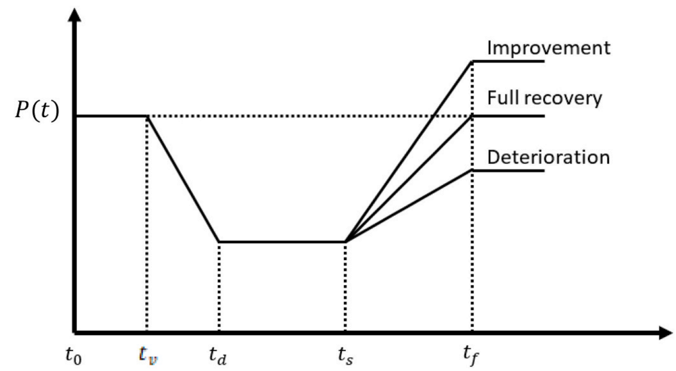

Fig. 1. Resilience formulation based on formulation by Henry and Ramirez-Marquez [42].

3.3. Resilience-informed decision support

The notion of resilience in this formulation recognizes that as a consequence of disasters and disaster response operations, the performance of the network may be fully recovered, deteriorated to a performance level below pre-disaster levels, or rebuilt to an improved level of performance (see Fig. 1). This formulation is adapted from [42] by considering multiple networks instead of figures of merit and represented as the ratio of recovery and maximum loss:

$$R_h(t_r|e_l) = \frac{P(t_r|e_l) - P(t_d|e_l)}{P(t_0) - P(t_d|e_l)}, \forall e_l \in E \tag{5}$$

In this formulation, we assume that the function *P(t)* represents a measurement of performance that is relevant and applicable to the studied network. The numerator of equation (5) represents the recovery between the time when the disruption has stabilized and any time $t_r \in (t_d, t_f)$. The denominator of the equation captures the total drop in performance. As such, the division between these metrics captures the capacity of the system to make a full recovery, partial or improve with respect to the original state. While the lower bound of $P(t)$ is assumed to be zero, the upper-most value is not bounded to account for potential levels of improvement beyond the pre-event performance level. In many cases, however, such as in the numerical experiment of this paper, $P(t)$ is bounded at 100 because a full network recovery is represented as $P(t) = 100$.

In general, a measurement of performance could include the quality of infrastructure [43], number of nodes in operation, percentage of functioning nodes, fractional size of the largest component, or deviation from benchmark (number of nodes operating vs. goal). The methodology remains flexible to accommodate more application-specific measurements of performance. For example, a performance measurement for energy infrastructure may include the percentage of customers with service, the number of businesses with service, or the square miles of land with service.

3.4. Modeling scenarios for resilience characteristics and network recovery

This step consists of applying risk management principles to multi-network resilience modeling. While the resilience formulation described in the previous section uses a single line to represent *P(t)*, the computational modeling approach can be interpreted as a single simulation of the stochastic model. Recall $E = \{e_1, e_2, \ldots, e_m\}$ as the set of uncertain scenarios, such that resilience $R_h(t_r|e_l)$ is computed. We treat E as a set of uncertain scenarios that can be used to model both the network disruption and the network recovery logic. These uncertain scenarios are modeled by either initially removing nodes (as a network disruption) or adding network nodes sequentially (as a network recovery). To accommodate for variability that can be observed across scenarios, E, it is necessary to compute $P(t)$ as an average, or median, or weighted performance level across a fixed number of scenarios.

The selection of uncertain scenarios should be determined by the context of the application area, the studied system, the decision-makers, and relevant stakeholders. The key to this decision-making process is not to focus on a limited set of feasible scenarios, but instead to model many relevant and widely ranging scenarios to understand the impact of uncertainties on system objectives, with a larger goal of guiding interventions for improved resilience.

While resilience measurements can help us understand how a network recovers, it is also important to understand the relationship *among* networks as they recover. This can help guide system investments toward recovery behaviors that can improve resilience for the combination of studied networks, instead of for only a single network. We test network interdependency over time by computing the interdependency index, D_{hl}, using the correlation coefficient for each pairwise comparison of interdependent networks over time, as shown in Equation (6). This is computed for each pairwise comparison, network h, such that $h = 1...p$ and network l, such that $l = 1...p$. The term n represents the number of observations, which is computed as the number of time steps represented as model iterations between the original network disruption and the full recovery to the pre-disruption performance level.

$$D_{hl} = \frac{n\left(\sum_{h,l} P_h(t)P_l(t)\right) - \left(\sum_h P_h(t)\right)\left(\sum_l P_l(t)\right)}{\sqrt{\left[n\sum_h P_h(t)^2 - \left(\sum_h P_h(t)\right)^2\right] * \left[n\sum_l P_l(t)^2 - \left(\sum_l P_l(t)\right)^2\right]}} \tag{6}$$

Decision-makers may use the interdependency index, D, to understand the relationship between the placement or number of internetwork connections and the recovery characteristics of the interdependent networks. Relevant decision support issues (DSIs) related to this interdependency index for some particular network structures and applications include the following:

DSI #1: Can interventions on a network provide benefits to connected networks, thereby benefiting the larger studied system? If network structures exhibit a sufficiently high interdependency index, D, this could imply that prioritizing interventions on one network can have an indirect benefit to another network. However, this high level of interdependency can also contribute to the cascading of failures in the event of a disruption.

DSI #2: Which networks benefit from the addition of new network connections to adjoining networks? If the interdependency index, D, increases as the number of connections between networks increases, decision-makers may further prioritize the study of placement for new network connections. Conversely, there may be reasons to decrease the number of network connections if decision-makers find the level of interdependency to be unacceptable for CIs in cases of cascading failures.

DSI #3: Which uncertainties matter the most? If the interdependency index, D, varies greatly among the studied scenarios, it may be necessary to further study which scenarios matter the most and why. This can help guide further study towards understanding the impact of particular disruptive events and help target decision-making exercises to understand how those situations influence system investment.

We make no claim about which decision support issue is most important or relevant for any particular type of network, application, or decision-making context. The issues may differ based on the stakeholders and decision-makers involved, as well as whether these network owners are incentivized to invest in a coordinated manner. Issues may also differ based on whether the studied network is considered a CI. For example, critical energy infrastructure managers may benefit from the study of all three DSIs. Conversely, the managers of privately owned non-critical infrastructure, such as those related to manufacturing networks, may not be incentivized to study the influence of their network on others, as described in DSI #1 and DSI #2. Instead, such decision makers may be more concerned with understanding the circumstances under which their network recovery would be acceptable, as described in DSI #3.

