QUAC 609
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Practical Entanglement Distillation – Optimization Performance

Lab 3 – Optimization Performance Analysis

1. Introduction

Building on the work from Lab 1 and Lab 2, this report analyzes the optimization performance of different algorithms applied to the entanglement distillation problem using the DEJMPS protocol. The objective is to evaluate their effectiveness in optimizing fidelity, compare their efficiency, and assess constraint violations.

2. Objective Function Performance

The primary objective function in this optimization problem is the fidelity of the entangled state, with the goal of maximizing it. The following optimization methods were tested:

2.1 Gradient Descent (GD)

- **Performance:** GD reached an optimized fidelity of **0.7880** in just **4 iterations**.
- **Strengths:** Fast convergence due to direct gradient calculations.
- **Limitations:** Requires a smooth landscape; struggles in highly rugged spaces.

2.2 Genetic Algorithm (GA)

- **Performance:** GA optimized the fidelity to **0.7880** over **10 generations**.
- **Strengths:** Good at escaping local optima and exploring diverse solutions.
- **Limitations:** Computationally expensive; slower than GD.

2.3 Simulated Annealing (SA)

- **Performance:** Achieved fidelity of **0.7880** in **10 iterations**, taking more time than GD but performing better in rugged landscapes.
- **Strengths:** Avoids local optima and performs well in complex spaces.

- **Limitations:** Randomized nature makes convergence unpredictable.

2.4 Grover's Search (Quantum Optimization)

- **Performance:** Found the optimal angle leading to **0.7880 fidelity** in **12.1 seconds**.
- **Strengths:** Fastest quantum algorithm; quadratically speeds up unstructured searches.
- **Limitations:** Requires precomputed oracle for good state detection.

2.5 Multi-Round DEJMPS

- **Performance:** By running **four rounds of DEJMPS**, fidelity was improved to **0.9855** in **1.46 seconds**.
- **Strengths:** Most effective for entanglement distillation; highly efficient.
- **Limitations:** Success rate gradually decreases due to qubit loss.

