上海交通大學
SHANGHAI JIAO TONG UNIVERSITY

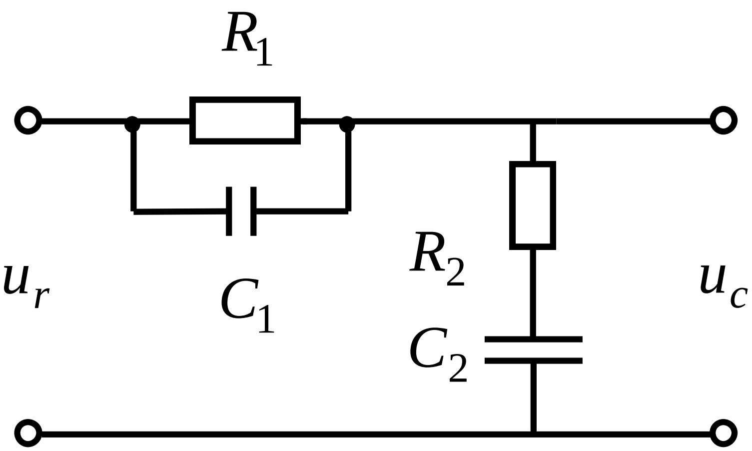
Chapter 7 System Compensation
(Linear Control System Design)

Instructor: Prof. Cailian Chen

7.4 Phase Lead-lag Compensation

- **If the system is unstable, the desired compensator could be applied to satisfy steady-state performance and transient performance.**
- **Only one kind of compensator is difficult to satisfy all the performance criteria. The phase lead-lag compensator is a good choice.**
- **The break frequency of lead compensator should be set in the medium frequency part.**
- **The break frequency of lag compensator should be set in the lower frequency part.**
- **The system gain should be designed properly.**

$$G_c(s)=\frac{U_c(s)}{U_r(s)}=\frac{R_2+\dfrac{1}{sC_2}}{\dfrac{1}{\dfrac{1}{R_1}+sC_1}+R_2+\dfrac{1}{sC_2}}$$

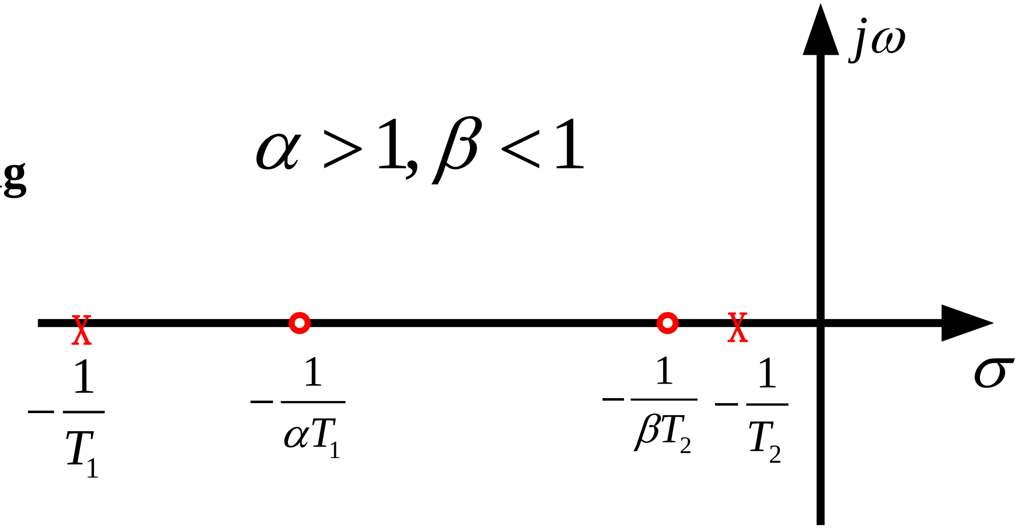


$$=\frac{(R_1C_1s+1)(R_2C_2s+1)}{R_1C_1R_2C_2s^2+(R_1C_1+R_2C_2+R_1C_2)s+1}$$

$$=\frac{\boxed{(\alpha T_1s+1)}\boxed{(\beta T_2s+1)}}{\boxed{(T_1s+1)}\boxed{(T_2s+1)}}$$

Phase Lead **Phase Lag**

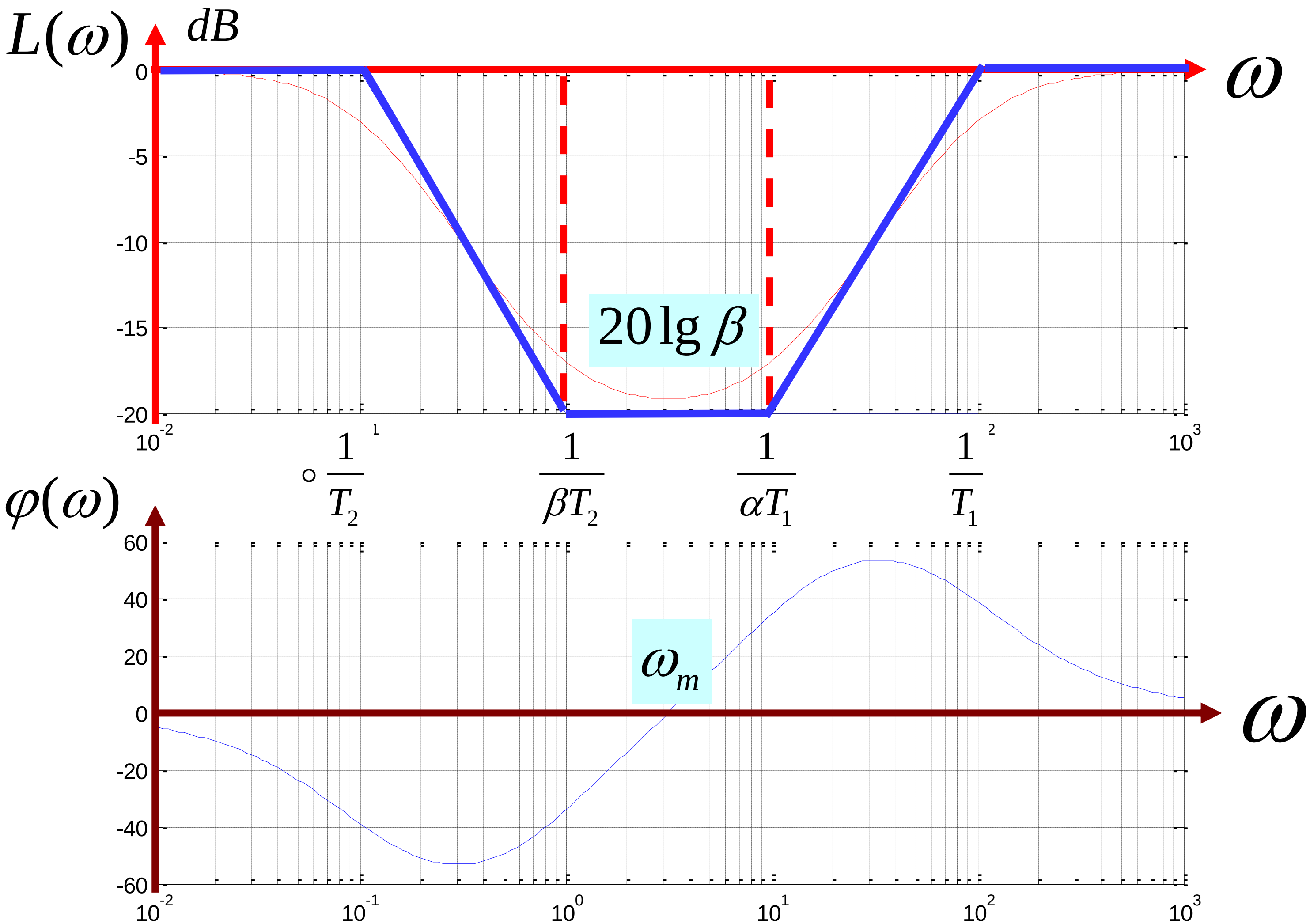
$$\alpha>1,\beta<1$$



where $T_1+T_2=R_1C_1+R_1C_2+R_2C_2$

$T_1T_2=R_1R_2C_1C_2$

$$\alpha=\frac{R_1C_1}{T_1},\beta=\frac{R_2C_2}{T_2},\alpha\beta=1$$



Frequency response of phase lead-lag compensator

The frequency ω_m where phase of compensator is zero

$$\varphi(\omega_m) = arctg\,\alpha T_1\omega_m + arctg\,\beta T_2\omega_m - arctg\,T_1\omega_m - arctg\,T_2\omega_m = 0$$

$$\Rightarrow \quad arctg\frac{(\alpha T_1 + \beta T_2)\omega_m}{1-\alpha\beta T_1T_2\omega_m^2} - arctg\frac{(T_1+T_2)\omega_m}{1-T_1T_2\omega_m^2} = 0$$

$$\Rightarrow \quad T_1T_2\omega_m^2 = 1 \qquad \boxed{\omega_m = \frac{1}{\sqrt{T_1T_2}}}$$