4. Numerical experiments

This section delivers a set of numerical experiments using two grids for networks with the objective of validating the results of the model in a controlled environment. The grids are symmetric $n \times n$, where n is the number of nodes in both vertical and horizontal axes. Nodes are numbered from the bottom up, and from left to right. The networks are interdependent; there are connections in both directions between the nodes of the networks. The matrix I_{hl} is randomly generated with a fixed number of connections between the networks, and nodes are connected to a single node in the other network. The interdependency component $G(x_h, x_l)$ is set to a binary functionality; nodes in one network will become functional only when their respective dependency in the other network is also recovered. Relatively more complex interactions (e.g., multiple connections per node, or partial functionality loss) can be implemented, but the purpose of these experiments is to show how interactions between the networks influence the recovery (or breakdown) of interdependent networks. In other words, single node connection means that, for example, a substation serves a single cell phone tower. If multiple cell towers are connected to a single substation, then it is multiple connections. This creates a vulnerability for the system; the failure of that substation will produce multiple failure points for the communications network. We do not see this case regularly nowadays. On the other hand, one could consider multiple connections to a single dependent node. For example, multiple substations are connected to the cell tower introducing redundancy to the system. Although this makes the system more resilient, it is normally prohibitively more expensive. It is normally reserved for key critical infrastructure facilities. Finally, partial functionality loss refers to when the loss of a service reduces the capacity of the facility, but not completely. A good example is a hospital that has a generator to cover the “critical load.” The critical load is the part of the hospital that cannot lose functionality (e.g., operating rooms). If there is a power outage, the generator will immediately take over the critical load of the hospital, but the non-essential parts of the hospital will lose power.

Fig. 2 shows four examples of network disruption and recovery scenarios. This figure shows the recovery of both networks after a major catastrophic failure, where Network 1 has been completely destroyed. For illustration purposes, the figure considers one pair of sample networks with varying numbers of connections. Parameters and assumptions for these experiments are.

- $n = 20$ for a total of 400 nodes in each network;
- The correspondence of network lines shows the influence of the number of connections between the two networks;
- Fig. 2A assumes 10 connections between the two networks;
- Fig. 2B assumes 25 connections between the two networks;
- Fig. 2C assumes 50 connections between the two networks;
- Fig. 2D assumes 100 connections between the two networks; and
- Network 1 is completely destroyed and starts recovery in time 0. Network 2 is partially destroyed due to interdependencies.

The model assumes that all nodes of Network 1 are destroyed. When a node in Network 1 is restored, connected nodes in Network 2 are recovered in the following time step; which, in turn, allows for the recovery of connected nodes in Network 1. This process is what we

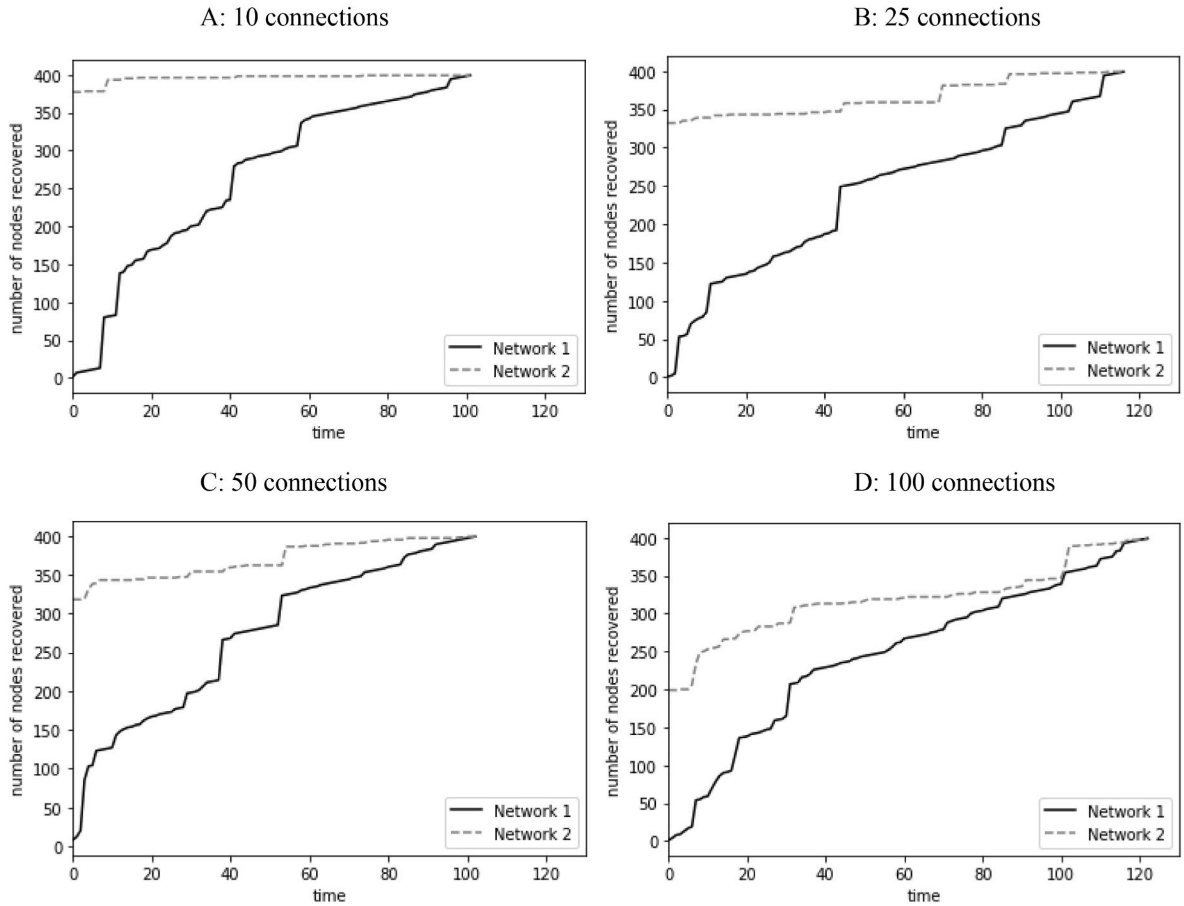

Fig. 2. Numerical experiments studying the relationship between network disruption and recovery behavior.

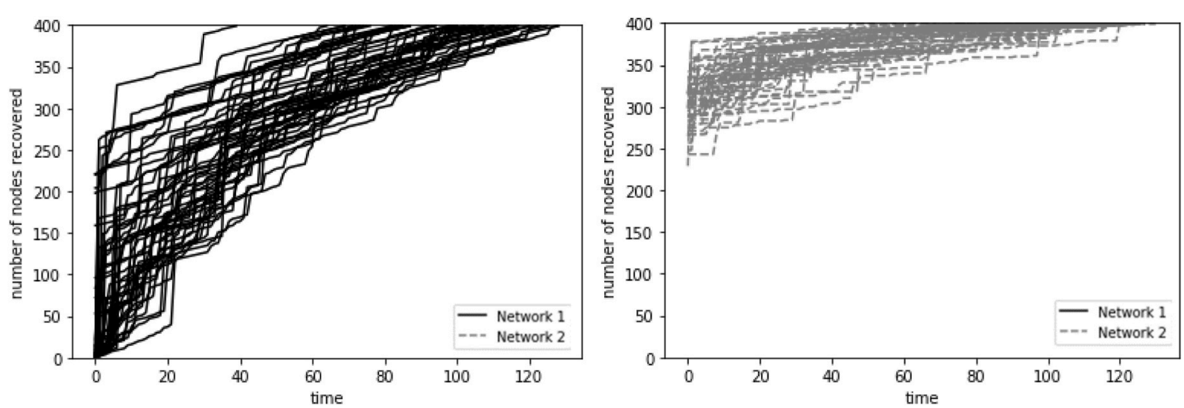

Fig. 3. Numerical experiments studying the relationship between network disruption and recovery behavior.

