Assignment 7

MA08 Applied Algebra Deadline 05:00 PM, Wednesday, 20190717

1. Decide whether the indicated operations of addition and multiplication are defined (closed) on the set, and give a ring structure. If a ring is not formed, tell why. If a ring is formed, state whether the ring is commutative, whether it is a field.

   (a) $n\mathbb{Z}$, $n \in \mathbb{N}$ with the usual addition and multiplication. (Hint: $(n\mathbb{Z}, +)$ are commutative groups.)

   (b) $\mathbb{Z}^+$ with the usual addition and multiplication.

   (c) The set of all pure imaginary numbers $G = \{ri \mid r \in \mathbb{R} \text{ and } r \neq 0\}$ with the usual addition and multiplication.

2. Describe all units in the ring $\mathbb{Z}$.

3. Let R be a commutative ring with unity and let $a_1, a_2, \cdots, a_n \in R$. Prove that $I = \langle a_1, a_2, \cdots, a_n \rangle = \{r_1 a_1 + r_2 a_2 + \cdots + r_n a_n \mid r_i \in R\}$ is an ideal of R. (It is called the ideal generated by $a_1, a_2, \cdots, a_n$.) (Hint: Use Definition 9.12. As reference, see the definition of 'ring with unity' in appendix of the updated slides of Lecture 9.)

   Notice: Please write Your Name and Student ID when you submit.