02244 Logic for Security
Security Protocols
The Lazy Intruder

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Dolev-Yao Closure: Summary

Dolev-Yao rules

$$\frac{}{M \vdash m} \text{ if } m \in M \text{ (Axiom)}$$

$$\frac{M \vdash m_1 \quad \ldots \quad M \vdash m_n}{M \vdash f(m_1, \ldots, m_n)} \text{ if } f/n \in \Sigma_p \text{ (Compose)}$$

$$\frac{M \vdash \langle m_1, m_2 \rangle}{M \vdash m_i} \text{ (Proj}_i\text{)} \qquad \frac{M \vdash \{\!|m|\!\}_k \quad M \vdash k}{M \vdash m} \text{ (DecSym)}$$

$$\frac{M \vdash \{m\}_k \quad M \vdash \mathrm{inv}(k)}{M \vdash m} \text{ (DecAsym)} \qquad \frac{M \vdash \{m\}_{\mathrm{inv}(k)}}{M \vdash m} \text{ (OpenSig)}$$

The compose rule is for all public functions Σ_p,
including $\{\!| \cdot |\!\}_{\cdot}$ $\{\cdot\}_{\cdot}$ $\langle \cdot, \cdot \rangle$

Automating Dolev-Yao

Step 1: Composition only—Solution

Composition Only

$$\frac{}{M \vdash_c m} \text{ if } m \in M \text{ (Axiom)}$$

$$\frac{M \vdash_c m_1 \quad \ldots \quad M \vdash_c m_n}{M \vdash_c f(m_1, \ldots, m_n)} \text{ if } f \in \Sigma_p \text{ (Compose)}$$

Decision procedure for $M \vdash_c m$

1. Check if $m \in M$; if so return yes.
2. Otherwise, let $m = f(t_1, \ldots, t_n)$
   - If f is not public return no.
   - Otherwise recursively check whether:

     $$M \vdash_c t_1 \text{ and } \ldots \text{and } M \vdash_c t_n$$

     Return yes if all these return yes, and no otherwise.

Automating Dolev-Yao

Step 2: Analysis—solution

Analysis Steps

- If $\{|m|\}_k \in M$ and $M \vdash_c k$ then add m to M.
- If $\{m\}_k \in M$ and $M \vdash_c \mathsf{inv}(k)$ then add m to M.
- if $\{m\}_{\mathsf{inv}(k)} \in M$ then add m to M.
- If $\langle m_1, m_2 \rangle \in M$ then add m_1 and m_2 to M.
- Repeat until no new messages can be added.

Automating Dolev-Yao

To check $M \vdash m$:

- Perform the Analysis Steps procedure, augmenting M with all derivable messages.
- Now it suffices to check $M \vdash_c m$.

Properties of the algorithm for checking $M \vdash m$:

- Soundness: if algorithm says "yes", then $M \vdash m$.
- Completeness: if $M \vdash m$, then the algorithm says "yes".
  - ★ This is quite tricky to prove.
- Termination: the algorithm never runs into an infinite loop.

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- Termination: the algorithm never runs into an infinite loop.
  - ★ Procedure for $\vdash_c$ terminates since it goes to smaller terms
  - ★ Analysis terminates because it only adds proper subterms of an existing term. This cannot go on forever, since there are only finitely many subterms.

Negative Question

Can we thus prove also statements of the form $M \nvdash m$

...that a m cannot be derived from M?

Example

$$M = \{\, k_1, \{\!|m_1|\!\}_{k_1}, m_2, \{\!|m_3|\!\}_{k_2} \,\} \nvdash m_3$$

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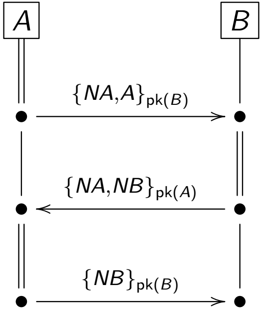
$$M = \{\, k_1, \{\!|m_1|\!\}_{k_1}, m_2, \{\!|m_3|\!\}_{k_2} \,\} \nvdash m_3$$

- Yes, due to completeness when our algorithm answers "no", we know there is no derivation for m.