characterize as cascading recoveries. The process is continued until all nodes in both networks are recovered. The recovery scenarios, E, are based on a variety of model simulations involving random node recoveries within Network 1. While more refined logic-patterns for node recovery may be used, the purpose of this analysis is to develop a high-level understanding of network behavior, allowing for more refined logic-patterns to remain as future work. In addition, we assume that $P(t)$ represents the number of nodes that are connected to the source. When a node is not connected, it is assumed to be destroyed. There are three ways that a node can be recovered, when: (1) using the model logic (here, a random node is selected for recovery); (2) a connected supporting node in another network has been recovered; or (3) a connected supporting node in the same network has been recovered. Fig. 2 shows examples of relationships between the number of connections and recovery behavior; the number of connections from each network is the same. Each plot shows the number of functional nodes on the y-axis and time (or model iterations) on the x-axis. Each plot starts at time = 0, the time at which a 100 % destruction of Network 1 occurs. Combined, the figures show that when the number of connections is increased, the combined networks can recover in a coordinated manner. The figure shows that when a low number of connections are formed, such as in Fig. 2A with 10 connections, the destruction of Network 1 causes a relatively small disruption to Network 2. Conversely, when a relatively higher number of connections are formed, such as in Fig. 2D with 100 connections, the destruction of Network 1 causes a relatively large disruption to Network 2. Generally, the performance of the two

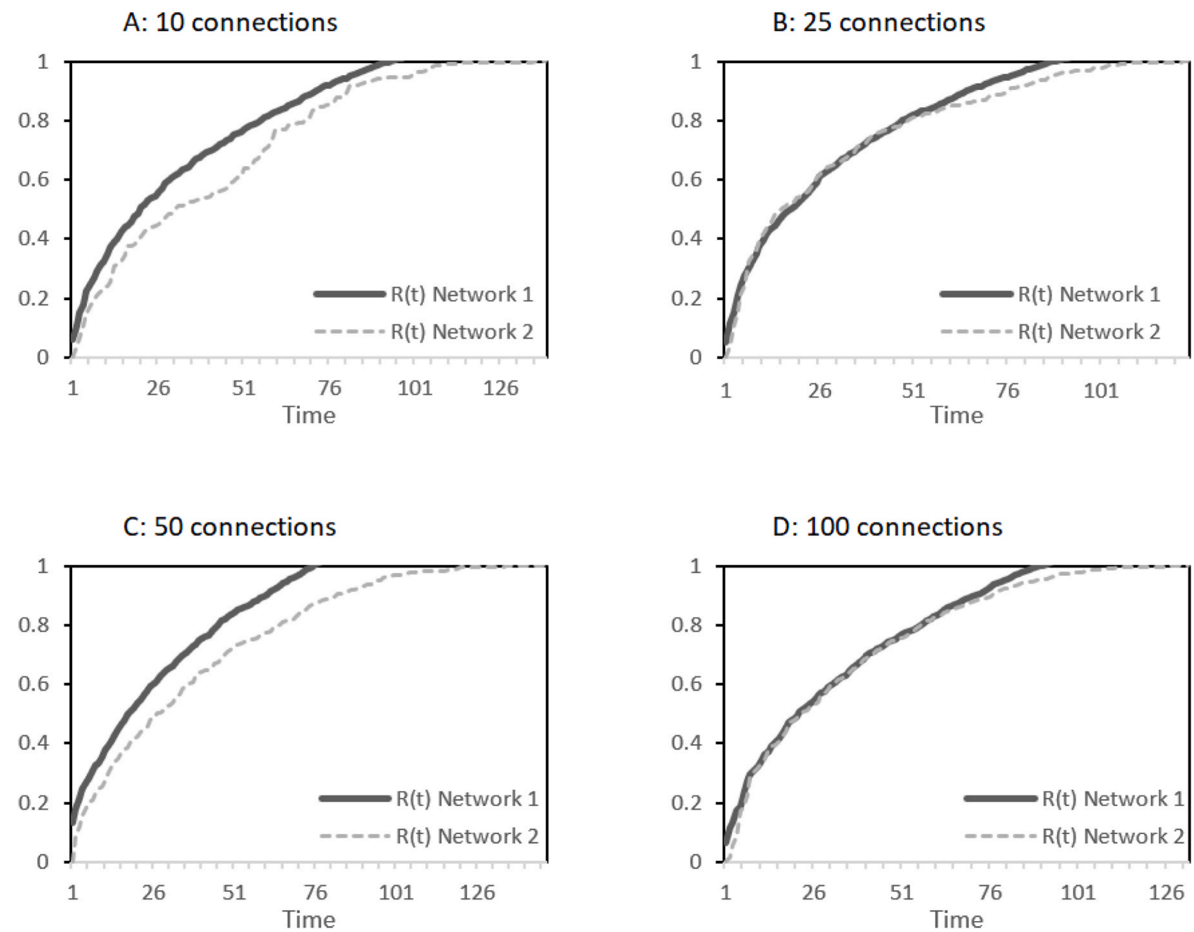

Fig. 4. Resilience as a function of time for numerical experiments studying the relationship between network disruption and recovery behavior.

networks visibly appears to be more closely associated as the number of connections increases.

Fig. 3 shows the recovery of the same interdependent networks, each with 50 randomly assigned connections, representing a variety of scenarios of connections between networks and the recovery process. This time, the figure shows a "cloud" of results, instead of one recovery of the networks. The cloud view of the network recovery is an important aspect of resilience measurement because it highlights uncertainty, variability, and potential vulnerabilities in network performance, with each recovery line representing a single recovery scenario. While the heaviest sections of the cloud represent the most likely outcome across many scenarios, the outliers also represent a less frequently observed, but possible, outcome. The figures show a generally positive number of nodes recovered as time progressed. However, the variability among the scenarios suggests that while a relatively quick recovery is anomalous, there are many possible scenarios representing a much slower recovery process. From a decision-making perspective, learning about the range and extreme cases of possible disruptive events affecting interdependent networks offers great value in enabling preparation for not only the average disruption, but also those events that will have the most severe consequences.

Fig. 4 shows $R(t)$ for the numerical experiments, using the formulation shown in Equation (5). The figures show that resilience for Network 1 and for Network 2 exhibit similar behaviors. However, Network 1 reaches full resilience considerably sooner than Network 2. This is given by the lower damage produced in Network 1. There is relatively less stable behavior in Fig. 4A, relating to the case of 10 connections between the networks. This is because the recovery of Network 2 was slower than that of Network 1, which is understandable because Network 2 having so few connections implies that it has fewer opportunities to achieve node recovery. Conversely, Fig. 4D shows a relatively fast recovery for Network 2 compared to Network 1. This provides evidence that, in this connection structure, Network 2 benefits in its recovery from increased connections to Network 1.

