Solution for Assignment of Chapter 7

1. Given the open-loop transfer function of a unity-feedback system by

$$G(s) = \frac{40}{s(s+4)},$$

determine what kind of cascade compensator should be applied such that the gain crossover frequency the compensated system is ω_c=10rad/s and ω_c=4 rad/s respectively. Moreover, the system needs to satisfy the following requirements:

(a) The steady-state error with respect to the ramp input $u(t) = At$ is less than $0.1A$;

(b) Phase margin is not less than 45°.

Solution:

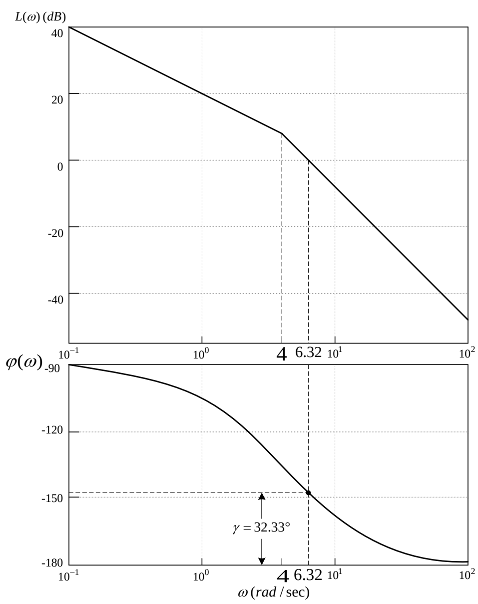
(1) $G(s) = \dfrac{40}{s(s+4)} = \dfrac{10}{s(0.25s+1)}$

$K_v = \dfrac{1}{e_{ss}} = 10 = \lim_{s\to 0} sG(s)$

The uncompensated system satisfies the condition (a).

(2) Draw the Bode diagram for the uncompensated system, and get $\gamma = 32.33^\circ$,

$\omega_c = 6.32 rad/\sec$



(3) It is required that phase margin is not less than 45°. It is easily satisfied by applying lead compensation. When we use lead compensator, the gain crossover frequency will be increased.

If the new gain crossover frequency is ω_{c1}=10rad/s>ω_c, the lead compensation can be applied.

If the new gain crossover frequency is ω_{c1}=4rad/s<ω_c, the lead compensation cannot be applied. The lag compensation can be designed to satisfy the required conditions.

2. Design the compensator for the system in Question 1 by using the Bode diagram method.

Solution:

Case 1: For the required new gain crossover frequency ω_{c1}=10rad/s

(1) Since the uncompensated system satisfies the requirement of the condition (a), the gain of compensator is 1.

The designed lead compensation is $\alpha G_c(s) = \dfrac{1+\alpha Ts}{1+Ts}$.

The log-magnitude of the uncompensated system at the new desired gain crossover frequency ω_{c1}=10rad/s is $L(\omega_{c1}) = -7.96dB$.

(2) It follows from $10\lg\alpha = 7.96dB$ that $\alpha = 6.25$.

$$\frac{1}{T} = \sqrt{\alpha}\omega_{c1} = 2.5\times 10 = 25$$
$$\frac{1}{\alpha T} = \frac{\omega_{c1}}{\sqrt{\alpha}} = \frac{10}{2.5} = 4$$

The compensator is $\alpha G_c(s) = \dfrac{0.25s+1}{0.04s+1}$.

(3) Confirmation

The compensated system is $\alpha G(s)G_c(s) = \dfrac{250(s+4)}{s(s+4)(s+25)}$. The new phase margin is $\gamma = 81^{\circ}$. It satisfy the all required conditions.

Case 2: For the required new gain crossover frequency ω_{c1}=4rad/s

(1) The open-loop gain satisfies the requirement on the steady-state error. When we choose ω_{c1}=4rad/s, the new phase margin is $\gamma = 46^{\circ}$. The condition (b) is thus satisfied.

(2) In order to make the new gain crossover frequency be ω_{c1}=4rad/s, it must be

$$20Lg\beta = -L(\omega)\big|_{\omega=\omega_{c1}=4} = -5dB \cdot$$

$$\beta = 10^{-L(\omega_{c1})/20} = 10^{-0.25} = 0.56$$

$$\frac{1}{\beta T} = 0.1\omega_c = 0.4$$

$$\frac{1}{T} = 0.4\beta = 0.22$$

$$G_c(s) = \frac{Ts+1}{\beta Ts+1} = \frac{2.5s+1}{4.54s+1}$$

(3) Confirmation

The compensated system is $G(s)G_c(s) = \dfrac{22(s+0.4)}{s(s+4)(s+0.22)}$.

The resulting gain crossover frequency is $\omega_c = 4rad/\sec$, and the phase margin is $\gamma = 45^{\circ}$. All the requirements are satisfied.

43度