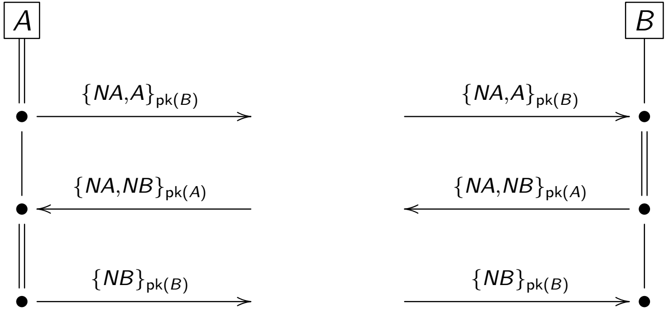
Needham-Schroeder Public-Key Protocol [1978]

```
Protocol: NSPK

Types: Agent A,B;
       Number NA,NB;
       Function pk,h

Knowledge: A: A,pk(A),inv(pk(A)),B,pk(B),h;
           B: B,pk(B),inv(pk(B)),A,pk(A),h
Actions:
A->B: {NA,A}(pk(B))  # A generates NA
B->A: {NA,NB}(pk(A)) # B generates NB
A->B: {NB}(pk(B))

Goals:
h(NA,NB) secret between A,B
```

NSPK as A Message Sequence Chart

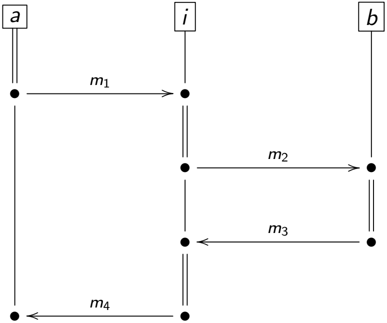


NSPK as Roles / Strands



- For each Role of the protocol, a program that sends and receives messages (over possibly insecure network)
- Strand: concrete execution of a role: all variables (here A, B, NA, NB) instantiated with concrete values
  - ★ or a prefix thereof (an agent might not finish)

Attacks



An attack is a strand space where the following conditions are met:

- Messages sent by honest agents are received by i
- Messages received by honest agents are sent by i who can compose the message from the messages he has received so far.

- The successful completion violates a goal of the protocol.

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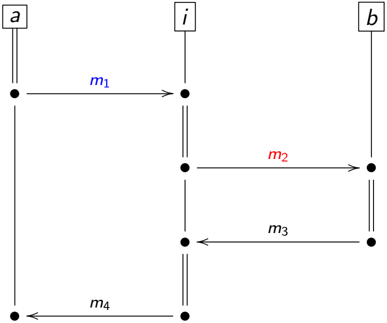

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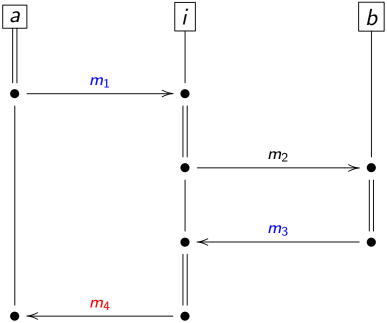

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- Messages sent by honest agents are received by i
- Messages received by honest agents are sent by i who can compose the message from the messages he has received so far.
  - ★ In the example: $\{m_1\} \vdash m_2$ and $\{m_1, m_3\} \vdash m_4$.
- The successful completion violates a goal of the protocol.

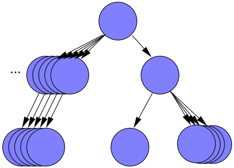
Infinite State Space

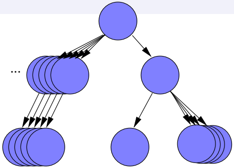
Problem 1: at any time, any number of people can run the protocol in parallel. (Think of TLS...)

- For now we bound the number of sessions: only finitely many strands of honest agents
- Later: how to verify for unbounded sessions

Problem 2: at any time the intruder has an inifinite choice of message they can construct and send to an agent.

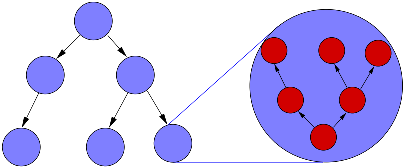
- We will solve this problem with a constraint approach: the lazy intruder.





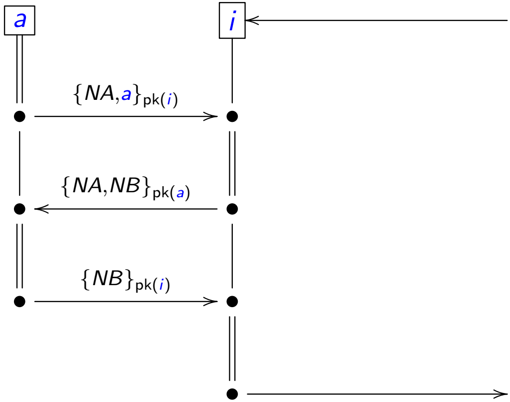
Even for bounded sessions we have an infinite tree of reachable states, i.e., how the intruder can interact with honest agents.