Table 2
Interdependency index for numerical experiments studying the relationship between network disruption and recovery behavior.

Network Connections	Interdependency Index			
Number	Average	Standard Deviation	Max	Min
10 Connections	0.889	0.098	0.984	0.581
25 Connections	0.925	0.052	0.985	0.768
50 Connections	0.950	0.041	0.988	0.766
100 Connections	0.954	0.049	0.990	0.733

We also study the interdependency index, D_{ij}, for each pairwise comparison of interdependent networks over time, as shown in Table 2. The interdependency index is shown across the 50 studied scenarios, with the table showing the average, standard deviation, maximum, and minimum across the scenarios. While a D_{ij} value close to 1 implies the highest level of interdependency, all four connection structures studied involve high levels of average interdependency. Most notably, the average interdependency increased as the number of connections increased. When considering a relatively low number of connections, the lack of inter-network relationships results in a relatively low average interdependency index. This is because Network 2 was not as severely disrupted when Network 1 was destroyed, due to the low number of connections. Also, the recovery of Network 1 was relatively quicker than that of Network 2. Conversely, when considering a relatively high

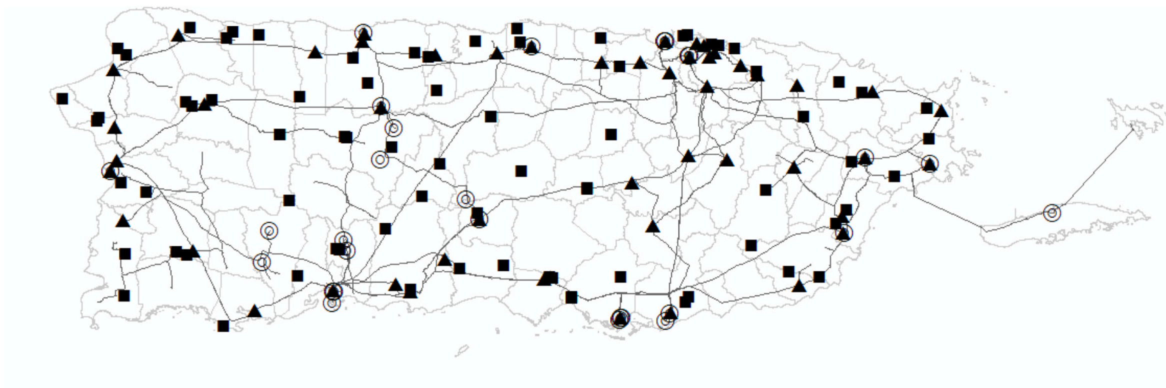

Fig. 5. Map of power and communications networks of Puerto Rico: triangles are transmission centers; targets are power plants; lines are power transmission lines; and squares are communications network nodes.

number of connections, redundancies in the inter-network connections contributed to a more severe initial disruption to Network 2, and also a relatively faster recovery of Network 2, resulting in a relatively higher average interdependency index.

The results of the numerical experiment allow for some insight into the following decision support issues:

DSI #1: Can interventions on a network provide benefits to connected networks, thereby benefiting the larger studied system? In the case of this numerical experiment, a connection structure of 100 connections results in the highest average interdependency index ($\boldsymbol{D_{hl}}$) among all studied connection structures. However, even the 50 connections structure shows a comparable average interdependency index. This information could support a prioritization of interventions with a goal of 50 or more connections, leading to the highest studied level of indirect benefit to connected networks.

DSI #2: Which networks benefit from the addition of new network connections to adjoining networks? In the case of this numerical experiment, the average interdependency index rapidly increased between the 10 and 50 connection structures, but then exhibited only a minor subsequent increase at 100 connections, suggesting that there is a non-linear pattern in changes to the interdependency index as the number of connections between networks increases. If a decision-maker chooses to avoid the cascading of failures, it is necessary to analyze critical nodes in a low number of connections made between the networks. If, on the other hand, the decision-maker chooses to increase the interdependency index ($\boldsymbol{D_{hl}}$) it is possible to strategically choose the connections that will have the highest impact on the interdependency between networks.

DSI #3: Which uncertainties matter the most? In this numerical experiment, the uncertainties studied involve the connection structure between the studied networks. The question remains for the decision-maker to determine whether there was an unacceptable variability among the interdependency index ($\boldsymbol{D_{hl}}$) among the studied network connection structures and scenarios. In cases where the scenarios represent actual disruptive events, there could be more reason for deeper study into the results of specific scenarios.

The next section focuses on a real-world case study in critical interdependent infrastructures in Puerto Rico.

5. Case study: interdependent infrastructures in Puerto Rico

This case study analyzes network resilience for the United States territory of Puerto Rico. The island's infrastructure is vital for the health and safety of the territory's three million residents [44] and for the island's manufacturing industry, which is critical for the global healthcare industry [45]. The infrastructure network interdependencies of Puerto Rico were involved in a massive humanitarian disaster in 2017 following Hurricane Maria. Hurricane Maria made landfall as a Category 4 storm, causing devastating flooding, mudslides, and storm surges. In addition to the thousands of deaths [7] resulting from the storm, original estimates concluded that this hurricane was the third costliest in United States history [8].

Network interdependencies are an important issue in Puerto Rico for several reasons. First, this is an island territory, making it challenging to rely on outside infrastructure resources. It is also heterogeneous, with large city centers, rural areas, and varying geographies/ecologies. In addition, the territory has faced severe economic limitations in terms of system investments due to financial instability [46].

Overall, the loss of infrastructure services was found to be a major issue in Hurricane Maria's recovery process, as described by Ref. [7]. The study found a strong association between remoteness and length of time without electricity, cellular phone coverage, and other key services. It is important to note that [7] found that services to households either recovered quickly or encountered very long delays. Thus, the identification of those network nodes that are the likeliest candidate for the longest delays has significant risk management implications.

In this case study, we focus our attention on two of the most critical networks in the aftermath of Hurricane Maria: electricity and communications. Electricity was a major concern; there are estimates of households on average spending 84 days without power [7]. Of course, electricity is also necessary to support basic medical services, sanitation, manufacturing, and other critical services. Communications was also a major concern, as households also spent on average 41 days without cellular coverage [7]. Communication network performance is necessary for emergency response and the coordination of recovery activities. We assume that these two networks, electricity and communications, are interdependent.

Both electricity and communications are assumed to operate with binary logic, such that nodes can be operational or not operational. We also assume that energy is a directed network, with future research work addressing more refined network representation that considers voltage and Ohm's law principles. Additionally, future work would need to take a more directed approach to understand and model the network at the substation level, considering various types of control: human control operators and automated control. Additionally, as described in the Conclusions section, there is opportunity for future work to adapt the methodology presented here toward assumptions of partial operability of nodes.

