Quant 1-1

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Probability Theory (2, 3)

Three axioms

- nonnegativity
- normalization
- additivity

$\rightsquigarrow$ Check these when you see probability distributions

Key concepts

Conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Law of total probability

- Let $A_1, \dots, A_n$ be disjoint events that form a partition of the sample space and assume P(A_i) > 0 for all i

$$\begin{aligned} P(B) &= P(A_1 \cap B) + P(A_2 \cap B) + \cdots + P(A_n \cap B) \\ &= P(B|A_1)P(A_1) + \cdots + P(B|A_n)P(A_n) \\ &= \sum_{i=1}^{n} P(B|A_i)P(A_i) \\ &= P(B|A)P(A) + P(B|A^C)P(A^C) \end{aligned}$$

- Example: quiz 1-extra

Bayes rule

- Use the conditional probability law and the law of total probability

$$\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ &= \frac{P(A)P(B|A)}{P(B)} \\ &= \frac{P(A)P(B|A)}{\sum_{i=1}^{n} P(B|A_i)P(A_i)} \\ &= \frac{P(A)P(B|A)}{P(B|A)P(A) + P(B|A^C)P(A^C)} \end{aligned}$$

- Example: slide 2-23, quiz 1-3

Independence

- Intuition: Events A and B are independent if knowing whether A occurred provides no information about whether B occurred
- rolling two fair dice, getting one for one die and getting six for the other one

$$P(A \cap B) = P(A)P(B) \Leftrightarrow A \perp\!\!\!\perp B$$

- Implication

$$P(A) = P(A|B) = P(A|B^C)$$
$$P(A \cap B) = P(A)P(B) = P(A|B)P(B) = P(A|B^C)P(B^C)$$

- Analogous concept: conditional independence
- Example: slide 1-26, quiz 1A

Random variables and distributions

Random variables

- Definition: A random variable is a real valued function that maps the sample space (Ω) to the real numbers ($\mathbb{R}$)

Probability law

- Definition: The probability law $P_X(x)$ of a random variable X is a real-valued function that assigns probabilities to each of its possible values $x \in X$. i.e.) $P_X : x \to [0, 1]$. It encodes our knowledge or belief about the likelihood of the outcomes in Ω.
- If random variables are discrete, we call it the probability mass function
- If random variables are continuous, we call it the probability density function

Cumulative function

- A cumulative distribution function $f_Y(y)$ of a random variable Y is a non-decreasing function that gives you the probability that $Y \leq y$: $F_Y(y) = Pr(Y \leq y)$
- Discrete: $F_Y(y) = \sum_{\forall Y' \leq y} P_Y(Y')$
- Continuous: $F_Y(y) = \int_{-\inf}^{y} f_Y(Y')dY'$ where $f_Y(y)$ is the probability density function for a random variable Y

Dice example