Idea: symbolic states



- Each state is an attack scenario: a sequence of interactions with honest agents, leaving undetermined what exactly the intruder sends.
- Then each state is a constraint solving problem: "Can the intruder generate all messages that the attack scenario has?"
- This will be a backward search: we start at the complete attack scenario and try to see how the intruder could have constructed this.

Example: Can the Intruder Play a Protocol Role?



Example:

- NSPK with $A = a$ and $B = i$

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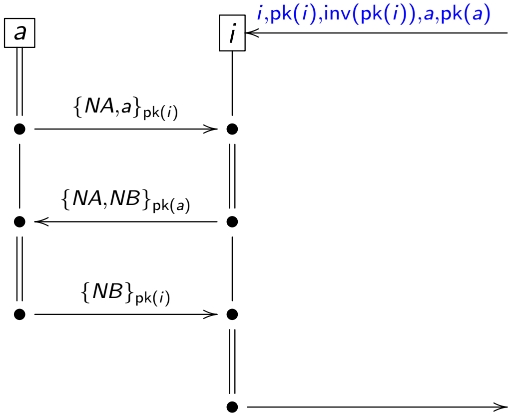

Example:

- NSPK with $A = a$ and $B = i$
- i needs corrsp. knowledge of role B: $i, \mathsf{pk}(i), \mathsf{inv}(\mathsf{pk}(i)), a, \mathsf{pk}(a)$

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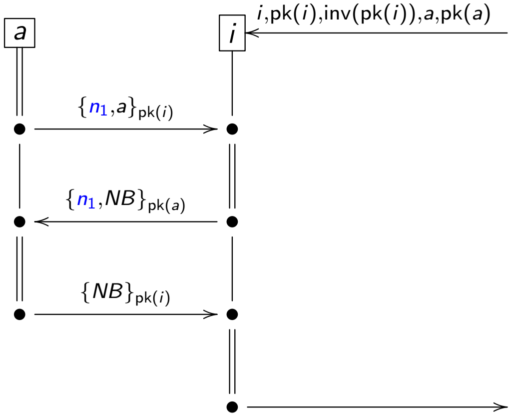

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- a uses a fresh $NA = n_1$.

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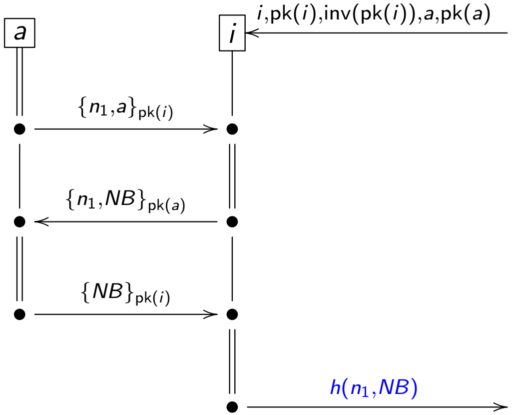
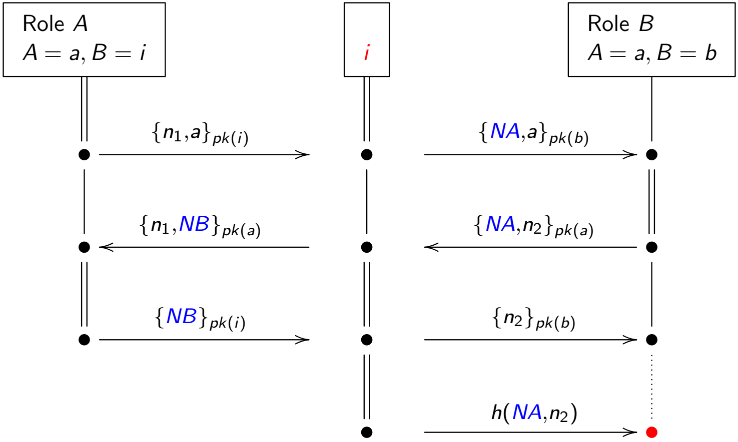


Example:

- NSPK with $A = a$ and $B = i$
- i needs corrsp. knowledge of role B: $i, \mathrm{pk}(i), \mathrm{inv}(\mathrm{pk}(i)), a, \mathrm{pk}(a)$
- a uses a fresh $NA = n_1$.
- Afterwards, i should be able to construct the shared key $h(NA, NB) = h(n_1, NB)$

The Lazy Intruder Attacking

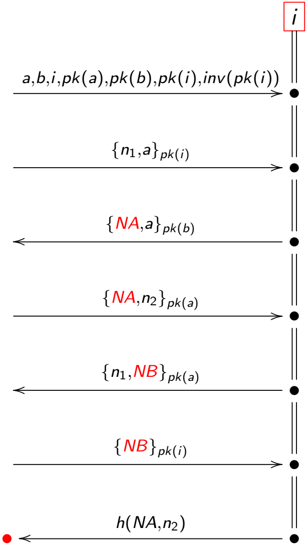
Scenario



The challenge labeled •:

- Can the intruder produce the secret session key $h(NA, n2)$?