5.1. Individual network operation

This section describes the multi-network infrastructure in Puerto Rico. The network data was collected between March and June 2018, during informal interviews with Puerto Rico infrastructure managers. All data sources were public and collected from government web pages. The data sources are.

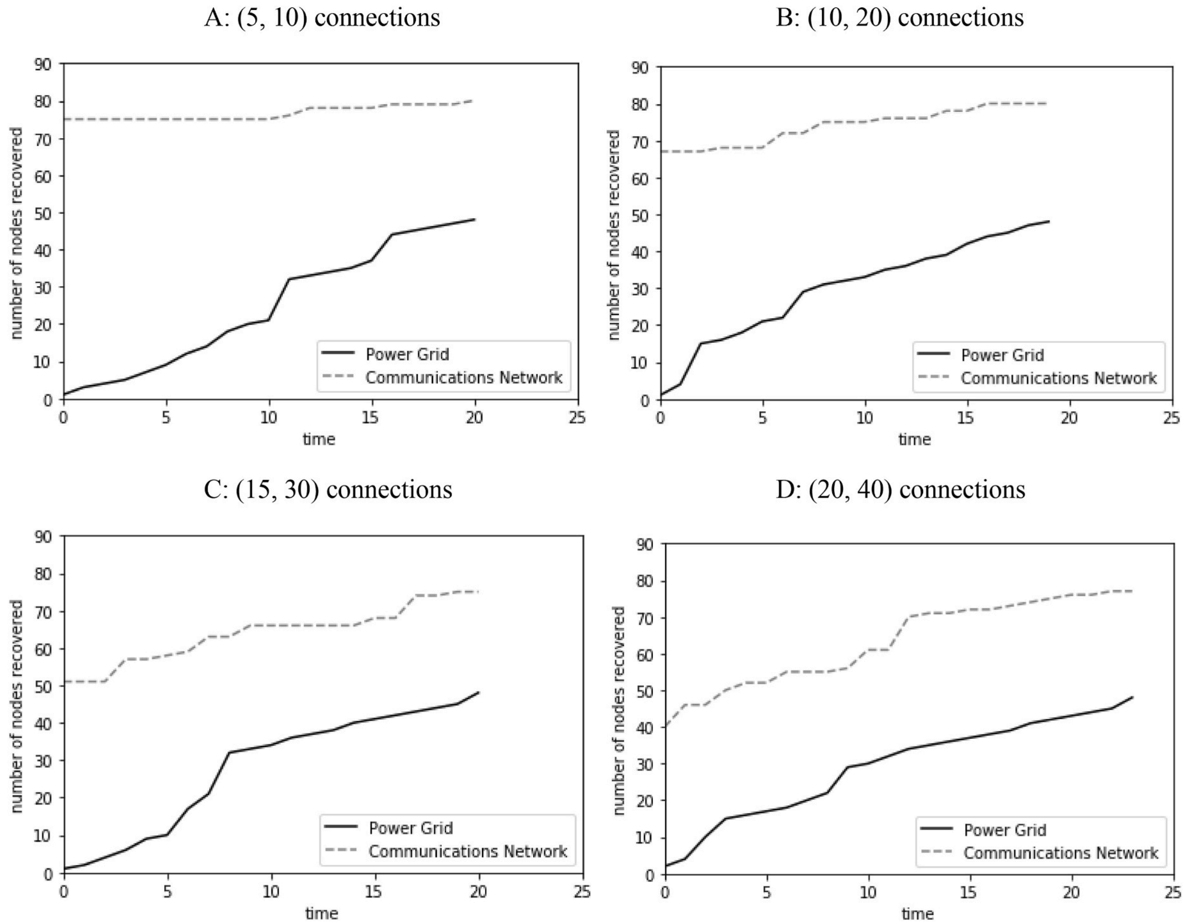

Fig. 6. Numerical experiments studying the relationship between network disruption and recovery behavior for the case study of Puerto Rico. Note: (a, b) denotes the number of connections from Power Grid to the Communications Network as a and from the Communications Network to the Power Grid as b.

- Energy network: Geospatial data showing the location of each transmission center and power station on the island.
- Communication network: Geospatial data showing the location of each cell phone tower on the island.

While the primary intent of this case study is to demonstrate the application of the modeling approach, several assumptions are being made to represent each infrastructure network. First, we assume that each network contains the same number of nodes as exists in the case study data, as shown in Fig. 5. For the energy network, we assume there are 49 transmission centers and 15 power stations. For the communications network, we assume there are 79 cellular communication towers. As detailed information about infrastructure and dependencies is protected for reasons of national security [47], we do not use the data to assume any node relationships, while noting that the methods are generalizable enough to be used for more specific/detailed datasets for those who have access to more confidential infrastructure information.

We conduct two sets of analyses using the Puerto Rico dataset. First, we adopt a scenario-based methodology that uses randomly assigned connections. Second, we use spatial modeling to assume connections.

5.2. Dependencies and interdependencies

We model the multi-network operation using the case study data as interdependent networks. Energy nodes are reliant on the functionality of neighboring energy nodes; communication nodes are dependent on functioning connected energy nodes; energy nodes are dependent on functioning connected communication nodes, etc.

First, we model four connection structures. For each connection structure, we run 50 scenarios, which are interpreted as model iterations using a randomized connection structure. We choose the number of connections from each network so that they keep a proportion between the total number of nodes in each network. Since the number of nodes of the power network is approximately half the number of nodes in the communications network, the number of connections from each node follows the same pattern.

The results in Fig. 6 show the average across all scenarios. As in the numerical experiments in Section 3, the interdependence between networks is expressed in the recovery of both networks. They follow a similar pattern of recovery. In fact, the impact on the communications network in presence of a full destruction of the power grid network is greater when the number of connections increases. It is important to note that, in our case, we assume the full destruction of one of the networks to represent what happened in a case of catastrophic failure. This was, indeed, what happened in Puerto Rico. In the worst case, in Fig. 6D, more than half of the communications network is down. The time to recover from the disruption is also affected by the increase in the number of connections. In fact, while Fig. 6A and B take about 20 iterations to recover, the highest number of connections in Fig. 6D–with around 50 % of nodes connected from each network–implies an increase in time of almost 20 % to recover. This is a particularly interesting finding from a practical perspective. Infrastructure recovery in Puerto Rico did indeed take a long time. These numerical experiments can be considered a “best-case” scenario for the recovery process. Assuming that every day after Hurricane Maria the authorities were able to recover one node in the power network, it would have taken a minimum of 20 days to recover the interdependent networks. The official report from the Autoridad de Energía Eléctrica (AEE) in Puerto Rico was that the