$\omega < \omega_m$ **The compensator has the phase lag characteristics.**

$\omega > \omega_m$ **The compensator has the phase lead characteristics.**

Example 7.3: Given the unity-feedback system with the open-loop transfer function

$$G_0(s) = \frac{K_v}{s(\frac{1}{6}s+1)(\frac{1}{2}s+1)}$$

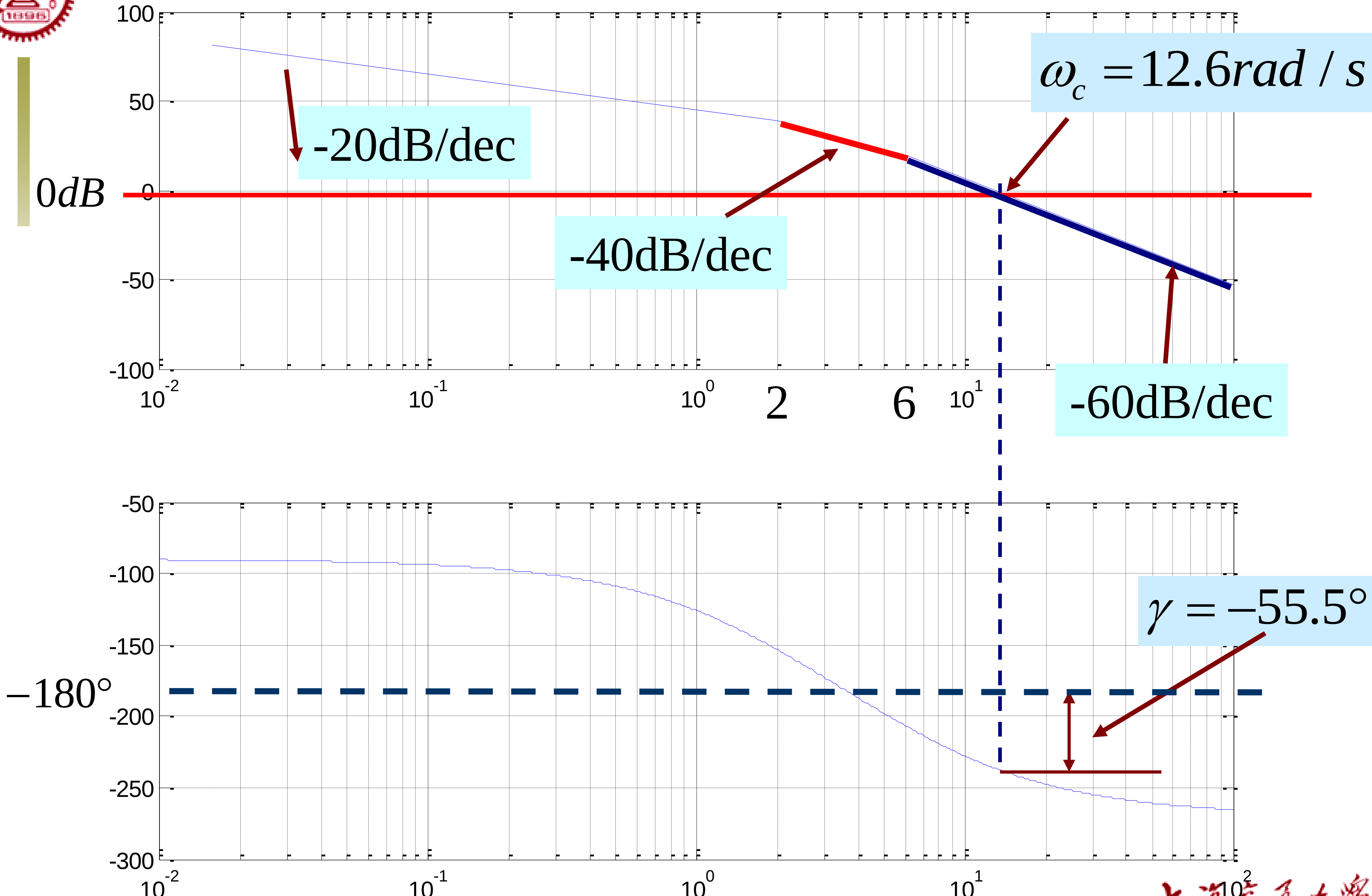
please design a compensator to satisfy the following criteria:

① **The steady-state error is not bigger than** 1° **for a ramp input** $180^\circ/s$

② **Phase margin** $45^\circ \pm 3^\circ$

③**Gain margin is not less than 10dB;**

④**The settling time of time response is not bigger than 3sec.**



$\omega_c = 12.6 rad/s$
-20dB/dec
-40dB/dec
-60dB/dec
0dB
2
6
$-180°$
$\gamma = -55.5°$
100
50
0
-50
-100
-150
-200
-250
-300
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}

The frequency-domain criteria and time-domain criteria

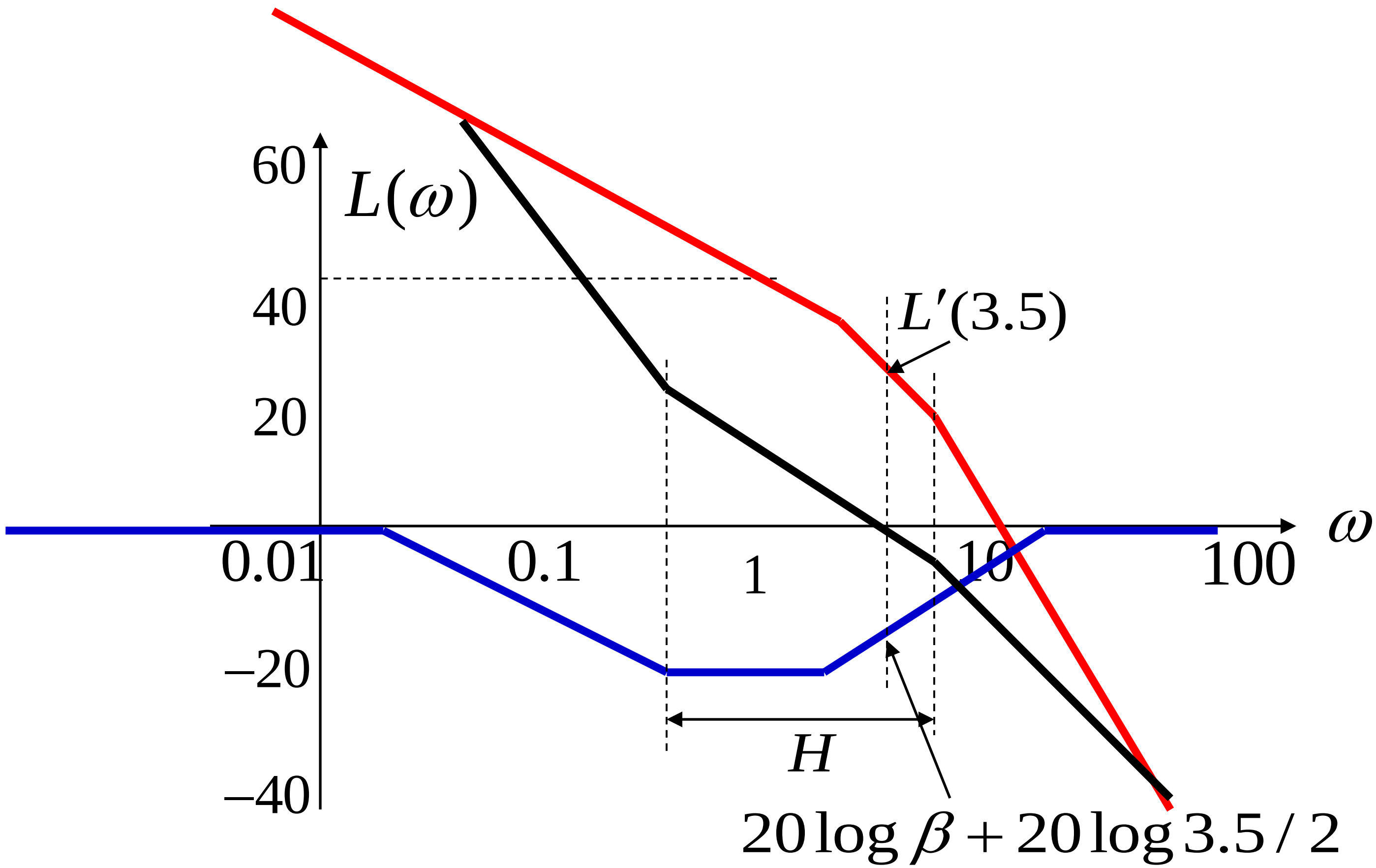
Resonance peak $$M_r = \frac{1}{\sin\gamma}$$