Choose and Check



- For simplicity we display here only outgoing and incoming messages of the intruder.

Q:
Is the protocol itself a public knowledge?
Or how practically we can create this tree

Needham-Schroeder-Lowe [1996]

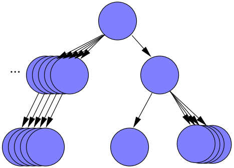
```
Protocol: NSL

Types: Agent A,B;
       Number NA,NB;
       Function pk,h

Knowledge: A: A,pk(A),inv(pk(A)),B,pk(B),h;
           B: B,pk(B),inv(pk(B)),A,pk(A),h
Actions:
A->B: {NA,A}(pk(B))
B->A: {NA,NB,B}(pk(A))
      # Inserted B's name into the message
A->B: {NB}(pk(B))

Goals:
h(NA,NB) secret between A,B
```

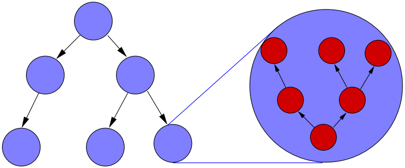
- Without the lazy approach, we would get an infinite search tree because the intruder has often an infinite choice of messages to send.



- We avoid this by using the lazy intruder:

  Layer 1: a symbolic search tree

  Layer 2: constraint solving



Lazy Intruder: Summary

Constraint Solving

Go step by step through the constraint.

- For incoming messages: can a decryption rule be applied?
  If so, add the decrypted message also as an incoming message
  - ★ When the key contains variables this can be handled by adding the key as an outgoing message, i.e., require that the intruder can produce it.
- For outgoing messages
  - ★ If it is a variable: be lazy for now.
    - ▶ If it gets replaced, you need to come back here.
  - ★ Otherwise: check all following possibilities (backtracking!):
    - ▶ Compose: can a compose rule be applied?
    - ▶ Axiom: can the axiom rule be applied?
      This may require instantiating variables

Lazy Intruder: Summary

Theorem (Rusinowitch & Turuani 2001)

Protocol insecurity for a bounded number of sessions is NP-complete.

Proof Sketch.

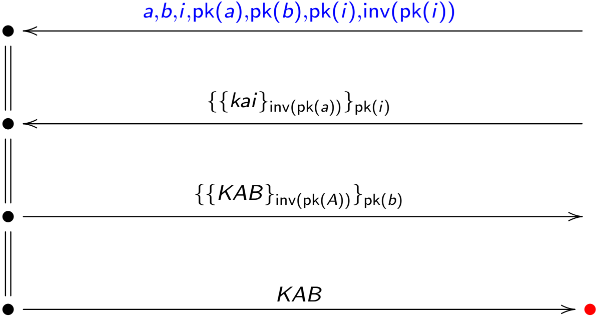
In NP: Guess a symbolic attack trace for the given strands and a sequence of reduction steps for the resulting constraints. Check that this sequence of reduction steps solves the constraint.

NP-hard: Polynomial reduction for boolean formulae to security protocols such that formula satisfiable iff protocol has an attack. □

Relevant Research Papers

- David Basin, Sebastian Mödersheim, and Luca Viganò. *OFMC: A symbolic model checker for security protocols.* International Journal of Information Security, 4(3), 2005.
- Gavin Lowe. *Breaking and Fixing the Needham-Schroeder Public-Key Protocol Using FDR*. Software Concepts Tools, 17(3), 1996.
- Jonathan K. Millen and Vitaly Shmatikov. *Constraint solving for bounded-process cryptographic protocol analysis*. Computer and Communications Security, 2001,
- Roger Needham and Michael Schroeder. *Using Encryption for Authentication in Large Networks of Computers.* Communications of the ACM, 21(12), 1978.
- Michaël Rusinowitch and Mathieu Turuani. *Protocol Insecurity with Finite Number of Sessions is NP-complete.* Computer Security Foundations Workshop, 2001.

Exercise



Exercise: show that this constraint has both

- a solution where $A = i$ (i.e., a normal execution)
- a solution where $A = a$ (i.e., an attack)