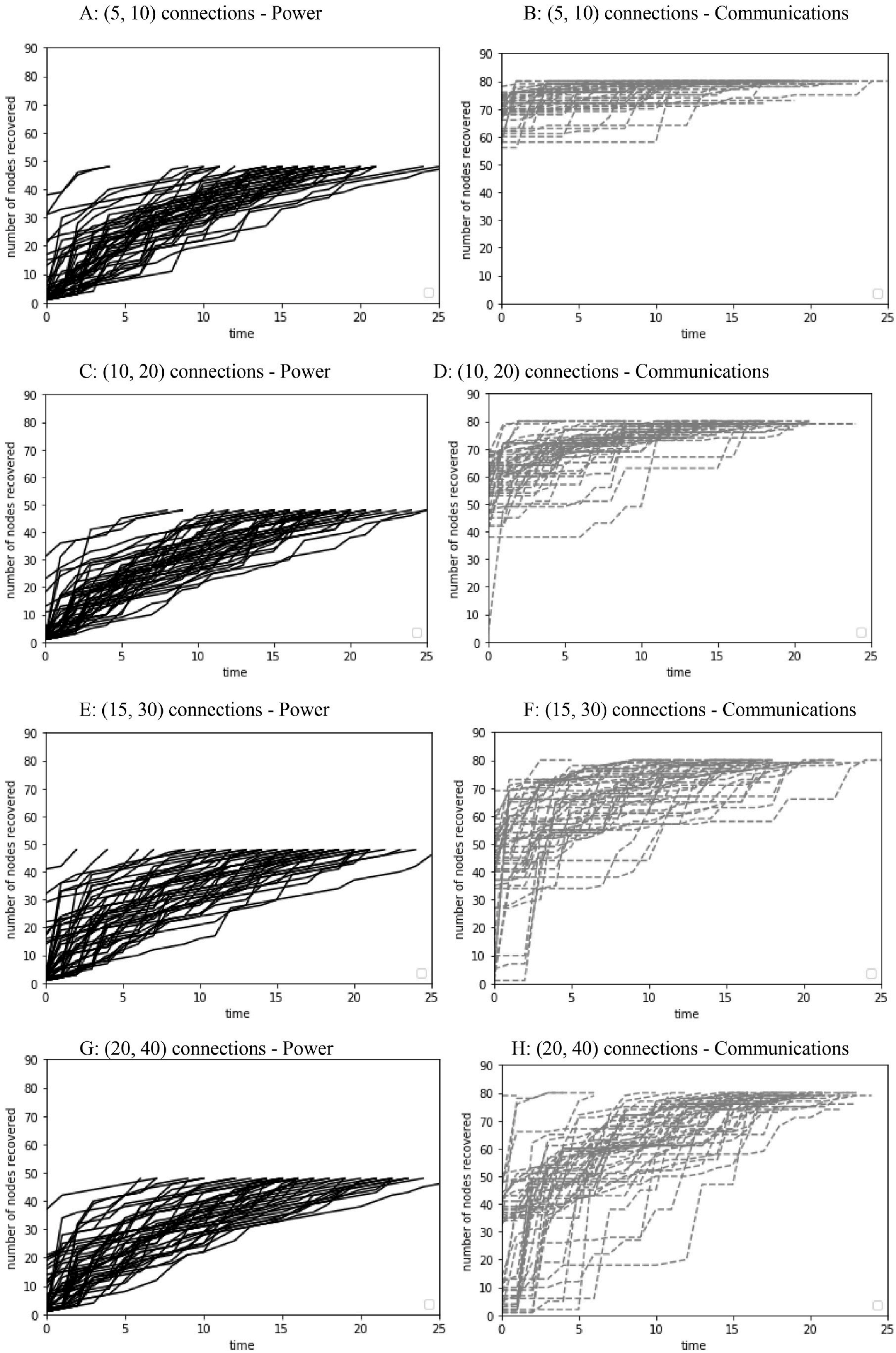

Fig. 7. Numerical experiments studying the relationship between network disruption and recovery behavior for the case study of Puerto Rico. Note: (a, b) denotes the number of connections from Power Grid to the Communications Network (as a) and from the Communications Network to the Power Grid (as b).

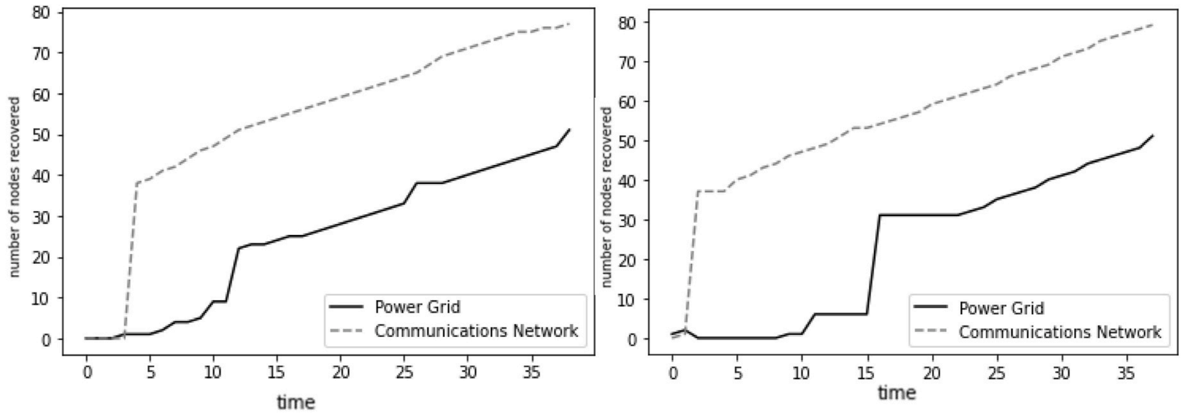

Fig. 8. Numerical experiments studying network resilience in Puerto Rico.

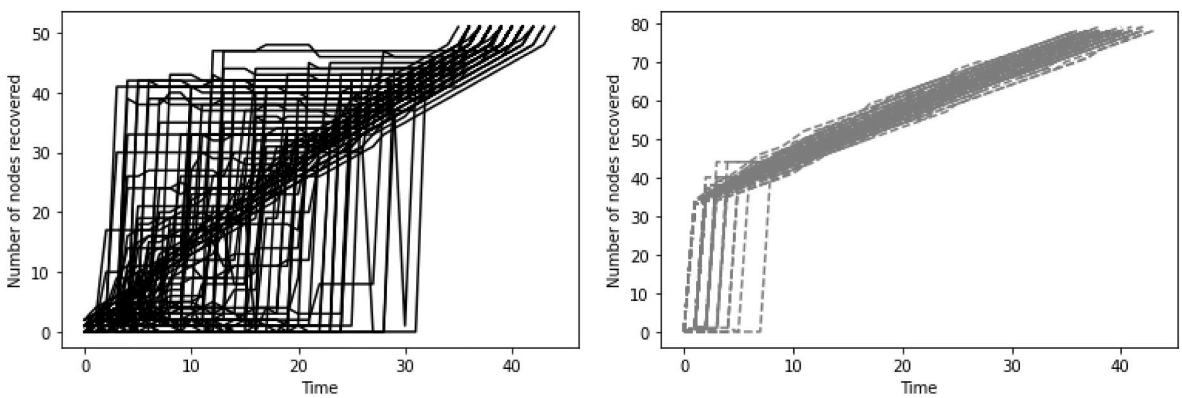

Fig. 9. Cloud representation for network resilience in Puerto Rico (left: power grid, right: communications network).

recovery took two and a half months. In reality, while the recovery process of the power network in Puerto Rico took months, some places in Puerto Rico did not have electricity even after a year.