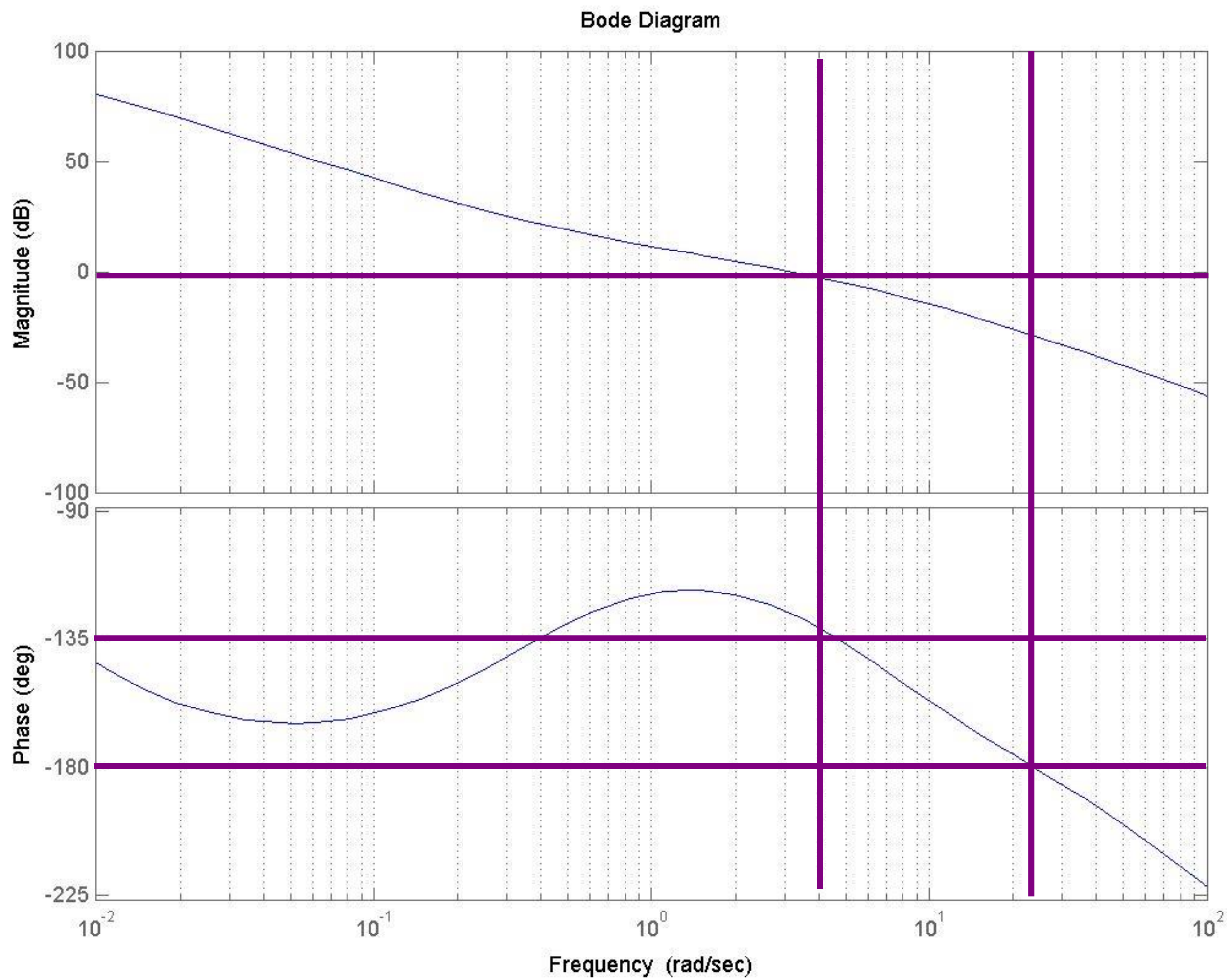
overshoot $$\sigma = 0.16 + 0.4(M_r - 1) \qquad 1 \le M_r \le 1.8$$

Settling time $$t_s = \frac{K\pi}{\omega_c}$$

$$K = 2 + 1.5(M_r - 1) + 2.5(M_r - 1)^2 \qquad 1 \le M_r \le 1.8$$

$$\omega_{c1} = 3.5\text{rad/s}$$


60
40
20
–20
–40
L(ω)
L′(3.5)
ω
0.01
0.1
1
10
100
H
20 log β + 20 log 3.5 / 2


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)
100
50
0
-50
-100
-90
-135
-180
-225
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}

Design rules for cascade phase lead-lag compensator:

① Determine open-loop gain K to satisfy the steady-state performance criteria;

② Draw Bode diagram for uncompensated system. Calculate gain crossover frequency ω_c, phase margin γ and gain margin GM.

③From the Bode diagram of uncompensated system, choose the break frequency where the slope of log-magnitude curve changes from -20dB/dec to -40dB/dec. The frequency is set as the phase lead break frequency $\frac{1}{\alpha T_1}$.

This method can reduce the order of the compensated system, and guarantee the slope at the medium-frequency part is -20dB/dec with enough bandwidth.

④ **From the requirement for transient response, choose the desired gain crossover frequency ω_{c1} and the parameters α and β.**

In order to the resulting gain crossover frequency is the desired one ω_{c1}, the following equation should be satisfied.

$$\boxed{20\lg\beta + L(\omega_{c1}) + 20\lg\alpha T_1\omega_{c1} = 0}$$

Amount of magnitude attenuation by phase lag compensator

The magnitude of uncompensated system

Magnitude change provided by phase lead compensator at the frequency ω_{c1}

⑤ In order to satisfy the phase margin requirement, set the break frequency of phase lag compensator

$$\boxed{\frac{1}{\beta T_2} = 0.1\omega_{c1}}$$

⑥ Confirmation if the compensated system satisfy all the requirements.

7.5 PID Control

Control rule for linear systems

- **PID: Proportional Integral Derivative is the basic rule for system control design.**
- **Inserting compensator can change the system's differential equation and then change the response.**
- **The proportional compensator can change the coefficients of differential equation and then change the assignment of poles and zeros. It then changes the system response.**
- **The compensator with differential and integral functions can change the transfer function to satisfy all the transient and steady-state responses.**

上海交通大學
SHANGHAI JIAO TONG UNIVERSITY

- PID, a particular control structure, has become almost universally used in industrial control.
- It is based on a particular fixed structure controller family, the so-called PID controller family.
- These controllers have proven to be robust and extremely beneficial in the control of many important applications.

PID stands for: **P** (*Proportional*)
I (*Integral*)
D (*Derivative*)

Historical Note

Early feedback control devices implicitly or explicitly used the ideas of proportional, integral and derivative action in their structures.

However, it was probably not until Minorsky's work on ship steering* published in 1922, that rigorous theoretical consideration was given to PID control.

This was the first mathematical treatment of the type of controller that is now used to control almost all industrial processes.

* Minorsky (1922) "Directional stability of automatically steered bodies", *J. Am. Soc. Naval Eng.*, 34, p.284.

The Current Situation

Despite the abundance of sophisticated tools, including advanced controllers, the Proportional, Integral, Derivative (PID controller) is still the most widely used in modern industry, controlling more than 95% of closed-loop industrial processes*

- Åström K.J. & Hägglund T.H. 1995, "New tuning methods for PID controllers", *Proc. 3rd European Control Conference*, p.2456-62;
- Yamamoto & Hashimoto 1991, "Present status and future needs: The view from Japanese industry", Chemical Process Control, *CPCIV, Proc. 4th Inter-national Conference on Chemical Process Control*, Texas, p.1-28.

