Assignment 9

MA08 Applied Algebra Deadline 05:00 PM, Friday, 20190726

1. Determine whether the element is an irreducible of the indicated ring.
   (a) -17 in $\mathbb{Z}$
   (b) $2x - 10$ in $\mathbb{Z}[x]$
   (c) $x^2 - 2$ in $\mathbb{R}$
2. Give at least two different associates of $2x - 7$ in (1) $\mathbb{Z}[x]$ and (2) $\mathbb{Q}[x]$. (Hint: Definition 9.7.)
3. Let D be a Euclidean domain and let v be a Euclidean valuation on D. Show that if a and b are associates in D, then $v(a) = v(b)$. (Hint: Definition 11.1(ii).)
4. Let $z = a + b\sqrt{3}i$ be in $\mathbb{Z}[\sqrt{3}i]$. If $a^2 + 3b^2 = 1$, show that z must be a unit. Show that the only units of $\mathbb{Z}[\sqrt{3}i]$ are 1 and -1. (Hint: Find z^{-1} according to the definition of unit.)
5. The Gaussian integers, $\mathbb{Z}[i]$, are a UFD. Factor the element 5 in $\mathbb{Z}[i]$ into a product of irreducibles. (Hint: $5 = (a + bi)(c + di)\cdots$, where $a, b, c, d, \cdots \in \mathbb{Z}$).

   Notice: Please write Your Name and Student ID when you submit.