A more complete picture emerges from the cloud analysis of the 50 recovery scenarios, as shown in Fig. 7. The variability increases when the number of interdependencies increases, as shown in Fig. 7G and H. A fraction of scenarios recover the power network in a few iterations, while the vast majority of scenarios take a longer time to recover the network.

Second, we model connection structures when assume node relationships based on spatial distances. Fig. 8 show two iterations of the recovery of the communications network when the power network is being recovered. One interesting pattern observed is the quick recovery at around 5 time periods. After that, the recovery of the network stabilizes. This pattern becomes clear when we analyze the "cloud" of 100 iterations of the implementation of the methodology as shown in Fig. 9. In this figure, the communications network is substantially more stable in its recovery than the power network in the island. This was observed on the ground after the 2017 hurricane season where the power grid kept failing after being recovered. Different situations happened to the communications network.

5.3. Resilience-informed decision support

Fig. 10 shows the resilience modeling for the four connection structures related to the Puerto Rico case study, as described in the previous section. Again, we see similar network recovery behavior of R (t) across the studied connection structures. The (10,20) connections structure shown in 10B appears to show the most similarity in behavior to the two studied networks. The (5,10) and (15,30) connection structures show that R(t) for Network 2 falls below that of Network 1 for the majority of time steps. In the (20,40) connections structure, the R(t) for Network 2 falls above that of Network 1. This may imply that the redundancies in network connectivity in the (20,40) connections structure enable Network 2 to recover faster. Conversely, the relatively low number of connections in the (5,10) connections structure causes Network 2 to undergo a relatively slow recovery, particularly in early time-steps.

Fig. 11 shows the resilience behavior for the network structure assuming node connections are based on spatial distances among nodes. Here, R(t) for Network 2 follows a more linear pattern and sits below R (t) for Network 1 for the majority of time steps. Because the connections in this case are not random, the linear behavior of R(t) for Network 2 is influenced by the spatial relationships in the network design.

5.4. Modeling scenarios for resilience characteristics and network recovery

Table 3 studies the interdependency index, D_{hl} for the case study. Again, all four connection structures studied involve high levels of average interdependency. As also shown in the numerical experiments of Section 3, the interdependency did not linearly increase as the number of connections increased. For the (5,10) connections structure, the relatively low average interdependency index results from a relatively low number of connections to aid in the recovery process. For the (20,40) connections structure, the relatively low interdependency index results from redundancies in inter-network relationships, which allows Network 2 to recover more quickly than Network 1. This demonstrates that a relatively high number of connections could work in favor of system recovery, as redundancies in the connection structure can allow

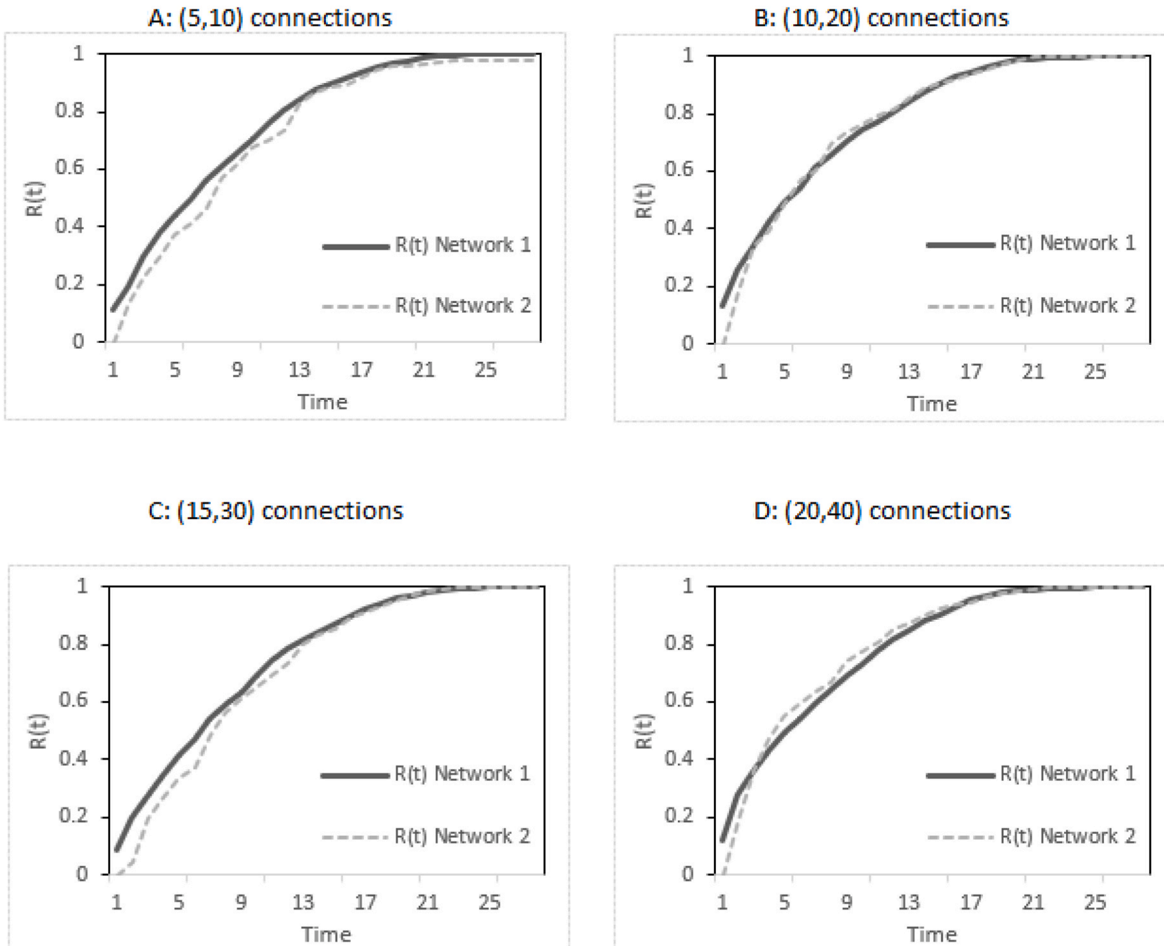

Fig. 10. Resilience behavior for four network structure scenarios for the case study of Puerto Rico Note: (a, b) denotes the number of connections from Power Grid to the Communications Network (as a) and from the Communications Network to the Power Grid (as b).