The standard form PID are:

Proportional only $$C_P(s) = K_p$$

Proportional plus Integral $$C_{PI}(s) = K_p\left(1 + \frac{1}{T_r s}\right)$$

Proportional plus derivative $$C_{PD}(s) = K_p\left(1 + \frac{T_d s}{\tau_D s + 1}\right)$$

Proportional, integral and derivative $$C_{PID}(s) = K_p\left(1 + \frac{1}{T_r s} + \frac{T_d s}{\tau_D s + 1}\right)$$

An alternative series form is:

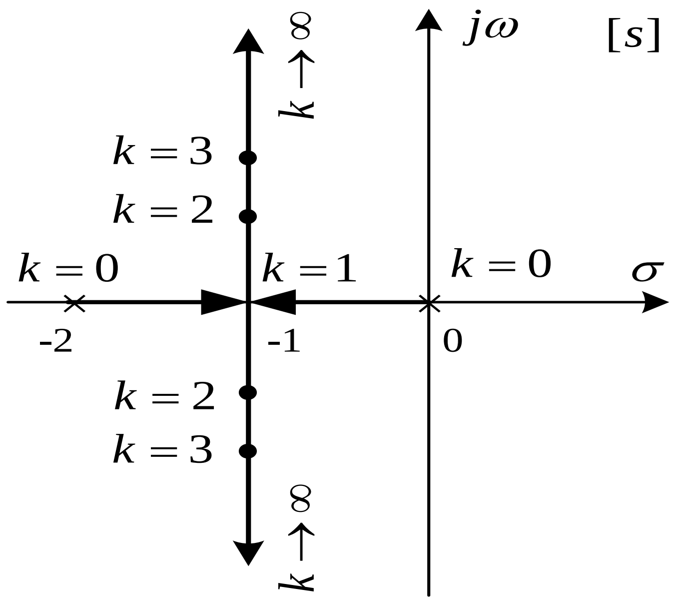
$$C_{series}(s) = K_s \left(1 + \frac{I_s}{s}\right)\left(1 + \frac{D_s s}{\gamma_s D_s s + 1}\right)$$

Yet another alternative form is the, so called, parallel form:

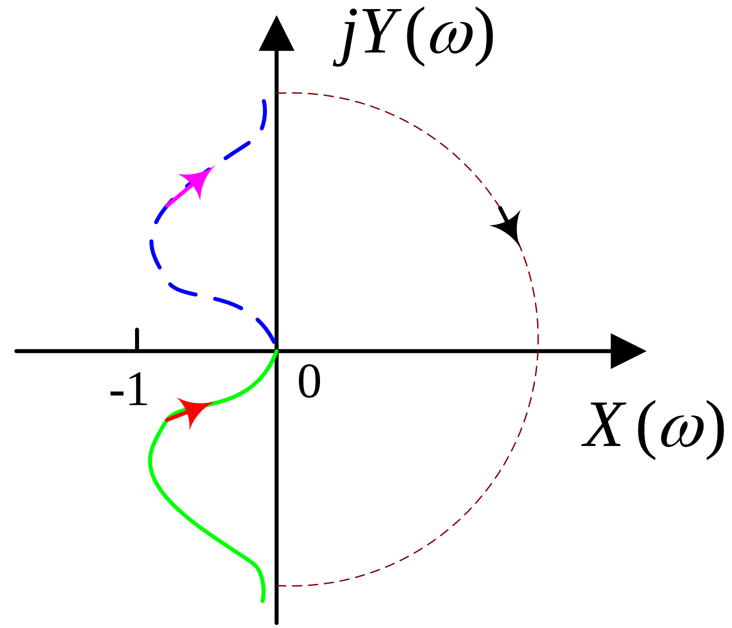
$$C_{parallel}(s) = K_p + \frac{I_p}{s} + \frac{D_p s}{\gamma_p D_p s + 1}$$

1.Proportional (P) control

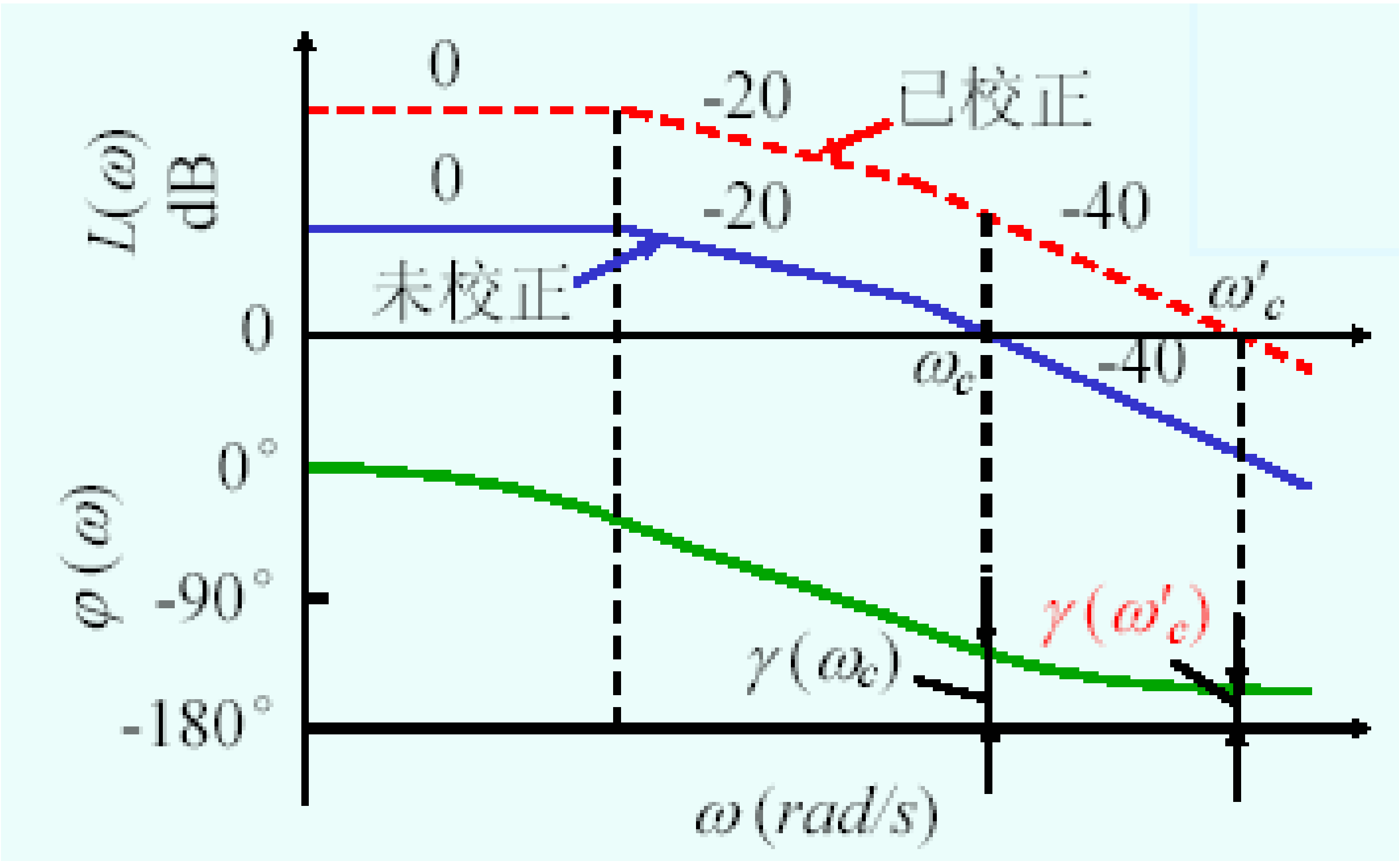
- **Gain of the P controller** $\boxed{G_c(s) = K_p}$



Change the closed-loop root.



The relative stability can be reduced by increasing the gain of controller.


0
-20
已校正
0
-20
-40
$L(\omega)$ dB
未校正
ω'_c
0
ω_c
-40
0°
$\varphi(\omega)$
-90°
$\gamma(\omega'_c)$
$\gamma(\omega_c)$
-180°
$\omega\,(rad/s)$

Comments

1. **Proportional controller is actually an amplifier with tunable gain.**
2. **By increasing K_p the open-loop gain can be increased to reduce the steady-state error.**

