Solution for Assignment of Chapter 6

6-1 Using the straight-line approximations, plot the Bode Diagram (Log-magnitude diagram and Log-phase diagram) of the systems described by the following open-loop transfer functions. Determine the Gain crossover frequency ω_c and Phase crossover frequency ω_g.

(1) $G(s)=\dfrac{20}{s(s+10)(s+20)}$

(2) $G(s)=\dfrac{5}{s(0.01s^2+0.1s+1)}$

(3) $G(s)=\dfrac{40}{s(s-10)(s+20)}$

Solution:

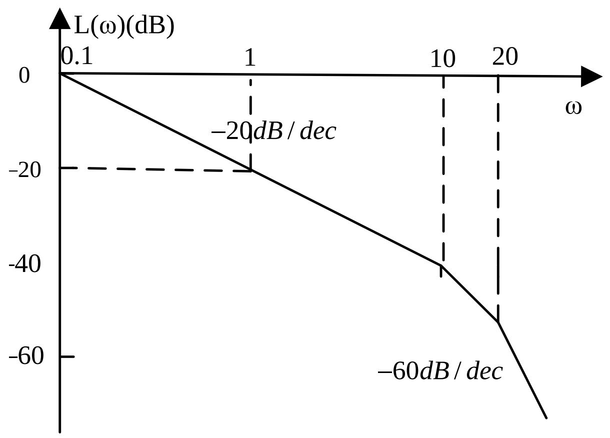
(1) $G(s)=\dfrac{20}{s(s+10)(s+20)}=\dfrac{1}{10}\dfrac{1}{s(s/10+1)(s/20+1)}$

The break frequencies are $\omega_1=10, \omega_2=20$

In the low frequency part, it is a straight line with the slop of -20dB/dec. It goes through the point of $\omega=1$, L(1)=-20dB.

At $\omega_1=10$, the slope changes to -40dB/dec.

At $\omega_2=20$, the slope changes to -60dB/dec

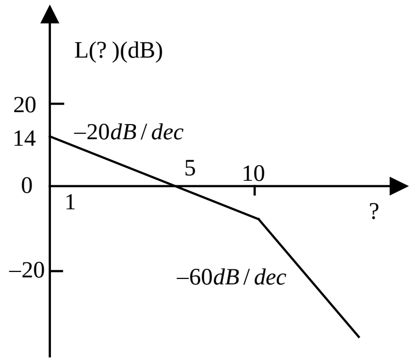


$\omega_c=0.1$

It follows from $\varphi(\omega_g)=-90^\circ-\arg tg0.1\omega_g-\arg tg0.05\omega_g=-180^\circ$ **that** ω_g **= 14.4.**

(2) $G(s)=\dfrac{5}{s(0.01s^2+0.1s+1)}=\dfrac{5}{s((\frac{s}{10})^2+s/10+1)}$

$\omega=1$, $20\lg K=20\lg(5)=14$

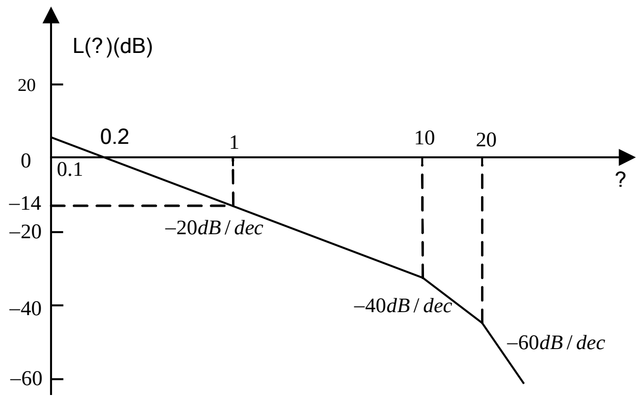


$\omega_c = 5$

ω_g **=10**

(3) $G(s)=\dfrac{40}{s(s-10)(s+20)}=\dfrac{0.2}{s(s/10-1)(s/20+1)}$

$20\lg K=20\lg(0.2)=-14$



ω_c **= 0.2**

There does not exist ω_g **.**

6-2 Please sketch the Nyquist Diagram of the systems described by the following open-loop transfer functions.

