Love Differential Equation

Tomohiro Nakamura
email: tnakamura@ucsb.edu
perm: 9940974
(Dated: March 14, 2020)

Differential Equation is a powerful tool to model how systems change over time. In this paper, I propose the differential equation architectures for love and see this model simulates the love dynamics well. Previous research suggested the linear differential equation model for love between two people. Here, I generalize this model to the loves between n people and incorporate the idea of popularity, gender, and capacity into this model, which are impossible to consider in two people's love. Also, I will show this initial problem is well posed. The code is available at `https://github.com/tomotomonakanaka/LoveDifferentialEquation`

I. INTRODUCTION

Elishakoff (2019) induced differential equations that model two people's love based on the background of William Shakespeare's Romeo and Juliet. Let Romeo's feeling at time t be R(t) and Juliet's feeling at time t be J(t). The love interaction is formulated as:

$$\frac{dR}{dt} = \alpha_R J \tag{1}$$

$$\frac{dJ}{dt} = \alpha_J R \tag{2}$$

I extend this differential equations to simulate n people's love.

II. MODEL

To construct the model, first we define the feeling from i to j. Let us denote the i's feeling to j at time t by $x_{i,j}(t)$. Now, we assume that the change of i's feeling only depends on the feeling from i to j and the feeling from j to i. The ordinary differential equation is written like

$$\frac{dx_{i,j}}{dt} = \alpha_{i,j} x_{i,j} + \beta_{i,j} x_{j,i} \tag{3}$$

which is naturally induced from Eq. (1) and Eq. (4). $\alpha_{i,j}$ and $\beta_{i,j}$ are constants and defined by popularity and love constitution.

A. Popularity

To model the popularity of the person, we define

$$\alpha_{i,j} = \alpha'_{i,j} + p_j \tag{4}$$

$$\beta_{i,j} = \beta'_{i,j} + p_j \tag{5}$$

$\alpha'_{i,j}$ and $\beta'_{i,j}$ are i's love constitution, and p_j is the popularity of j. If p_j is big then j's popularity is high.

B. Capacity

In the real world, people cannot love all people. Therefore, we introduce the concept of capacity of love to this model.

$$\frac{dx_{i,j}}{dt} = \alpha_{i,j} x_{i,j} + \beta_{i,j} x_{j,i} + sgn(x_{i,j})\gamma_{i,j}(1 - |x_i|) \tag{6}$$

where $|x_i| = \sqrt{\sum_j x_{i,j}^2}$ and $\gamma_{i,j}$ is a constant.

C. Gender

To model gender, we initialize $x_{i,j}$ as

$$x_{i,j}^{(0)} = \begin{cases} 0 & (g(i) = g(j)) \\ f_{i,j} & (otherwise) \end{cases} \tag{7}$$

where

$$g(k) = \begin{cases} 0 & (k\ is\ male) \\ 1 & (k\ is\ female) \end{cases} \tag{8}$$

D. Matrix Form

If we rewrite Eq. (6) in matrix form,

$$\frac{dX}{dt} = A \otimes X + B \otimes X^T + sgn(X) \otimes \Gamma \otimes (\mathbf{1} - X_2) \tag{9}$$

where
$X_{i,j} = x_{i,j}$, $sgn(X)_{i,j} = sgn(x_{i,j})$, $A_{i,j} = \alpha_{i,j}$, $B_{i,j} = \beta_{i,j}$, $\Gamma_{i,j} = \gamma_{i,j}$, $X_{2,i,j} = |x_i|$

III. PROOF OF WELL POSEDNESS

Let $f_{i,j}$ be

$$f_{i,j}(t, x_{1,1}, x_{1,2}, ..., x_{n,n})$$
$$= \alpha_{i,j} x_{i,j} + \beta_{i,j} x_{j,i} + sgn(x_{i,j})\gamma_{i,j}(1 - \sqrt{\sum_{j=1}^{n} x_{j,i}^2})$$

Check if the function $f_{i,j}$ defined on the set

$$
\begin{aligned}
D = \{&(t, x_{1,1}, x_{1,2}, ..., x_{n,n})| \\
&0 < t < \infty \ and \ 0 < x_{i,j} < \infty, \\
&for \ each \ i = 1, 2, ..., n \ and \ j = 1, 2, ..., n\}
\end{aligned}
$$

satisfies Lipschitz condition on D.
Proof:

$$
\begin{aligned}
&|f_{i,j}(t, u_{1,1}, u_{1,2}, ..., u_{n,n}) - f_{i,j}(t, z_{1,1}, z_{1,2}, ..., z_{n,n})| \\
&\leq |\alpha_{i,j}||u_{i,j} - z_{i,j}| + |\beta_{i,j}||u_{j,i} - z_{j,i}| \\
&\quad + |\gamma_{i,j}|| \sqrt{\sum_{k=1}^{n} u_{i,k}^2} - \sqrt{\sum_{k=1}^{n} z_{i,k}^2}| \\
&\leq |\alpha_{i,j}||u_{i,j} - z_{i,j}| + |\beta_{i,j}||u_{j,i} - z_{j,i}| \\
&\quad + |\gamma_{i,j}| \sum_{k=1}^{n} |u_{i,k} - z_{i,k}| \\
&\leq max(|\alpha_{i,j}| + |\gamma_{i,j}|, \ |\beta_{i,j}| + |\gamma_{i,j}|) \sum_{k=1}^{n} |u_{i,k} - z_{i,k}|
\end{aligned}
$$

Therefore, $f_{i,j}$ satisfies Lipshitz condition, and this differential equation is well-posed.

IV. EXPERIMENT

A. Parameters

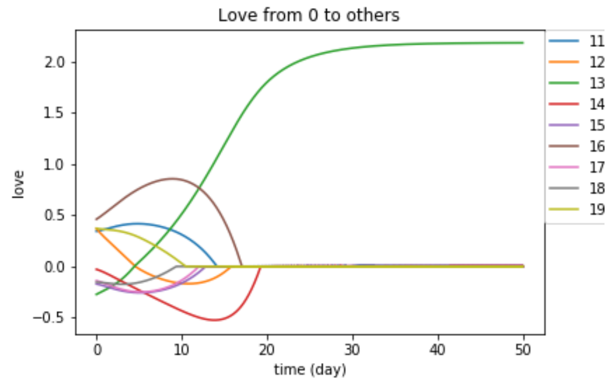
In the experiment, I set the parameters like below.
n = 20
{1,2,3,4,5,6,7,8,9} is a set of male.
{10,11,12,13,14,15,16,17,18,19} is a set of female.
$f_{i,j} \sim Uniform(-1, 1)$
$\alpha'_{i,j} \sim Uniform(-0.1, 0.1)$
$\beta'_{i,j} \sim Uniform(-0.1, 0.1)$
$p_j \sim Uniform(-0.05, 0.05)$
$\gamma_{i,j} = 0.4$
$f_{i,j}$ is regularized to satisfy $\sqrt{\sum_{j=1}^{n} x_{i,j}^{(0)2}} = 1$.

B. The difinition of falling in love

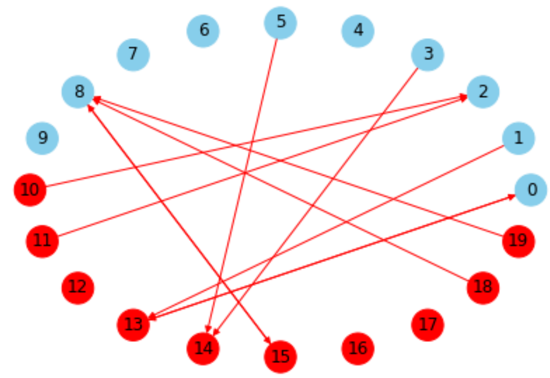
If $x_{i,j}(t) \geq 0.7$, i is falling in love to j

C. Result

I use Euler method to simulate 20 people's love. Below graph is the result of 0's feeling in 5000 steps. (100 step is one day.) According to our definition, 0 loved 16 after 10 days, but 0 finally loved 13.

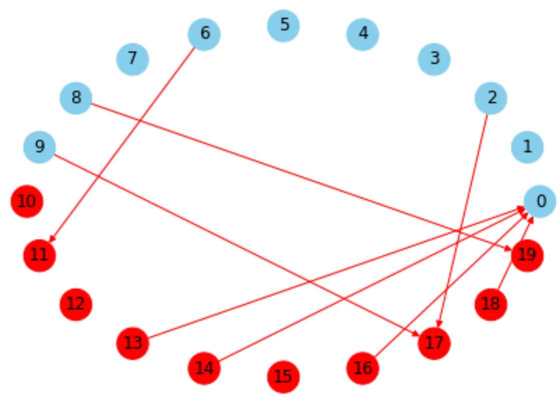


Below graph is the love graph after 50 days.



D. Popularity

To see if p_j model the j's popularity, I set $p_0 = 0.2$, which is pretty high popularity. The result is the below graph. We can see that 0 is popular in this community.



V. CONCLUSION

I formulate the love differential equations based on Romeo and Juliet Equations and incorporate the ideas of popularity, capacity and gender. I show that this model simulate the n people love well.

[1] I. Elishakoff, Differential equations of love and love of differential equations, Journal of Humanistic Mathematics **9**, 229 (2019).