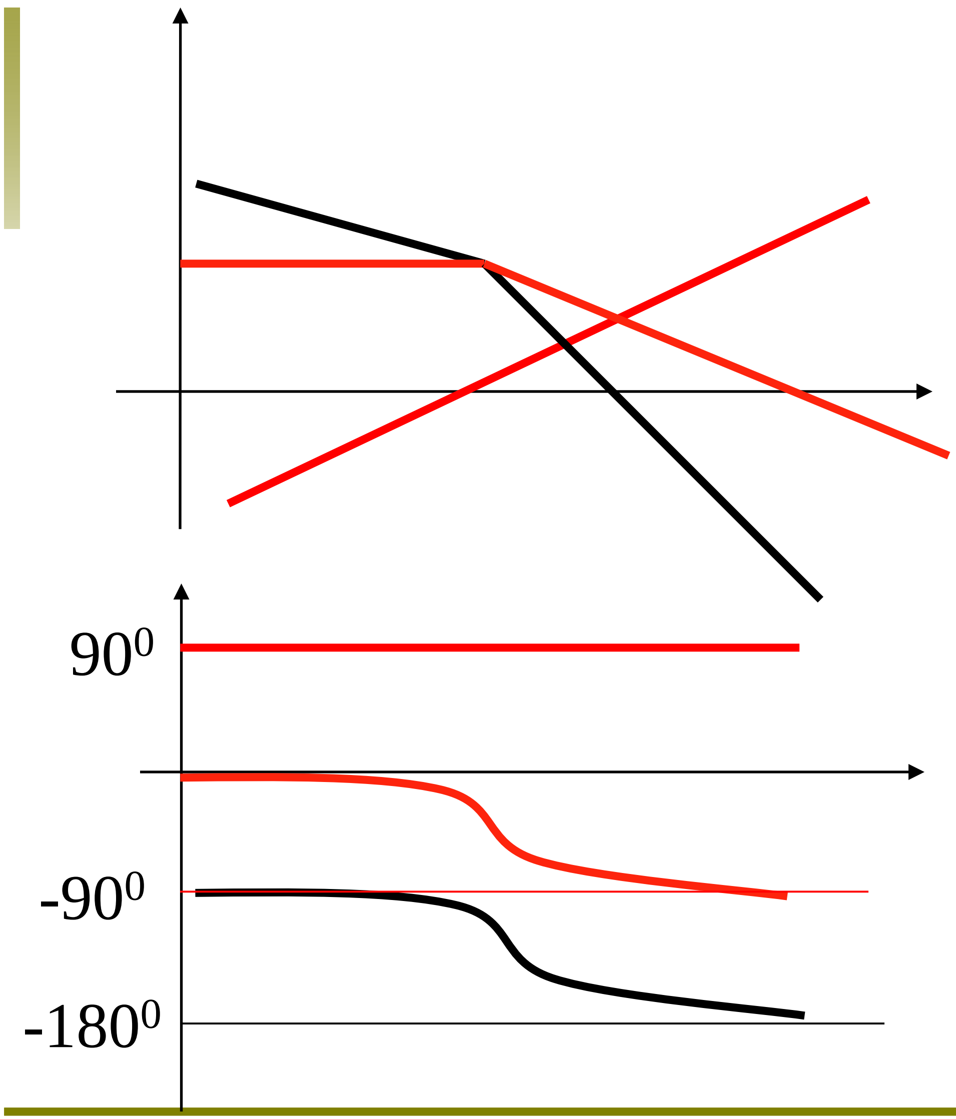
   But it also reduces the relative stability, or even deteriorates the stability of the system.
3. **Seldom used by itself.**

2、Proportional-derivative (PD) control

$$\boxed{G_c(s) = K_p(1 + T_d s)}$$

$$\boxed{m(t) = K_p e(t) + K_p T_d \frac{de(t)}{dt}}$$

where K_p is the gain for proportional part, T_d is the time constant for derivative part. K_p and T_d are both to be determined.



Derivative control can increase gain crossover frequency and phase margin, reduce overshoot and settling time, and improve the transient response.

The derivative controller without proportional part normally amplifies the high-frequency disturbance.

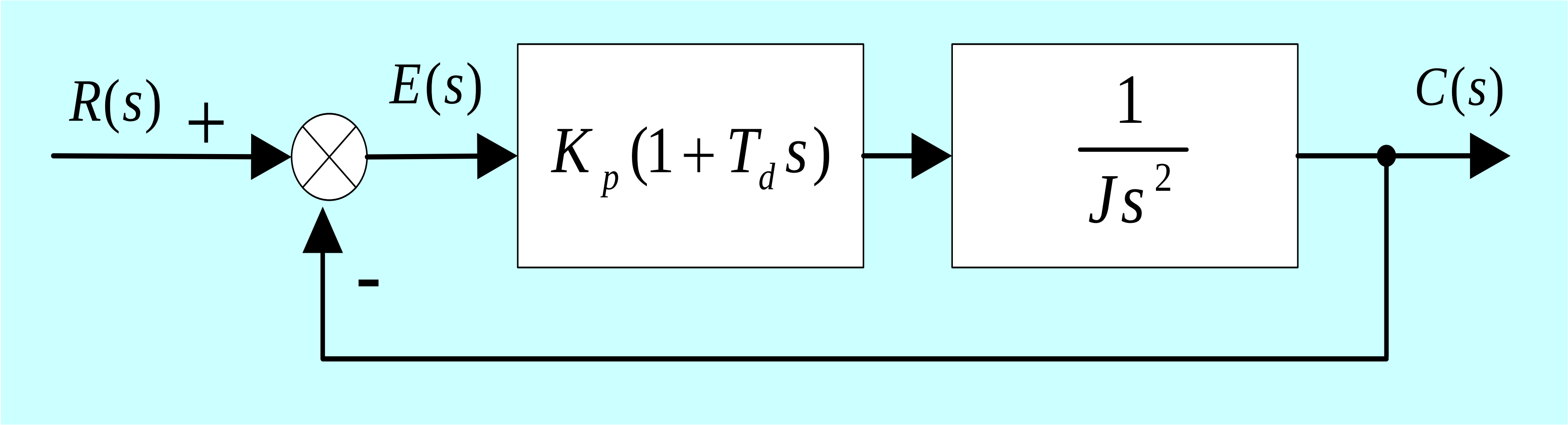
Not used by itself.

上海交通大学
SHANGHAI JIAO TONG UNIVERSITY

Comments:

1. **The derivative part of PD controller can reflect the change of input signal. It helps to amend the input signal.**
2. **It can improve the stability by increasing the damping ratio.**
3. **It provide an open-loop zero at $-1/T_d$. It increase the phase margin and thus improve the transient response performance.**
4. **Derivative control only takes effect on the transient process. It is sensitive to noise. It does not affect the steady-state performance.**
5. **Derivative control is normally in the form of PD or PID controller in the practical applications.**

Example 7.4: The proportional-derivative control system is given in the following figure. Please analyze the effects of PD controller on the system performance.



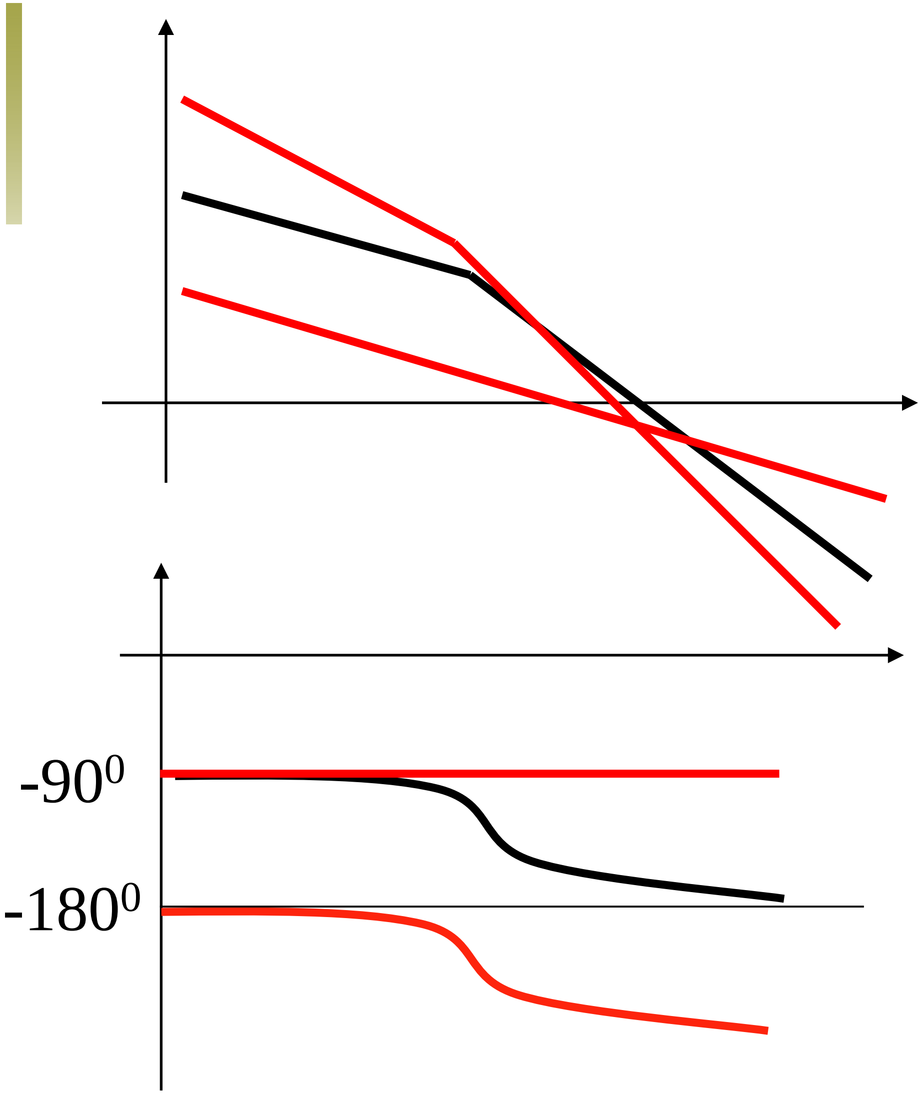
3、Integral (I)Control

$$G_c(s) = \frac{1}{T_i s}$$

$$m(t) = \frac{1}{T_i}\int_0^t e(t)dt$$

where T_i is the time constant to be determined.