(1) $G(s)=\dfrac{100}{s(s+10)(s+20)}$

(2) $G(s)=\dfrac{10}{s^3(s+1)(s+2)}$

(3) $G(s)=\dfrac{10(s-1)}{s(s+2)}$

Solution:

(1) $G(s)=\dfrac{100}{s(s+10)(s+20)}=\dfrac{0.5}{s(0.1s+1)(0.05s+1)}$

$$G(j\omega)=\frac{100}{s(s+10)(s+20)}=\frac{0.5}{j\omega(0.1j\omega+1)(0.05j\omega+1)}$$

$$=\frac{-3000}{900\omega^2+\left(200-\omega^2\right)^2}-j\frac{100\left(200-\omega^2\right)}{900\omega^3+\omega\left(200-\omega^2\right)^2}$$

$\mathrm{Re}[G(j\omega)]<0$ for any ω

$\varphi(\omega)=-90^\circ-arctg0.1\omega-arctg0.05\omega$

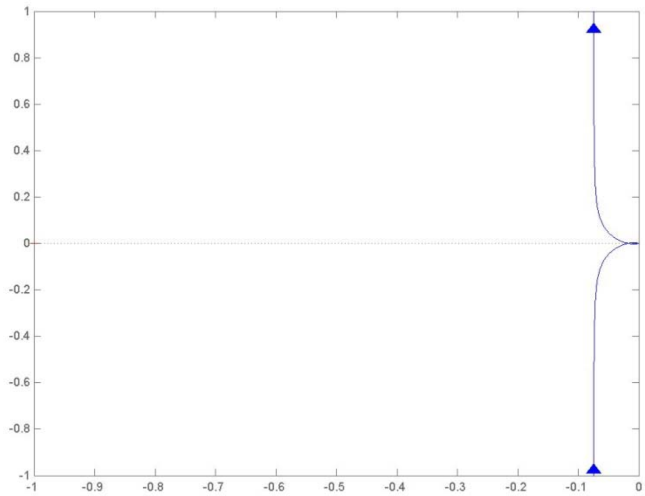
For $\omega=0$, $A(\omega)=\infty$, $\varphi(\omega)=-90^\circ$

For $\omega=\infty$, $A(\omega)=0$, $\varphi(\omega)=-270^\circ$

Let $\mathrm{Im}[G(j\omega)]=0$ and get $\omega=10\sqrt{2}$.

Substitute $\omega=10\sqrt{2}$ into $\mathrm{Re}[G(j\omega)]$ and get the interception with the real axis

$\mathrm{Re}[G(j\omega)]=-\dfrac{1}{60}$



Note: The above Nyquist diagram is drawn by Matlab. It is only needed to sketch for positive frequency, i.e. $\omega = [0, +\infty)$. The interception with the real axis should be calculated.

(2) $G(s) = \dfrac{10}{s^3(s+1)(s+2)} = \dfrac{5}{s^3(s+1)(0.5s+1)}$

$$G(j\omega) = \frac{30}{\omega^2(1+\omega^2)(4+\omega^2)} + j\frac{10(2-\omega^2)}{\omega^3(1+\omega^2)(4+\omega^2)}$$

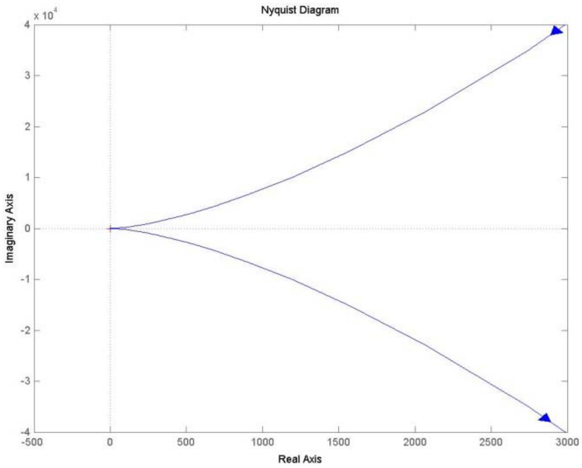
$\mathrm{Re}[G(j\omega)] > 0$ for any positive ω

$\varphi(\omega) = -270^\circ - arctg\omega - arctg0.5\omega$

For $\omega = 0$, $A(\omega) = \infty$, $\varphi(\omega) = -270^\circ$

For $\omega = \infty$, $A(\omega) = 0$, $\varphi(\omega) = -450^\circ$

The interception with the real axis is $\mathrm{Re}[G(j\omega)] = \dfrac{5}{6}$ for $\omega = \sqrt{2}$