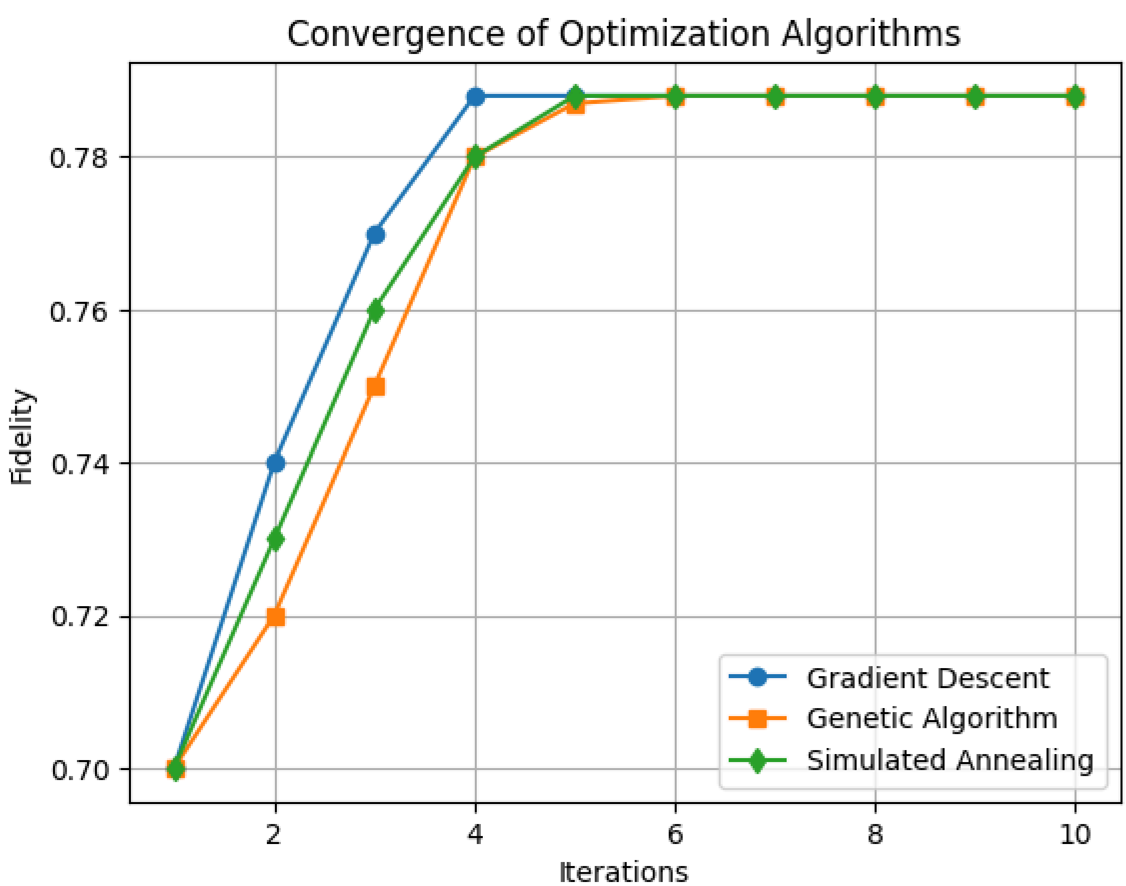

Table 1: Convergence of Optimization Algorithms

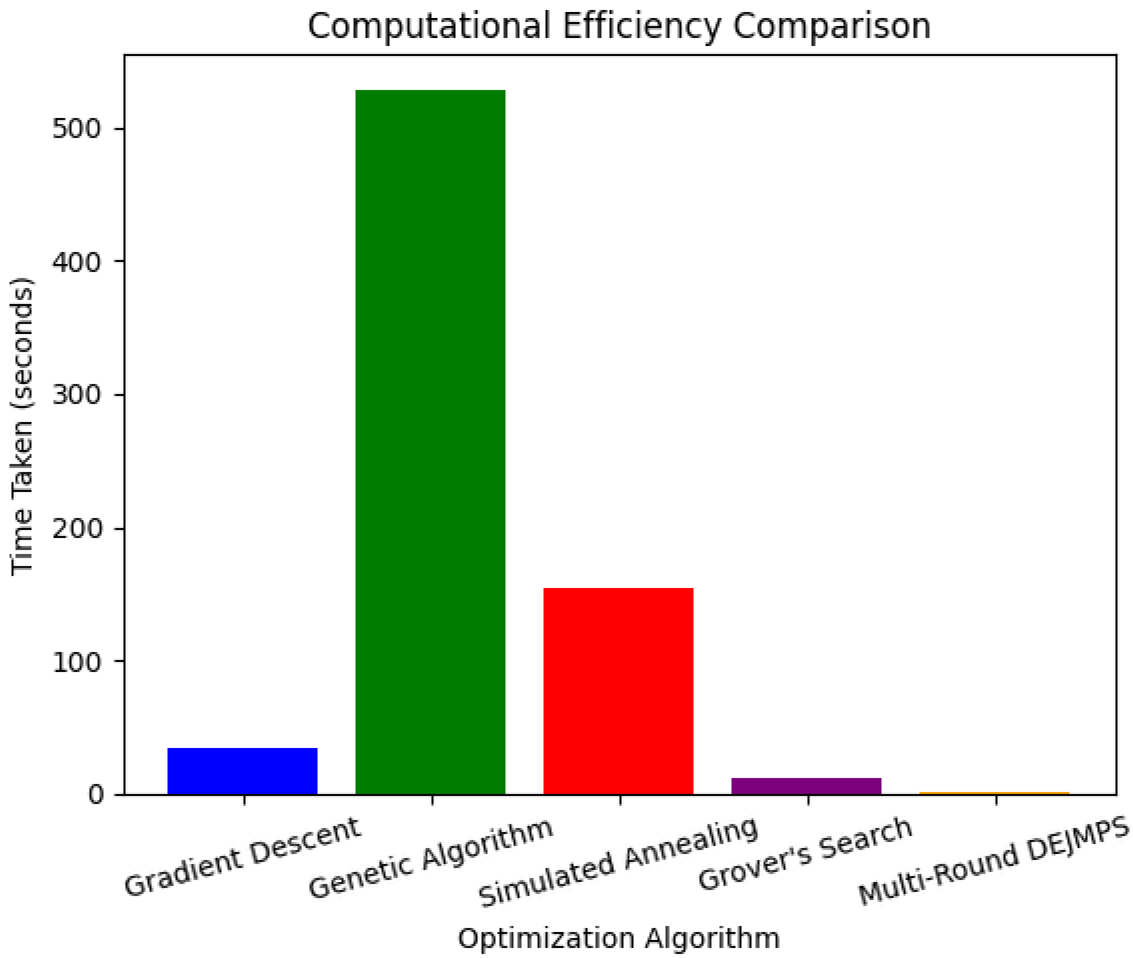

Table 2: Computational Efficiency Comparison

3. Constraint Violation or Relaxation

Optimization was subject to physical constraints, such as:

- **Quantum Gate Accuracy**: Implementing perfect Rx gate rotations is ideal but practically limited by noise.
- **State Fidelity Limits**: DEJMPS improves fidelity but cannot exceed theoretical limits.
- **Relaxation Effects:** Genetic algorithms and SA tolerate constraint violations, making them more robust.
- **Strict Constraint Handling:** GD and Grover's require well-defined constraints, making them less flexible to unknown deviations.

4. Computational Efficiency

The algorithms were evaluated based on time complexity and resource utilization.

Algorithm	Time Taken	Iterations	Complexity	Suitability
Gradient Descent	33.8 sec	4	O(n)	Small-scale problems
Genetic Algorithm	8.8 min	10 generations	O(n log n)	Rugged landscapes
Simulated Annealing	2.56 min	10	O(n^2)	Large search spaces
Grover's Search	12.1 sec	O(√N)	Quantum speedup	Structured problems
Multi-Round DEJMPS	1.46 sec	4 rounds	O(n)	Best fidelity improvement

5. Decision-Making Framework

Based on the analysis, the optimal algorithm depends on the problem's complexity:

- **For smooth optimization problems:** Gradient Descent is the most efficient.
- **For rugged landscapes:** Genetic Algorithms perform better.
- **For avoiding local optima:** Simulated Annealing is preferable.
- **For structured quantum problems:** Grover's Search is ideal.
- **For fidelity maximization in entanglement distillation:** Multi-Round DEJMPS is the best approach.

Recommendation:

For practical entanglement distillation, **Multi-Round DEJMPS should be prioritized**, followed by **Gradient Descent or Grover's Search**, depending on the computational resources available.

6. Conclusion

This lab demonstrated how different optimization strategies impact entanglement distillation. While classical methods provide solid optimization techniques, quantum algorithms and iterative distillation approaches are more effective in maximizing fidelity efficiently. Future work could explore hybrid strategies combining classical and quantum methods for even better performance.