Due to the integration function, the output of the controller is not equal to zero even the input signal is disappear.



Integral control can improve the disturbance attenuation. With the proportional part, it can reduce the steady-state error.

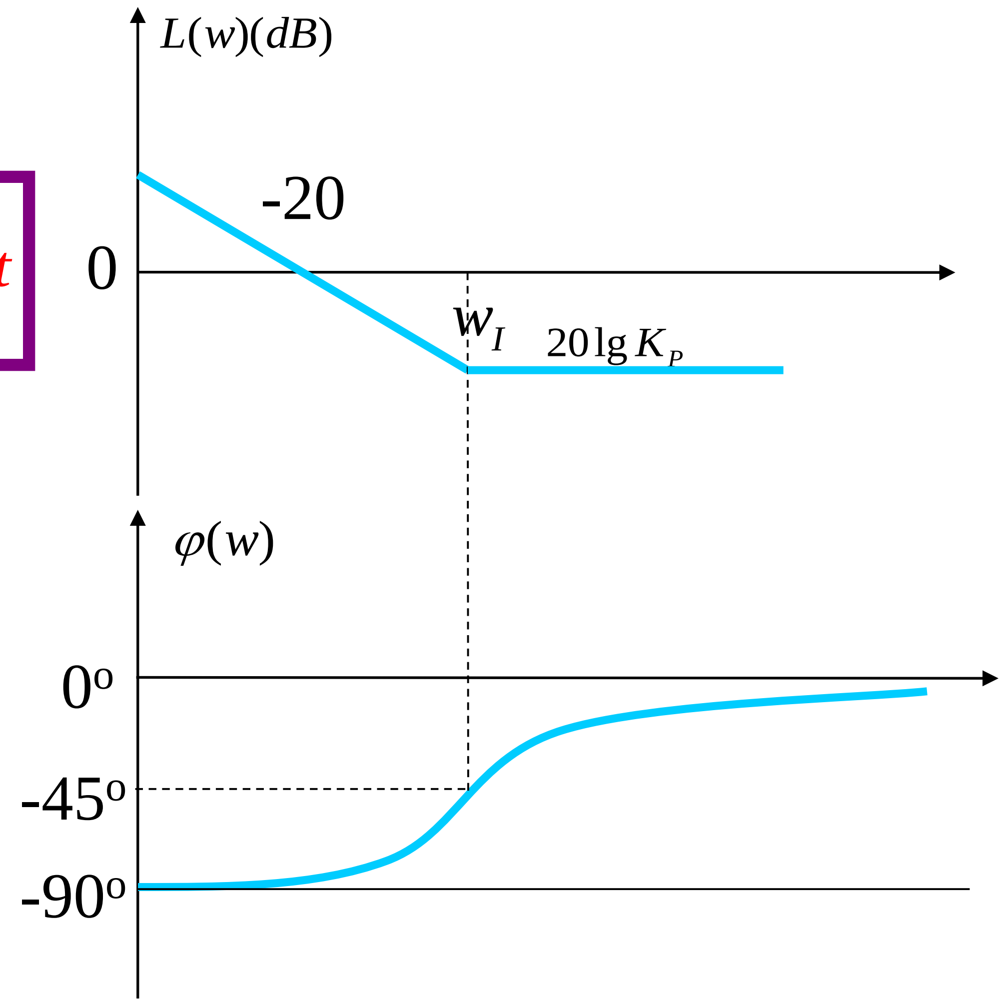
The integral control reduce the phase margin and is normally not used by itself.

4、Proportional-Integral(PI)Control

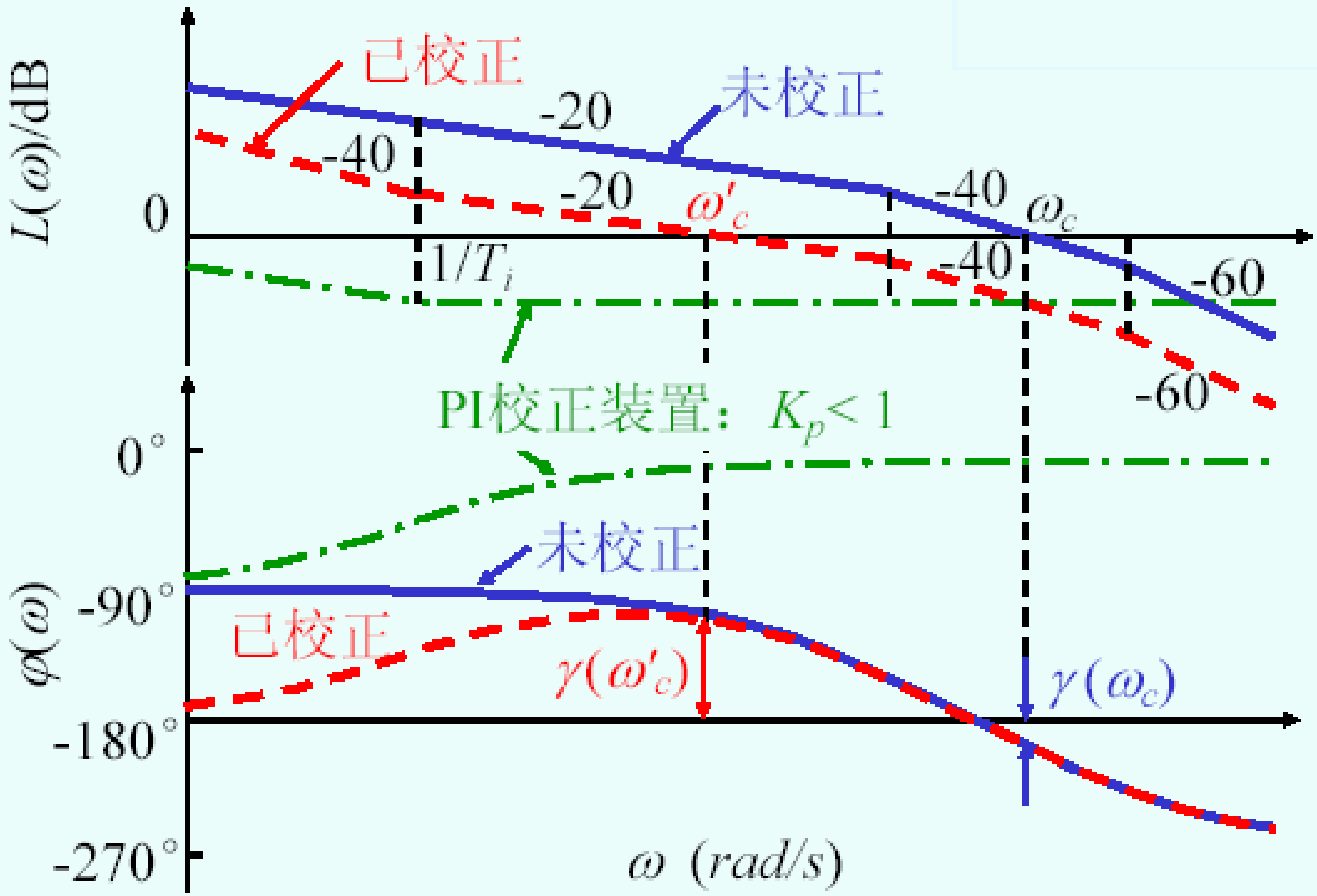
$$G_c(s) = K_p(1 + \frac{1}{T_i s})$$

$$m(t) = K_p e(t) + \frac{K_p}{T_i}\int_0^t e(t)dt$$

where K_p and T_i are to be determined.