(3) $G(s) = \dfrac{10(s-1)}{s(s+2)} = \dfrac{5(s-1)}{s(0.5s+1)}$

$$G(j\omega) = \frac{30}{\omega^2+4} + j\frac{20-10\omega^2}{\omega^3+4\omega}$$

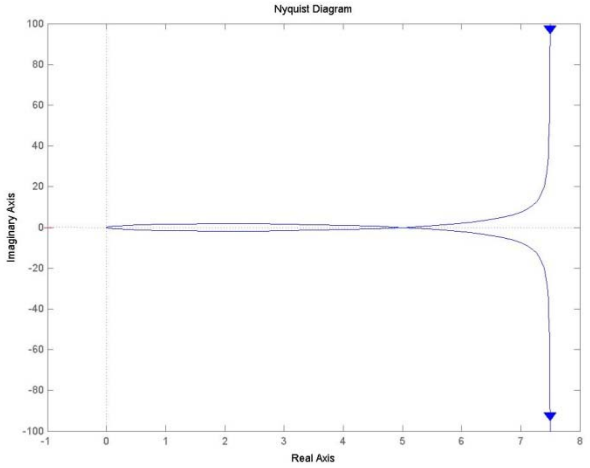
$\mathrm{Re}[G(j\omega)] > 0$ for any positive ω

$$\varphi(\omega) = -90^\circ + 180^\circ - arctg\omega - arctg0.5\omega = 90^\circ - arctg\omega - arctg0.5\omega$$

For $\omega = 0$, $A(\omega) = \infty$, $\varphi(\omega) = 90^\circ$

For $\omega = \infty$, $A(\omega) = 0$, $\varphi(\omega) = -90^\circ$

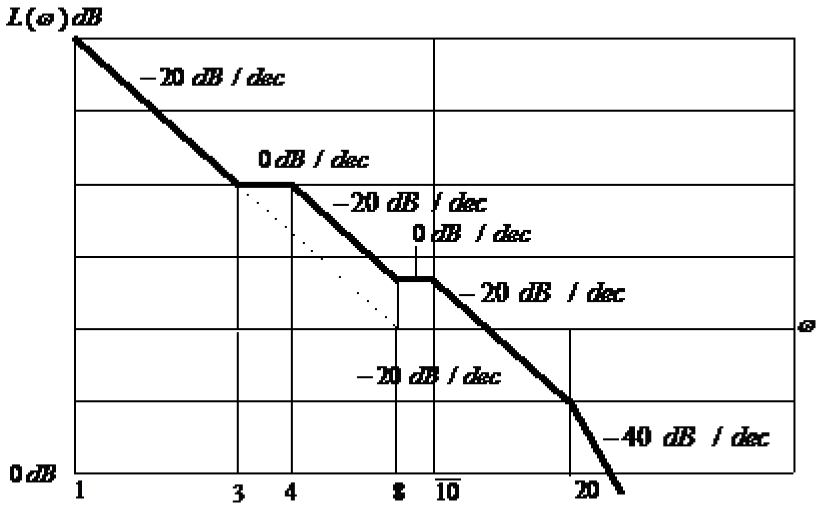
The interception with the real axis is $\text{Re}[G(j\omega)] = 5$ for $\omega = \sqrt{2}$



6-3 Consider the unity-feedback control system described by the open-loop transfer function

$$G(s) = \frac{K(s+8)(as+1)}{s(0.1s+1)(0.25s+1)(bs+1)}.$$

Its Bode diagram is shown in the following figure. Determine the values of K, a and b.



Solution:

$$G(s)=\frac{K(s+8)(as+1)}{s(0.1s+1)(0.25s+1)(bs+1)}=\frac{8K(\frac{1}{8}s+1)(as+1)}{s(0.1s+1)(0.25s+1)(bs+1)}$$

It follows from the figure that $1/b$ a=3，a=1/3;　1/b=20，　b=1/20.

$$20\lg(8K)=3\times 20\lg\frac{8K}{8}=60\lg K$$

$$K=2\sqrt{2}$$

Note: If an extra condition is given in the figure that $L(\omega)=40$ at $\omega=1$, we can have 20lg8K=40,8k=100,K=12.5.

6-4 By using Bode Diagram based stability analysis, please judge if the systems (1)-(3) in 6-1 are stable. If they are stable, please determine the phase margin γ and gain margin GM .