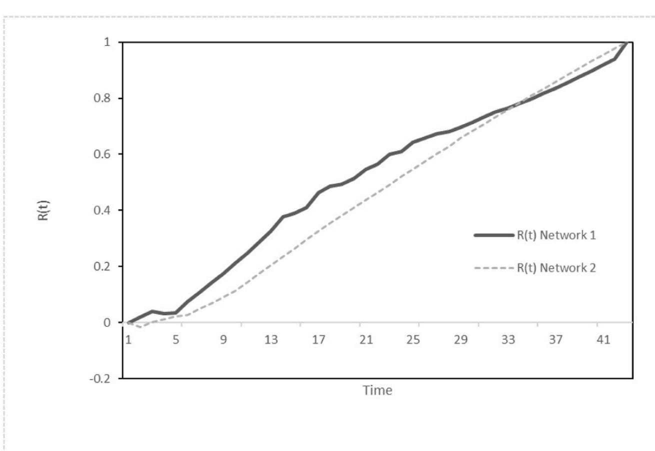

Fig. 11. Resilience behavior for network structure based on spatial distance between nodes for the case study of Puerto Rico.

for more effective dependent network recovery.

Table 3 also characterizes outlier behavior for the interdependency index. While all four studied connection structures showed relatively similar variability across scenarios, the maximum and minimum values across scenarios provide some insight. Across all four connection structures, there was one scenario that involved an interdependency index that was near a value of 1.

Table 3
Interdependency index for numerical experiments studying the relationship between network disruption and recovery behavior.

Network Connections	Interdependency Index			
Number	Average	Standard Deviation	Max	Min
(5,10) Connections	0.865	0.087	0.967	0.584
(10,20) Connections	0.892	0.086	1.000	0.614
(15,30) Connections	0.916	0.083	0.980	0.510
(20,40) Connections	0.884	0.098	0.979	0.568

In the case of assuming node relationships based on spatial proximity, the interdependency index was 0.988, which far exceeds the scenarios shown in Table 3.

Next, we revisit the decision support questions.

DSI #1: Can interventions on a network provide benefits to connected networks, thereby benefiting the larger studied system? In the case study, the mid-range number of connections, at (15,30), showed the highest interdependency index, suggesting that the Network 2 recovery more closely follows the Network 1 recovery under this connection structure. This suggests that interventions can appropriately be prioritized for connecting both networks. Additionally, Fig. 10 showed that the resilience $R(t)$ of Network 2 surpassed that of Network 1 under the highest (20,40) connection structure.

DSI #2: Which networks benefit from the addition of new network connections to adjoining networks? The case study behavior showed a relatively high average interdependency index for the mid-range number of connections between Network 1 and 2, as shown in the (15,30) connections structure. While this high average interdependency index can be beneficial in cases of network recovery, it can also cause failures to cascade in the event of network disruption.

DSI #3: Which uncertainties matter the most? While the average interdependency index behaved similarly among the studied scenarios, there were several scenarios that require further study. For example, under the (10,20) connections structure, the highest interdependency index was one. This requires further study, allowing decision-makers to anticipate some set of conditions under which this level of interdependence exists. Conversely, all four connection structures showed a minimum interdependency index of between 0.51 and 0.62. Under these scenarios, network connectivity did not facilitate similar co-recovery behavior between networks. This also requires further study, as this implies that investments in network intervention do not necessarily always aid the multi-network recovery process.

The results of this case study are the first step in understanding the behavior of resilience in interdependent networks for Puerto Rico, and elsewhere. Research is still needed to understand what the optimal number of connections is, and how to prioritize investment in the management of network interdependencies. We have introduced a preliminary problem structure and model that can help facilitate such further research. The results would be useful for long-term planning exercises for interdependency management, which often occur after a major disaster, such as the 2017 Hurricane Maria in Puerto Rico.

6. Conclusions

This paper presents an approach to model risk and resilience for multilayered infrastructure networks. The approach allows for concurrent resilience modeling of many types of networks, such as supply chains, communication, water, and healthcare. The approach leverages understanding of network dependencies and interdependencies, such that a disruption in one network can potentially have a cascading impact on connected networks. From the perspective of resilience, this paper recognizes that interdependencies among networks suggest that recovery in a particular network may be contingent upon recovery in a connected network. Thus, interdependencies can be leveraged for the purpose of aiding or improving network resilience across all modeled networks. This is what we denominate “cascading recoveries.” The paper also recognizes that dependent and interdependent networks may have multiple functions, users, owners, and associated stakeholders, thereby requiring the use of a decision-support system for prioritizing network interventions. The work of this paper will be useful to provide both public and private-sector infrastructure managers a risk-based assessment of network interventions.

The major difference between this paper and more traditional resilience modeling is that we consider multi-network interdependency in gradual network recoveries, with particular interest in decision-making and network design-related factors, and how they relate to network resilience features.

The methods developed were demonstrated on the infrastructure network of Puerto Rico, which was devastated by Hurricane Maria in 2017. Network dependencies both caused disruptions to cascade, and also played a role in the overall recovery process. The case study showed that the most appropriate number of network connections depends on the needs of various decision-makers, recognizing that increased interdependence among networks can cause failures to cascade across networks, while also allowing recoveries to cascade across networks.

This work makes several contributions. First, the approach provides a data-informed representation of multiple interdependent networks, allowing for multiple types of infrastructure to be modeled concurrently. Second, this work is the first to provide a framework for understanding interdependencies in resilience characteristics. These resilience characteristics allow decision-makers to explore which network connection structures are appropriate for the studied application, an understanding of which facilitates long-term planning exercises that allow decision-makers to prioritize investments for those network connection structures that will aid in post-disaster recovery and also manage the cascading of failures.

There are several opportunities for future work. First, there is an opportunity to apply and test the methodology to assumptions for additional infrastructure types. While the case study involved the study of electricity and communications networks, which were assumed to operate with binary logic, with nodes either operational or not operational, future work could expand the methodology to include the ability to address other types of operational logic, such as partial service or “brownout” electricity coverage. Second, there is an opportunity to test network behavior for more specific network recovery scenarios. While the case study and numerical experiments considered randomized scenarios involving connection structures, there is need to understand or generalize the impact of more specific scenarios, such as geographically-informed node recovery, rule-based logic for network recovery, or the use of various probability models within the node-recovery logic. Third, future work can also address more refined decision-support questions. For example, it is important to understand: which uncertainties, E, are the best/worst for $R_i(t)$; which nodes are of highest/lowest priority; which uncertainties matter the most; and whether the resilience, $R_i(t)$, is more sensitive to some uncertainties, E, versus others. Decision-makers can use this information to direct their investments toward more refined study of particular uncertainties, those which are most impactful toward resilience. This may lead to the inclusion of additional uncertainties to model in later iterations of the methodology presented in this paper. Finally, there is an opportunity to use the results of this paper to create a more refined risk-based decision-support methodology to model the most appropriate protective investments. For example, decision support can be used to identify the highest priority networks and nodes. This could be supported by surveying infrastructure managers to understand the importance of various network performance indicators and criteria in selecting the most appropriate investments.

CRediT authorship contribution statement

Felipe Aros-Vera: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Formal analysis, Data curation, Conceptualization. **Shital Thekdi:** Writing – review & editing, Writing – original draft, Visualization, Validation, Methodology, Formal analysis, Conceptualization.

Data availability

Data will be made available on request.

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