❑ $K_p < 1$

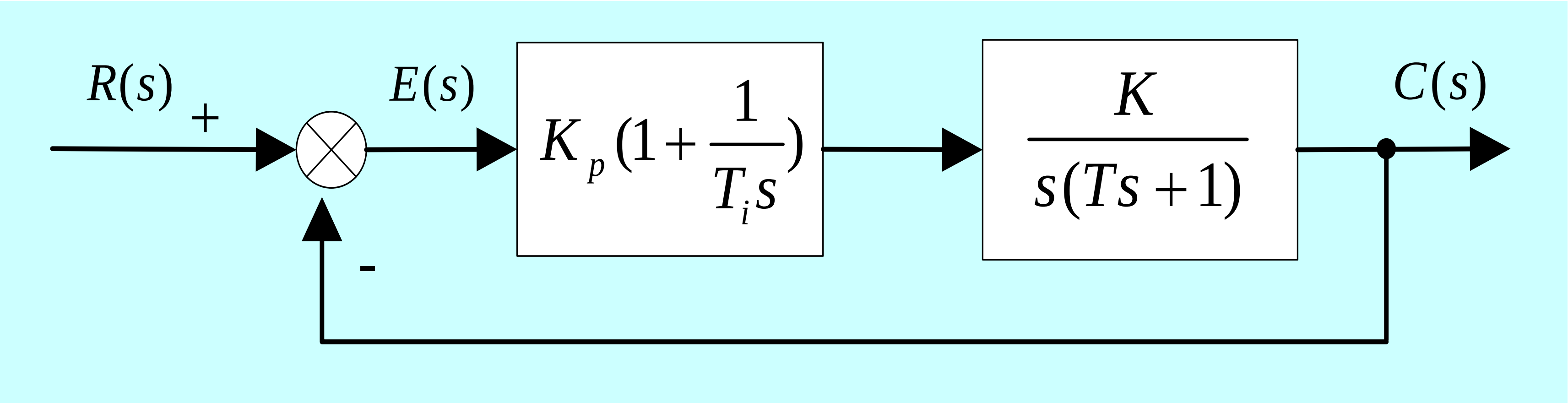


L(ω)/dB
已校正
未校正
-20
-40
-20
0
ω'c
-40
ωc
1/Ti
-40
-60
-60
PI校正装置：Kp< 1
0°
φ(ω)
-90°
未校正
已校正
γ(ω'c)
γ(ωc)
-180°
-270°
ω (rad/s)

讨论：

- **PI controller provides an open-loop pole at the origin as well as an zero in the left-hand side of *s* plane.**
- **The extra pole can increase the type of the open-loop system. It then reduces or eliminates the steady-state error. It improves the steady-state performance.**

Example 7.5: The proportional-integral control system is given by the following figure. Please analyze the improvement of steady-state performance by the PI controller.



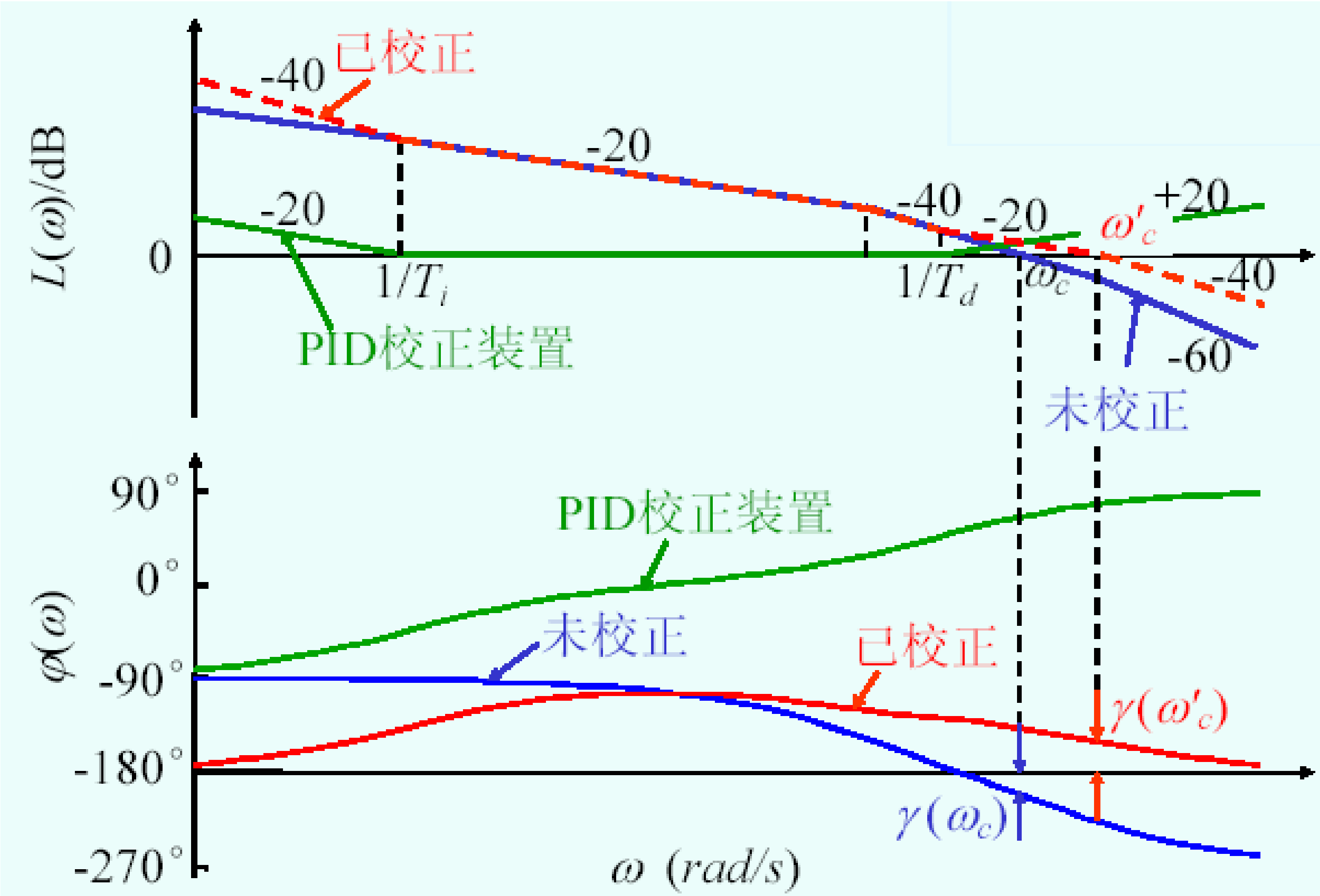
5、Proportional-integral-derivative (PID) Control

$$G_c(s)=K_p[1+1/(T_is)+T_ds]$$
$$=\frac{K_p}{T_i}\cdot\frac{T_iT_ds^2+T_is+1}{s}$$

$$m(t)=K_pe(t)+\frac{K_p}{T_i}\int_0^t e(t)dt+K_pT_d\frac{de(t)}{dt}$$

If $4T_d/T_i<1$, $G_c(s)=\dfrac{K_p}{T_i}\cdot\dfrac{(T_1s+1)(T_2s+1)}{s}$

$$T_1=\frac{T_i}{2}\left(1+\sqrt{1-\frac{4T_d}{T_i}}\right)\qquad T_2=\frac{T_i}{2}\left(1-\sqrt{1-\frac{4T_d}{T_i}}\right)$$



已校正
-40
-20
-20
L(ω)/dB
0
1/T_i
-40
-20
+20
ω'_c
ω_c
1/T_d
-40
-60
PID校正装置
未校正
90°
0°
-90°
-180°
-270°
φ(ω)
PID校正装置
未校正
已校正
γ(ω'_c)
γ(ω_c)
ω (rad/s)

Remarks:

- By using PID controller，the type of the system is increased by 1. Moreover, it provides two more negative real zeros.
- Compared with PI controller, PID controller can not only improve the steady-state performance, but also provide one more negative real zero. It thus can outperform to improve system's transient response.
- The PID controller is widely used in industrial process control. The parameters are determined after field debugging.

The Characteristics of P, I, and D Controllers

Note that these correlations may not be exactly accurate, because *Kp*, *Ki*, and *Kd* are dependent of each other. In fact, changing one of these variables can change the effect of the other two. For this reason, the table should only be used as a reference when you are determining the values for *Ki*, *Kp* and *Kd*.

Response	Rise Time	Overshoot	Settling Time	SS Error
K_P	Decrease	Increase	Small Change	Decrease
K_I	Decrease	Increase	Increase	Eliminate
K_D	Small Change	Decrease	Decrease	Small Change

PID Control and Lead-Lag Compensation

- Lead/Lag compensation is very similar to PD/PI, or PID control.
- The lead compensator plays the same role as the PD controller, reshaping the root locus to improve the transient response.
- Lag and PI compensation are similar and have the same response: to improve the steady state accuracy of the closed-loop system.
- Both PID and lead/lag compensation can be used successfully, and can be combined.

Compliment Content

Lead Compensation Techniques Based on the Root-Locus Approach

- From the performance specifications, determine the desired location for the dominant closed-loop poles.