Solution:

(1) Stable, $\gamma=89.14^{\circ}$　　gain margin $K_g=200$ or GM=46dB

(2) Stable, $\gamma=50.3^{\circ}$　　GM=6dB, $K_g=2$

(3) Unstable, $\gamma=$ -89.4°　　GM=Inf

6-5 Please use Nyquist criterion to judge if the systems in 6-2 are stable. Please determine Phase Mar[illegible]gin GM .

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(1) Stable, $\gamma=85.7^{\circ}$　　$K_g=60$, GM=35.6dB

(2) Unstable, $\gamma=$ -93°　　$K_g=\infty$

(3) Unstable, $\gamma=$ -107.3°　　$K_g=\infty$

6-6 Consider the unity-feedback control system described by the open-loop transfer function

$$G(s)=\frac{K_r}{s(s+10)}.$$

(1) If the overshot of the closed-loop system satisfies $M_p\le 5\%$, please determine the open-loop gain (Bode's gain);

(2) resonant peak $M_{p\omega}$ of the closed-loop system;

(3) resonant frequency ω_r of the closed-loop system;

(4) band-width ω_b;

(5) unit step response of the closed-loop system.

Solution:

(1) $\Phi(s)=\dfrac{G(s)}{1+G(s)H(s)}=\dfrac{K_r}{s^2+10s+K_r}=\dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

Kr/10

It follows from $M_p=e^{-\pi\varepsilon/\sqrt{1-\varepsilon^2}}<5\%$ that $\varepsilon>0.69$.

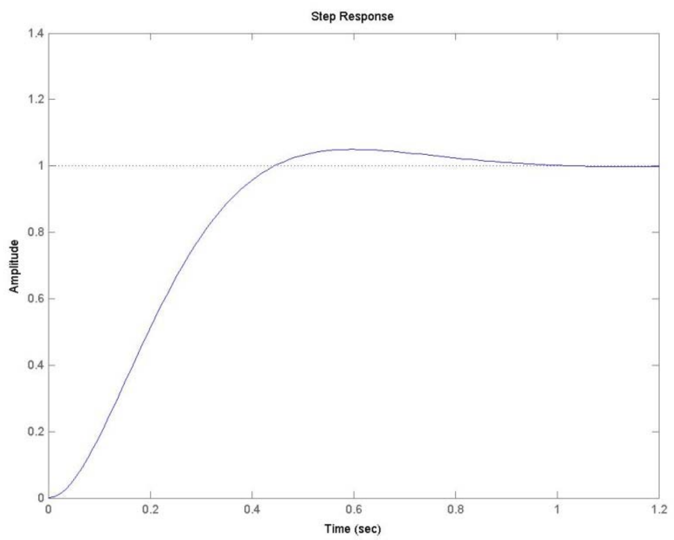
Since $2\xi\omega_n=10$, it yields $\omega_n<7.25$ and thus $K_r=\omega_n^2<52.6$

(2) $M_{p\omega}=\dfrac{1}{2\varepsilon\sqrt{1-\varepsilon^2}}=1$

(3) $\omega_r=\omega_n\sqrt{1-2\varepsilon^2}=1.585$

(4) $\omega_b=7.43$

(5)



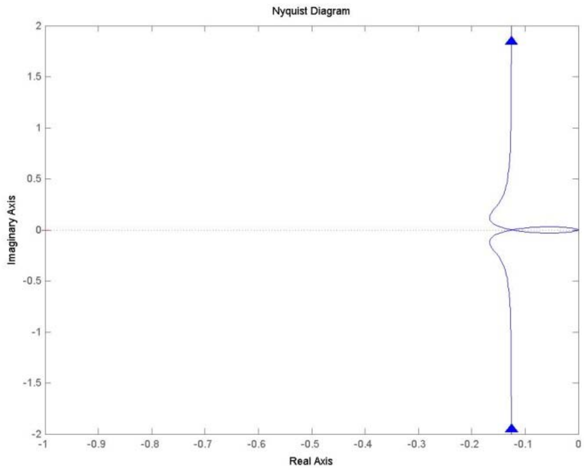
6-7 Sketch the Nyquist diagram of the systems described by the following open-loop transfer functions. From the obtained Nyquist diagrams, find the range of K for which the systems are stable.

(1) $G(s)=\dfrac{K}{s(s^2+2s+4)}$

(2) $G(s)=\dfrac{K(s+1)(s-2)}{s^2(s+4)(-s+1)}$

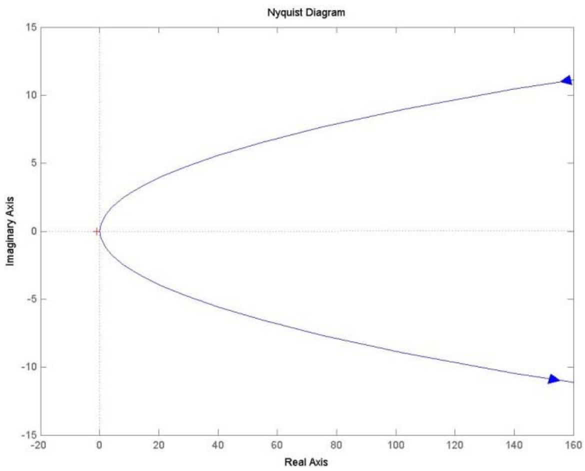
Solution:

(1) Please sketch the Nyquist diagram step by step by yourself. It is omitted here.



The interception with the real axis is -k/8. If *-K/8>-1*, i.e. K<*8*, the closed-loop system is table.

(2)



P=1, N=1, Z=P+N=2

The system is unstable for any *K*.

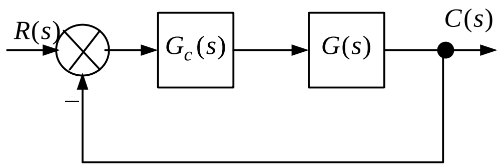
6-8 A control system is shown in the block diagram (a), and $G(s)$ and $G_c(s)$ are both of minimum phase. The Log-magnitude diagrams of $G(s)$ and $G_c(s)G(s)$ are given in Figure (b). Please determine

(1) the transfer function of $G_c(s)$;

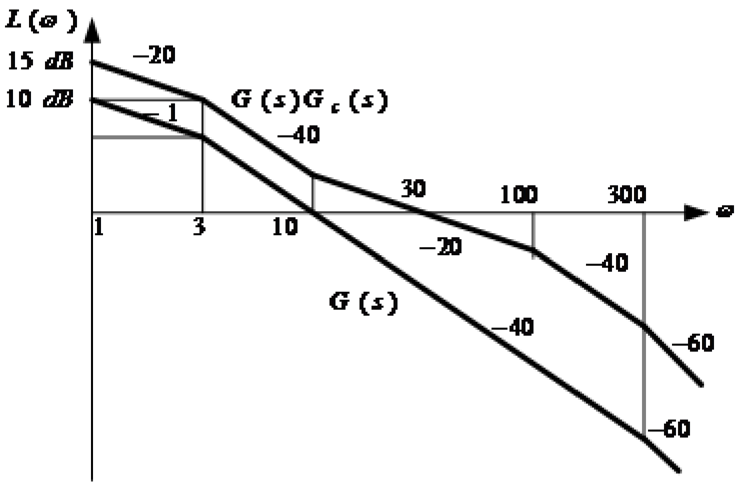
(2) steady-state error constants K_p, K_v and K_a of $G(s)$ and $G_c(s)G(s)$, respectively;

(3) phase margin of $G(s)$ and $G_c(s)G(s)$;

(4) the system overshoots with and without $G_c(s)$, respectively.



(a)



(b)

Solution:

(1) $L(1)=20\lg K=10$， $K=\sqrt{10}$

$$G(s)=\frac{\sqrt{10}}{s(\frac{1}{3}s+1)(\frac{1}{300}s+1)}$$

(2) For $G(s)$, we have $K_p=\infty$， $K_v=\sqrt{10}$， $K_a=0$

For $G(s)G_c(s)=\dfrac{10^{\frac{3}{4}}(0.1s+1)}{s(\frac{1}{3}s+1)(0.01s+1)(\frac{1}{300}s+1)}$, we have

$K_p=\infty$， $K_v=10^{\frac{3}{4}}$， $K_a=0$

(3) For $G(s)$, $\omega_c = 3.08$, and the phase margin is $\gamma = 43.66^\circ$.

For $G(s)G_c(s)$, $\omega_c = 5.48$, and the phase margin is $\gamma = 53.3^\circ$

(4) The overshoot of the system $G(s)G_c(s)$ is smaller than that of $G(s)$ since γ is increased. The transient response is improved.