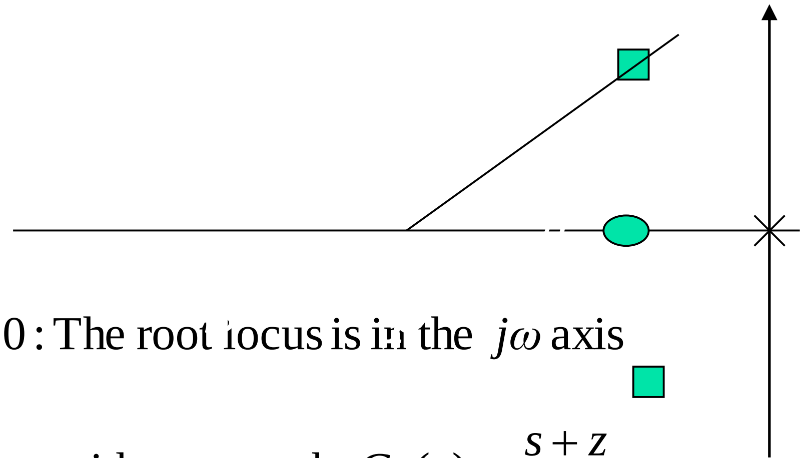
- By drawing the root-locus plot of the uncompensated system ascertain whether or not the gain adjustment alone can yield the desired closed-loop poles. If not calculate the angle deficiency. This angle must be contributed by the lead compensator.

- If the compensator is required, place the zero of the phase lead network directly below the desired root location.
- Determine the pole location so that the total angle at the desired root location is 180^o and therefore is in the compensated root locus.

- Assume the transfer function of the lead compensator.

- Determine the open-loop gain of the compensated system from the magnitude conditions.

Lead Compensator using the Root Locus



$$GH(s)=\frac{K_1}{s^2};1+GH(s)=1+\frac{K_1}{s^2}=0:\text{The root locus is in the } j\omega \text{ axis}$$

We desire to compensate this system with a network, $G_c(s)=\dfrac{s+z}{s+p}$

$$Ts\le 4s;P.O\le 35\%;\xi \text{ should be} \ge 0.32;T_s=\frac{4}{\xi\omega_n}=4;\xi\omega_n=1$$

We will choose a desired dominant root location as $r_1,\hat{r}_1=-1\pm j2$

We place the zero of the compensator directly below the desired location at $s=-z=-1$

$$\phi=-2\times116+90=-142^o;-180^o=-142-\theta_p;\theta_p=38^o;G_c(s)=\frac{s+1}{s+3.6}$$

$$GH(s)G_c(s)=\frac{K_1(s+1)}{s^2(s+3.6)};K_1=\frac{(2.23)^2(3.25)}{2}=8.1$$

Adding Lead Compensation

The lead compensator has the same purpose as the PD compensator: to improve the transient response of the closed-loop system by reshaping the root locus. The lead compensator consists of a zero and a pole with the zero closer to the origin of the *s* plane than the pole. The zero reshapes a portion of the root locus to achieve the desired transient response. The pole is placed far enough to the left that it does not have much influence of the portion influenced by the zero.

$$\text{Consider } G_p = \frac{10}{s(s+1)}$$

$$\text{Design Specifications : P.O} \le 20\%;\ t_p \le 1.0\text{s}$$

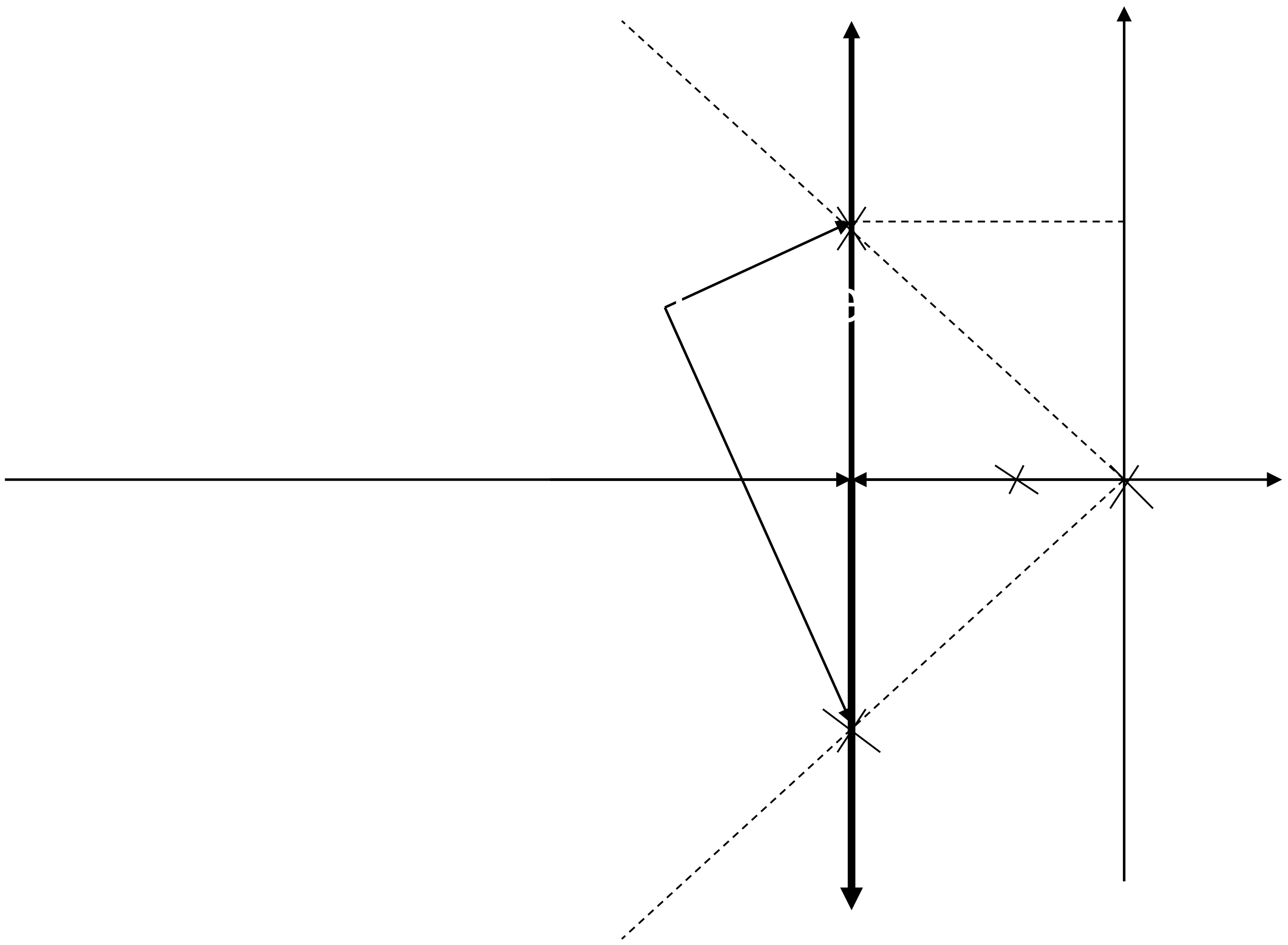
To achieve the desired tp, we place the closed-loop poles at $s = -3 \pm j3$.

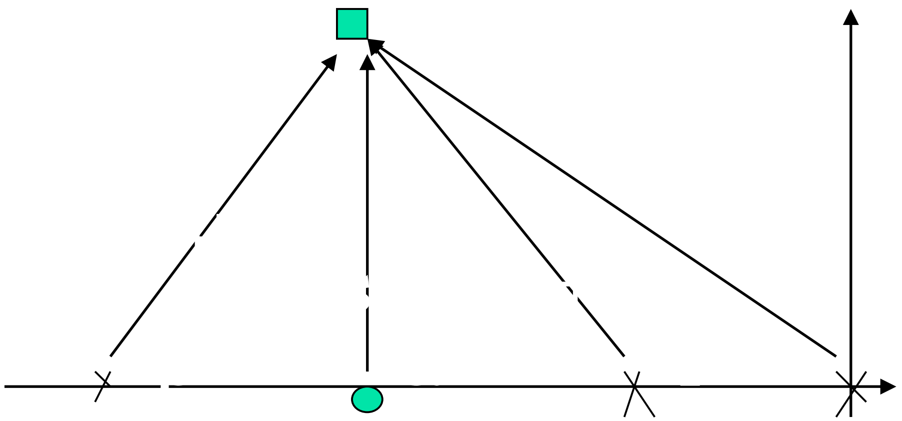
$\xi = 1/\sqrt{2}$; Expect P.O to be 5%; The general formular for the compensator is

$$G_c(s) = \frac{K_c(s+a)}{s+b};0\langle a\langle b$$

$$\angle G_c(s)G_p(s)_{s=-3\pm j3} = -180$$

Root Locus for Simple Gain Compensator





$$\alpha-\beta-\theta_1-\theta_2=180^o;\alpha-\beta=\theta_1+\theta_2-180^o=78.7^o$$

$$\text{Fix s at -3};\beta=90\text{-}78.7^o=11.3^o;b=3+\frac{3}{\tan 11.3^o}=3+15=18$$

$$G_c(s)=\frac{K_c(s+3)}{s+18};Kc=\left(\frac{|s||s+1||s+18|}{10|s+3|}\right)_{s=3+j3}=7.8$$

$$G_c(s)=\frac{7.8(s+3)}{s+18}$$